CHAPTER *1* CURVES

The center line of a road consists of series of straight lines interconnected by curves that are used to change the alignment, direction, or slope of the road. Those curves that change the alignment or direction are known as horizontal curves, and those that change the slope are vertical curves.
The initial design is usually based on a series of straight sections whose positions are defined largely by the topography of the area. The intersections of pairs of straights are then connected by horizontal curves. Curves can be listed under three main headings, as follows:

      (1) Horizontal curve
      (2) Vertical curves

1.1 Horizontal Curves

When a highway changes horizontal direction, making the point where it changes direction a point of intersection between two straight lines is not feasible. The change in direction would be too abrupt for the safety of modem, high-speed vehicles. It is therefore necessary to interpose a curve between the straight lines. The straight lines of a road are called tangents because the lines are tangent to the curves used to change direction.
The smaller the radius of a circular curve, the sharper the curve. For modern, high-speed highways, the curves must be flat, rather than sharp. The principal consideration in the design of a curve is the selection of the length of the radius or the degree of curvature. This selection is based on such considerations as the design speed of the highway and the sight distance as limited by headlights or obstructions (see Fig. 1).
The horizontal curve may be a simple circular curve or a compound curve. For a smooth transition between straight and a curve, a transition or easement curve is provided. The vertical curves are used to provide a smooth change in direction taking place in the vertical plane due to change of grade.
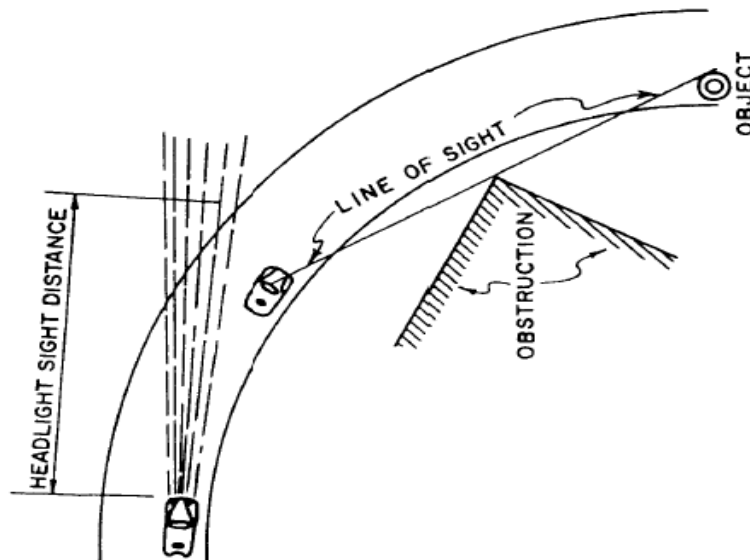

Figure 1.—Lines of sight.

1.1.2 Types of Horizontal Curves

There are four types of horizontal curves. They are described as follows:

A. *Simple*. The simple curve is an arc of a circle (view A, fig. 2). The radius of the circle determines the sharpness or flatness of the curve.

B. *Compound.* Frequently, the terrain will require the use of the compound curve. This curve normally consists of two simple curves joined together and curving in the same direction (view B, fig. 2).

C. *Reverse.* A reverse curve consists of two simple curves joined together, but curving in opposite direction. For safety reasons, the use of this curve should be avoided when possible (view C, fig. 2).

D. *Spiral*. The spiral is a curve that has a varying radius. It is used on railroads and most modem highways. Its purpose is to provide a transition from the tangent to a simple curve or between simple curves in a compound curve (view D, fig. 2).
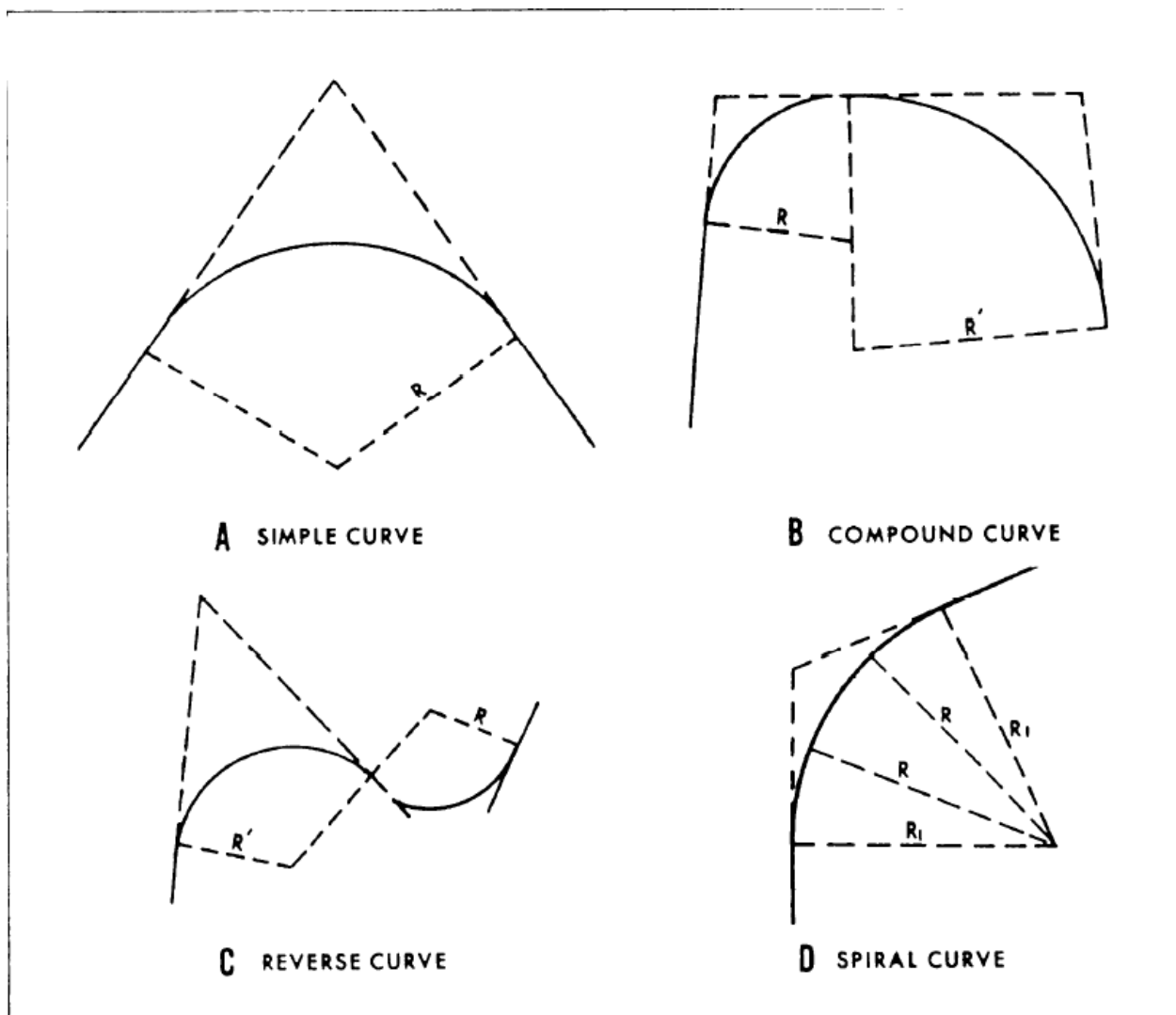

Figure 2.—Horizontal curves.

(A) Horizontal curve or Circular curves of constant radius.

A simple circular curve shown in Fig., consists of simple arc of a circle of radius R connecting two straights lines, intersecting at PI, called the point of intersection (P.I.), having a deflection angle $\triangle$. The distance E of the midpoint of the curve from P I is called the external distance. The arc length from T1 to T2 is the length of curve, and the chord T1T2 is called the long chord. The distance M between the midpoints of the curve and the long chord, is called the mid-ordinate. The distance T1 PI which is equal to the distance P IT2, is called the tangent length.

1.1.3 Elements of Horizontal Curves

The elements of a circular curve are shown in figure 3. Each element is designated and explained as follows:

Point of Intersection (PI). The point of intersection is the point where the back and forward tangents intersect. Sometimes, the point of intersection is designated as V (vertex).

Deflection Angle (△). The central angle is the angle formed by two radii drawn from the center of the circle (O) to the PC and PT. The value of the central angle is equal to the I angle. Some authorities call both the intersecting angle and central angle either I or A.

Radius (R). The radius of the circle of which the curve is an arc, or segment. The radius is always perpendicular to back and forward tangents.

Point of Curvature (PC). The point of curvature is the point on the back tangent where the circular curve begins. It is sometimes designated as ***BC (beginning of curve)*** or ***TC (tangent to curve).***

<p align="center">Station P.C.= P.I. – T</p>

Point of Tangency (PT), The point of tangency is the point on the forward tangent where the curve ends. It is sometimes designated as EC (end of curve) or CT (curve to tangent).

<p align="center">Station P.T. = P.C.+ L</p>

Point of Curve. The point of curve is any point along the curve.

Length of Curve (L) . The length of curve is the distance from the PC to the PT, measured along the curve.

$$L = \Delta \times R \frac{2\Pi}{360}$$

Tangent Distance (T). The tangent distance is the distance along the tangents from the PI to the PC or the PT. These distances are equal on a simple curve.

$$T = R \tan \frac{\Delta}{2}$$

Long Cord (C). The long chord is the straight-line distance from the PC to the PT. Other types of chords are designated as follows:

       C The full-chord distance between adjacent stations (full, half, quarter, or one tenth stations) along a curve.

       C_1 The subchord distance between the PC and the first station on the curve.

C$_2$ The subchord distance between the last station on the curve and the PT.

$$C = 2R \sin \frac{\Delta}{2}$$

External Distance (E). The external distance (also called the external secant) is the distance from the PI to the midpoint of the curve. The external distance bisects the interior angle at the PI.

$$E = R \left[\left| \sec \frac{\Delta}{2} - 1 \right| \right]$$

Middle Ordinate (M). The middle ordinate is the distance from the midpoint of the curve to the midpoint of the long chord. The extension of the middle ordinate bisects the central angle.

$$M = R \left| 1 - \cos \frac{\Delta}{2} \right|$$

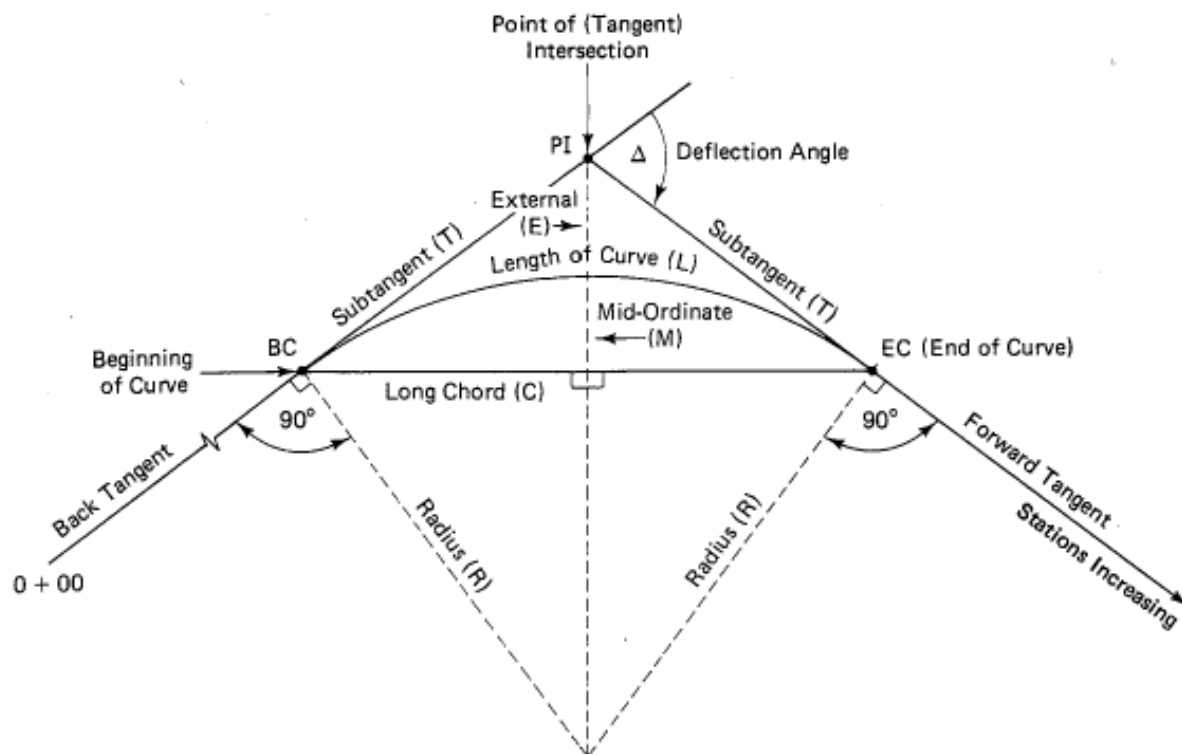**Degree of Curve.** The degree of curve defines the sharpness or flatness of the curve.

Figure 3 Elements of a Circular Curve

1.1.4 Horizontal Curve Layout

(A) Rectangular Offsets From The Tangent /Coordinate/ Method

This method is also suitable for short curve and, as in the previous method, no attempt is made to keep the chord of equal lengths.

$R^2 = AB^2 + AO^2$ (Pythagoras' theorem),

$= y^2 + (R - x)^2$

There fore $x = R - \sqrt{(R^2 - y^2)}$

$= R - R(1 - y^2/R^2)^{1/2}$

Expand $(1 - y^2/R^2)^{1/2}$ using Binomial theorem

Then $x = R - R(1 - 1/2\ y^2/R^2 + \dots)$
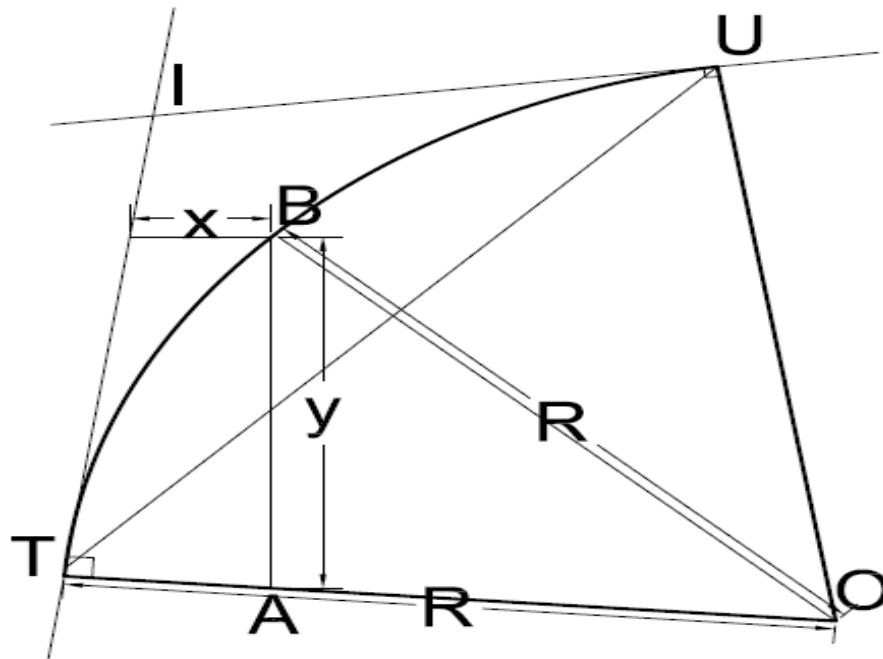
$= y^2/2R$ Approximately

Figure 4 Rectangular Offset

(B)   *Polar Staking / Deflection Method/*

Polar staking methods have become increasingly popular, especially with the availability of electronic tachometers. A simple method can be derived using the starting point of the circle Є is equal to the angle between the tangents and chord.

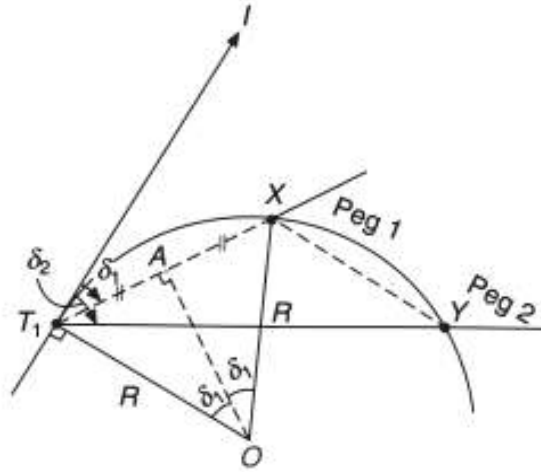For equal arc lengths the polar staking elements are determined with respect to the tangent.

Figure 5 Polar Staking

By radians, arc length $T_1X = R2\delta_1$

$$\therefore \delta_1 \text{ rad} = \frac{\text{arc } T_1X}{2R} \approx \frac{\text{chord } T_1X}{2R}$$

$$\therefore \delta_1^\circ = \frac{\text{chord } T_1X \times 180^\circ}{2R \cdot \pi} = 28.6479\frac{\text{chord}}{R} = 28.6479\frac{C}{R}$$

1.2 Vertical curves

Once the horizontal alignment has been determined, the vertical alignment of the section of highway can be addressed. Again, the vertical alignment is composed of a series of straight-line gradients connected by curves, normally **parabolic** in form. These vertical parabolic curves must therefore be provided at all changes in gradient. **The curvature will be determined by the design speed, being sufficient to provide adequate driver comfort with appropriate stopping sight distances provided.**
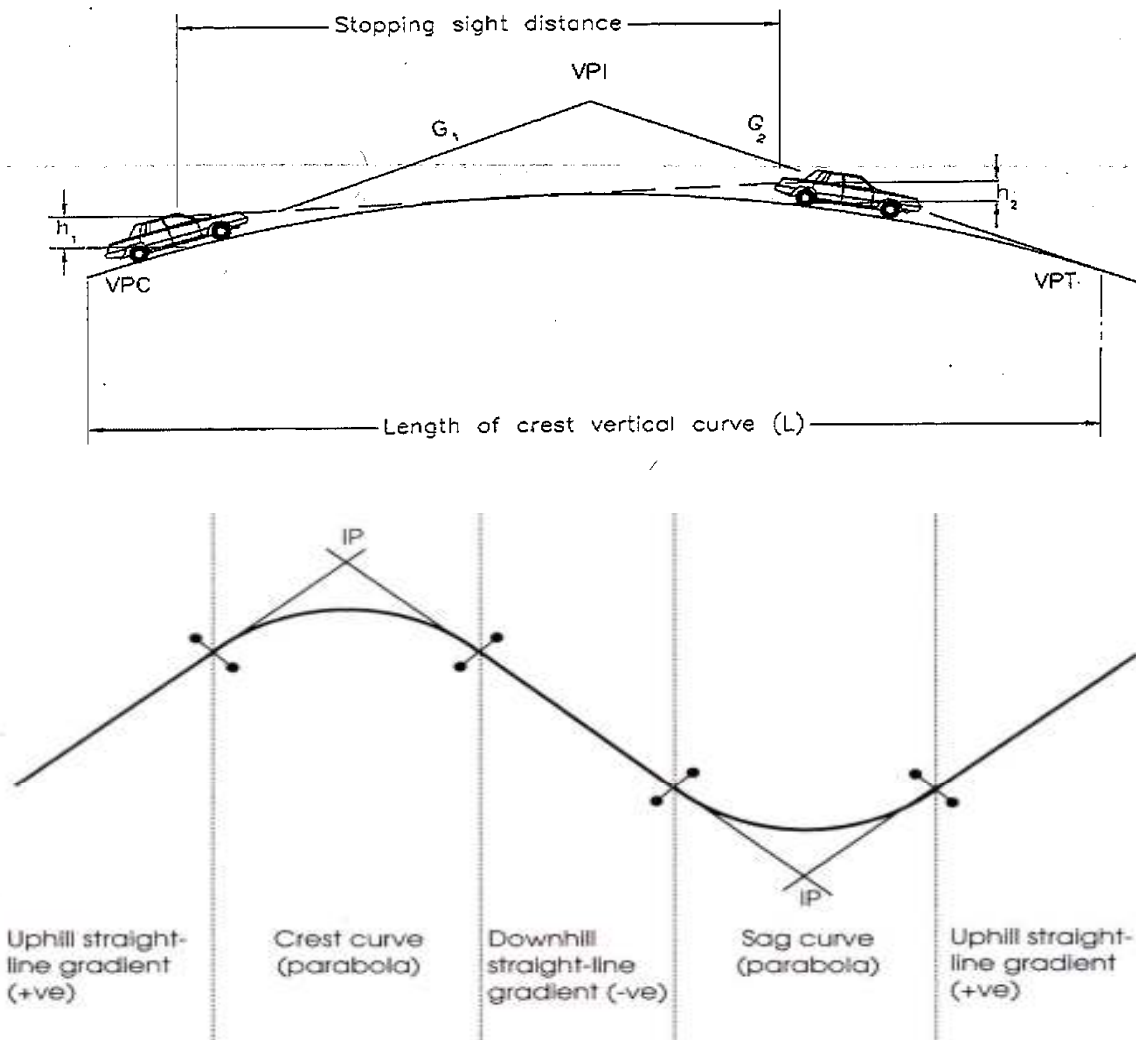

Figure 6: Example of typical vertical alignment

Vertical curves should be simple in application and should result in a design that is **safe and comfortable in operation, pleasing in appearance, and adequate for drainage.**

The major control for safe operation **on crest vertical curves** is the provision of **ample sight distance for the design speed**; while research has shown that vertical curves with limited sight distance do not necessarily experience safety problems, it is recommended that all vertical curves should be designed **to provide at least stopping sight distances**. Wherever practical, more liberal stopping sight distances should be used. Furthermore, additional sight distance should be provided at decision points.

For driver comfort, the rate of change of grade should be kept within tolerable limits. This consideration is most important **in sag vertical curves where gravitational and vertical centripetal forces act in opposite directions**. Appearance also should be considered in designing vertical curves. A long curve has a more pleasing appearance than a short one; *short vertical curves may give the appearance of a sudden break in the profile due to the effect of foreshortening.*

The vertical offset from the tangent grade at any point along the curve is proportional of the vertical offset at the VPI, which is AL/800. The quantity **L/A, termed "K"**, is useful in determining the horizontal distance from the Vertical Point of Curvature (VPC) to the high point of Type I curves or to the low point of type III curves.
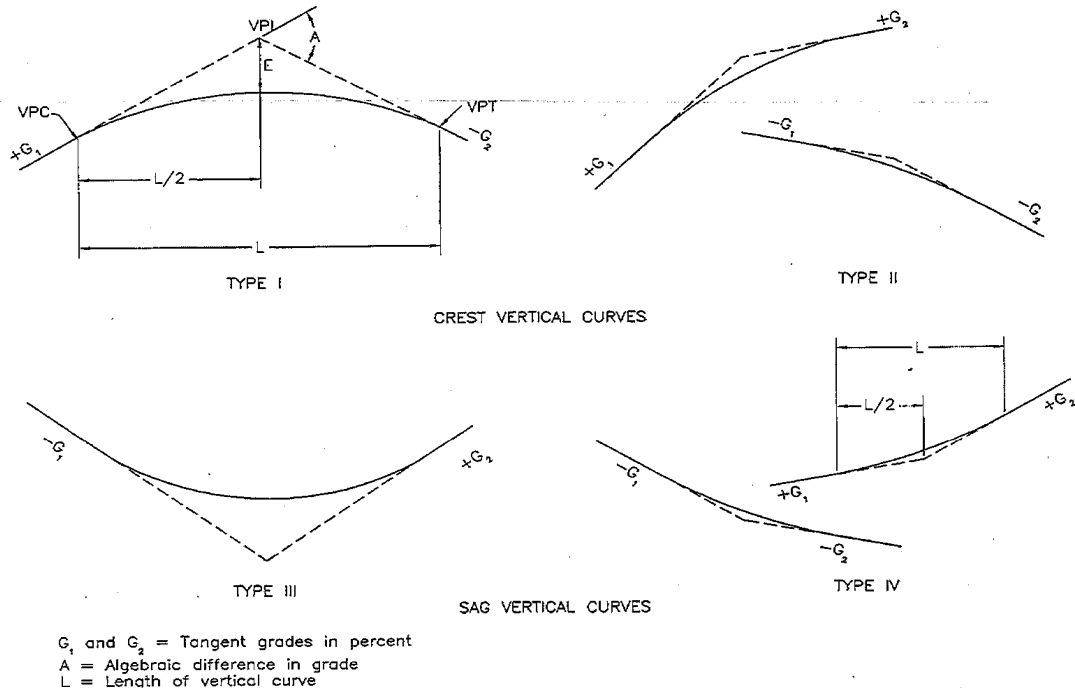

G_1 and G_2 = Tangent grades in percent
A = Algebraic difference in grade
L = Length of vertical curve

Figure 7- Types of vertical curves

1.2.1 Elements of Vertical Curves

Figure 8 shows the elements of a vertical curve. The meaning of the symbols and the units of measurement usually assigned to them follow:

PVC Point of vertical curvature; the place where the curve begins.

PVI Point of vertical intersection; where the grade tangents intersect.

PVT Point of vertical tangency; where the curve ends.

POVC Point on vertical curve; applies to any point on the parabola.

POVT Point on vertical tangent; applies to any point on either tangent.

g_1 Grade of the tangent on which the *PVC* is located; measured in percent of slope.

g_2 Grade of the tangent on which the *PVT* is located; measured in percent of slope.

G The **algebraic difference** of the grades:

$$G = g_2 - g_1$$

Where in plus values are assigned to uphill grades and minus values to downhill grades.

L Length of the curve; the **horizontal** length measured in 20- 100 meters stations from the *PVC* to the PVT. This length may be computed using the formula **L = G/r,** where *r* is the rate of change (usually given in the design criteria).

When the rate of change is not given, *L* (in stations) can be computed as follows: for a summit curve, **L = 125** x G/4; for a sag curve, **L = 100** x G/4. If *L* does not come out to a whole number of stations using these formulas, then it is usually extended to the nearest whole number.

l_1 Horizontal length of the portion of the *PVC* to the *PVI;*

l_2 Horizontal length of the portion of the curve form the *PVI* to the *PVT;*

e Vertical (external) distance from the *PVI* to the curve. This distance is computed using the formula **e = LG/8,** where

  L is the total length in stations and G is the algebraic difference of the grades in percent.

  x The Horizontal distance from the *PVC* to any *POVC* or *POVT* back of the *PVI,* or the distance from the PVT to any *POVC* or *POVT* ahead of the *PW,* measured in feet.

  Y Vertical distance (offset) from any *POVT* to the corresponding *POVC,* measured in feet; which is the fundamental relationship of the parabola that permits convenient calculation of the vertical offsets.
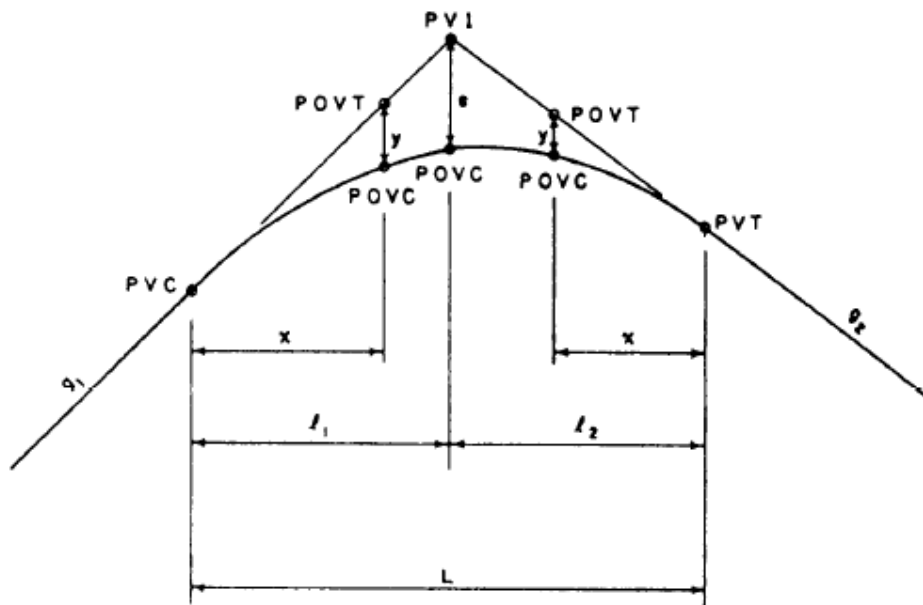
Figure 8.—Elements of a vertical curve.

Parabolic formula:

Referring to Fig. 8, the formula for determining the co-ordinates of points along a typical vertical curve is:

$$y = \left[\frac{G2 - G1}{2L} \right] x^2 \qquad \text{(a)}$$

Where: G1and G2 are the gradients of the two straights being joined by the vertical curve in question.

- *L is the vertical curve length*

- X and y are the relevant co-ordinates in space

Proof:

If Y is taken as the **elevation** of the curve at a point x along the parabola and k is a constant, then:

$\dfrac{d^2Y}{dx^2} = k$ Integrating this equation gives: $\dfrac{dY}{dx} = kx + C$ (b)

Examining the boundary conditions: When x = 0: $\dfrac{dY}{dx} = G1$ (c)

(G1 being the slope of the first straight line gradient) $\dfrac{dY}{dx} = G2$

Therefore: G1 = C and when x = L,   (d)

(G2 being the slope of the second straight line gradient)

$G2 = kL + C = kL + G1$ and rearranging this equation

k = (G2-G1)/L   (e)

Substituting Equations (d) and (e) in to equation (b):

$$\frac{dY}{dx} = \left[\frac{G2-G1}{L}\right]x + G1$$ (f)

Integrating Equation (f);

$$Y = \left[\frac{G2-G1}{L}\right]\frac{x^2}{2} + G1x$$

From figure 2: G1= (y+Y)/x

Which gives
$$Y = \left[\frac{G2-G1}{L}\right]\frac{x^2}{2} + (y+Y)$$

and rearranging this equation gives

$$y = -\left[\frac{G2-G1}{2L}\right]x^2$$

Where x is the distance along the curve measured from the start of the vertical curve and y is the vertical offset measured from the continuation of the slope to the curve.

At the intersection point VPI: x= L/2

Therefore; $$e = -\left[\frac{G2-G1}{2L}\right]\left(\frac{L}{2}\right)^2 = y$$

= - (G2-G1) L/8

The co-ordinates of the highest/lowest point on the parabolic curve, frequently required for the estimation of minimum sight distance requirements, are:

$$x = \frac{L*G1}{G1-G2}$$ $$y = \frac{L*G1^2}{2(G1-G2)}$$

1.2.2 Field Stakeout of Vertical Cures

The stakeout of a vertical curve consists basically of marking the finished elevations in the field to guide the construction personnel. The method of setting a grade stake is the same whether it is on a tangent or on a curve, so a vertical curve introduces no special problem. As indicated before, stakes are sometimes set closer together on a curve than on a tangent. But that will usually have been foreseen, and the plans will show the finished grade elevations at the required stations. If, however, the field conditions do require a stake at an odd plus on a curve, you may compute the needed POVC elevation in the field using given on the plans and the computational explained in this chapter.
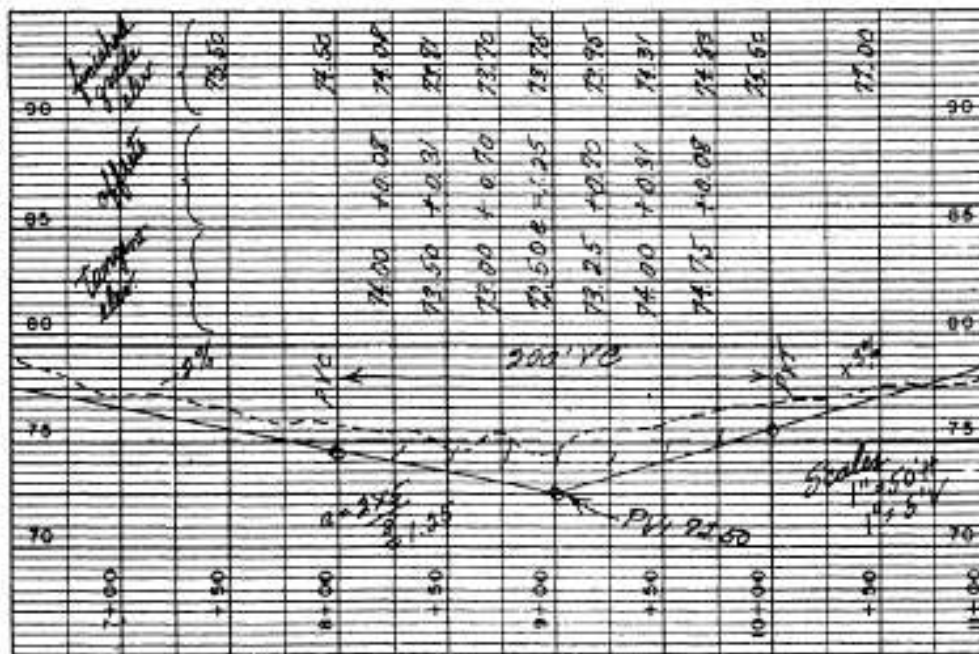

Figure 9 Profile work sheet.

CHAPTER 2   TRIANGULATION

2.1   Overview

Triangulation is a method of surveying in which the station points are located at the vertices of a chain or network of triangles. It deals with the establishment of network of points which form the framework up on which topographic mapping or engineering surveying can be based. In a triangulation system one line termed the base line is measured directly all other distances are derived by measuring the angles of the triangles and calculated the sides by trigonometry. Measuring distances was extremely difficult and time consuming. Therefore triangulation technique was developed to minimize the need for physically measuring distances on the ground. The basic mathematical formula used for determining the sides of the triangle is the law of sine, a/sinA=b/sinB=c/sinC

2.2   Triangulation System

Four types of systems that have been used are: -

1. Chain of single triangles
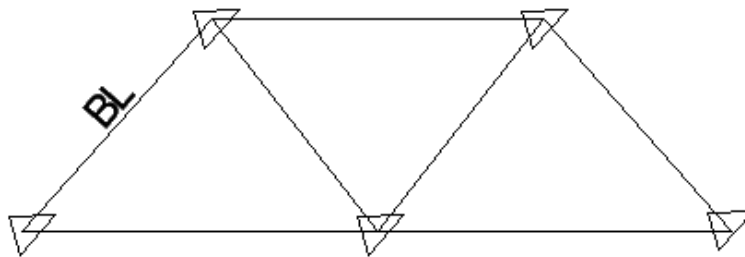
- This type of triangulation system doesn't provide the most accurate result.
- Employed in rather long and narrow surveys of low precision.
- There is only one route to compute the unknown side of a triangle. The means of checking in this system is very limited. That is one for sum of interior angles to 180^0 & calculation for the check of base line from trigonometry.

2. Chain of quadrilaterals formed with overlying triangles
- It is the most common triangulation system, best adapted to rather long & narrow surveys where a high degree of precision is required.
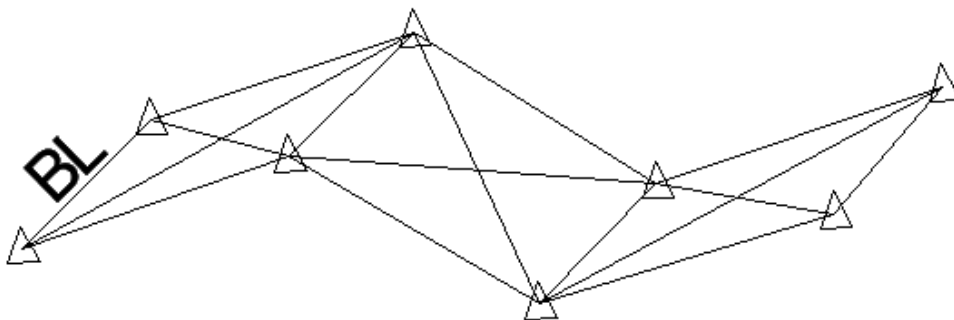
- The sides can be computed with different routes as well as different angles & triangles offering excellent checks for on the computation.

3. Chain of central point Figures

- When horizontal control is to be extended over a rather wide area involving a rather large no of points, as might be the case in Metropolitan areas, a chain of polygons or central point figures may be used. These figures are very strong & are often quite easy to arrange.

4. Central Point Figures with Extra Diagonal.

- This is a central point figure further strengthen by a diagonal as shown.
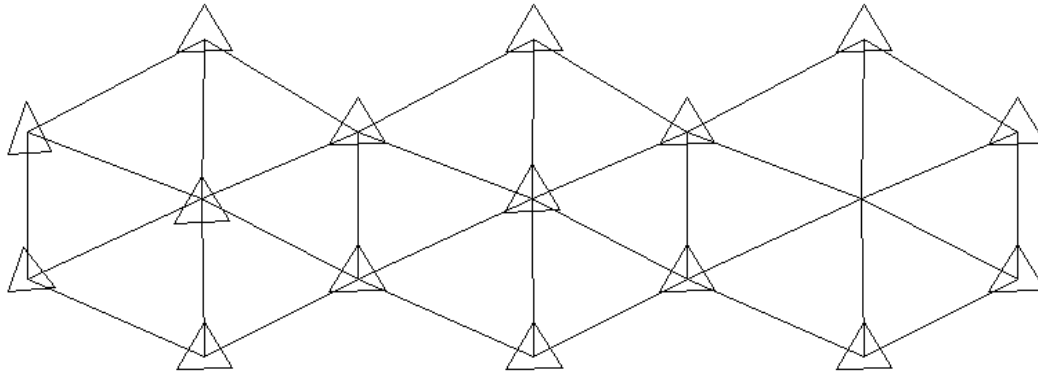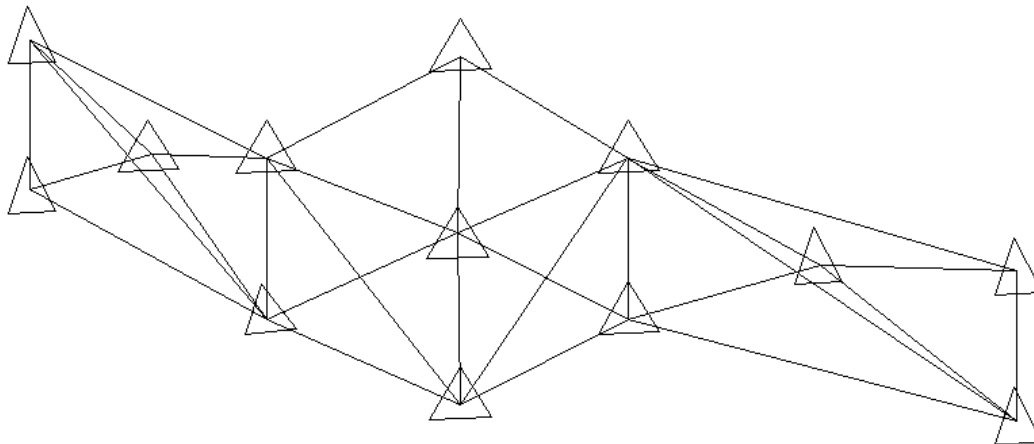
* The combination of the above systems can also be used.

Triangulation station

The area to be covered by triangulation scheme must be carefully studied to select the most suitable positions for the control stations. Existing maps, especially if contoured can be of great value since the size and shape of triangles formed by the stations can be difficult to visualize in the field.

The following consideration must be taken in to account for the choice of stations.
1. Every station must be visible from the adjacent stations.
2. The triangles formed thereby should be well conditioned, that is to say, as nearly equilateral as possible. No angles should be less than 30°, if possible.
3. The size of the triangles will depend on the configuration of the land, but they should be as large as possible.
4. The end purpose of the triangulation scheme must be kept in mind. Where choice of station sites exists, the once most suitable for correction to subsequent traverse and detail survey should be used.

2.3 Procedures in Triangulation

A triangulation survey usually involves the following steps
1. Reconnaissance: - meaning the selection of the most visible points for the station.
2. Signal erection
3. Measurement of angles
4. Determination of direction or Azimuth
5. Base line measurement
6. Computations

❖ Reconnaissance: -
The first consideration with regard to the selection of stations is intervisibility. An observation between two stations that are not intervisible is impossible. Next comes accessibility, a station that is inaccessible can not be occupied and between two stations and between two stations otherwise equally visible the one that provides the easier access is preferable.
The next consideration involves **strength of figure**: - In triangulation the distances will be computed by the sine law, the more nearly equal the angles of a triangle are the less will be the ratio of error in the sine computations. Values computed from the sines of angles near 0º and 180º are subjected to large ratios of error.
As a general rule, one selects stations that will provide triangles in which no angle is smaller than 30º or larger than 150º (closer to 180º).

❖ Signal Erection: -
After the stations have been selected, the triangulation signals or the triangulation towers will be erected. When the triangulation signals or towers are erected, it is imperative for these stations to be intervisible. It is also important that the target be large enough to be seen at a distance, which is the color of the signal must be selected for good visibility against the background where it will be viewed.

❖ Computation: -
Station adjustment, figure adjustment and computation of lengths.

2.4 Strength of figure

Strength of figure refers to the effect of the proportions of a triangle on the accuracy with which the length of the side can be computed. When small errors in angle measurement affect the computed distances to a lesser extent the figure is said to be strong.

For example to compute CD
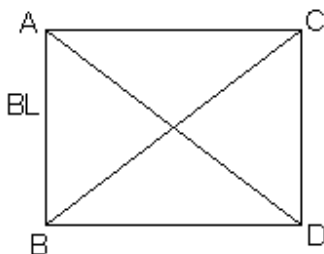
► Δ ABC ~ Δ ADC
Δ ABC ~ Δ CBD

$$\Delta\ ABD \sim \Delta\ ADC$$
$$\Delta\ ABD \sim \Delta\ BDC$$

accordingly which alternative is best is governed by the strength of figures.

In triangulation the length of the triangle sides are computed by using the law of sine. When triangles, which contain small angles are used the best results may not be obtained because of the sines of angles near zero and 180° is quite large as compared with the rate of change of angles near 90°.

Example: - $\Delta 1° \sim \text{Sin} 1° = 0.01745$
                 $\underline{\text{Sin} 2° = 0.03489}$
                   $\Delta 1° = 0.1745$ (Higher discrepancy)
         $\Delta 1° \sim \text{Sin} 89° = 0.99984$
            $\underline{\text{Sin} 90° = \qquad\quad 1}$
                 $\Delta 1° = 0.0001523$

Therefore the strongest chain of triangle is the one whose distance angles are near 90°.
There is a measure of testing by which the strength of figure is measured. The measure of the strength of figure with respect to length is evaluated quantitatively in terms of a factor '**R**'. The lower the value of **R** the stronger the figure will be.

    $R= \dfrac{D\text{-}C}{D} * \sum (\delta_A{}^2 + \delta_A\delta_B + \delta_B{}^2)$

Where   R   = strength of figure,
         D   = the number of directions observed in both directions, excluding the base line.
         δ_A = the tabular difference for 1'' in the logarithmic sine of distance angle A** in the sixth decimal place.
         δ_B = the tabular difference for 1'' in the logarithmic sine of distance angle B** in the sixth decimal place.
            ** Distance angle A = angle opposite to known side
            ** Distance angle B = angle opposite to the side which is to be computed.

         C   = the number of geometric conditions.
           = (n'- s'+1) + (n-2s+3)
               n = total number of lines
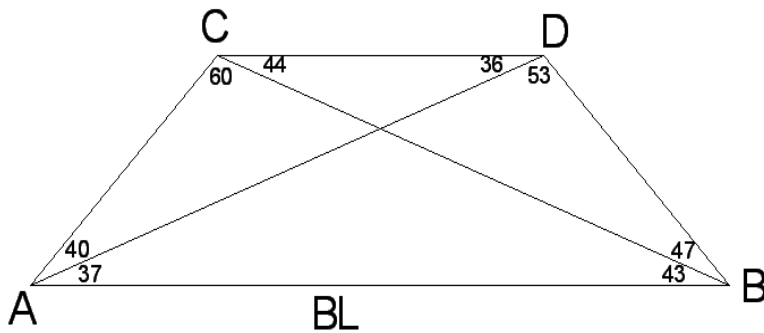   s = total number of stations
               n' = total number of lines observed in both directions (lines observed in one direction only are usually shown by broken lines)
               s' = total number of stations occupied

Example: - It is desired to compute the strength of figure of the given quadrilateral for computation of the side CD from the known side AB (base line AB)

Solution: -

The length of line CD can be computed in 4 different routes.

Case	Triangles	Known Side	Computed Side	Distance Angle		$\Sigma\ \delta_A^2+\delta_A\delta_B+\delta_B^2$
				A	B	
I.	ACB	AB	AC	60	43	9.32
	ACD	AC	CD	36	40	21.97
II.	ABD	AB	AD	53	90	2.52
	ACD	AD	CD	104	40	5.25
III.	ABC	AB	BC	60	77	2.30
	BCD	BC	CD	89	47	3.93
IV.	ABD	AB	BD	53	37	14.76
	BCD	BD	CD	44	47	12.89

Therefore,

$R_1 = 3.740$ Case III

$R_2 = 4.663$ Case II

And hence, Case III shall be used to determine the length of CD.

2.5   Triangulation Adjustment

Before the length computation begins it is necessary to make triangulation adjustment.
Triangulation adjustment is needed because the net must give geometric consistency. Corrections have to be applied to the measured values to satisfy the set of conditions from the geometry. It is logical to use corrections that have the greatest probability of being the true corrections.

The method of least squares will give the most probable corrections that satisfy the conditions imposed. This method involves so many computations that approximate methods are desirable. To be acceptable, an approximate method must result in corrections that approach as nearly as possible the results of the least square solutions.

The approximate adjustment of a triangulation system consists of:
   A. **Station Adjustment**: correction of the angles at each station so as to satisfy any geometric
                       conditions existing among the measured angles.

B. Figure Adjustment: adjustment of each figure (triangle, quadrilateral etc) so as to make it a perfect geometric polygon.

It consists of two parts, namely:

i. Angle adjustment

ii. Side adjustment

Following discusses the adjustments for a chain of triangles and quadrilateral systems.

1. Adjustment of chain of single Triangles

A. Station Adjustment
 - The sum of the angles around each point should exactly be 360^0.

B. Figure Adjustment
 - The sum of the measured angles in each triangle should exactly be 180^0.

 * Compare the check Base Line from measurement & calculation.

 - For the angles about a point the difference between the sum of the angles and $360°$ is balanced equally between the numbers of measured angles.

 - In the same fashion for the difference between the sums of the angles in each triangle $180°$ is balanced equally between the angles.

 - If the adjustment of the station is disturbed during figure adjustment then adjust the station by adding the difference to the exterior angle of the station.

Example: -

 Make the necessary station and figure adjustments for the figure shown below.
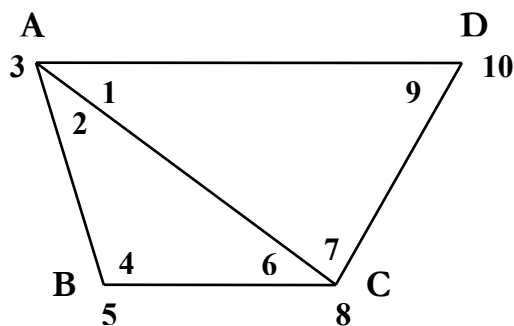
Measured Angles

1.	41°	16'	10"		6.	35	07	00
2.	53	36	20		7.	78	42	30
3.	265	07	45		8.	246	11	00
4.	91	16	10		9.	60	01	05
5.	268	43	30		10.	299	59	15

Solution: -

A. Station Adjustment

Station	Angle No	Measured Angle			Adjustment Angle		
A	1	41	16	10	41	16	05
	2	53	36	20	53	36	15
	3	265	07	45	265	07	40
	Σ	**360**	**00**	**15**	**360**	**00**	**00**
B	4	91	16	10	91	16	20
	5	268	43	30	268	43	40
	Σ	**359**	**59**	**40**	**360**	**00**	**00**
C	6	35	07	00	35	06	50
	7	78	42	30	78	42	20
	8	246	11	00	246	10	50
	Σ	**360**	**00**	**30**	**360**	**00**	**00**
D	9	60	01	05	60	00	55
	10	299	59	15	299	59	05
	Σ	**360**	**00**	**20**	**360**	**00**	**00**

B. Figure Adjustment

Triangle	Angle No	Angle after Station Adjustment			Angle after Figure Adjustment		
ABC	2	53	36	15	53	36	27
	4	91	16	20	91	16	32
	6	35	06	50	35	07	01
	Σ	179°	59'	25"	180°	00'	00"
ACD	1	41	16	05	41	16	18
	7	78	42	20	78	42	34
	9	60	00	55	60	01	08
	Σ	179°	59'	20"	180°	00'	00"

➢ **Adjustment on exterior angle.**

Since the change of interior angles will affect the station adjustment, we have to add the difference to the exterior angle.

For e.g. consider sum of angles 1, 2, and 3

$$\Sigma = 41°16'18" + 53°36'27" + 265°07'40" = 360°00'25"$$
$$\underline{- \ 360°00'00"}$$
$$\underline{- \ 00°00'25"}$$

Then angle 3 becomes:

$$3 = 265°07'40" - 00°00'25" \ = 265°07'15"$$

Lecture Note: - CENG 2803: SURVEYING II

2. Adjustment of a quadrilateral system

    A. Angle Condition
      - The sum of the interior angles must equal $(\eta-2)(180)$ where η is the number of sides.
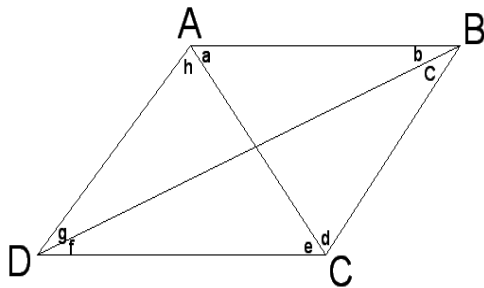
    B. Side Condition
      - The sine of each angle must be proportional to the length of the opposite side of that triangle.

The angles of the quadrilateral could be adjusted by using the 1st approximate method.

Adjustment of quadrilateral by 1st approximate method.

Consider the quadrilateral shown below:

    A. Angle Condition

    i. The sum of all interior angles of a triangle must be equal to 180^0.

        ➤ **ΣAi = 180^0**

    a+b+c+d= 180^0   **a+f+g+h= 180^0**
    c+d+e+f = 180^0   **g+h+a+b= 180^0**

    ii. The sum of interior angles of a quadrilateral must be equal to 360^0.
        $(\eta-2)(180)$ = (4-2) X 180= 360^0

        i.e., a+b+c+d+e+f+g+h =360^0.
    iii. The sum of opposite angles (at the intersection of the diagonals) should be equal as they are opposite angles.

        a+b = e+f
        g+h = c+d

  B. Side Condition
      Consider △ ABD and △BCA.

       → $\dfrac{AD}{\sin b} = \dfrac{AB}{\sin g}$ ⇒ $AB = AD\ \dfrac{\sin g}{\sin b}$ .... (1)

       → $\dfrac{AB}{} = \dfrac{BC}{}$ ⇒ $BC = AB\ \underline{\sin a}$

Sin d   Sin a   Sin d

$$\Rightarrow BC = AD \; \frac{Sin\,g}{Sin\,b} * \frac{Sin\,a}{Sin\,d} \quad ……… (2)$$

Consider △ ACD and △BCD

$$\frac{CD}{Sin\,h} = \frac{AD}{Sin\,e} \qquad \Rightarrow CD = AD \; \frac{Sin\,h}{Sin\,e} \qquad ……… (3)$$

$$\frac{CD}{Sin\,c} = \frac{BC}{Sin\,f} \qquad \Rightarrow BC = CD \; \frac{Sin\,f}{Sin\,c}$$

$$\Rightarrow BC = AD \; \frac{Sin\,h}{Sin\,e} * \frac{Sin\,f}{Sin\,c} \quad ……… (4)$$

From eqn (2) & (4), we have

$$AD \; \frac{Sin\,g}{Sin\,b} * \frac{Sin\,a}{Sin\,d} \; = \; AD \; \frac{Sin\,h}{Sin\,e} * \frac{Sin\,f}{Sin\,c}$$

$$\Rightarrow \frac{Sin\,b * Sin\,d * Sin\,f * Sin\,h}{Sin\,a * Sin\,c * Sin\,e * Sin\,g} = 1.0$$

$$\Rightarrow Log \frac{(Sin\,b * Sin\,d * Sin\,f * Sin\,h)}{(Sin\,a * Sin\,c * Sin\,e * Sin\,g)} = 0.0$$

$$\Rightarrow log\,Sin\,b + log\,Sin\,d + log\,Sin\,f + log\,Sin\,h - [log\,Sin\,a + log\,Sin\,c + log\,Sin\,e + log\,Sin\,g] = 0$$

This is the side condition (trigonometric condition) that has to be satisfied for quadrilateral.

Steps in the adjustment of a quadrilateral.

1) Correct each of the eight angles so that their sum will be exactly 360°.
2) Adjust the opposite angles so that their sum should be equal.
3) Record the *Log Sin* for every alternate angle.
4) For each angle, record the logarithmic sine difference for a change of 1 second.
5) Find the difference in the sum of the *Log Sin*.
6) Divide the difference in the sum of the *Log Sin* with the sum of *Log Sign* for the change of 1''.
7) The outcome in the 6th step gives the correction value to the angles in seconds to be applied. Add this value to each of the four angles, the sum of whose *Log Sin* is smaller, and subtract it from the rest. And then the corrected values will be there.

Example:

⊠ **Computation of Lengths**

Once the base line is measured, the angles are measured and adjusted; the other two sides of a triangle are computed by the law of sine. In computing the sides of the triangles in a quadrilateral, the solution of using two triangles is sufficient to compute the positions of the forward triangulation stations. The two triangles chosen must be the strongest routes through the quadrilateral.

Example:

CHAPTER 3 CONTOUR LINES AND DIGITAL TERRAIN MODEL

3.1 Overview
Contours and Contour Lines

A contour is an imaginary line that connects points of equal elevations on the ground surface. A line joining several closely spaced ground points of equal elevation on a drawing is called a contour line. Thus contours on the ground may be represented be contour lines on the map. On a given map successive contour lines represent elevations differing by a fixed vertical distance called the contour interval. The choice of this contour interval depends upon the following factors.

   i.   The nature of the ground

        In a flat and uniformly sloping country, the contour interval is small, but in mountainous region the contour interval should be large otherwise the contours will come too close to each other.

   ii. The purpose and extent of the survey

        Contour interval is small if the area to be surveyed is small and the maps are required to be used for the design work or for determining the quantities of earth work etc. Wider interval shall be used for large area and comparatively less important works.

   iii. Scale of the maps

        The contour interval should be in the reverse ratio to the scale of the map i.e. the smaller the scale the greater the contour interval.

   iv. Time and expense of field and office work

        The smaller the contour interval the greater is the amount of field work and plotting work.

The choice of map scale also depends upon;

   i.   The clarity with which features can be shown.
   ii. The cost: - the larger the scale the higher the cost.
   iii. The contour interval

Typical map scales, map uses ad corresponding contour interval are shown in the table below.

Map Scale	Typical Use	Contour Interval for mountainous terrain
1:1000	Design	0.25m
1:2000	Design	0.50m
1:5000	Planning	1m
1:10000	Planning	2m

1:25000	Regional Planning	2.5m
1:50000	Regional Planning	5m
1:100000	Regional Planning	5 or 10m
1:250000	Regional Planning	10m
1:500000	State/National	20 r 50m

3.2.1 General Rules for Contour

The principal characteristics of contour lines are as follows:

i. All points on contour lines have same elevation.

ii. Flat ground is indicated where the contour lines are widely separated and steep ground is indicated where contour lines run close together.

iii. The contour lines connecting points on uniform slopes are spaced uniformly.
iv. Contours crossing a human-made horizontal surface (roads, railroads) will be straight parallel lines as they cross the facility.

v. A series of closed contour lines on the map represent a hill if the higher values are inside and a depression if the higher values are outside.

vi. Contour lines across ridge or valley lines are at right angles. If the higher values are inside the bend or loop, it indicates a ridge and if the higher values are outside the bend, it represents a valley.

vii. Contour line cannot end anywhere but close on themselves either within or outside the limits of the map.

viii. Contour lines cannot merge or cross one another.

3.2.2   Methods of Contouring

There are mainly two methods of contouring:

a. **Direct Method**

- In this method the contours to be located are directly traced out in the field by locating and making a number of points on each contour.

- These method is very slow and tedious as a lot of time is wasted in searching points on the same contour. Compared to the other, it is very accurate and suitable for small areas where great accuracy is required.

b. **Indirect Method**

- In this method, the points located and surveyed are not necessary on the contour lines. Spot elevations will be taken along a series of lines laid out over the area, e.g. grid or cross section. The contours are then determined by interpolation based on the spot elevations.

- This method is less tedious, cheaper & quicker than the direct method but they are less accurate when compared to the direct method.

3.2.3 Interpolation of Contour

The process of spacing the contours proportionally between the plotted ground points established by indirect method is termed as interpolation of contour.

3.2.4   Measuring Slope from Contour

By measuring the distance between two points (ΔH) on two contour lines, whose elevations are, of course, known, the slope of the line connecting these two points can be calculated.

To layout a highway, railway, canal or any other communication line at a constant gradient, the alignment can easily plotted on a map.

⊠ **Uses of Contour Lines (Contour Map)**

✻ Enable an engineer to approximately select the most economical and suitable site for an engineering work such as reservoir, dam, runways, highways etc.

✻ Drawing of a section/profile.

✻ Help in computation of quantities of earthwork.

✻ Determination of catchment areas

✻ Determination of Reservoir Capacity

3.2   Digital Terrain Model (DTM)

3.3.1 Overview

➢ A digital terrain model represents the relief of the earth's surface by a set of x, y, z date

points where x is the horizontal x-position [e.g.   x longitude, y is the horizontal y-position

(e.g. latitude) and z is the altitude of the earth's surface [e.g. elevations above see level). It is collection of terrain data as a sequence of discrete [x, y, z] data points.

➢ The points are usually horizontally regular spaced in the form of a square grid.

➢ Just as there are engineering design criteria for selecting a contour interval to represent terrain for a given application, so to similar criteria are used to select point spacing so that the DTM adequately represents the terrain. These criteria depend on the potential uses for the data, accuracy requirements, the terrain character and other factors. The advantages of the regular grid layout a simplified data collection routine and ease of data access by subsequent programs.

➢ As the scale dose for maps the grid cell size determines the resolution and degree of generalization of the DTM

➢ The grid call size of DTM varies from a few meters (high resolution DTM, mostly available only for small areas) to medium resolution [20 to 100m) and low resolution DTM such as the 30 arc sec DTM with 925mm grid cell size which covers the entire world

➢ Digital terrain model should not be confused with digital elevation model (DEM).

➢ DEM is only the more general expression for digital surface data but one must define the kind of surface the elevation date are for :e.g. DEM   of the vegetation surface are for: e.g. DEM of the vegetation surface, DEM of groundwater surface or DEM of the relief of the earth surface which is also called digital terrain model.

3.3.2 DTM CREATION

There are two steps necessary for the creation of a DTM.

1. Collecting original x, y, z coordinates.
2. Interpolations of a regular grid DTM

   1. Collecting original x, y, z co-ordinates the altitude data of the earth's surface can be collected from the following data source.

    I.   Digitalization of contour lines from topographic maps.

    II.   Stereoscopic measurements from aerial photos

    III.   Stereoscopic measurements from (optical) satellite data

- DTM of regulation surface gives already regular spaced data

    IV.   Radar satellite data

- gives already regular spaced data

    V.   Laser scanning measurements

    VI.   Field measurements or ground surveys

2. Interpolation of a regular grid DTM.

    1.   The x, y, z co-ordinates collected from the above sources are usually irregularly spaced.

    2. The data have to be transformed into regular space grid data using different interpolation algorithms.

    3. The chosen grid cell size usually depends mostly on the density of the collected irregular spaced data.

    4. The quality of the DTM is mainly determined by the density and accuracy of the collected original altitude data.

3.3.4 DERIVATIONS FROM DTM

    i.   Calculation of local morphometric values for each grid cell of DTM such as slope angle and curvature.

    ii.   Calculation of complex morphometric values for each grid cell of the DTM such as measures of drainage basins, distance to flow lines and water sheds etc.

    iii.   Derivation of morphographic features: linear features such as crest lines, flow lines and edges; area features such as valley grounds summit areas subdividing of slopes etc.

    iv.   Visualization of the relief, computing contour lines, analytic hill shading and 3d-views.

3.3.5 APPLICATION OF DTM

- ✓ DTM were first used for engineering purposes such as planning of rods, railroads, channels and water dams or for calculation of mass (soil) movements for bigger hooking areas.
- ✓ DTM is a base component of all modern geographical information systems or spatial information systems.

UNCERTANITY AND ERRORS IN DTMs

1. The accuracy of a DTM depends on the accuracy of its sources data and on the model resolution.

2. Two DTMs produced from the same data will not contain the same information if their resolution and sampling strategies are different.

3. A DTM may contain badly formed links or spot height errors which must be corrected by hand or automatically prior to use.

CHAPTER 4 PHOTOGRAMMETRY

4.1 Introduction

- The photogrammetry has been derived from three Greek words:
  - o Photos: means light
  - o Gramma: means something drawn or written
  - o Metron: means to measure
- This definition, over the years, has been enhanced to include interpretation as well as measurement with photographs.

Definition
The art, science, and technology of obtaining reliable information about physical objects and the environment through process of recording, measuring, and interpreting photographic images and patterns of recorded radiant electromagnetic energy and phenomenon (American Society of Photogrammetry, Slama).

- Originally photogrammetry was considered as the science of analysing only photographs.
- But now it also includes analysis of other records as well, such as radiated acoustical energy patterns and magnetic phenomenon.

Definition of photogrammetry includes two areas:

(1) Metric:

It involves making precise measurements from photos and other information source to determine, in general, relative location of points. Most common application: preparation of plannimetric and topographic maps.

(2) Interpretative:

It involves recognition and identification of objects and judging their significance through careful and systematic analysis. It includes photographic interpretation which is the study of photographic images. It also includes interpretation of images acquired in Remote Sensing using photographic images, MSS, Infrared, TIR, SLAR etc.

Metric photogrammetry is classically divided into two, terrestrial photogrammetry and aerial photogrammetry.
Definitions Aerial Photogrammetry
Photographs of terrain in an area are taken by a precision photogrammetric camera mounted in an aircraft flying over an area.
Terrestrial Photogrammetry
Photographs of terrain in an area are taken from fixed and usually known position or near the ground and with the camera axis horizontal or nearly so.
Photo-interpretation

Aerial/terrestrial photographs are used to evaluate, analyse, and classify and interpret images of objects which can be seen on the photographs.

Applications of photogrammetry Photogrammetry has been used in several areas. The following description give an overview of various applications areas of photogrammetry (Rampal, 1982)

(1) Geology:

Structural geology, investigation of water resources, analysis of thermal patterns on earth's surface, geomorphological studies including investigations of shore features.

- engineering geology
- stratigraphics studies
- general geologic applications
- study of luminescence phenomenon
- recording and analysis of catastrophic events
- earthquakes, floods, and eruption.

(2) Forestry: Timber inventories, cover maps, acreage studies

(3) Agriculture Soil type, soil conservation, crop planting, crop disease, crop-acreage.

(4) Design and construction Data needed for site and route studies specifically for alternate schemes for photogrammetry. Used in design and construction of dams, bridges, transmission lines.

(5) Planning of cities and highways New highway locations, detailed design of construction contracts, planning of civic improvements.

(6) Cadastre Cadastral problems such as determination of land lines for assessment of taxes. Large scale cadastral maps are prepared for reapportionment of land.

(7) Environmental Studies Land-use studies.

(8) Exploration To identify and zero down to areas for various exploratory jobs such as oil or mineral exploration.

(9) Military intelligence Reconnaissance for deployment of forces, planning manoeuvres, assessing effects of operation, initiating problems related to topography, terrain conditions or works.

(10) Medicine and surgery Stereoscopic measurements on human body, X-ray photogrammetry in location of foreign material in body and location and examinations of fractures and grooves, biostereometrics.

(11) Miscellaneous Crime detection, traffic studies, oceanography, meteorological observation, Architectural and archaeological surveys, contouring beef cattle for animal husbandry etc.

4.2 Classification of Photographs The following paragraphs give details of classification of photographs used in different applications (1) On the basis of the alignment of optical axis

(a) *Vertical* : If optical axis of the camera is held in a vertical or nearly vertical position.
(b) *Tilted* : An unintentional and unavoidable inclination of the optical axis from vertical produces a tilted photograph.
(c) *Oblique* : Photograph taken with the optical axis intentionally inclined to the vertical. Following are different types of oblique photographs:

(i) High oblique: Oblique which contains the apparent horizon of the earth.
(ii) Low oblique: Apparent horizon does not appear.
(iii) Trimetrogon: Combination of a vertical and two oblique photographs in which the central photo is vertical and side ones are oblique. Mainly used for reconnaissance.
(iv) Convergent: A pair of low obliques taken in sequence along a flight line in such a manner that both the photographs cover essentially the same area with their axes tilted at a fixed inclination from the vertical in opposite directions in the direction of flight line so that the forward exposure of the first station forms a stereo-pair with the backward exposure of the next station.

Comparison of photographs

Type of photo	Vertical	Low oblique	High oblique
Characteristics	Tilt $< 3^\circ$	Horizon does not appear	Horizon appears
Coverage	Least	Less	Greatest
Area	Rectangular	Trapezoidal	Trapezoidal
Scale	Uniform if flat	Decreases from foreground to background	Decreases from foreground to background
Difference with map	Least	Less	Greatest
Advantage	Easiest to map	-	Economical and illustrative

4.3 Aerial Photography

Aerial photographs are pictures of ground features taken from air craft equipped with specially designed cameras and films for air use. The pictures are taken in a set of run with at least 60% overlap between two consecutive photos in the flight duration and 20% - 30% side lap between adjacent lines.
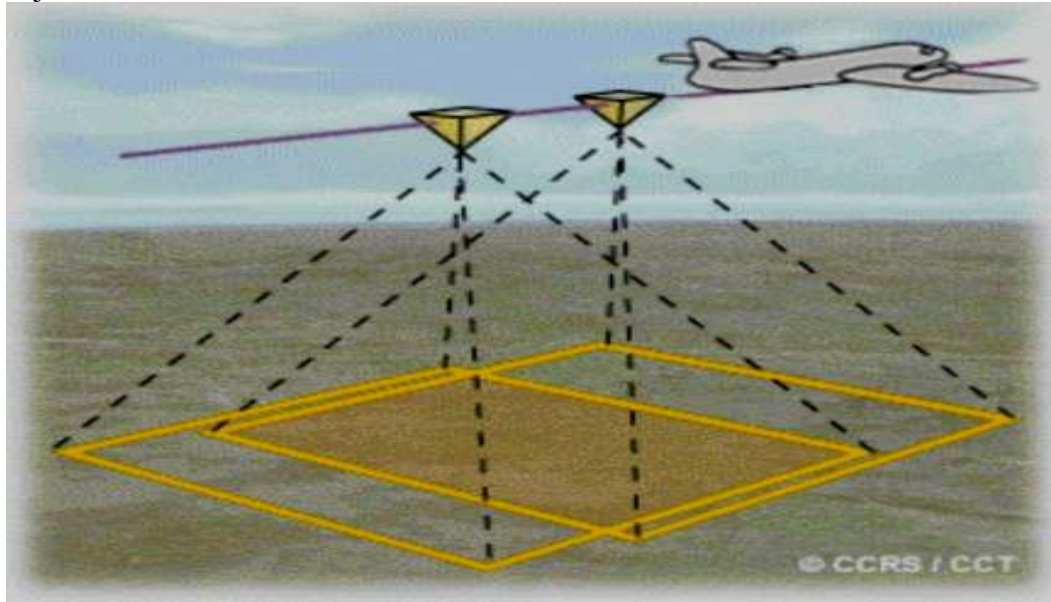

Figure1: - Showing an overlap (min. 60%) between two photo shoots.

The overlap between successive pictures serves three primary purposes. Each object on the ground appears on two consecutive photos, this arrangement enables

    i)   To study overlapping photographs in three dimensions using stereoscopes.
    ii) Allows only the central portions of the photographs to be used in mosaic construction, elimination to a great extent the effect of relief displacement.
         Mosaic is the aerial view of a given area obtained by joining together a net of overlapping vertical photographs.
    iii) The small overlap area between alternate photographs allows horizontal control to be extended along the strip.

4.4 Photographic Scale (Scale of Aerial Photograph)

The scale of an aerial photograph is the dimension less ratio of distance on the photograph to a corresponding distance on the ground. It is commonly expressed in fraction of 1:10000, 1:20000, 1:50000 and so on. Consider the following figure of a truly vertical aerial photograph.
Due to perspective geometry of photographs, the scale of photograph varies as a function of focal length, flying height, and the reduced level of terrain over a certain reference datum. In figure 2, for a vertical photograph, L is exposure station, f is its focal length, H is the flying height above datum, h represents the height of ground point A above datum. Point A is imaged as a in the photograph. From the construction and using similar triangles Loa and LO_AA, we can write the following relations (Wolf and Dewitt, 2000)
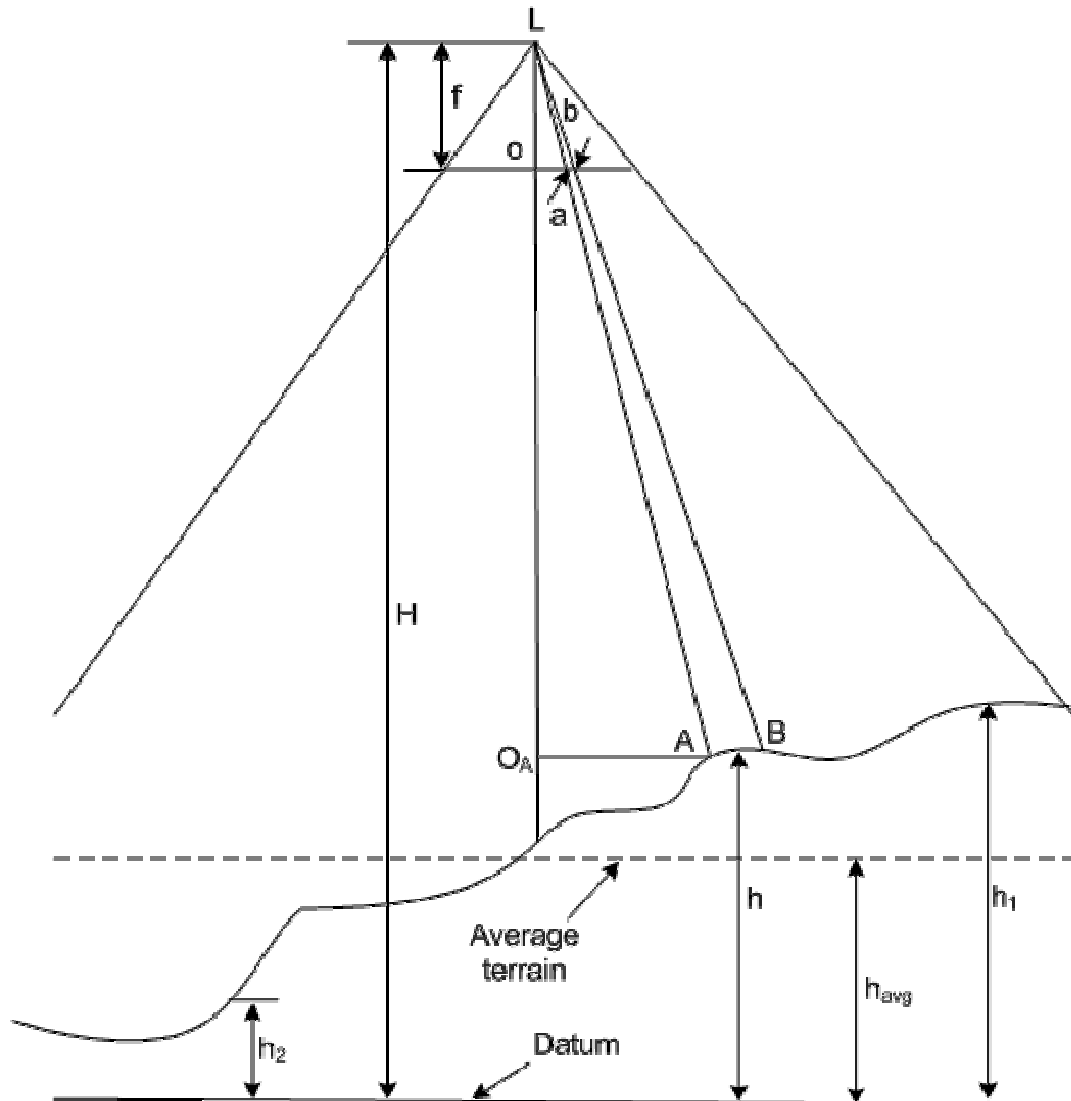

Figure 2:   Scale for vertical photo

h_{avg} Average terrain elevation
S_{avg} *Best single scale to use for a photo or group of photographs*

$$Scale = \frac{ao}{AO_A} = \frac{f}{H - h_A}$$

$$Datum\ scale = S_d = \frac{f}{H}$$

$$Average\ scale = S_{avg} = \frac{f}{H - h_{avg}}$$

Determination of Scale of photograph Scale of photograph can be determined by various methods such as

1. By using known full length and altimeter reading, the datum scale can be found.
2. Any scale can be determined if h_{avg} known. h_{avg} can be obtained from a topographic map.
3. By comparing length of the line on the photo with the corresponding ground length. To arrive at fairly representative scale for entire photo, get several lines in different area and the average of various scales can be adopted.
4. Use the formula

$$\frac{Photo\ scale}{Map\ scale} = \frac{Photo\ distance}{Map\ distance}$$

4.5 Relief Displacement

Unevenness of terrain causes radial relief displacement. Relief displacement is the difference between the actual position of the object on the photograph and its position if it were on the reference plane (datum).

- In figure, L is the perspective center of the camera system. A is the point on ground at an elevation of h with respect to the datum. a is the image of ground point on photograph. a' is the location of projected point A' on the datum. These figures indicate that although point A is vertically above point B, their images are not coinciding and are displaced on photographic plane due to relief.
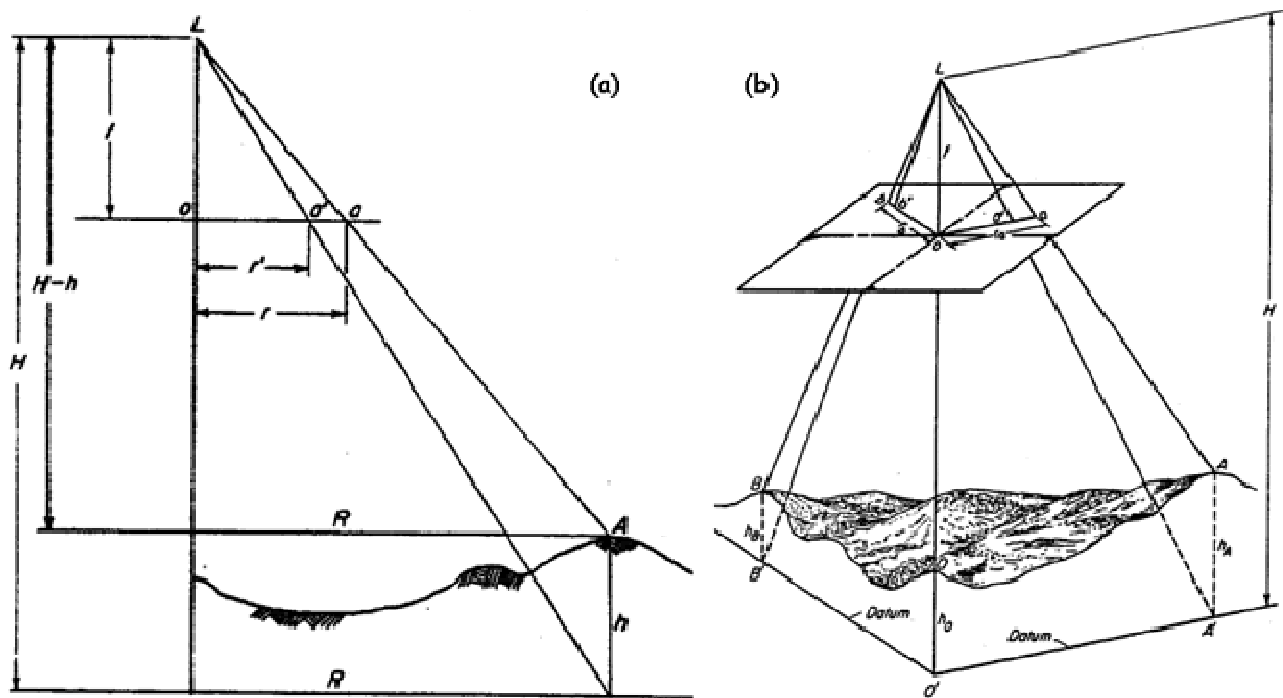
Click on the image for larger view
Figure 4: (a) Relief displacement on vertical photo (b) Radial nature of relief displacement (Moffit, 1959)

- The displacement of the point a on the photograph from its true position, due to height, is called the height or relief displacement or relief distortion (RD). This distortion is due to the perspective geometry.
- It can also be noticed form these figures that the relief displacement is radial from nadir point. In case of vertical photographs, the nadir point and the principal point coincide. Hence, in this case relief displacement can be considered to be radial from the principal point also. The following derivation using figure 4(a) provides the magnitude of relief distortion

$\Delta r = r * h/H$

-Where Δr - relief displacement of the point in question

r - radial distance measured from the principal point out to the image of the point

h - elevation of the point with reference to datum

H - flight height above the datum

NOTE: -

1. Points above the average terrain elevation are displaced out ward from the center of the photograph and points below the average elevation are displaced in ward towards the center along a radial line from the center of the vertical photograph.

2. The relief displacement equation can be used to determine the height of the object for situations when the image of both the top & bottom of an object can be observed on a photograph and the top of the object occurs vertically above the bottom.

3. If a photograph contains all the corners of a tract of land and that the elevations of these corners are known, the relief displacement of each point can be computed provided the flying height above sea level has been determined by measuring the radial distance to each point. The positions of the points are then corrected by the amount of relief displacement and the new positions are therefore at a common datum. The corrected lines may be scaled and the lengths of the sides, the angles between sides, and the track can be determined.

4. The relief displacement, which is nothing more than a manifestation of perspective in an aerial photograph, is of rather minor consequence in a photograph taken at a great altitude over the ground with very little relief. However, if a photograph is to be used as a map substitute then the effect of relief displacement substitute must be recognized, especially where the photograph is large and there is relatively large terrain relief

4.6 Stereoscopy and Parallax

Stereoscopy

Stereoscopy is any technique capable of recording three-dimensional visual information or creating the illusion of depth in an image. The illusion of depth in a photograph, movies, or other two-dimensional image is created by presenting a slightly different image to each eye. Many 3D displays use this method to convey images. Depth perception is the mental process of determining relative distances of object from the observer to the impression received via the eyes. Numerous impressions are received that serves as clues to depth. Those clues that concern photogrammetery are

(a) Head parallax – is the apparent relative movement of object at different distances from the observer when the observer moves.

(b) Accommodation – the lens of the eye can be flatted or made more convex in accordance with the requirement placed on it. This process is termed as Accommodation.

(c) Convergence – means directing the lines of sight of the two eyes at a certain point.

(d) Retinal Disparity – since the two eyes are at different positions, the pictures they receive differ slightly in their horizontal position. This difference between the images on the retina is called retinal disparity or horizontal disparity.

Stereo-pair: - since a photograph is a perspective projection, it represents geometrically the line of view seen by one eye. When two photographs are made of the same object from different positions and then arranged so that the right-hand photograph is seen by the right eye, retinal disparity is established and the observer can distinguish depth. Two such photographs are called stereo pairs.

An instrument used for 3-D viewing of a stereo pairs is called **stereoscope**.

Parallax

The change in position of an image from one photograph to the next caused by the air crafts motion is termed as the stereoscopic parallax (x-parallax). The parallax of any point is directly related to the elevation of the point.