CABLE

Cables and arches are closely related to each other and hence they are grouped in this course in the same module. For long span structures (for e.g. in case bridges) engineers commonly use cable or arch construction due to their efficiency. In the first lesson of this module, cables subjected to uniform and concentrated loads are discussed. In the second lesson, arches in general and three hinged arches in particular along with illustrative examples are explained.

Fundamental Characteristic of Cable & Arch

Cables

- ✓ Carry applied loads & develop mostly tensile stresses
- ✓ Loads applied through hangers
- ✓ Cables near the end supporting structures experience bending moments and shear forces

Arches

- ✓ carry applied loads and develop mainly in-plane compressive stresses;
- ✓ Loads applied through ribs
- ✓ Arch sections near the rib supports and
- ✓ Arches, other than three-hinged arches, experience bending moments and shear forces

Example

Cable type structures - Suspension roof, suspension bridges, cable cars, guy-lines, transmission lines, etc.

Arch type structures - Arches, domes, shells, vaults

More information related to the topic

Structure may be classified into rigid and deformable structures depending on change in geometry of the structure while supporting the load. Rigid structures support externally applied loads without appreciable change in their shape (geometry). Beams trusses and frames are examples of rigid structures. Unlike rigid structures, deformable structures undergo changes in their shape according to externally applied loads. However, it should be noted that deformations are still small. Cables and fabric structures are deformable structures. Cables are mainly used to support suspension roofs, bridges and cable car system. They are also used in electrical transmission lines and for structures supporting radio antennas. In the following sections, cables subjected to concentrated load and cables subjected to uniform loads are considered.

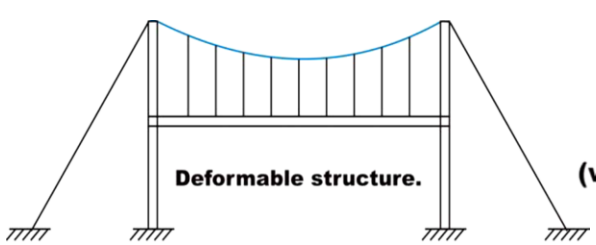


Fig-1

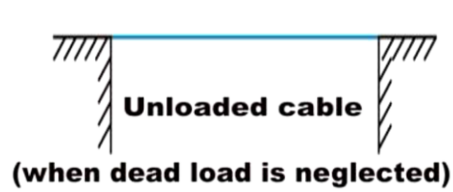


Fig-2

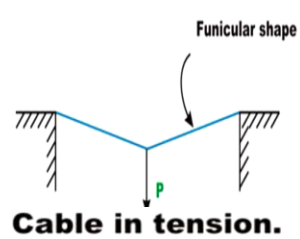
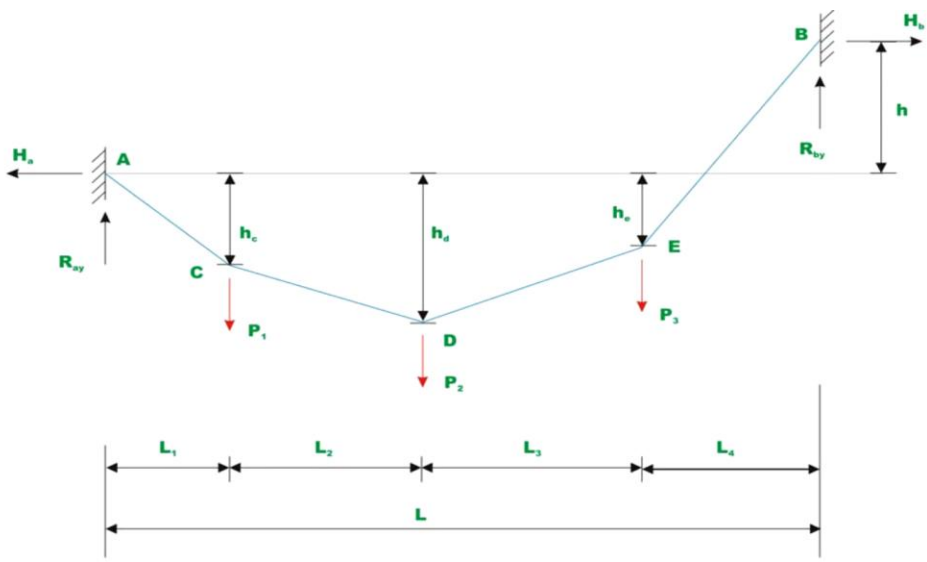


Fig-3

The shape assume[illegible]pe or a chain (with no stiffness [illegible] he action of external loa[illegible] hung from two supports is known as a funicular shape. Cable is a funicular structure. It is easy to vis[illegible]at a cable hung from two supports subjected to external load must be in tension. Now let us modify our definition of cable. **A cable may be defined as the structure in pure tension having the funicular shape of the load**.

Cable subjected to Concentrated Loads

As stated earlier, the cables are considered to be perfectly flexible (no flexural stiffness) and inextensible. As they are flexible they do not resist shear force and bending moment. It is subjected to axial tension only and it is always acting tangential to the cable at any point along the length. If the weight of the cable is negligible as compared with the externally applied loads then its self weight is neglected in the analysis. In the present analysis self weight is not considered.



Cable subjected to concentrated load.

Consider a cable as loaded in the Fig above. Let us assume that the cable lengths L_1, L_2, L_3, L_4 and sag at **C, D, E (h_c, h_d, h_e)** are known. The four reaction components at *A* and *B*, cable tensions in each of the four segments and three sag values: a total of eleven unknown quantities are to be determined. From the geometry, one could write two force equilibrium equations ($\sum F_x=0$, $\sum F_y=0$) at each of the point *A,B,C,D* and *E* i.e. a total of ten equations and the required one more equation may be written from the geometry of the cable. For example, if one of the sag is given then the problem can be solved easily. Otherwise if the total length of the cable ***S*** is given then the required equation may be written as

$$S = \sqrt{L_1^{\,2} + h_c^{\,2}} + \sqrt{L_2^{\,2} + (h_d - h_c)^2} + \sqrt{L_2^{\,2} + (h_d - h_e)^2} + \sqrt{L_2^{\,2} + (h + h_e)^2}$$

Cable subjected to uniform load

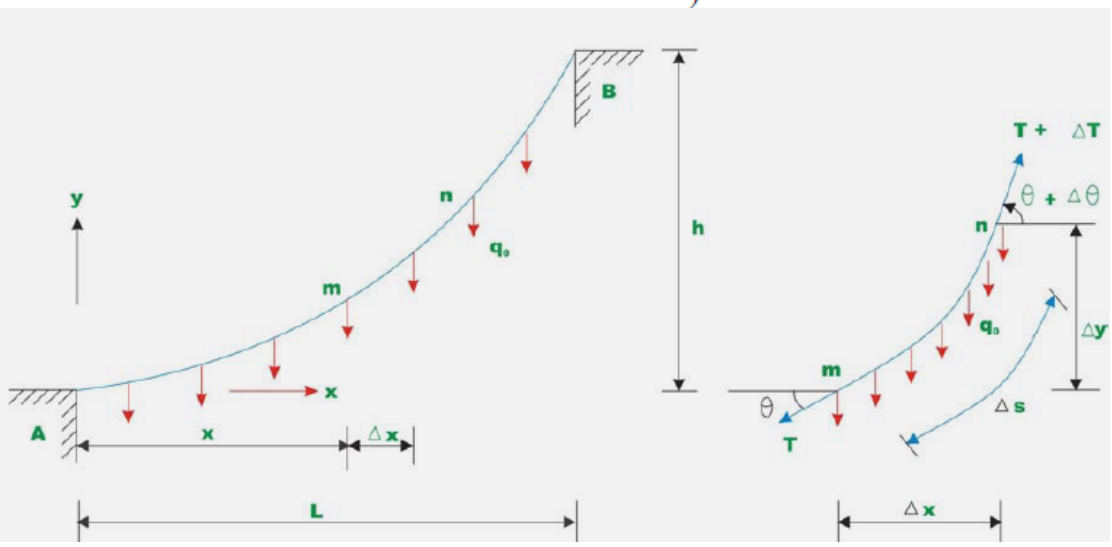
Cables are used to support the dead weight and live loads of the bridge decks having long spans. The bridge decks are suspended from the cable using the hangers. The stiffened deck prevents the supporting cable from changing its shape by distributing the live load moving over it, for a longer length of cable. In such cases cable is assumed to be uniformly loaded.

Consider a cable which is uniformly loaded as shown in Fig 1. Let the slope of the cable be zero at *A*. Let us determine the shape of the cable subjected to uniformly distributed load. Consider a free body diagram of the cable as shown in Fig 31.3b. As the cable is uniformly loaded, the tension in the cable changes continuously along the cable length. Let the tension in the cable at ***m*** end of the free body diagram be ***T*** and tension at the ***n*** end of the cable be (**T**+Δ**T**). The slopes of the cable at **m** and **n** are denoted by **θ** and **θ+Δθ** respectively. Applying equations of equilibrium, we get

$$\sum Fy = 0 \qquad -T\sin\theta + (T+\Delta T)\sin(\theta+\Delta\theta) - q_0(\Delta x) = 0 \qquad \text{... ... ... } (Eq^n\text{-1})$$

$$\sum Fx = 0 \qquad -T\cos\theta + (T+\Delta T)\cos(\theta+\Delta\theta) = 0 \qquad \text{... ... ... } (Eq^n\text{-2})$$

$$\sum Mn = 0 \qquad -(T\cos\theta)\Delta y + (T\sin\theta)\Delta x + (q_0\Delta x)\frac{\Delta x}{2} = 0 \qquad \text{... ... ... } (Eq^n\text{-3})$$



Cable subjected to uniformly distributed load.

Fig-1

Free-body diagram

Fig-2

Dividing equations *Eq^n-1, 2 & 3* by Δx and noting that in the limit as Δx → 0, Δy→0, Δθ →0 and ΔT → 0;

$$\lim_{\Delta x \to 0} \quad \frac{\Delta T}{\Delta x}\sin(\theta+\Delta\theta) = q_0$$

$$\frac{d}{dx}(T\sin\theta) = q_0 \qquad \text{... ... ... } (Eq^n\text{-4})$$

$$\frac{d}{dx}(T\cos\theta) = 0 \qquad \text{... ... ... } (Eq^n\text{-5})$$

$$\lim_{\Delta x \to 0} \quad -T\cos\theta\frac{\Delta y}{\Delta x} +$$

$$\frac{dy}{dx} = \tan\theta \qquad T\cos\theta = \text{constant}$$

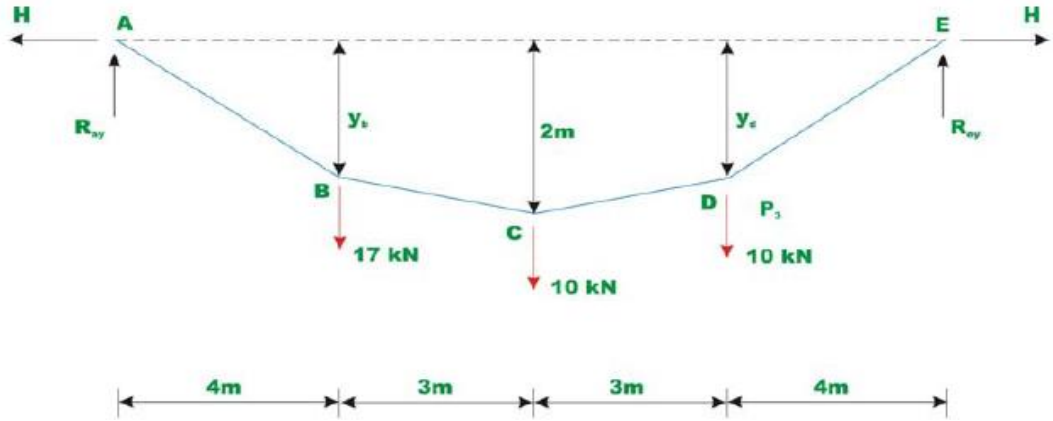
Integrating equation 5 we get,

At support (i.e., at x=0), **Tcosθ = H**

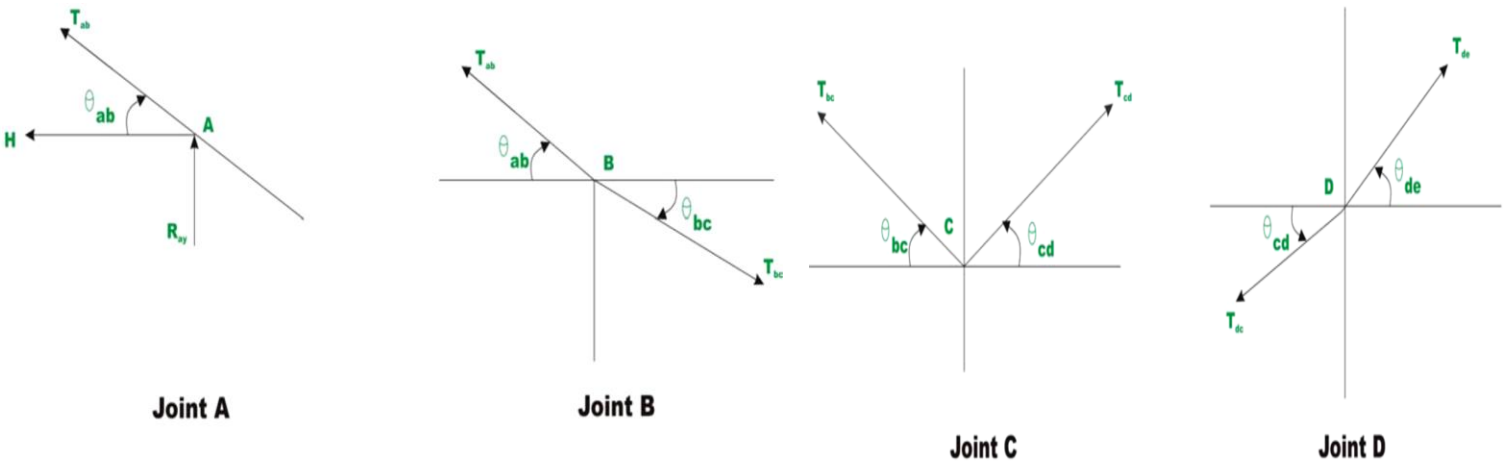
i.e. **horizontal component of the force along the length of the cable is constant.**

→Problem-1:

Determine reaction components at A and B, tension in the cable and the sag Y_B & Y_D of the cable shown in the following Fig. Neglect the self weight of the cable in the analysis.



Solution:



Since there are no horizontal loads, horizontal reactions at A and B should be the same. Taking moment about *E*, yields

$$R_{ay} \times 14 - 17 \times 20 - 10 \times 7 - 10 \times 4 = 0$$

$$R_{ay} = \frac{280}{14} = 20 \quad \text{kN}; \qquad R_{ey} = 37 - 20 = 17\text{kN}.$$

Now horizontal reaction *H* may be evaluated taking moment about point *C* of all forces left of *C*.

$$R_{ay} \times 7 - H \times 2 - 17 \times 3 = 0$$

$$H = 44.5 \text{ kN}$$

Taking moment about *B* of all the forces left of *B* and setting $\mathbf{M_B=0}$, we get

$$R_{ay} \times 4 - H \times y_B = 0; \qquad y_B = \frac{80}{44.50} = 1.798\,m$$

Similarly, $y_D = \dfrac{68}{44.50} = 1.528\,m$

To determine the tension in the cable in the segment *AB*, consider the equilibrium of joint *A*

$$\sum F_x = 0 \Rightarrow T_{ab}\cos\theta_{ab} = H$$

$$T_{ab} = \frac{44.5}{\left(3\Big/\sqrt{3^2+0.298^2}\right)} = 48.789 \text{ kN}$$

The tension T_{ab} may also be obtained as,

$$T_{ab} = \sqrt{R_{ay}{}^2 + H^2} = \sqrt{20^2 + 44.5^2} = 48.789 \quad \text{kN}$$

Segment bc

Applying equations of equilibrium,

$$\sum F_x = 0 \Rightarrow T_{ab}\cos\theta_{ab} = T_{bc}\cos\theta_{bc}$$

$$T_{bc} = \frac{44.5}{\left(3\Big/\sqrt{3^2+0.298^2}\right)} \cong 44.6 \quad \text{kN}$$

Segment cd

$$T_{cd} = \frac{T_{bc}\cos\theta_{bc}}{\cos\theta_{cd}} = \frac{44.5}{\left(3\Big/\sqrt{3^2+0.472^2}\right)} = 45.05 \quad \text{kN}$$

Segment de

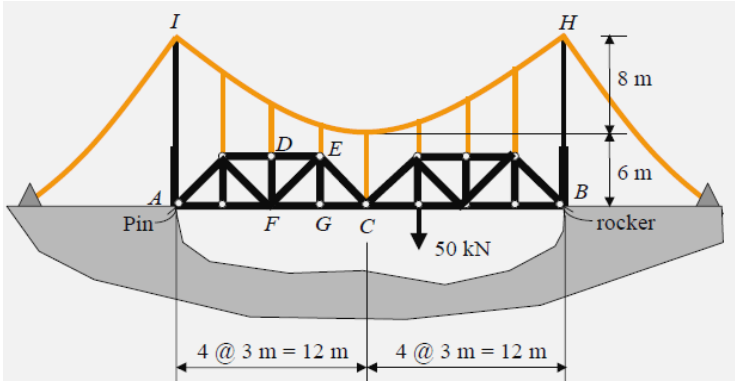
$$T_{de} = \frac{T_{cd}\cos\theta_{cd}}{\cos\theta_{de}} = \frac{44.5}{4\Big/\sqrt{4^2+1.528^2}} = 47.636 \quad \text{kN}$$

The tension T_{de} may also be obtained as,

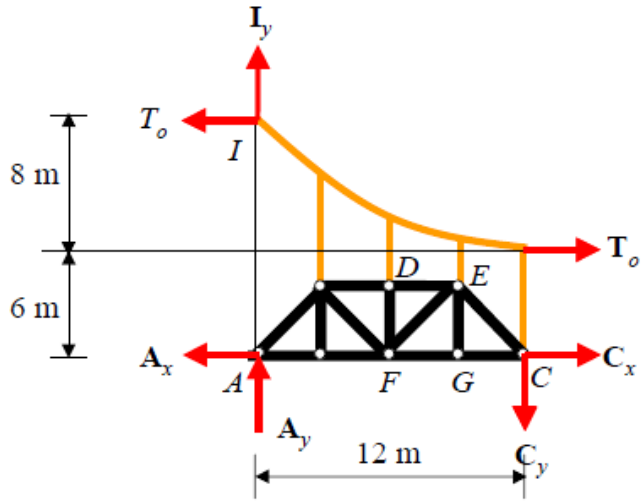
$$T_{de} = \sqrt{R_{ey}{}^2 + H^2} = \sqrt{17^2 + 44.5^2} = 47.636 \quad \text{kN}$$

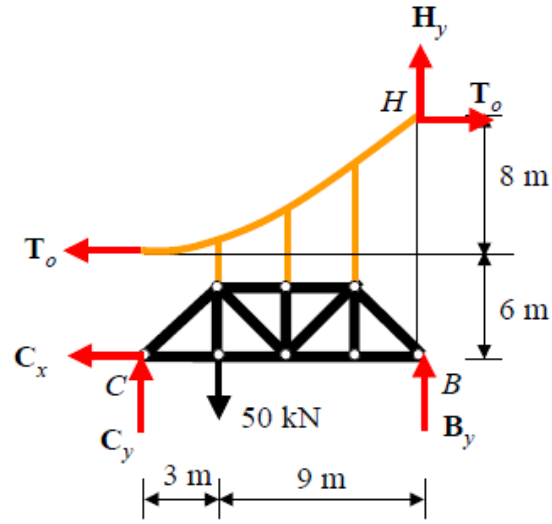
→Problem-2:

The suspension bridge in the figure below is constructed using the two stiffening trusses that are pin connected at their ends *C* and supported by a pin at *A* and a rocker at *B*. Determine the maximum tension in the cable *IH*. The cable has a parabolic shape and the bridge is subjected to the single load of 50 kN.



Solution:





$+\circlearrowleft \Sigma M_A = 0:$

$$-12C_y + 8T_o = 0$$

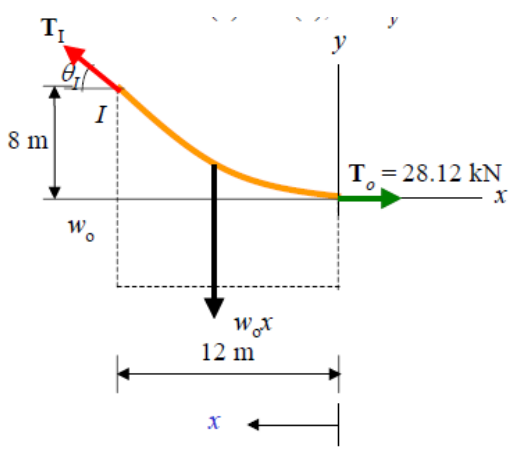
$$T_o = 1.5C_y \quad \text{----------(1)}$$

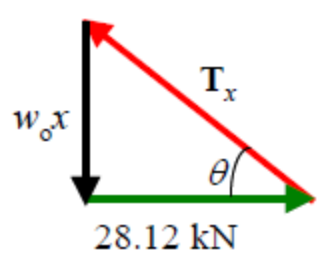
$+\circlearrowleft \Sigma M_B = 0:$

$$-12C_y + 50(9) - 8T_o = 0$$

$$T_o = -1.5C_y + 56.25 \quad \text{----------(2)}$$

From (1) and (2), $C_y = 18.75$ kN, $T_o = 28.125$ kN





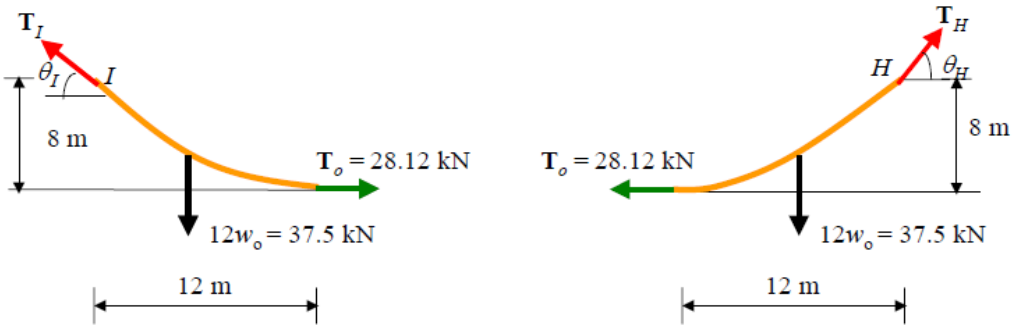
$$\frac{dy}{dx} = \tan\theta = \frac{w_o x}{28.12}$$

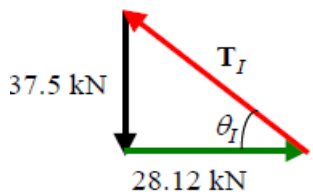
$$y = \int \frac{w_o x}{28.12} dx$$

$$y = \frac{w_o x^2}{28.12} + \cancel{C_1}^{\,0}$$

$$8 = \frac{w_o (12)^2}{2(28.12)}$$

$$w_o = 3.125 \text{ kN/m}$$



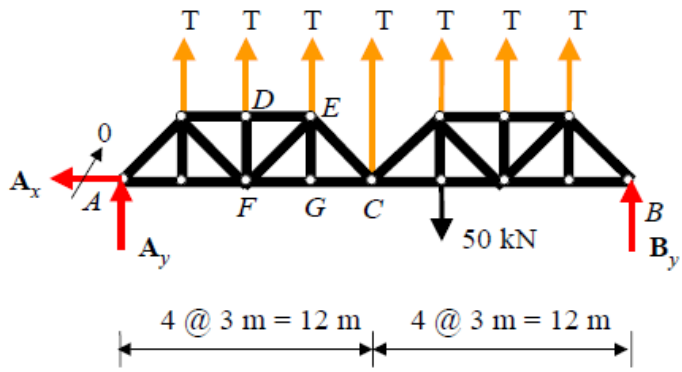


$$T_I = \sqrt{(37.5)^2 + (28.12)^2}$$

$$= 46.88 \text{ kN}$$

$$T_{max} = T_I = T_H = 46.88 \text{ kN}$$

$$T_{min} = T_o = 28.12 \text{ kN}$$



$$T = w_o \times 3 = 3.125 \times 3$$

$$= 9.375 \text{ kN}$$

$$\circlearrowleft + \ \Sigma M_A = 0: \quad 9.375(3+6+9+12+15+18+21) - 50(15) + B_y(24) = 0$$

$$B_y = -1.56 \text{ kN}, \downarrow$$

$$+\uparrow \ \Sigma F_y = 0: \quad A_y + 7(9.375) - 50 - 1.56 = 0$$

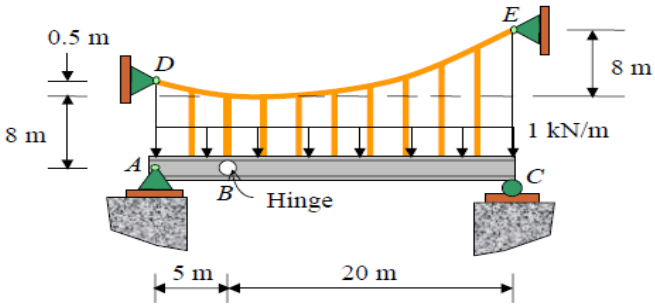
$$A_y = -14.07 \text{ kN}, \downarrow$$

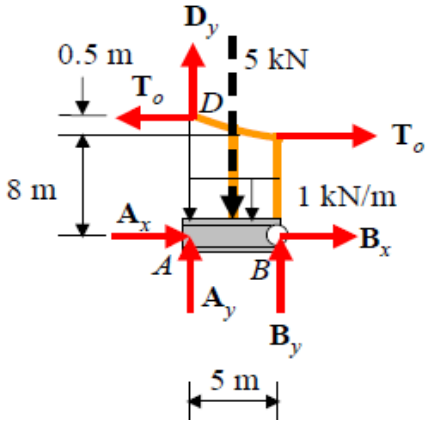
→Problem-3:

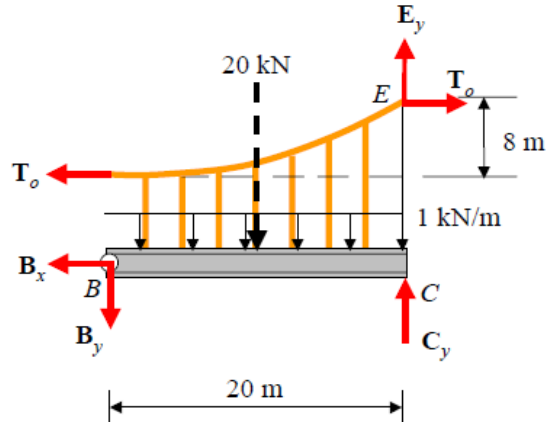
For the structure shown:

(a) Determine the **maximum tension** of the cable

(b) Draw **quantitative shear & bending-moment diagrams** of the beam.







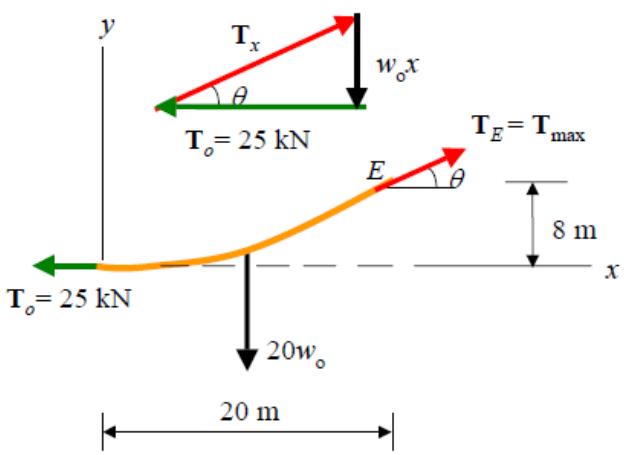
$+\circlearrowleft \Sigma M_A = 0:$

$$B_y(5) - 5(2.5) + T_o(0.5) = 0$$

$+\circlearrowleft \Sigma M_C = 0:$

$$B_y(20) + 20(10) - T_o(8) = 0$$

From (1) and (2), $B_y = 0$, $T_o = 25$ kN



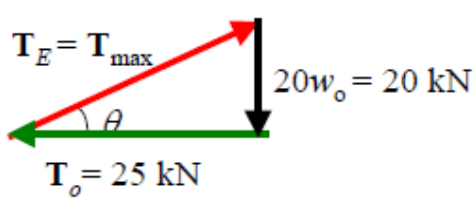
$$\frac{dy}{dx} = \tan\theta = \frac{w_o x}{25}$$

$$y = \int \frac{w_o x}{25} dx$$

$$= \frac{w_o x^2}{2(25)} + \cancel{C_1}^{\,0}$$

$$8 = \frac{w_o (20)^2}{2(25)}$$

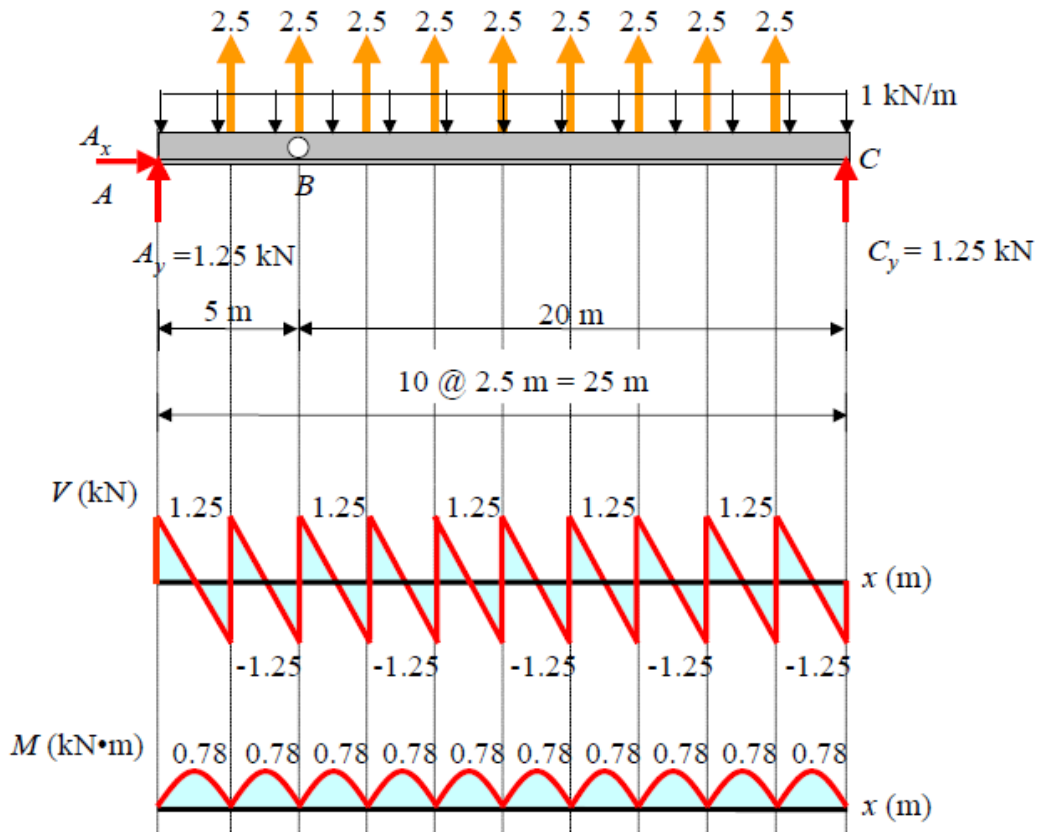
$$w_o = 1 \text{ kN/m}$$



$$T_{max} = T_E = \sqrt{(25)^2 + (20)^2}$$

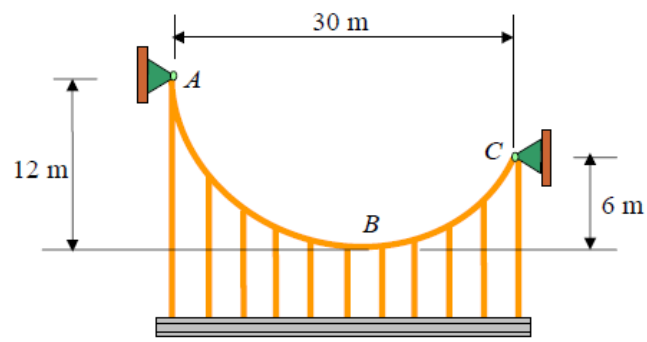
$$T_{max} = 32.02 \text{ kN}$$

$T = w_o(2.5\text{ m}) = (1\text{kN/m})(2.5\text{ m}) = 2.5\text{ kN}$

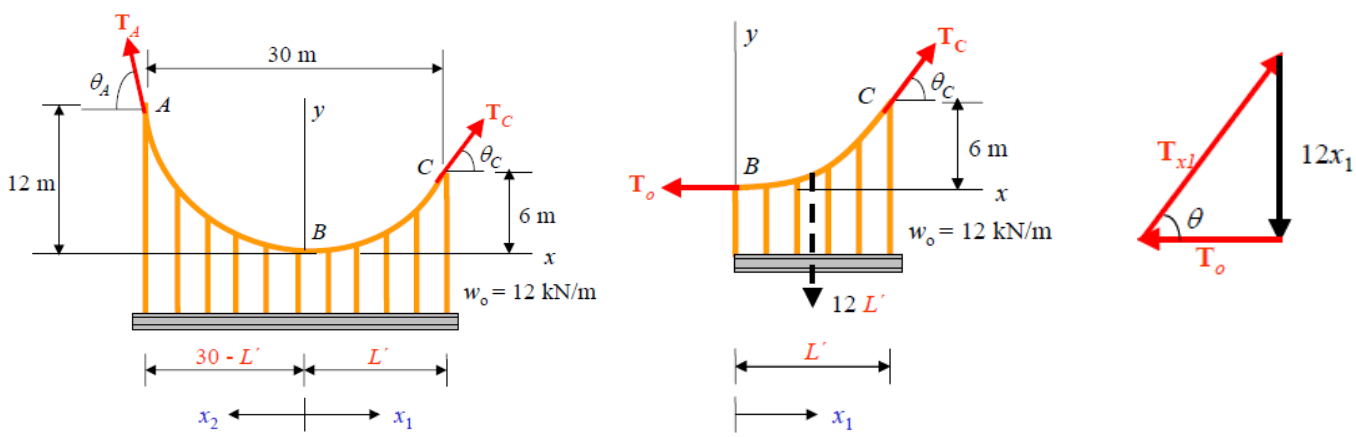


→Problem-4

The cable shown supports a girder which weighs 12kN/m. Determine the tension in the cable at points *A*, *B*, and *C*.



Solution:



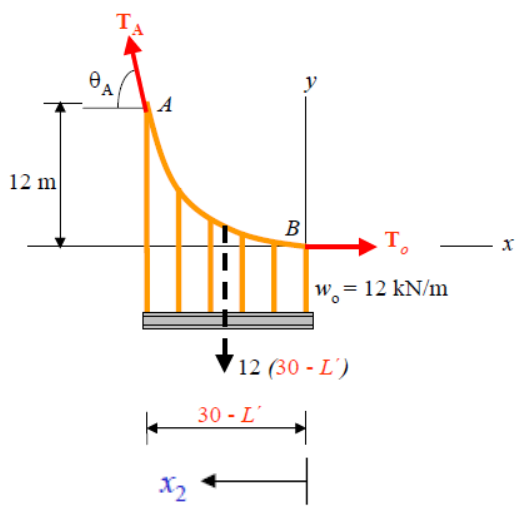
$$\frac{dy_1}{dx_1} = \tan\theta = \frac{12x_1}{T_o}$$

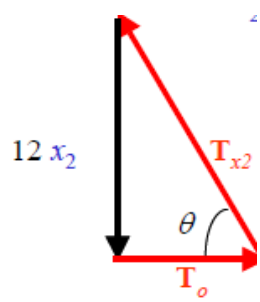
$$y_1 = \int \frac{12x_1}{T_o} dx_1$$

$$y_1 = \frac{12x_1^2}{2T_o} + \cancel{C_1}^{\,0}$$

$$6 = \frac{12L'^2}{2T_o}$$

$$T_o = L'^2 \qquad \text{----------(1)}$$





$$\frac{dy_2}{dx_2} = \tan\theta = \frac{12x_2}{T_o}$$

$$y_2 = \int \frac{12x_2}{T_o} dx_2 = \frac{12x_2^2}{2T_o} + \cancel{C_2}^{\,0}$$

$$y_2 = \frac{12x_2^2}{2T_o}$$

$$\cancel{12} = \frac{\cancel{12}(30 - L')^2}{2T_o}$$

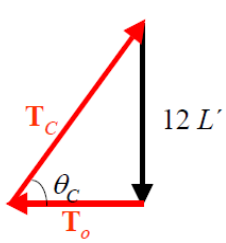
$$1 = \frac{(30 - L')^2}{2T_o} \qquad \text{----------(2)}$$

$$T_o = L'^2 \qquad \text{----------(1)}$$

$$1 = \frac{(30 - L')^2}{2T_o} \qquad \text{----------(2)}$$

From (1) and (2), $L' = 12.43$ m, $T_o = 154.5$ kN

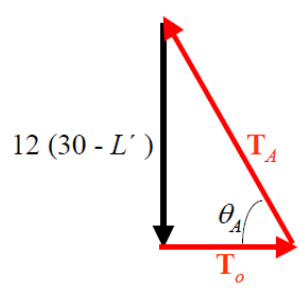
$$T_B = T_o = 154.5 \text{ kN}$$



$$T_C = \sqrt{T_o^2 + (12L')^2}$$

$$= \sqrt{(154.50)^2 + (12 \times 12.43)^2}$$

$$= 214.8 \text{ kN}$$



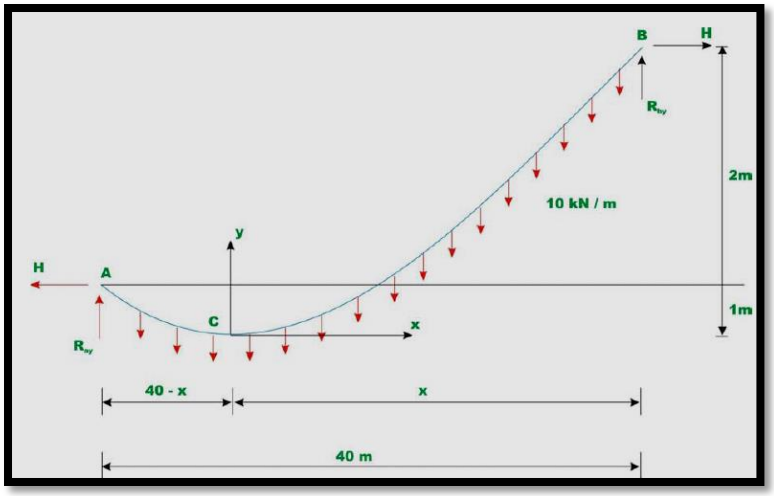
$$T_A = \sqrt{T_o^2 + [12(30 - L')]^2}$$

$$= \sqrt{(154.50)^2 + [12(30 - 12.43)]^2}$$

$$= 261.4 \text{ kN}$$

→Problem-5:

A cable of uniform cross section is used to span a distance of 40m as shown in Fig. The cable is subjected to uniformly distributed load of 10 kN/m. run. The left support is below the right support by 2 m and the lowest point on the cable *C* is located below left support by 1 m. Evaluate the reactions and the maximum and minimum values of tension in the cable.



Solution:

Assume the lowest point *C* to be at distance of *x* m from *B*. Let us place our origin of the co-ordinate system *xy* at C.

$$y_a = 1 = \frac{q_0(40-x)^2}{2H} = \frac{10(40-x)^2}{2H} \qquad \text{... ... ... } (Eq^n\text{-1})$$

$$y_b = 3 = \frac{10x^2}{2H} \qquad \text{... ... ... } (Eq^n\text{-2})$$

Where y_a and y_b be the co-ordinates of supports *A* and *B* respectively. From equations 1 and 2, one could evaluate the value of *x*.

$$10(40-x)^2 = \frac{10x^2}{3} \quad \Rightarrow \quad x = 25.359\,m$$

From equation 2, the horizontal reaction can be determined.

$$H = \frac{10 \times 25.359^2}{6} = 1071.80 \text{ kN}$$

Now taking moment about *A* of all the forces acting on the cable, yields

$$R_{by} = \frac{10 \times 40 \times 20 + 1071.80 \times 2}{40} = 253.59 \text{ kN}$$

Writing equation of moment equilibrium at point *B*, yields

$$R_{ay} = \frac{40 \times 20 \times 10 - 1071.80 \times 2}{40} = 146.41 \text{ kN}$$

Tension in the cable at supports *A* and *B* are

$$T_A = \sqrt{146.41^2 + 1071.81^2} = 1081.76 \text{ kN}$$

$$T_B = \sqrt{253.59^2 + 1071.81^2} = 1101.40 \text{ kN}$$

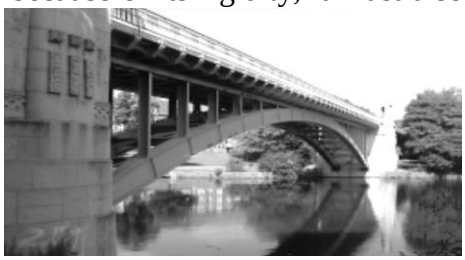
The tension in the cable is maximum where the slope is maximum as $\boldsymbol{T \cos\Theta = H}$. The maximum cable tension occurs at *B* and the minimum cable tension occurs at *C* where $\frac{dy}{dx} = \Theta = 0$; and $T_c = H = 1071.81$ kN.

ADDITIONAL CONSIDERATIONS FOR CABLE SUPPORTED STRUCTURES

- **Forces on cable bridges**: Wind drag and lift forces - Aero-elastic effects should be considered (vortex-induced oscillations, flutter, torsional divergence or lateral buckling, galloping and buffeting).
- **Wind tunnel tests**: To examine the aerodynamic behavior
- **Precaution to be taken against**: Torsional divergence or lateral buckling due to twist in bridge; Aero-elastic stability caused by geometry of deck, frequencies of vibration and mechanical damping present; Galloping due to self-excited oscillations; Buffeting due to unsteady loading caused by velocity fluctuations in the wind flow

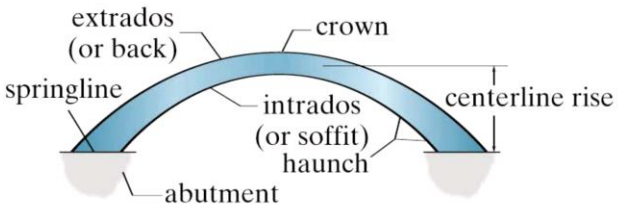
ARCHES

Arches can be used to reduce the bending moments in long-span structures. Essentially, an arch acts as an inverted cable, so it receives its load mainly in compression although, because of its rigidity, it must also resist some bending and shear depending upon how it is loaded and shaped.

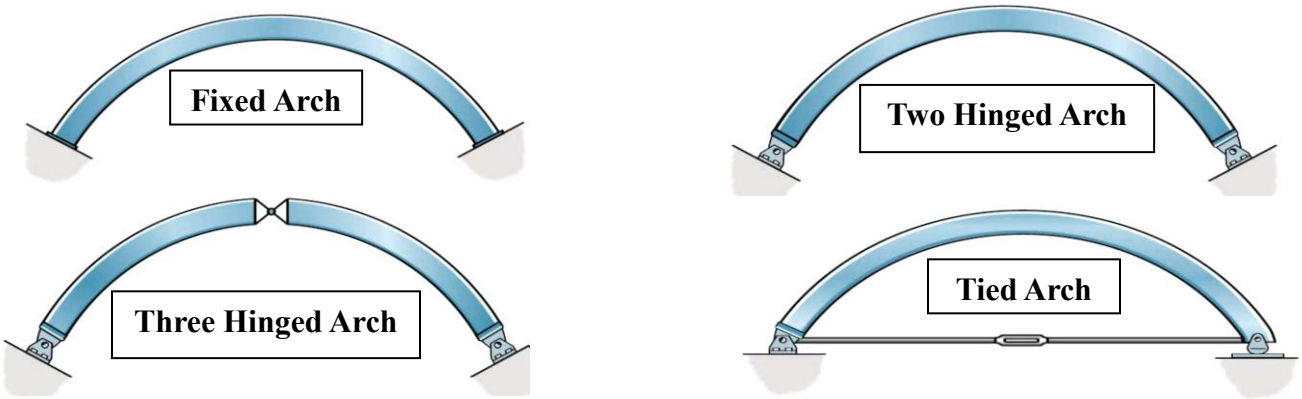


In particular, if the arch has a parabolic shape and it is subjected to a uniform horizontally distributed vertical load, then only compressive forces will be resisted by the arch. Under these conditions the arch shape is called a funicular arch because no bending or shear forces occur within the arch.

Different terms and types of Arches



Depending on its uses, several types of arches can be selected to support a loading:



Fixed Arch:

A *fixed arch* is often made from reinforced concrete. Although it may require less material to construct than other types of arches, it must have solid foundation abutments since it is indeterminate to the third degree and, consequently, additional stresses can be introduced into the arch due to relative settlement of its supports.

Two Hinged Arch:

A *two-hinged arch* is commonly made from metal or timber. It is indeterminate to the first degree, and although it is not as rigid as a fixed arch, it is somewhat insensitive to settlement. We could make this structure statically determinate by replacing one of the hinges with a roller. Doing so, however, would remove the capacity of the structure to resist bending along its span, and as a result it would serve as a curved beam, and not as an arch.

Three Hinged Arch:

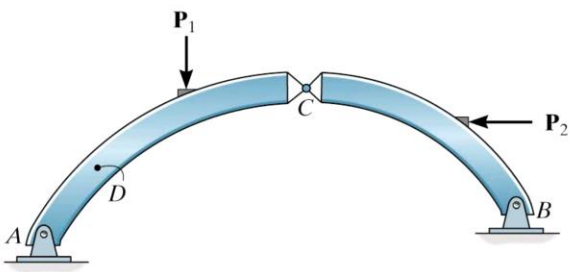
A *three-hinged arch* which is also made from metal or timber, is statically determinate. Unlike statically indeterminate arches, it is not affected by settlement or temperature changes.

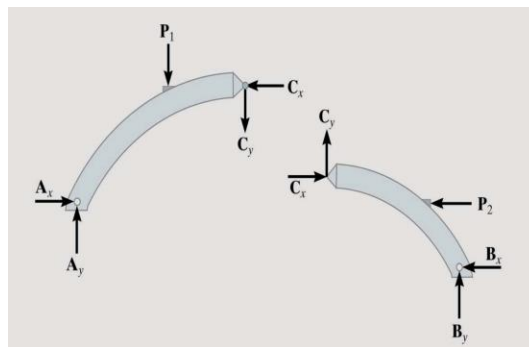
Tied Arch:

If two and three-hinged arches are to be constructed without the need for larger foundation abutments and if clearance is not a problem, then the supports can be connected with a tie rod. A tied arch allows the structure to behave as a rigid unit, since the tie rod carries the horizontal component of thrust at the supports. It is also unaffected by relative settlement of the supports.

Three-Hinged Arches

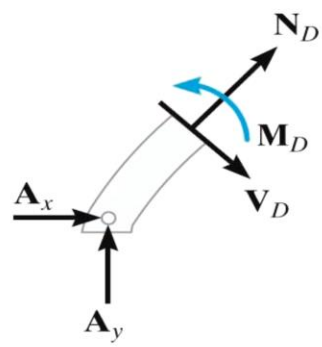
- ✓ The third hinge is located at the crown & the supports are located at different elevations
- ✓ To determine the reactions at the supports, the arch is disassembled





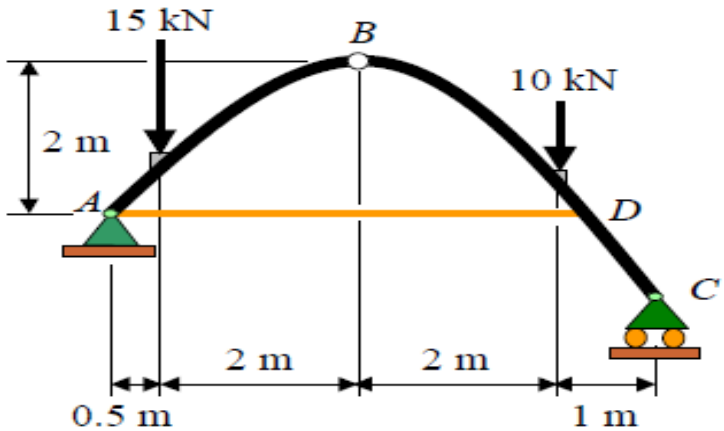
In order to determine the reactions at the supports, the arch is disassembled and the free-body diagram of each member. Here there are six unknowns for which six equations of equilibrium are available. One method of solving this problem is to apply the moment equilibrium equations about points *A* and *B*. Simultaneous solution will yield the reactions C_x and C_y. The support reactions are then determined from the force equations of equilibrium.

Once all support reactions obtained, the internal normal force, shear, and moment loadings at any point along the arch can be found using the method of sections. Here, of course, the section should be taken perpendicular to the axis of the arch at the point considered.



→Problem-6:

The tied three-hinged arch is subjected to the loading shown. Determine the components of reaction at A and C and the tension in the cable.



Solution:

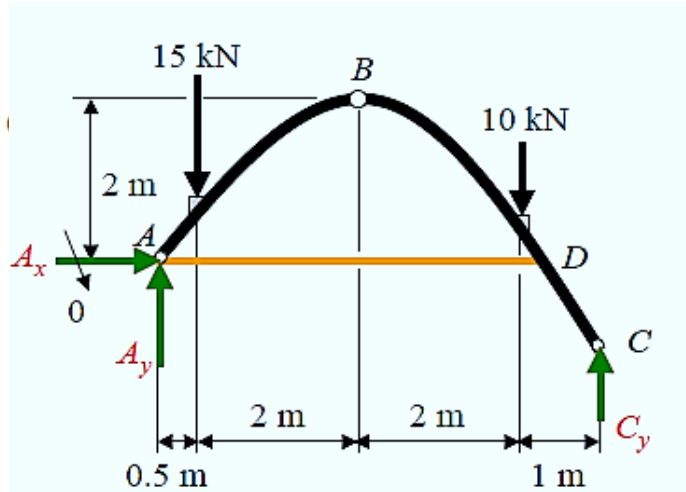
Entire arch :

$\circlearrowleft + \; \Sigma M_A = 0: \qquad C_y(5.5) - 10(4.5) - 15(0.5) = 0$

$C_y = 9.545 \text{ kN}$

$+\uparrow \; \Sigma F_y = 0: \qquad A_y - 15 - 10 + 9.545 = 0$

$A_y = 15.46 \text{ kN}$

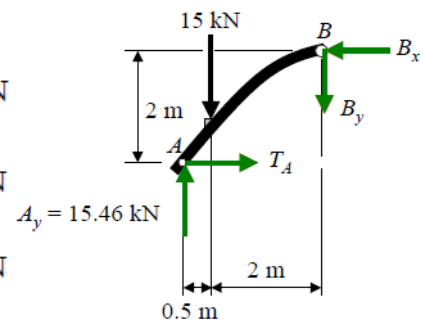


Member *AB* :

$\circlearrowleft + \; \Sigma M_B = 0: \qquad 15(2) - 15.455(2.5) + T_A(2) = 0 \qquad T_A = 4.319 \text{ kN}$

$+\uparrow \; \Sigma F_y = 0: \qquad 15.455 - 15 - B_y = 0 \qquad B_y = 0.455 \text{ kN}$

$\xrightarrow{+} \; \Sigma F_x = 0: \qquad 4.319 - B_x = 0 \qquad B_x = 4.319 \text{ kN}$



Member *AB* :

$\xrightarrow{+} \; \Sigma F_x = 0: \qquad 4.319 - T_D = 0 \qquad T_D = 4.319 \text{ kN}$