CHAPTER 5

COLUMN

5.1 INTRODUCTION

Columns are structural elements used primarily to support compressive loads. All practical columns are members subjected not only to axial load but also to bending moments, about one or both axes of the cross section. The bending action may produce tensile forces over a part of the cross sections, even in such cases; columns are generally refereed to as compression members.

5.2 TYPES OF COLUMN

According to the Ratio of Height to Least Lateral Dimension: *(ACI Code 10.11.5 and ACI Code 10.12.2)*

Short column (Figure 5.1a): When

1. $\frac{kl_u}{r} \leq 34\text{-}12(M_1/M_2)$, where (M_1/M_2) is not taken less than -0.5.

2. $\frac{kl_u}{r} \leq 22$, when member is braced against sideways.

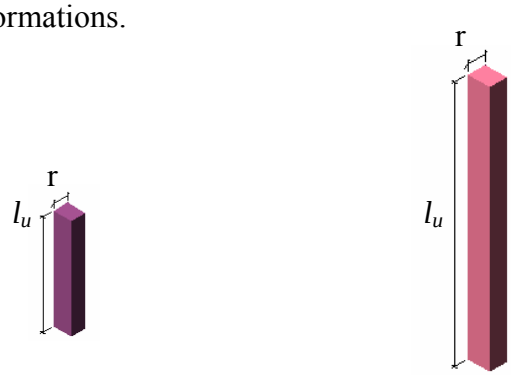
Strength is governed by the strength of the material and dimension of the cross section

Slender Column (Figure 5.1b): When

$$\frac{kl_u}{r} \geq 100$$

Where, r = radius of gyration

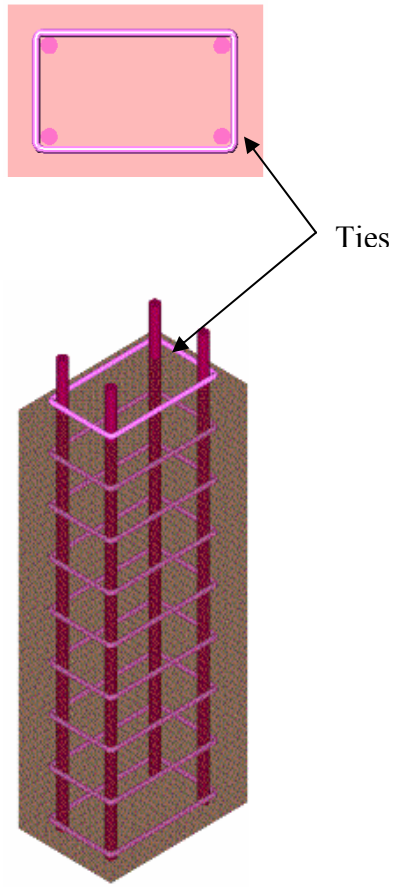
Strength is influenced by slenderness, which produces additional bending because of transverse deformations.



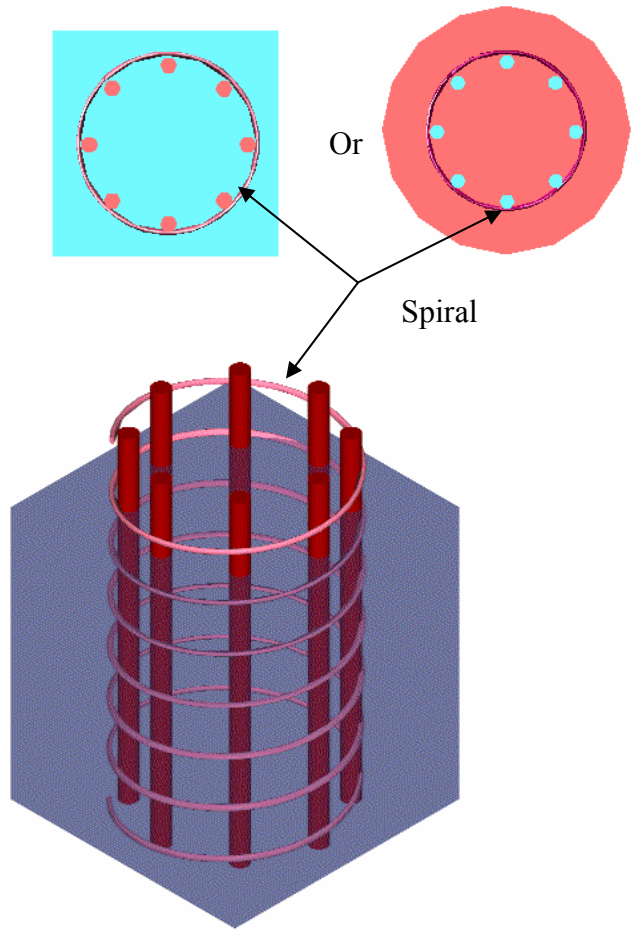
(a) Short column (b) Slender column

Figure 5.1: Types of column

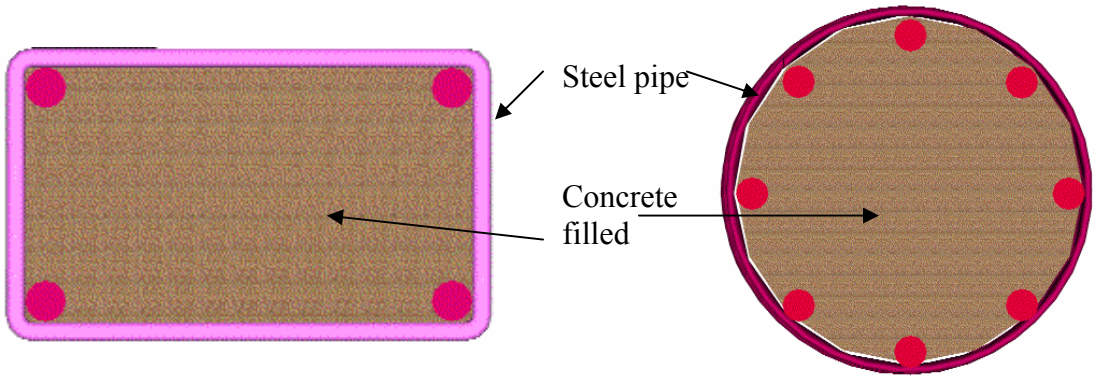
According to the Geometry of Cross Section:



(c) Tied Column



(d) Spiral Column



(e) Composite Column

Figure 5.1: Types of column (continued)

5.3 SHORT COLUMN

5.3.1 CONCENTRICALLY LOADED COLUMN

5.3.1.1 Design of Tied Column

Step 1: Determination of Factored Load

Load contribution to the column: (See Figure 5.2)

- *For columns of intermediate floor:* load from upper floor through the column above it + slab and beam loads from corresponding floor tributary area.
- *For columns of top floor*: slab and beam loads from adjacent tributary area

$P_u = 1.4\ D + 1.7\ L$ (ACI Code-00)

$P_u = 1.2\ D + 1.6\ L$ (ACI Code-02)

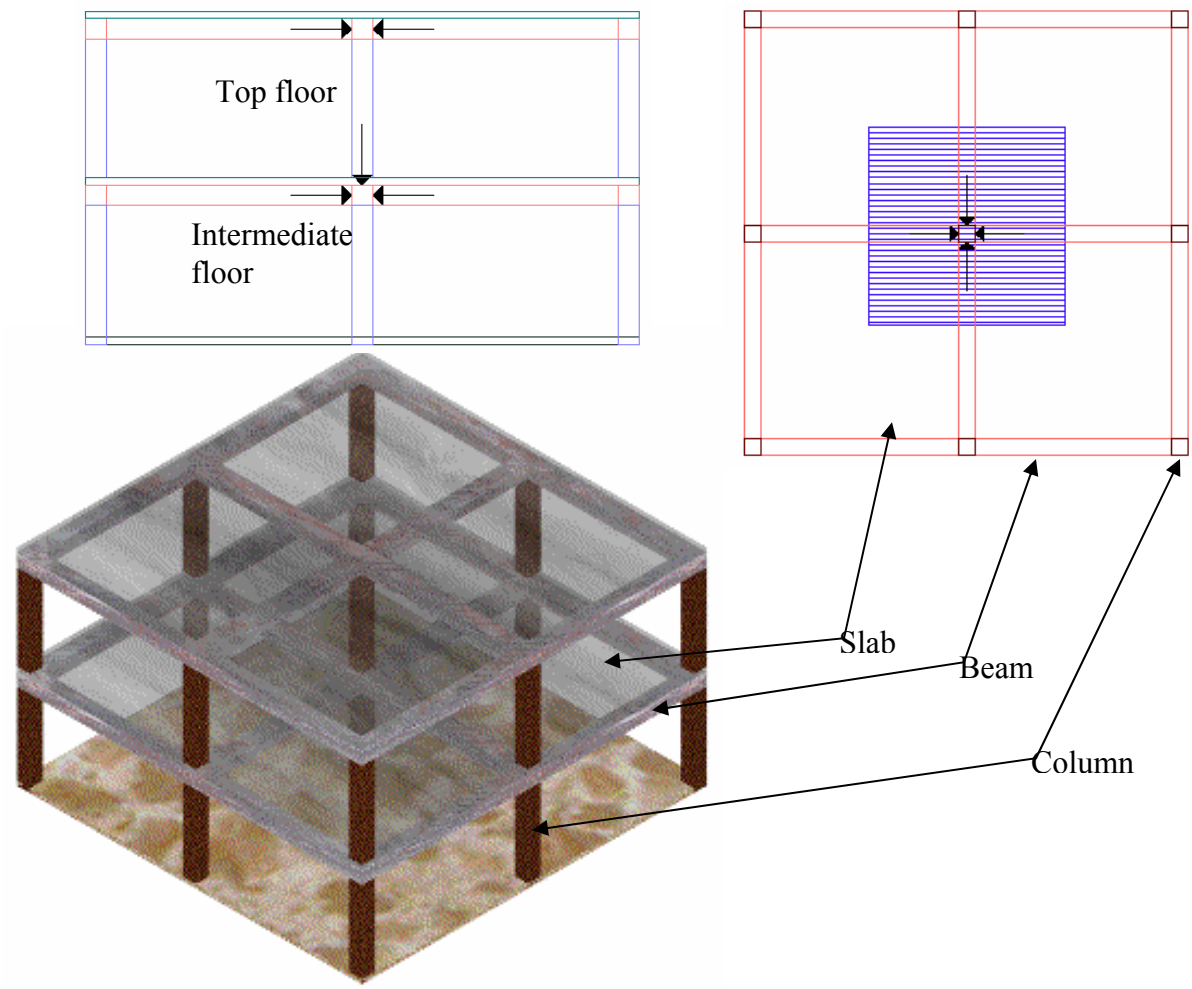


Figure 5.2: Live Load contribution to a column (*ACI Code-8.9*)

Step 2: Steel Ratio Assumption

Assumption of steel ratio depends on designer's experience.
According to *ACI Code 10.9.1*
$0.01 \leq \rho_g \leq 0.08$

Step 3: Determination of Concrete Gross Area

According to *ACI Code 10.3.6.1*, the design strength for tied column

$$\varnothing P_{n,max} = 0.80\varnothing\,[0.85f'_c(A_g - A_{st}) + f_y A_{st}] \qquad (5\text{-}1)$$

Where, $\varnothing = 0.65$ *(ACI Code-02),* consider $P_u = \varnothing\, P_n$
Equation (4-1) can be rewritten as:

$$P_u = 0.80\, A_g\, \varnothing\, [0.85\, f'_c\, (1-\rho_g) + \rho_g\, f_y] \qquad (5\text{-}2)$$

Using equation (5-2) A_g is determined

Step 4: Selection of Column Size

ACI Code Commentary R 10.8 eliminates the minimum sizes for compression members to allow wider utilization of reinforced concrete compression members in smaller sizes for lightly loaded structures. With the A_g as obtained in step 3 select a square (h^2) or rectangular (b x h) section, such that the side dimensions are integer multiplier.

Step 5: Check for Steel Ratio

Putting the selected A_g in equation (5-2), ρ_g is calculated.
If $\rho_g = 0.01 \approx 0.08$; Design is ok

Otherwise change the column dimension and repeat this step.

Step 6: Calculation of Reinforcement

Total reinforcement, $A_{st} = A_g * \rho_g$
Now choose required bar size satisfying the *ACI Code 10.9.2* requirements i.e. minimum number of bars $\geq$ 4 Nos.

Step 7 : Selection of Ties

Reinforcement for ties should be selected in accordance with *ACI Code 7.10.5.1:*

- Use ties of # 3 bar for longitudinal bar up to # 10
- Use ties of # 4 bar for longitudinal bar of # 11, 14, 18 and bundled bar

Step 8 : Determination of Vertical Spacing of Ties

According to *ACI Code 7.10.5.2* the vertical spacing shall be smallest of:

- 16 d_b of longitudinal reinforcement
- 48 d_b of tie bar
- Least dimension of column section

Step 9 : Arrangement of Ties

ACI Code 7.10.5.3 specifies the arrangement of ties:

The ties shall be so arranged that every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie having an included angle of not more than 135°.No bar shall be farther than 6 in. clear on either side from such a laterally supported bar. Where the bars are located around the periphery of a circle, complete circular ties may be used. Details in Figure 5.3.

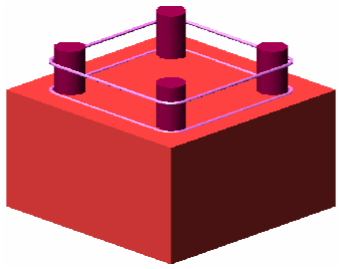
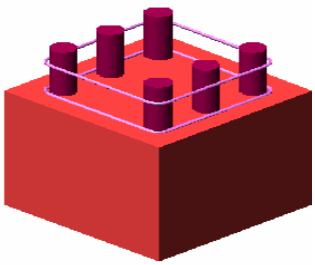
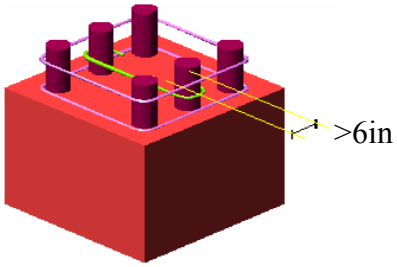
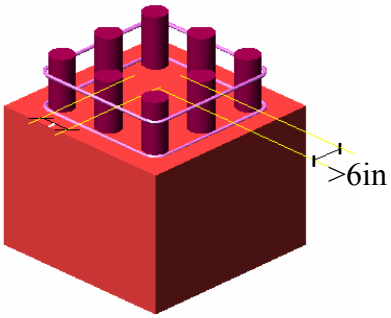
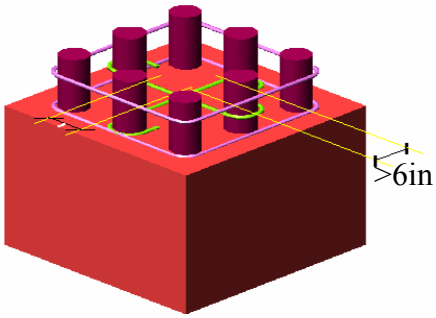
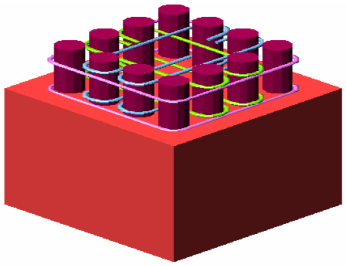
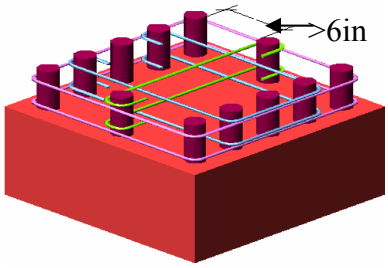
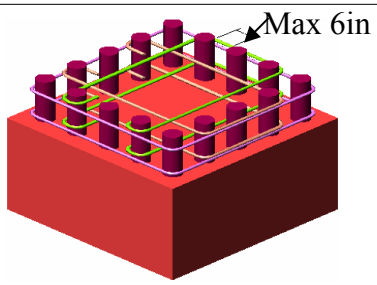
4 bar		
6 bar		
8 bar		
12 bar		
16 bar		

Figure 5.3: Arrangement of column ties

Step 10 : Concrete Protection for Reinforcement

Details are shown in Figure 5.4:

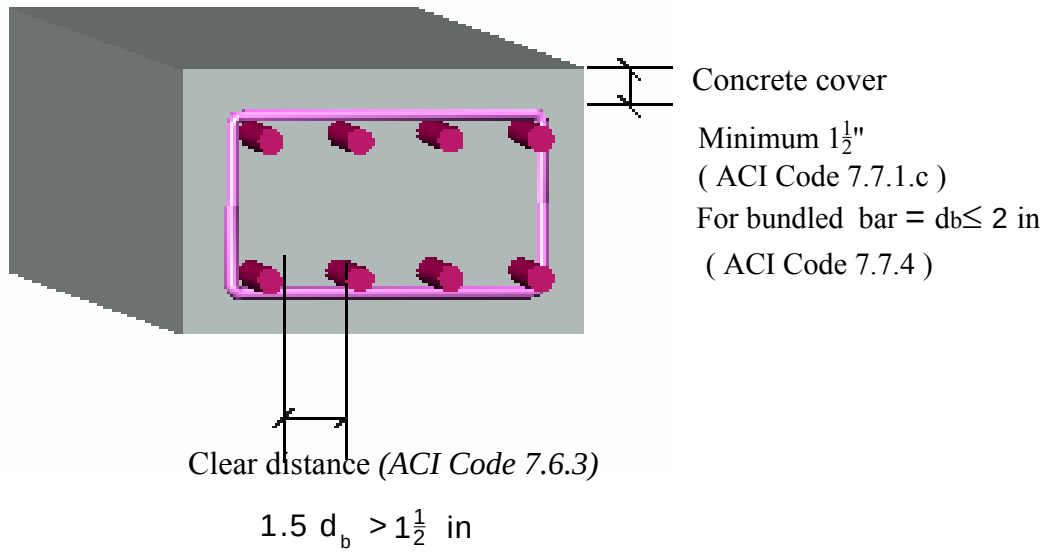


Figure 5.4 : Concrete cover in column

Step 11 : Bar Splicing in Columns

The main vertical reinforcement in columns is usually spliced just above each floor, or at alternate floors.
According to *ACI Code 12.17.1*, lap splices, mechanical splices, but welded splices and bearing splices shall be used.

Lap Splices:

Code requirements for lap splices are given in Table 5.1. Details of lap splices at typical interior column are shown in Figure 5.4. In addition, for offset bar following should be observed:

ACI Code 7.8.1.4, offset bars shall be bent before placement in the forms.

ACI Code 7.8.1.5, where column face is offset 3 in. or greater, longitudinal bars shall not be offset bent. Separate dowels, lap spliced with the longitudinal bars adjacent to the offset column faces shall be provided. Details in Figure 5.5.

Mechanical and Welded Splices:

According to ACI Code 12.14.3, a full mechanical splice or welded splice shall develop at least 125% of the specified f_y of the bar.

Table 5.1: Requirements for lap splices

Longitudinal Bars in Compression				**Reduction by** *(ACI Code 12.17.2.4* and *12.17.2.5)*
All bars of equal sizes (ACI Code 12.16.1)	$f_c' > 3000$ psi	$f_y \leq 60{,}000$ psi	$l_s = 0.0005\ f_y d_b \geq 12$ in.	*For Tied:* Effective area $\geq$ 0.0015hs $l_s = l_s * 0.075$
		$f_y > 60{,}000$ psi	$l_s = (0.0009\ f_y - 24) * d_b \geq 12$ in.	*For Spiral:* $l_s = l_s * 0.75$
	$f_c' < 3000$ psi	l_s increased by 33 %		
All bars of different sizes (ACI Code 12.16.2)	l_s = larger of the following : (a) l_s of smaller bar (b) l_d of larger bar			
Longitudinal Bars in Tension				
$0 \leq f_s \leq 0.5 f_y$ (ACI Code 12.17.2.2)	More than one- half of the bars are spliced at any section		Class B splice $l_s = 1.3 l_d$	
	Half or fewer bars are spliced at any section		Class A splice $l_s = 1.0 l_d$	
$f_s > 0.05 f_y$ (ACI Code 12.17.2.3)			Class B splice $l_s = 1.3 l_d$	
	l_d is defined in section 2			

Note:

For Larger Bar *(ACI Code 12.16.2)* **:** Lap splices are generally prohibited for No. 14 or No.18 bars, however, for compression only, lap splices are permitted for No14, or No. 18 bar to No. 11 bars or smaller bars.

For Bundled Bar *(ACI Code 12.4.1)*:
For bars in 3 bar bundles l_s is increased by 20%.
For bars in 4 bar bundles l_s is increased by 33%.

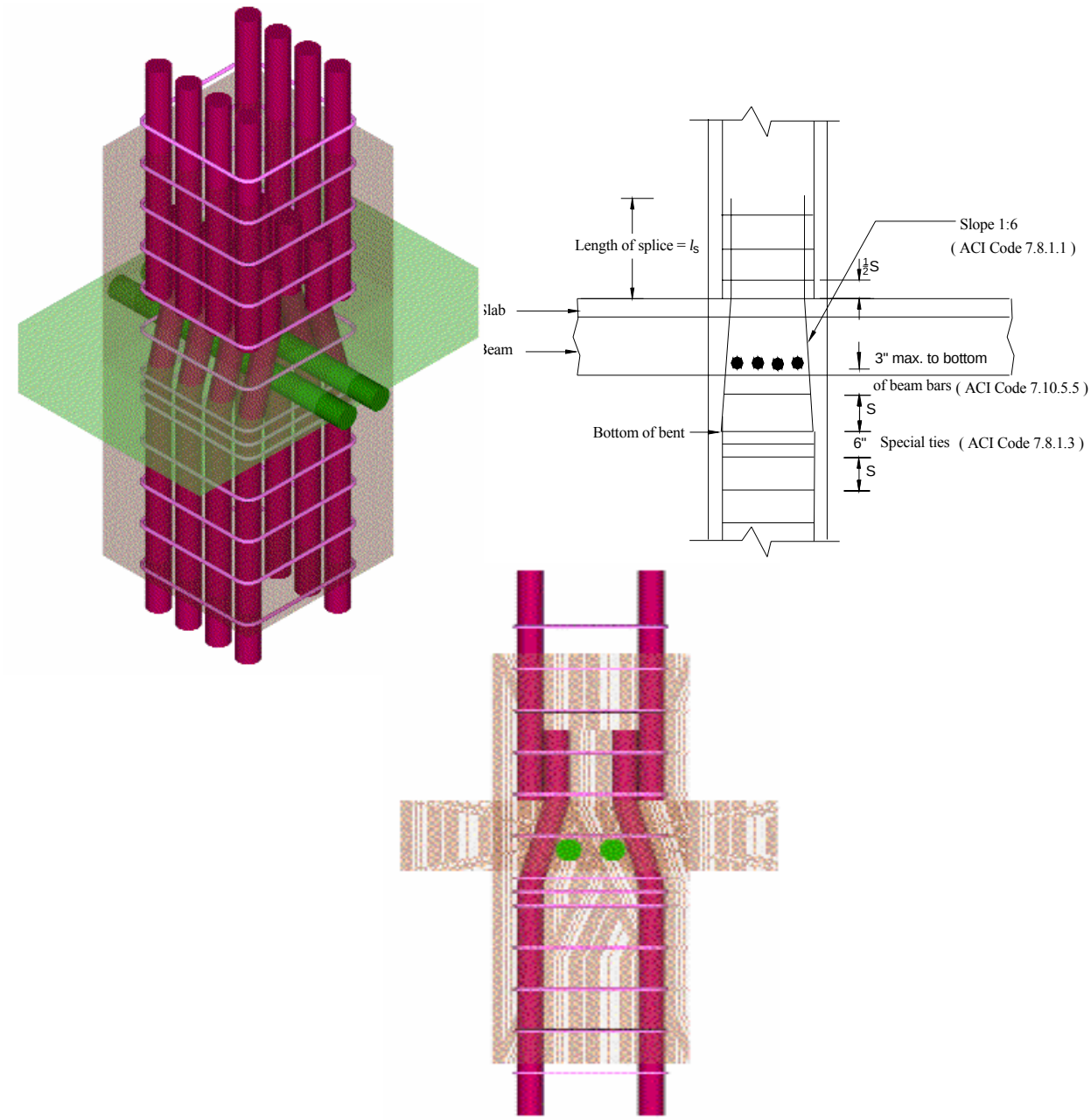


Figure 5.5: Lap splice details at typical interior column

5.3.1.2 Design of Spiral Column

Step 1 : Determination of Factored Load

Load contribution to the column similar to step 1 of section 5.3.1.1.

$P_u = 1.4\ D + 1.7\ L$ *(ACI Code-00)*
$P_u = 1.2\ D + 1.6\ L$ *(ACI Code-02)*

Step 2: Steel Ratio Assumption

According to *ACI Code 10.9.1*
$0.01 \leq \rho_g \leq 0.08$

Step 3: Determination of Concrete gross Area

According to *ACI Code 10.3.6.1*, the design strength for spiral column

$$\varnothing P_{n,max} = 0.85\varnothing\ [0.85\ f'_c\ (A_g - A_{st}) + f_y\ A_{st}] \quad (5\text{-}3)$$

Where $\varnothing = 0.70$, consider $P_u = \varnothing\ P_n$ *(ACI Code-9.3.2.1)*

Equation (5-3) can be rewritten as:

$$P_u = 0.85\ A_g\ \varnothing\ [0.85\ f'_c\ (1-\rho_g) + \rho_g\ f_y] \quad (5\text{-}4)$$

Using equation (5-4), A_g is determined

Step 4 : Selection of Column Size

$$D = \sqrt{\frac{4A_g}{\pi}}$$

Select D as whole number (say, D = 30 in.)

Step 5 : Check for Steel Ratio

Using selected D, A_g is recalculated.

$$A_g = \frac{\pi D^2}{4}$$

Putting this A_g, in equation (5-4), ρ_g is calculated
If $\rho_g = 0.01 \approx 0.08$, Design is ok

Otherwise change the column diameter and repeat this step.

Step 6 : Calculation of reinforcement

$A_{st} = A_g * \rho_g$
Now choose required bar size satisfying the *ACI Code 10.9.2* requirement
i.e. minimum number of bars should not be less than 6.

Step 7: Check for Clear Distance Between Longitudinal Reinforcement

.ACI Code 7.6.3 specify: (see Figure 5.6)
Clear distance = [π d_c - (N* dia of longitudinal bar)] / (N-1)
Clear distance > 1.5 d_b in.
But > 1.5 in.

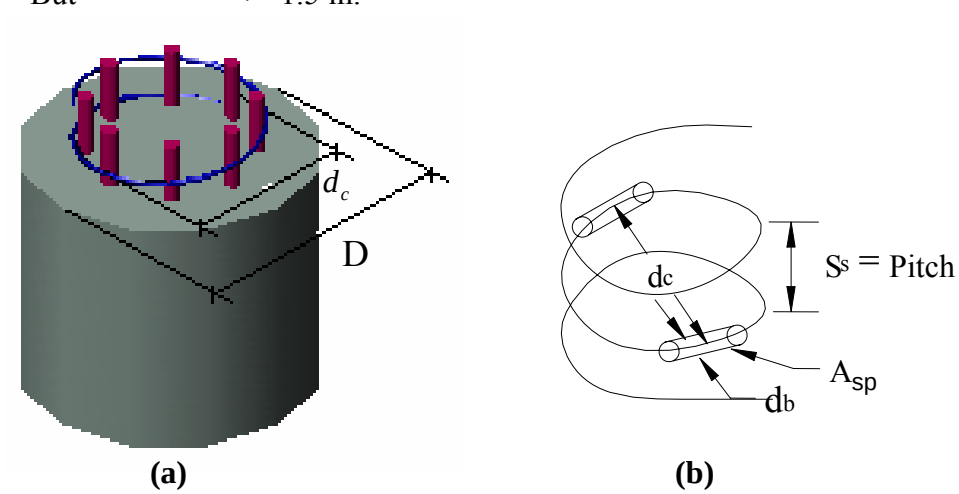


Figure 5.6 : Clear distance and pitch for spiral column

Step 8 : Selection of Spirals

ACI Code 7.10.4.2 specifies that spirals shall consist of a continuous bar or wire not less than ¾ inch in diameter.

Step 9: Determination of Spacing of Spiral

According to *ACI Code 10.9.3* pitch of spiral:

$$Ss = \frac{8.90 A_{sp} f_y}{d_c \left(\frac{A_g}{A_c} - 1 \right) f'_c} \tag{5-5}$$

But 1 in. $\leq$ Ss $\leq$ 3 in *(ACI Code 7.10.4.3)*

Step 10 : Placement of spirals

Placing of spirals is shown in Figure 5.7 and 5.8

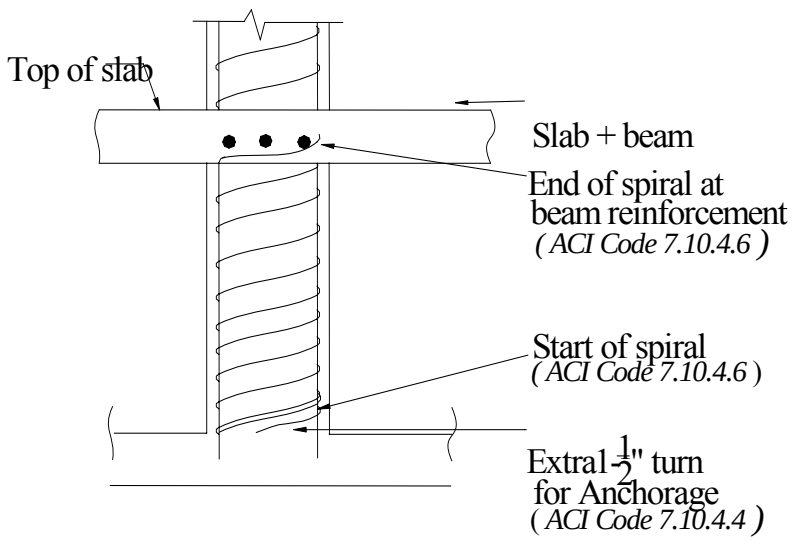


Figure 5.7 : Spirals in a column supporting slab with beam

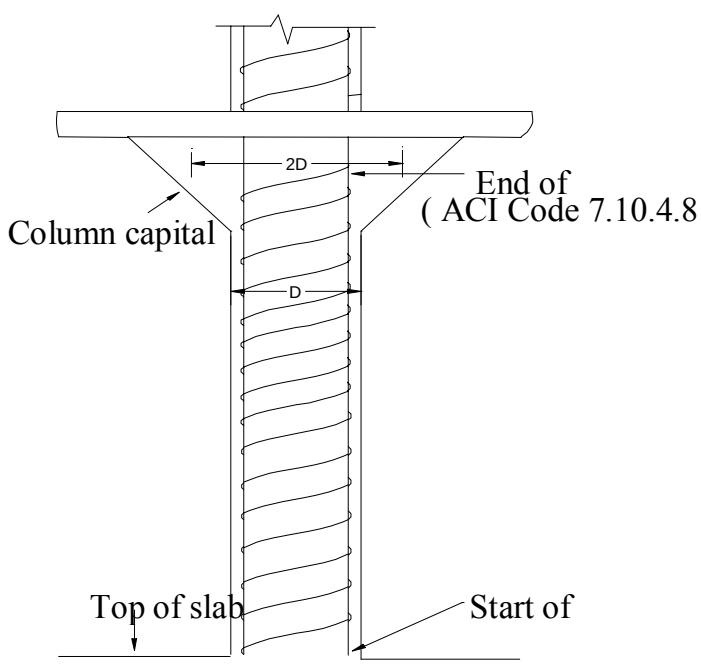
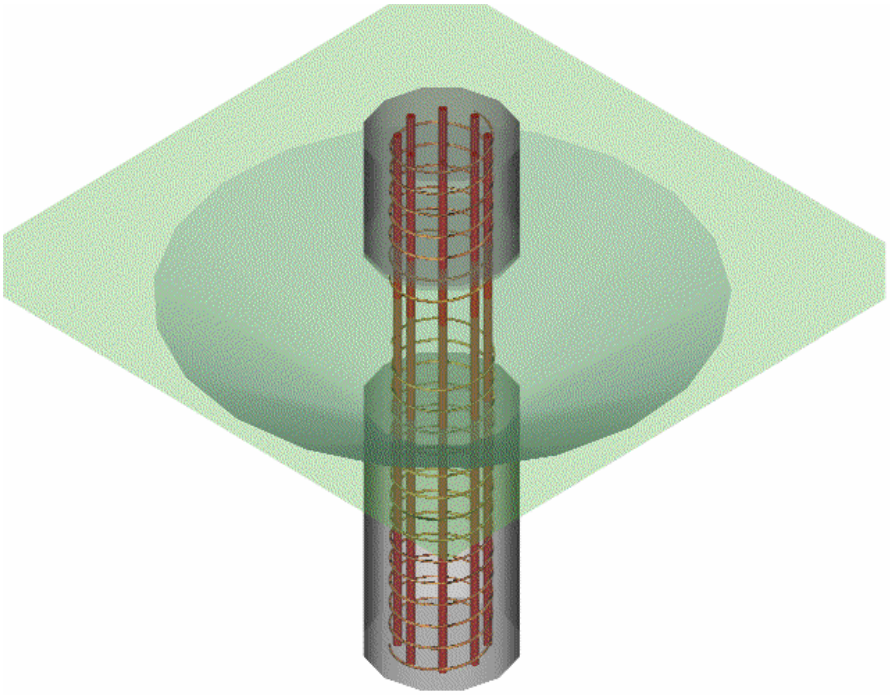


Figure 5.8: Placement of spirals in column with capital



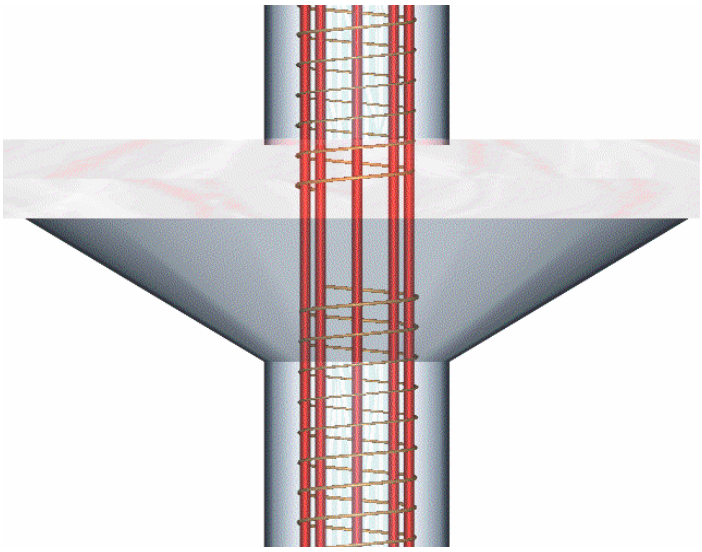


Figure 5.8: Placement of spirals in column with capital (continued)

Step 11 : Splicing of Spiral Reinforcement

Spiral reinforcement shall be spliced, if needed, in accordance with *ACI Code 7.10.4.5*. Length of lap splices is given in Table 5.2.

Table 5.2 : Length of lap splice for spiral reinforcement

1.	Deformed uncoated bar or wire	48 d_b
2.	Plain uncoated bar or wire	72 d_b
3.	Epoxy-coated deformed bar or wire	72 d_b
4.	Plain uncoated bar or wire with a standard tie hook at ends of lapped spiral reinforcement	48 d_b
5.	Epoxy- coated deformed bar or wire with a standard tie hook at ends of lapped spiral reinforcement	48 d_b
Length of splice should be $\geq$ 12 in.		

Step 12: Concrete Protection for Reinforcement

Same as mentioned in Step 10 of section 5.3.1.1.

Step 13: Bar Splicing in Column

The main vertical reinforcement in column is spliced in the same manner mentioned in step 11 of section 5.3.1.1.

5.3.2 ECCENTRICITY LOADED COLUMN WITH UNIAXIAL BENDING

5.3.2.1 Strength Interaction Diagram

Strength Interaction Diagram defines the failure load and failure moment for a given column for the full range of eccentricities from zero to infinity. For any eccentricity, there is a unique pair of values of P_n, M_n that will produce the state of incipient failure. The pair of values can be plotted as a point on a graph relating P_n and M_n, such as shown in Figure 5.9.

5.3.2.2 Design Procedure For Eccentricity Loaded Column

The approach to design of compression members in uniaxial bending may be divided into three categories: (Figure 5.9)

Design for Region I : For member subject to small or negligible bending moment, maximum permitted axial strength $P_{n.\max}$ governs. Then design as concentrically loaded column. Minimum eccentricities of loading:

e = 0.10 h for tied column
e = 0.05 h for spiral column

Design for Region II : Section is compression- controlled (e< e_b)

Design for Region III : Section is tensioned- controlled (e> e_b or $\varepsilon_t > \varepsilon_y$).

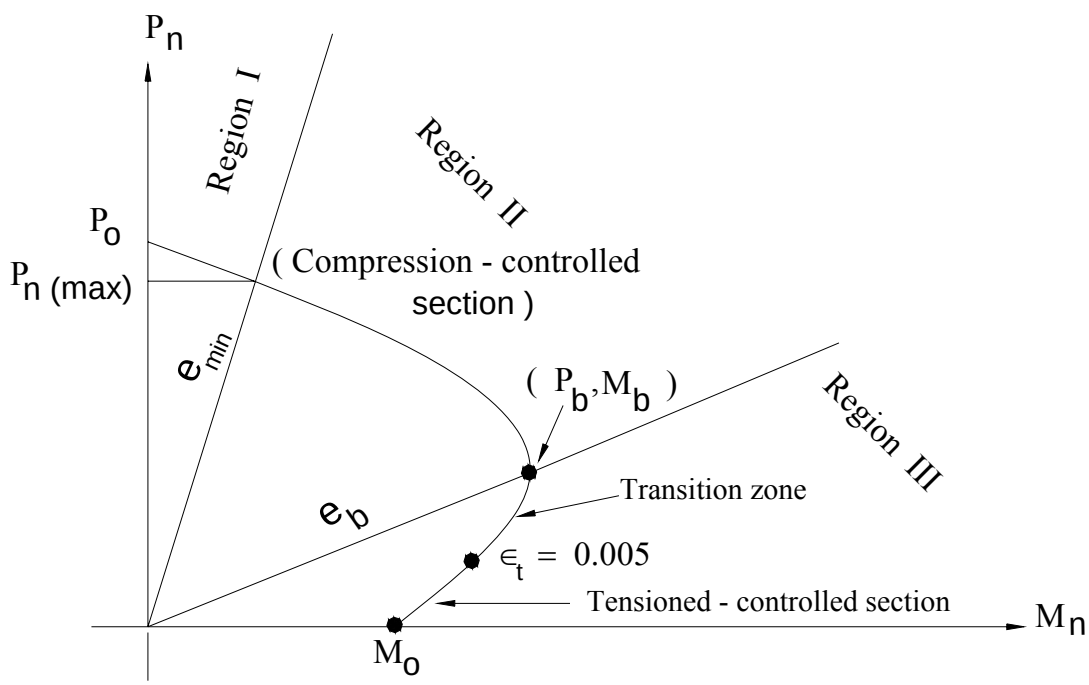


Figure 5.9: Interaction diagram for nominal column strength in combined bending and axial load

5.3.2.3 Design of Eccentrically Loaded Column with Uniaxial Bending, Compression -Controlled Section (Region II - $e_{min} < e < e_b$).

Step1 : Determination of Factored Load and Moment

$P_u = 1.4D + 1.7L$ (ACI Code-00)
$P_u = 1.2D + 1.6L$ (ACI Code-02)

$M_u = 1.4M_D + 1.7\,M_L$ (ACI Code-00)
$M_u = 1.2M_D + 1.6\,M_L$ (ACI Code-02)

Where: M_D = moment due to dead load
M_L = moment due to live load

Step 2 : Computation of Required Nominal Strength

$$P_n = \frac{P_u}{\phi}$$

$M_n = \dfrac{M_u}{\phi}$, Where $\varnothing = 0.65$ for tied column and, 0.70 for spiral column

Step 3 : Computation of Eccentricity

$$e = \frac{M_n}{P_n}$$

Step 4 : Check for Eccentricity

By approximation, $e_{min} = 0.1h$, for tied column
$= 0.05h$, for spiral column

Now $h = \dfrac{e}{0.10}$ in. or $\dfrac{e}{0.05}$ in.

- If section choosen > h, design in *Region I.*
- If section choosen < h, design in *Region II.*

Step 5 : Determination of Maximum Limit of Size of the Column for Compression – Controlled Section

Considering balanced strain condition $P_n = P_b$

$$P_n = 0.85\, f_c^{'}\, \beta_1\, (bd)\, \frac{\in_u}{\in_u + \in_y} \qquad (5\text{-}6)$$

$$\varepsilon_u = 0.003 \;;\quad \varepsilon_y = \frac{f_y}{E_s}\;;\; E_s = 29000 \text{ ksi}$$

From equation (5-6), (bd) at balanced condition is obtained

$$Ag_{balanced} = bh = \frac{bd}{0.9} \text{ (see Figure 5.10)}$$

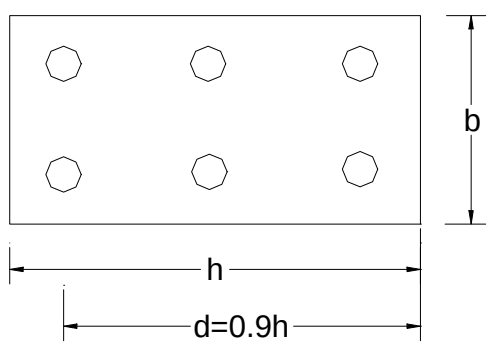


Figure 5.10 : Dimension of column

For compression - controlled section choose $A_g < Ag_{balanced}$

Step 6: Selection of the Size of the Column

Select steel ratio, ρ_g = 0.01 ~ 0.08 (selection of ρ_g depends on designer's experience, generally ρ_g = 0.02 to 0.03 for light structures)

Whitney Formula:

$$P_n = A_g \left[\frac{f'_c}{(3/\xi^2)(e/h) + 1.18} + \frac{\rho_g f_y}{(2/\gamma)(e/h) + 1} \right] \qquad (5\text{-}7)$$

Here: $\xi = \dfrac{d}{h}$ generally $\xi = 0.8 \sim 0.9$

$\gamma = \dfrac{d - d'}{h}$ $\gamma = 0.7 \sim 0.9$

From the chosen A_g in step 5, h is assumed (less than as mentioned at Step 4), e = 1.5 in to 2.0 in.; e = found in step 3 putting all the values in equation (5-6) A_g is obtained.

Size of column is selected as
$A_g = h^2$ for square section
$= b\ h$ for rectangular section

Step 7: Estimation of Reinforcement

$A_{st} = A_g * \rho_g$

Select the bar considering symmetric reinforcement

Step 8: Selection of ties.

Step 9: Determination of vertical spacing of ties.

Step 10: Arrangement of ties.

Step 11: Concrete protection for reinforcement.

Step12: Bar splicing in columns.

For Step 8 to Step 12 see section 5.3.1.1 or 5.3.1.2.

Step 13 : Check for Design

Adequacy of nominal strength of the selected column section can be checked by *Whitney Formula*. At first the balanced condition (P_b , M_b) must be determined for the selected section. (Figure 5.11)

- **Balanced Load**

$$P_b = 0.85\, f_c^{'} ab + A_s^{'} f_s^{'} - A_s f_s \qquad (5\text{-}8)$$

$$a = \beta_1 C_b = .85\,(d\ \frac{\in_u}{\in_u + \in_y}) \quad ; \ \varepsilon_y = \frac{f_y}{E_s} \quad ; \ f_s = f_y$$

$$f_s^{'} = \varepsilon_u E_s\,(\frac{C_b - d^{'}}{C_b}) \le f_y$$

For symmetric reinforcement $A_s^{'} = A_s$

- **Balanced Moment**

$$M_b = 0.85 f_c^{'} ab\,(\frac{h}{2} - \frac{a}{2}) + A_s^{'} f_s^{'} (\frac{h}{2} - d^{'}) + A_s f_s\,(d - \frac{h}{2}) \qquad (5\text{-}9)$$

$$e_b = \frac{M_b}{P_b}$$

Now check:

If $P_n > P_b$ and $e < e_b$

Then the section is compression controlled.

- **Check for Nominal Strength**

Whitney Formula:

$$P_{n,design} = \frac{bhf'_c}{\frac{3he}{d^2} + 1.18} + \frac{A'_s\ f_y}{\frac{e}{d - d'} + 0.5} \qquad (5\text{-}10)$$

$P_{n,design} > P_{n,required}$ design ok

Otherwise design should be revised

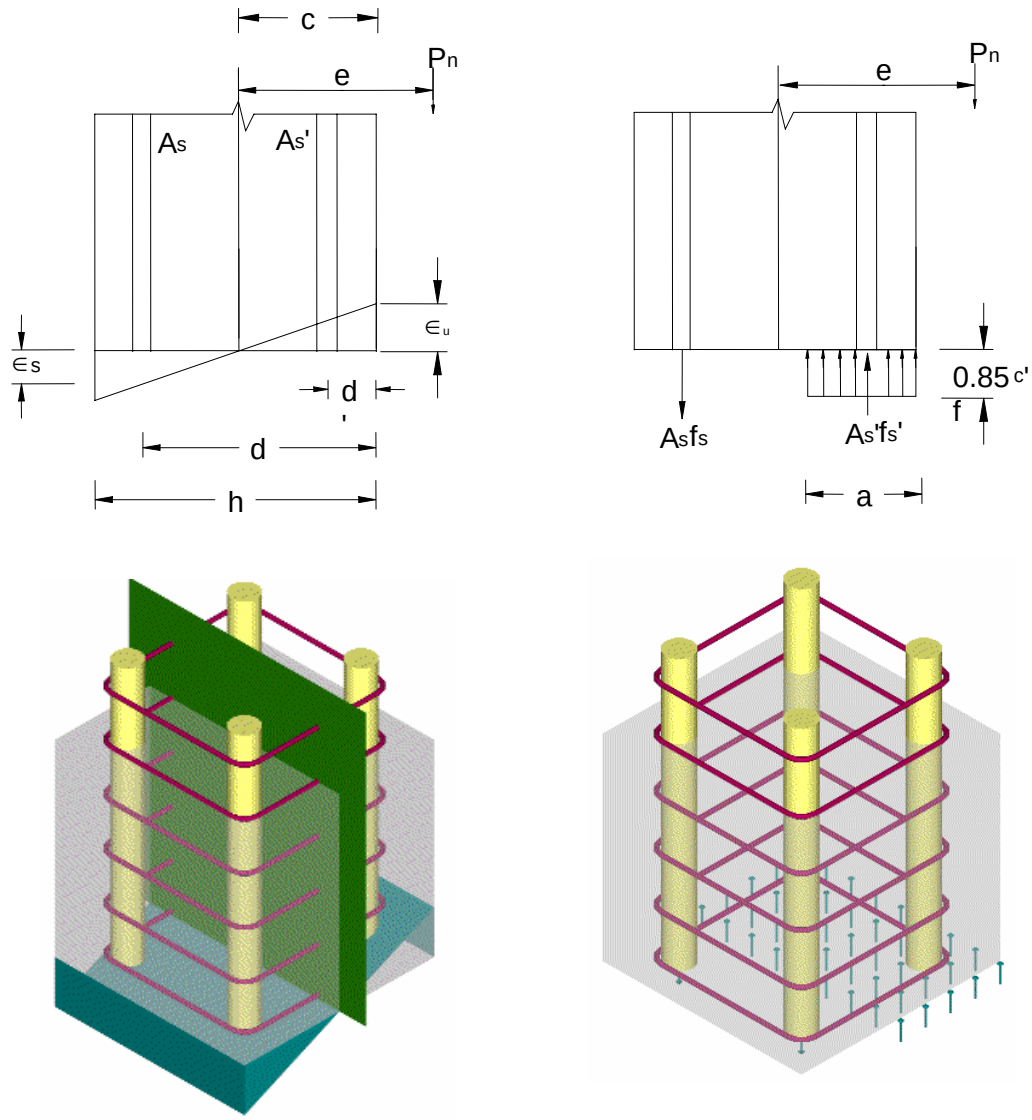


Figure 5.11: Column subject to eccentric compression

5.3.2.4 Design of Eccentrically Loaded Column with Uniaxial Bending-Transition Zone, Tension- Controlled Section

The *transition zone* is where e >e_b , but the net tensile strain ε_t at the extreme tension steel is less than 0.005. Where $\varepsilon_t \geq 0.005$ the section is *tension controlled*. The case may occur when dimension limitation is imposed. Important point to note that according to *ACI Code 9.3.2* for transition zone sections there is a variation in strength reduction factor ϕ for members:

- $f_y \leq 60{,}000$ psi and,
- $\dfrac{h - d' - d_s}{h} \geq 0.07$ and,
- Symmetric reinforcement

ϕ shall be permitted to be increased linearly to 0.9 as ϕP_n decreases from $0.10 f'_c A_g$ to zero.

For other members:

ϕ shall be permitted to be increased linearly to 0.9 as ϕP_n decreases from $0.1 f'_c A_g$ or ϕP_b, whichever is smaller, to zero.

Step 1: Determination of Factored Load and Moment

$P_u = 1.4D + 1.7L$ (ACI Code-00)
$P_u = 1.2D + 1.6L$ (ACI Code-02)

$M_u = 1.4M_D + 1.7 M_L$ (ACI Code-00)
$M_u = 1.2M_D + 1.6 M_L$ (ACI Code-02)

Where: M_D = moment due to dead load
M_L = moment due to live load

Step 2: Calculation of Required Nominal Strength

Assume a value for ϕ
For tied column $\phi = 0.65 \sim 0.90$
For spiral column $\phi = 0.70 \sim 0.90$

$\phi P_n = P_u \geq 0.10 f'_c A_g$, So $\phi = 0.65$

$$P_{n,required} = \frac{P_u}{\phi}$$

$$M_{n\ ,\ required} = \frac{M_u}{\phi}$$

Step 3 : Calculation of Eccentricity

$$e = \frac{M_n}{P_n}$$

Step 4 : Determination of Minimum Limit of the Column Section for Tension Controlled

The approximate section of the column as obtained in this step will be used to get a preliminary idea about the size of the column.

Considering balanced strain condition, $P_n = P_b$

$$P_n = 0.85\ f_c^{'}\ \beta_1\ (bd)\ \frac{\in_u}{\in_u + \in_y}$$

$$\varepsilon_u = 0.003\ ;\quad \varepsilon_y = \frac{f_y}{E_s}\ ;\ E_s = 29000 \text{ ksi}$$

From equation (5-6), (bd) at balanced condition is obtained

$$A_{g,\ balanced} = bh = \frac{bd}{0.9}$$

When limitation on width b of the column is imposed it may be required to choose

$A_g > A_{g,\ balanced}$; Thus $e > e_b$.

Step 5 : Selection of Preliminary Size of the Column

Select steel ratio, ρ_g = 0.01 ~ 0.08 (selection of ρ_g depends on designer's experience, generally ρg = 0.02 to 0.03 for light structures)

Approximate Formula:

$$P_n = 0.85\ f'_c\ bd\left[-\rho + 1 - \frac{e'}{d} + \sqrt{\left(1 - \frac{e'}{d}\right)^2 + 2\rho\left[(m-1)\left(1 - \frac{d'}{d}\right) + \frac{e'}{d}\right]}\right] \qquad (5\text{-}11)$$

For definition of various notations see Figure 5.12

Here: m = $\dfrac{f_y}{0.85 f_c'}$

ρ = $\dfrac{1}{2}\rho_g$

e' = eccentricity of axial load measured from centroid of tensile steel.

Assume $\dfrac{e'}{d} = \dfrac{(d - \frac{h}{2}) + e}{d}$; (say = 2.0)

And $\dfrac{d'}{d}$ = Assume from step 4 (say = 0.10)

From equation (5-11), required (bd) is obtained

$A_{g,\ preliminary}$ = bh = $\dfrac{bd}{0.9}$ is obtained.

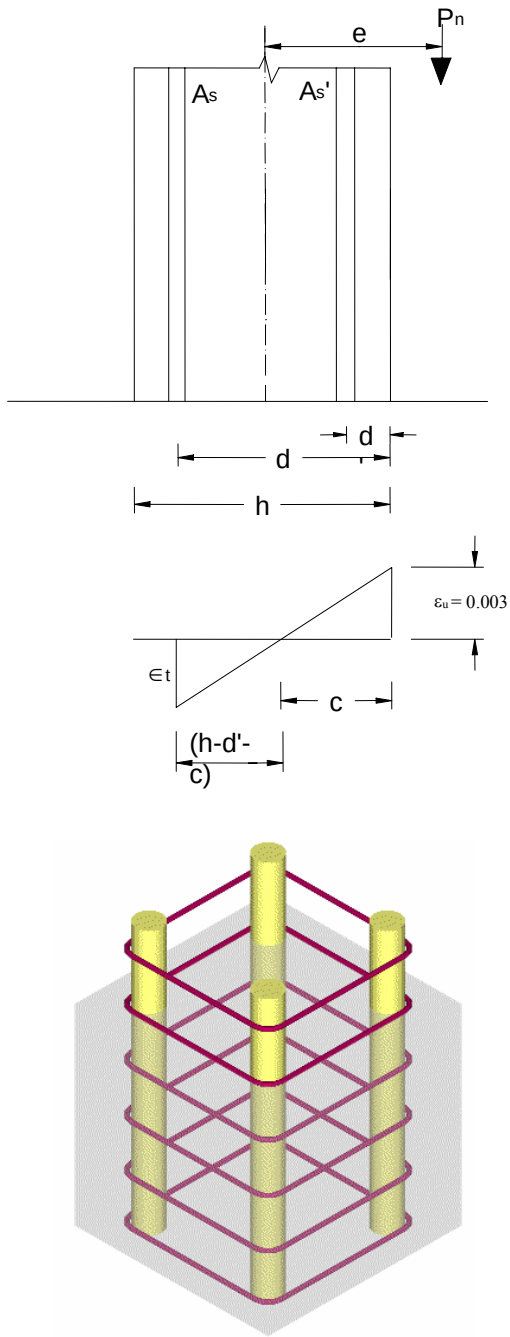
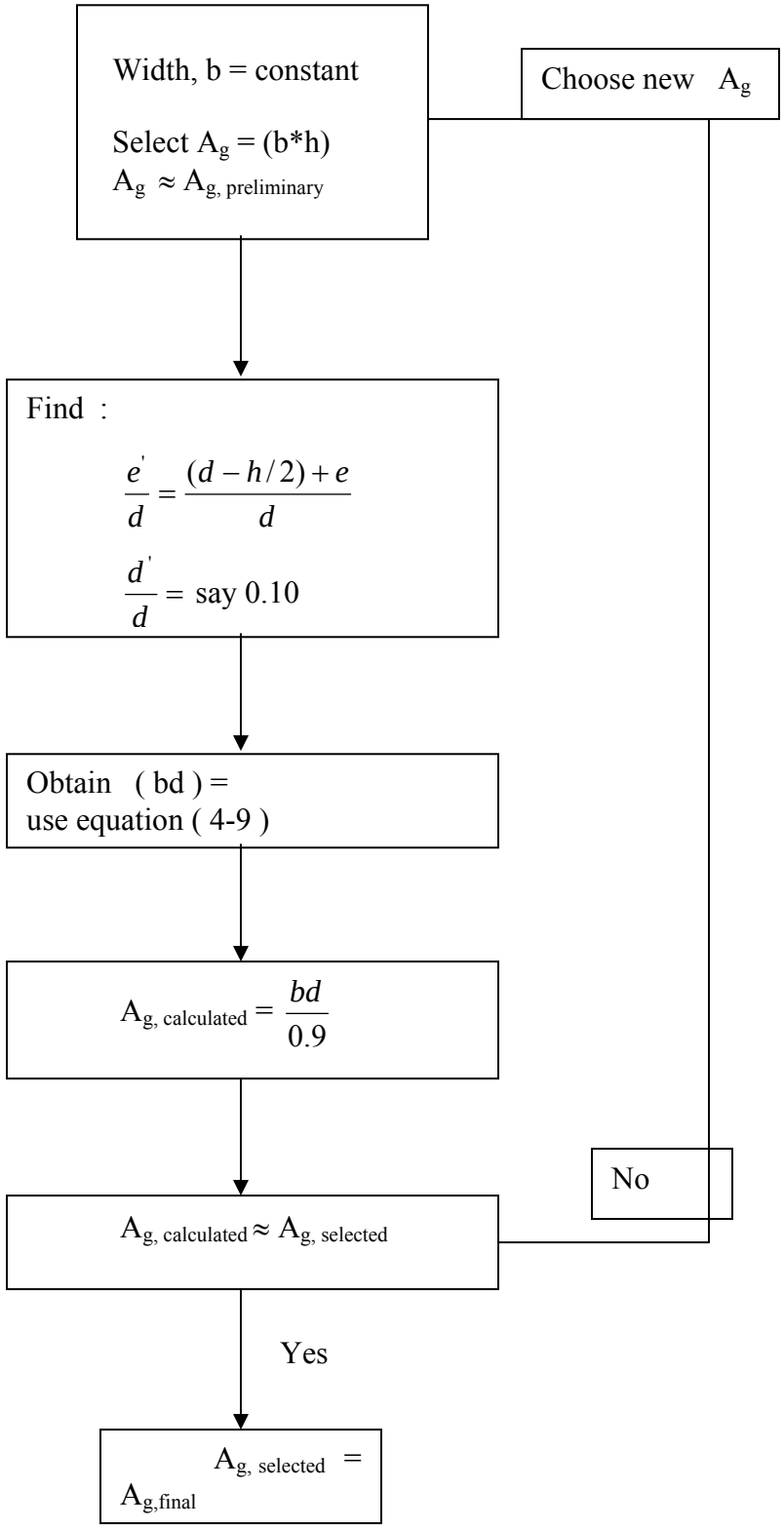


Figure 5.12 : Eccentrically loaded column with uniaxial bending for $e > e_b$

Step 6 : Selection of the Size of Column

Give trial with $A_g = A_{g,preliminaru}$



Step 7 : Estimation of Reinforcement

From equation of static

$P_n = 0.85\ f'_c\ ab$ (5-12)

$$P_n \left[\left(e - h/_2 \right) + \frac{a}{2} \right] = A_s\ f_y\ (d\text{-}d') \quad (5\text{-}13)$$

Using equation (5-12), value of **a** is obtained. From equation (5-13), A_s is obtained.

Total steel required: $A_{st} = 2 \times A_s$

Check for $\rho_g = \dfrac{A_{st}}{bh}$

Select the bar size considering symmetric reinforcement.

Step 8 : Check for ∅ Function

Compute the correct value of∅. Considering the selected section $A_g = b \times h$

Find $0.10\ f'_c\ A_g > \varnothing\ P_n$ or P_u

$$\gamma = \frac{h - d' - d_s}{h} \geq 0.70$$

$f_y \leq 60,00$ pii

Then with $\varnothing\ P_n = P_u$

$$\varnothing = 0.90 - 2.0 \left(\frac{P_u}{f'_c A_g} \right) \quad (5\text{-}14)$$

If ∅ as obtained using equation (5-14)>assume ∅ *(step-2),* nominal strength is ok.

Step 9 : Selection of ties.

Step 10 : Determination of vertical spacing of ties.

Step 11 : Arrangement of ties.

Step 12 : Concrete protection for reinforcement.

Step 13 : Bar splicing in column.

For **step 9** to **step 13** see section 5.3.1.1 or 5.3.1.2.

5.3.3 Practical Design Approach for Column Subject to Uniaxial Bending (use of design aids)

In practice, columns are rarely designed in details as discussed in preceding sections. Instead, the designers prefer to use the prepared charts as design aids. Design aids are available in *"Design Handbook, Vol-2, columns, ACI special publications, SP-17, 1990"*.Few design charts are attached here as Appendix F-1 through F-12. These 12 charts are prepared for f'_c = 4000 psi and f_y = 60,000 psi. Charts for other combinations of material strengths are avail able in above mentioned reference.

5.3.3.1 Design of a Column Section

Step 1 : Determination of Factored Load and Moment

$P_u = 1.4D + 1.7L$ (ACI Code-00)
$P_u = 1.2D + 1.6L$ (ACI Code-02)

$M_u = 1.4M_D + 1.7\ M_L$ (ACI Code-00)
$M_u = 1.2M_D + 1.6\ M_L$ (ACI Code-02)

Where: M_D = moment due to dead load
M_L = moment due to live load

Step 2 : Calculation of Eccentricity

$$e = \frac{M_u}{P_u}$$

Step 3 : Determination of Approximate Size of the Column by Using Charts

- Assume value for γ , $e/_h$ and ρ_g
  Generally, $\gamma = 0.45, 0.60, 0.75, 0.9$; $e/_h = 0.1, 0.2, 0.3, 0.4, 0.5, 1.0$

  For trial 1: assume γ = a mid value, say 0.75
  $e/_h$ = a value, say, 0.20
  $\rho_g = 0.02 \sim 0.03$
- Select the chart form *Appendix F* for corresponding size (rectangular/ square or circular) and γ.
- From the selected chart, find $\frac{\phi P_n}{A_s}$, ksi corresponding to $e/_h$ and ρ_g.
- Obtain approximate A_g from the value of $\frac{\phi P_n}{A_g} = \frac{P_u}{A_g}$
- Select the required dimension
  - For Square Column

    $h = \sqrt{A_g}$

  - For Rectangular Column

    $A_g = b*h$

    Then, $h = \frac{A_g}{b}$

Step 4 : Selection of the Size of Column

- Find γ : $\gamma = \frac{d-d'}{h} = \frac{h-2d'}{h}$; **h** as found in step 3 .
- Find $e/_h$, **e** as obtained in step 2.
- Select a new chart from *Appendix F* for corresponding γ .
- From the selected chart, find $\frac{\phi P_n}{A_y}$, ksi corresponding to $e/_h$ and ρ_g .
- Find A_g from the value of $\frac{\phi P_n}{A_y}$ or $\frac{P_u}{A_g}$

- Select the required dimension :
  - For square column
    $$h = \sqrt{A_g}$$
  - For rectangular column assume b
    $$h = \frac{A_g}{b}$$

Step 5 : Estimation of Steel Area

$A_{st} = \rho_g * A_g$

Chose bar size to have symmetric reinforcement detailing

Step 6: Checking of Nominal Strength

Equation (5-1) can be rewritten as

$$\phi P_{n,max} = 0.80\, \phi\, [0.85f'_c A_g + (f_y - 0.85f'_c) A_{st}]$$
$$\phi P_{n,max} > P_u \text{ design is ok}$$

Other steps are similar to previous section

5.3.3.2 Design of Reinforcement of a Given Section

Step 1 : Determination of Factored load and Moment

$P_u = 1.4D + 1.7L$ (ACI Code-00)
$P_u = 1.2D + 1.6L$ (ACI Code-02)

$M_u = 1.4M_D + 1.7\, M_L$ (ACI Code-00)
$M_u = 1.2M_D + 1.6\, M_L$ (ACI Code-02)

Where: M_D = moment due to dead load
M_L = moment due to live load

Step 2 : Computation of Design parameters

$$\frac{P_u}{A_g} = \frac{\phi P_n}{A_g}$$

$$\frac{M_u}{A_{gh}} = \frac{\phi M_n}{A_g h}$$

Step 3 : Selection of Chart

b, h are known, assume d′ find $\gamma = \frac{h - 2d'}{h}$

Select the chart from *Appendix-F* corresponding to γ

Step 4 : Determination of Steel Ratio

From the selected chart, coordinate corresponding to $\frac{\phi P_n}{A_g}$ and $\frac{\phi M_n}{A_g h}$ gives the value of $0.01 \leq \rho_g \leq 0.08$

Step 5 : Estimation of Reinforcement

$A_{st} = \rho_g * A_g$

Choose the bar size to have symmetric reinforcement. Other steps are similar to previous section.

5.3.4 BIAXIAL BENDING

Many columns are subject simultaneously to moments about both principal axes of the section. This special problem is often encountered in the practical design of corner columns, wall columns with heavy spandrel beams and columns supporting two-way construction. A symmetrically reinforcement concrete column section with biaxial bending is illustrated in Figure 5.13 and a typical set of interaction diagram for a given section appears in Figure 5.14.

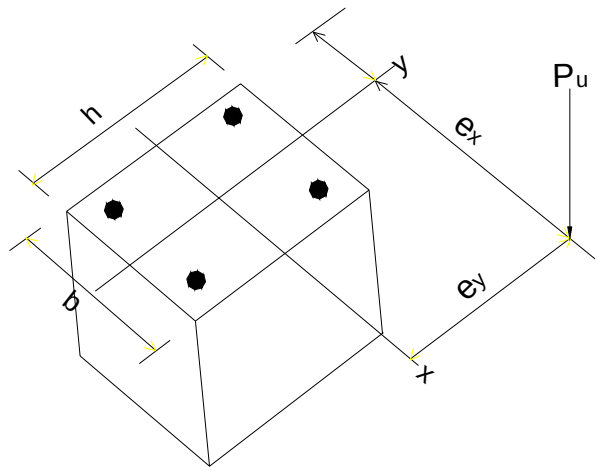


Figure 5.13 : Column section with biaxial bending

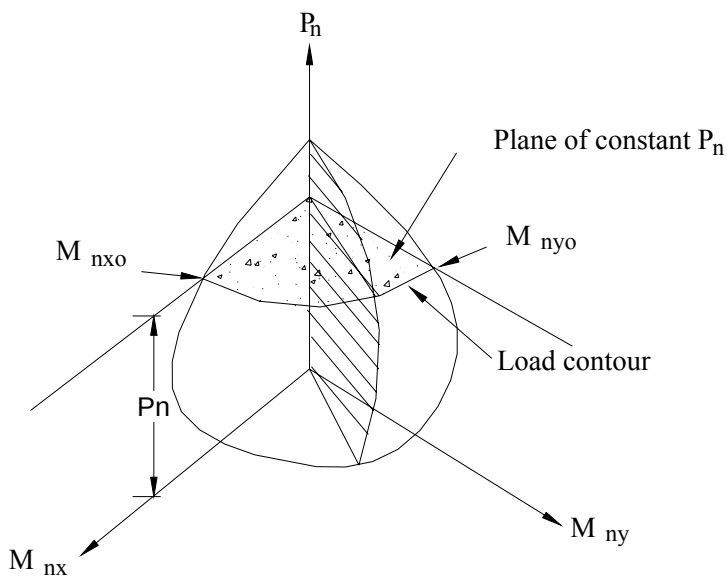


Figure 5.14 : Interaction surface for a column with biaxial bending

5.3.4.1 Design of Column for Biaxial Bending by Approximate Equivalent Uniaxial Method

Figure 5.15 shows an interaction line for a given rectangular column section with biaxial bending under constant P_u. Possible combinations of eccentricities for any point (e_y, e_x) on the line is the same as P_u for a point of application with uniaxial eccentricity e_o.

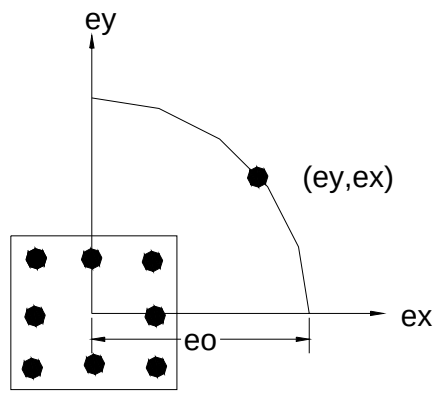


Figure 5.15 : Interaction line for column with biaxial bending under constant P_u

The equivalent uniaxial moment are given by:

- $M_{ny} + M_{nx}\left(\frac{b}{h}\right)\left(\frac{1-\beta}{\beta}\right) \approx M_{nyo}$ (5-15a)
- $M_{nx} + M_{ny}\left(\frac{h}{b}\right)\left(\frac{1-\beta}{\beta}\right) \approx M_{nxo}$ (5-15b)

When $M_{uy} > M_{ux}$ use eqn (5-15a) and $\frac{b}{h} \approx \frac{M_{uy}}{M_{ux}}$

When $M_{ux} > M_{uy}$, use eqn (5-15b) and $\left(\frac{h}{b}\right) \approx \left(\frac{M_{ux}}{M_{uy}}\right)$

Here, $\beta = 0.55$ to 0.65

Step 1: Determination of Factored Load and Moment

$P_u = 1.4D + 1.7L$ (ACI Code-00)
$M_{ux} = 1.4\ M_{x,dl} + 1.7\ M_{x,ll}$ (ACI Code-00)
$M_{uy} = 1.4\ M_{y,dl} + 1.7\ M_{y,ll}$ (ACI Code-00)

$P_u = 1.2D + 1.6L$ (ACI Code-02)
$M_{ux} = 1.2\ M_{x,dl} + 1.6\ M_{x,ll}$ (ACI Code-02)
$M_{uy} = 1.2\ M_{y,dl} + 1.6\ M_{y,ll}$ (ACI Code-02)

Step 2: Conversion to Approximate Equivalent Uniaxial Factored Moment

For: $M_{uy} > M_{ux}$ use equation (5-15 a)

$M_{ux} > M_{uy}$ use equation (5-15b)

Say equation (5-15b) to be applied

Equivalent $M_{nxo} \approx M_{nx} + M_{ny} \left(\frac{h}{b}\right)\left(\frac{1-\beta}{\beta}\right)$

Equivalent ϕ $M_{nxo} \approx M_{ux} + M_{uy} \left(\frac{M_{ux}}{M_{uy}}\right)\left(\frac{1-\beta}{\beta}\right)$

Equivalent ϕ M_{nxo} is determined

Step 3 : Computation of Equivalent Eccentricity for Uniaxial Bending

$e_o = \frac{\phi M_{nx0}}{p_u}$

The factored axial load = P_u
Equivalent uniaxial factored moment = ϕ M_{nxo} or ϕ M_{nyo}
Equivalent eccentricity = e_o

One can proceed with the design as uniaxial bending,

- *For compression-controlled section* – Step 5 to step 7 of section 5.3.2.3.
- *For tension-controlled section* - Step 4 to step 6 of section 5.3.2.4.
- By using design aids as given in appendix-F Step 3 to step 5 of section 5.3.3.

Check for Design

After determining the column section and reinforcement the adequacy for nominal strength should be checked by either of any methods:

- Reciprocal load method

- Load contour method

5.3.4.2 Design of Column for Biaxial Bending Using Reciprocal Load Method

Bresler's Reciprocal Load equation to estimate the nominal strength of biaxially loaded column as given in *ACI Code Commentary R 10.3.6* is:

$$\frac{1}{p_{ni}} = \frac{1}{P_{nxo}} + \frac{1}{P_{nyo}} - \frac{1}{P_o} \tag{5-16a}$$

Where,

P_{ni} = nominal axial load strength at given eccentricity along both axes
P_o = nominal axial load strength at zero eccentricity
P_{nxo} = nominal axial load strength at given eccentricity along x-axis
P_{nyo} = nominal axial load strength at given eccentricity along y- axis

Application of the equation (5-16a) requires design charts as available in *"Design Handbook, Vol-2, columns, ACI special publication, SP-17, 1990."* Few design charts are attached here as Appendix F.

Step 1 : Determination of Factored Load and Moment

$P_u = 1.4D + 1.7L$ (ACI Code-00)
$M_{ux} = 1.4\ M_{x,dl} + 1.7\ M_{x,ll}$ (ACI Code-00)
$M_{uy} = 1.4\ M_{y,dl} + 1.7\ M_{y,ll}$ (ACI Code-00)

$P_u = 1.2D + 1.6L$ (ACI Code-02)
$M_{ux} = 1.2\ M_{x,dl} + 1.6\ M_{x,ll}$ (ACI Code-02)
$M_{uy} = 1.2\ M_{y,dl} + 1.6\ M_{y,ll}$ (ACI Code-02)

Step 2 : Computation of Eccentricity

$$e_x = \frac{M_{ux}}{P_u}$$

$$e_y = \frac{M_{uy}}{P_u}$$

Step 3 : Determination of Load Parameter for Bending About X-Axis

Find $\gamma = \frac{h - 2d'}{h}, \frac{e_y}{h}$, here h = b

Select the suitable chart corresponding to γ from *Appendix F.*

For $\frac{e_y}{h}$, and ρ_g find:

$\frac{\phi P_{nxo}}{A_g}$, = say A =► $\phi P_{nxo} = A_g * A$

And $\frac{\phi P_o}{A_g}$, = say B =► $\phi P_o = A_g * B$

Step 4 : Determination of Load Parameter for Bending About Y - Axis

Find $\gamma = \frac{h-2d'}{h}, \frac{e_x}{h}$

Select the suitable chart from *Appendix F*

For $\frac{e_x}{h}$ and P_g find out

$\frac{\phi P_{ny0}}{A_g}$ = say C =► $\phi P_{nyo} = A_g * C$

And $\phi \frac{P_o}{A_g}$ = say B =► $\phi P_o = A_g * B$

Step 5 : Checking of Design Strength

Introducing ACI strength reduction factor equation (5-16a) can be rewritten as :

$$\frac{1}{\phi P_{ni}} = \frac{1}{\phi P_{nxo}} + \frac{1}{\phi P_{nyo}} - \frac{1}{\phi P_o} \qquad (5\text{-}16b)$$

Replacing all the values ϕP_{ni} is obtained

$\phi P_{ni} > P_u$ design is ok

Otherwise column section should be revised.

5.3.4.3 Design of Column for Biaxial Bending Using Load Contour Method

Bresler has suggested that the failure surface of Figure 5.14 is represented by a family of curves corresponding to constant values can be approximated by the following equation:

$$\left(\frac{\phi M_{nx}}{\phi M_{nxo}}\right)^{\alpha} + \left(\frac{\phi M_{ny}}{\phi M_{nyo}}\right)^{\alpha} = 1.0 \qquad (5\text{-}17)$$

Bresler indicates $\alpha = 1.15 \sim 1.55$

For practical purposes:

- For rectangular column, $\alpha = 1.5$
- For square column $\alpha = 1.5 \sim 2.0$

$M_{nx} = P_n e_y$
$M_{ny} = P_n e_x$
M_{nxo} = Uniaxial bending about X -axis
M_{nyo} = Uniaxial bending about Y - axis

Checking of the Adequacy of a Trial Design

Design data:
Sectional dimension = (b×h)
Axial load = P
Moments = M_x, M_y
Reinforcement detailing = A_{st}
Steel ratio = ρ_g
Necessitate the design chart of *Appendix F*

Step 1 : Determination of Factored Load and Moment

$P_u = 1.4D + 1.7L$ (ACI Code-00)
$P_u = 1.2D + 1.6L$ (ACI Code-02)

M_{ux} and M_{uy} are also determined
$\phi M_{nx} = M_{ux}$ and $\phi M_{ny} = M_{uy}$

Step 2 : Selection of Chart from Appendix F for Bending About X- Axis

Find $\gamma = \dfrac{h - 2d'}{h}, h = b$

Select chart corresponding to γ

Step 3 : Determination of Moment Parameter for Bending About X – Axis

Find: $\dfrac{\phi P_n}{A_g}$ and ρ_g

Intersection of these two points on the chart given the value of $\dfrac{\phi M_{nxo}}{A_g h}$

Determine ϕM_{nxo} = say A

Step 4 : Selection of Chart from Appendix F for Bending About Y- Axis

Find $\gamma = \dfrac{h - 2d'}{h}$

Select chart corresponding to γ

Step 5 : Determination of Moment Parameter for Bending About Y Axis

Find: $\dfrac{\phi P_n}{A_g}$ and

ρ_g (same as step 3)

Intersections of these two points give $\dfrac{\phi M_{nyo}}{A_g h}$

Determine ϕ M_{nyo} = say B

Step 6 : Checking of Design Strength

Replacing all the values in equation (5-17)

If $\left(\dfrac{\phi M_{nx}}{\phi M_{nxo}}\right)^{1.15} + \left(\dfrac{\phi M_{ny}}{\phi M_{nyo}}\right)^{1.15} \approx$ 1.0 design is ok

5.4 SLENDER COLUMN

4.4.1 General

A column is said to be *slender* if its cross-sectional dimensions are small compared to its length. The degree of slenderness is generally expressed in terms of the slenderness ratio l_u/r where

l_u = Unsupported length of compression member

$r = \sqrt{\dfrac{I}{A}}$ = radius of gyration of its cross section

I = moment of inertia

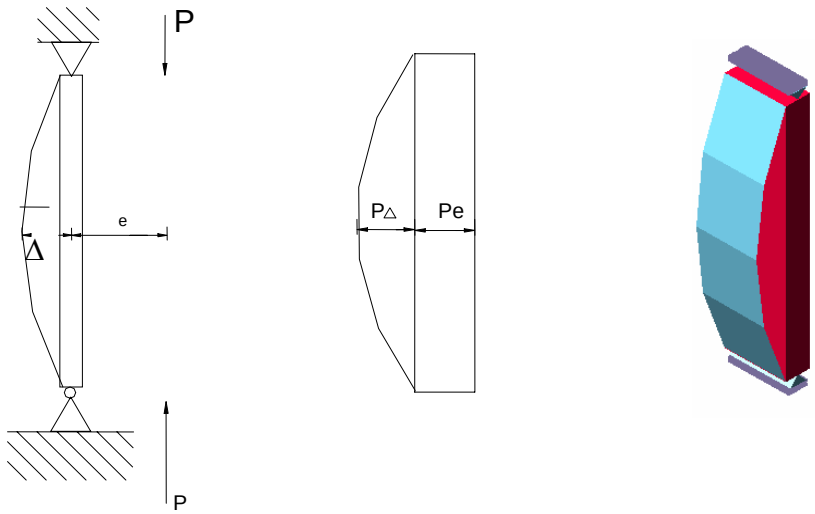
Most of the column is subjected to axial load as well as bending moment. These moments produce:

- Lateral deflection of a member between its ends and, or
- Relative lateral displacements of joints

Associated with these lateral displacements are known as *secondary moment* (pΔ effect), that's added to the *primary moments*. The effect of secondary moments is included by introducing moment magnification factor (δ) this effect is illustrated in Figure 4.16.

Hence:

$$\sum \text{(Primary moment + secondary moment)} = \delta^* \text{ primary moment}$$



(a) Uniaxial bending (b) Moment diagram (c) Uniaxial bending (3D)

Figure 5.16 : Slender column

ACI Code 10.11, 10.12, 10.13 present detail provisions for the design of slender column using *Moment Magnifier Method.*

5.4.2 Distinguishing Between Non Sway and Sway Frames

In designing a slender column, it is very important for the designer to identify whether the frame should be considered either non sway or sway.

ACI Code Commentary R 10.11.4 states that a compression member may be assumed non sway by inspection. if it is located in a story in which the bracing elements (shear walls, shear trusses or other types of lateral bracing) have such substantial lateral stiffness to resist the lateral deflections of the story, that any lateral deflection id not large enough to affect the column strength substantially

If not readily apparent by inspection, ACI Code give two possible ways of distinguishing these:-

- A story in a frame is said to be non-sway, if the increase in the lateral load moments resulting from PΔ effects does not exceed 5% of the first order moments *(ACI code 10.11.4.1)*

- A story is non-sway, if stability index *(ACI code 10.11.4.2)*

$$Q = \frac{\sum P_u \Delta_0}{V_u l_c} \leq 0.05 \qquad (4\text{-}18)$$

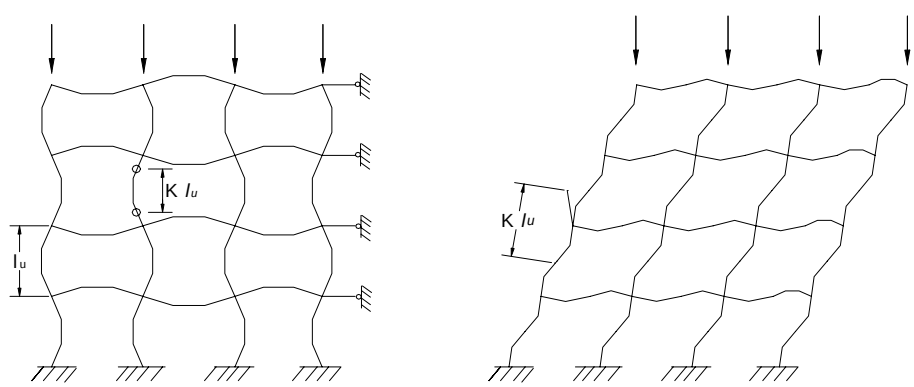
For Q > 0.05 sway frame analysis is required

Here:

$\sum P_u$ = Total factored vertical load for the story correspond to the equation,
$P_u = 0.75\ (1.4D + 1.7L + 1.7\ W)$ (*ACI code)*
V_u = The story shear
Δ_o = First order relative deflection between the top and bottom of that story due to V_u
l_c = length of column measured center to center of the joints

5.4.3 Buckling modes for non-sway and sway frames

Buckling modes for non-sway and sway frames are shown in Figure 5.17



(a) Non-sway frame (b) Sway frame

Figure 5.17: Buckling modes

5.4.4 Design of Non-sway (braced) frames by moment magnifier method

The overall design sequence for slender column is necessarily an iterative process because member sizes and reinforcement, unknown at the outset, affect such key parameters as moment of inertia, effective length factors and critical buckling loads.

Step 1 : Determination of Factored Load and Moment

$P_u = 1.4D + 1.7L$	(ACI Code-00)
$M_{ux} = 1.4\ M_{x,dl} + 1.7\ M_{x,ll}$	(ACI Code-00)
$M_{uy} = 1.4\ M_{y,dl} + 1.7\ M_{y,ll}$	(ACI Code-00)
$P_u = 1.2D + 1.6L$	(ACI Code-02)
$M_{ux} = 1.2\ M_{x,dl} + 1.6\ M_{x,ll}$	(ACI Code-02)
$M_{uy} = 1.2\ M_{y,dl} + 1.6\ M_{y,ll}$	(ACI Code-02)

Step 2 : Selection of a Trial Column Section

Assume short column behavier. Select a trial column section to carry the factored load = P_u and factored moment $M_u = M_2$, where M_2 is the largest end moment. To select a trial column section follow any procedure mentioned in section 5.3.

Step 3 : Determination of Mode of Analysis Either as Non-Sway Or Sway Frame

Check for non-sway frame and sway frame.

Step 4 : Check for Slenderness Effect

A check for the trial column should be given, whether the slenderness effect should be taken into account or not:

- Find l_u = unsupported length of the column
- Consider effective length factor k = 1.0
- Find r = radius of gyration

According to *ACI Code 10.11.2*

For rectangular member, r = 0.30h
For circular member, r = 0.25 D
h = overall dimension in the direction stability is being considered
d = diameter of circular member

ACI code 10.12.2 permits to ignore slenderness effect for non-sway compression member that satisfy:

$$\frac{Kl_u}{r} \leq \left[34 - \frac{12M_1}{M_2}\right] \tag{5-19}$$

Where, $\left[34 - \frac{12M_1}{M_2}\right] \leq 40$

M_1 = Smaller factored end moment, positive if the member is bent in single curvature and negative if bent in double curvature

M_2 = larger factored end moment, always positive

Comment:

- When slenderness effect is within allowable limit according to equation (4-19), no further analysis is required and the trial section will be selected, *i.e.* the column can be designed as short column.

- On the contrary, for considerable slenderness effect the design should continue with the next steps.

Step 5 : Determination of Refined Effective Length Factor (K)

K can be determined by any of the two methods *(ACI Commentary R10.12.1)*

- Using alignment chart given in Table 4.3a
- K may be table as the smaller of the following two expressions.

$$K = 0.7 + 0.05\,(\Psi_A + \Psi_B) \leq 1.0 \tag{5-20a}$$

$$K = 0.85 + 0.05\,\Psi_{min} \leq 1.0 \tag{5-20b}$$

Where Ψ_A and Ψ_B are the values of Ψ at the two ends of the column and Ψ_{min} is the smaller of the two values.

$$\Psi = \frac{\sum\left(E_c I_{column} / l_c\right)}{\sum\left(E_b I_{beam/l_n}\right)} \tag{5-21}$$

l_n = span length of beam measured center to center of the joints
$I_{column} = 0.7 I_g$ *(ACI Code 10.11.1)*
$I_{beam} = 0.35\ I_g$
I_g = moment of inertia of gross concrete section about centroidal axis
I_g for T beam = 2 * I_g for web (approximately)

Step 6 : Recheck for Slenderness Effect

With the refined value of k repeat step 4 for $\frac{Kl_u}{r} > \left[34 - \frac{12M_1}{M_2}\right]$, it is confirmed that slenderness effect must be considered

Step 7 : Check for Minimum Design Moment

According to *ACI Code 10.12.3.2* the factored moment M_2 shall not be taken less than

$$M_{2,\ min} = P_u\ (0.6+0.03h) \tag{5-22}$$

about each axis separately, where 0.6 and h are in inches

Step 8 : Computation of Equivalent Moment Correction Factor, C_m

According to *ACI Code 10.12.3.1*

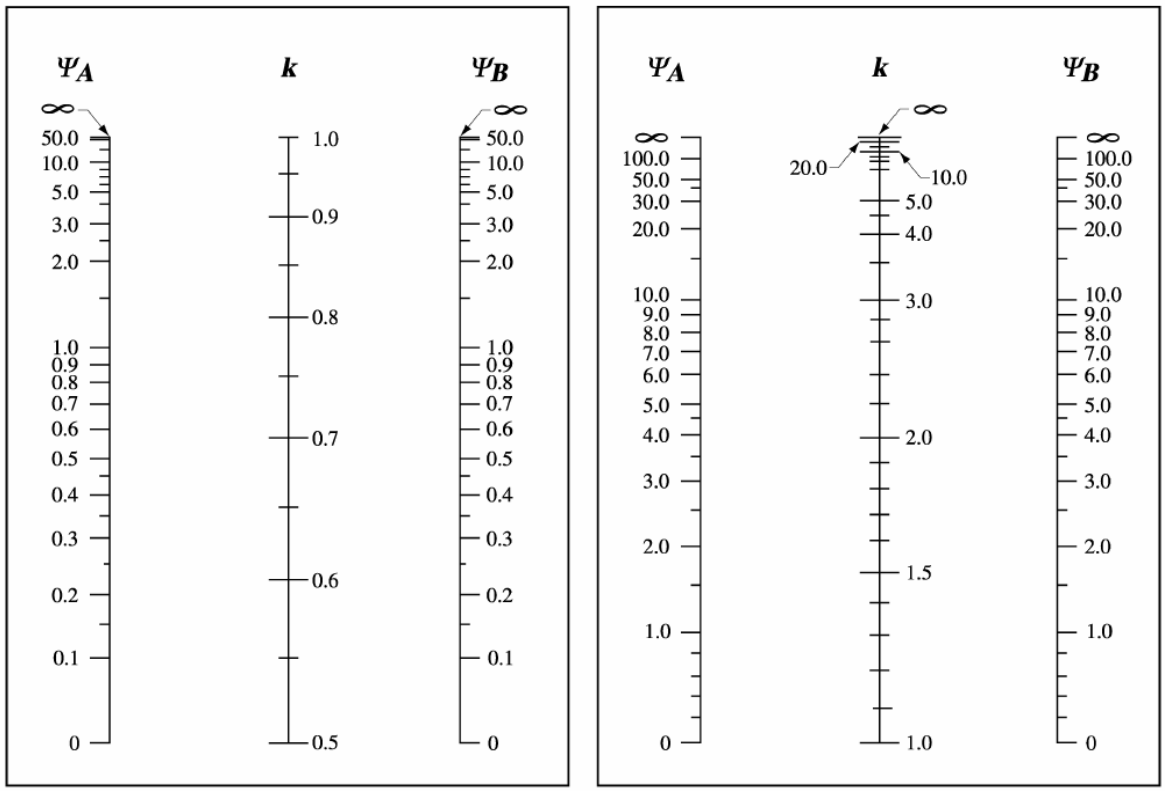
- For members without transverse load between support

$$C_m = 0.6 + 0.4\ \frac{M_1}{M_2} \geq\ 0.4 \tag{5-23}$$

- For members with transverse load between support C_m =1.0

According to *ACI code 10.12.3.2* for members $M_{2,\ min} > M_2$, C_m = 1.0 or based on the ratio of the computed end moment M_1 and M_2.

Table 5.3 : Alignment Chart *(ACI Code Commentary R 10.12.1)*



(a)
Nonsway Frames

(b)
Sway Frames

Step 9 : Computation of Column Flexural Stiffness, El

According to *ACI Code 10.12.3,* EI be determined by either:

$$EI = \frac{0.2E_c I_g + E_s I_{se}}{1+\beta_d} \tag{5-24a}$$

$$EI = \frac{0.4E_c I_g}{1+\beta_d} \tag{5-24b}$$

Where,

I_{se} = moment of inertia of reinforcement about centroidal axis of member cross section

β_d = (Maximum factored axial DL) / (Maximum factored axial total load)

Step 10 : Determination of Critical Buckling Load, P_c

According to *ACI Code 10.12.3*

$$P_c = \frac{\pi^2 EI}{(Kl_u)^2} \tag{5-25}$$

Step 11 : Calculation of Moment Magnification Factor, δ_{Ns}

According to *ACI Code 10.12.3*

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \tag{5-25a}$$

Step 12 : Calculation of Magnified Design Moment, M_c

According to *ACI Code 10.12.3*

$$M_c = \delta_{ns} M_2$$

M_2 should satisfy the criteria mentioned in step 7.

Step 13 : Check for Steel Ratio Considering Magnified Moment

Now the trial column section (bxh) and steel ratio ρ_g should be checked against P_u and M_c. Checking can be done by method mentioned in section 4.3 or by design chart given in Appendix F. Generally, even if the trial section is adequate, ρ_g need to be increased. Accordingly with increased ρ_g total steel area is obtained.

$$A_{st} = \rho_{g\,(increased)} * A_g$$

Then suitable bar size is chosen.

Comment:

If in step 13, column dimension needs to be revised, requite step 6 (for K) step 10 (for P_c), step 11 (for δ_{ns}) basing on new dimensions. Adequacy of new section should be checked again.

4.4.5 Design of sway (unbraced) frames by moment magnifier method

Column subject to side sway do not normally stand alone but are part of a structural system including floors and roof. A floor or roof is normally very stiff in its own plane. Consequently, all columns at a given story level in a structure are subject to essentially identical sway displacements. All columns at a given level must be considered together in evaluating slenderness effect relating to side sway. Sometime, A single column in an unbraced frame may buckle individually under gravity loads. For that particular case non- sway moment should also be considered.

Step 1: Determination of Factored Load

When wind load, W is included in the design, three possible factored load combinations are to be considered:

U = 1.4D + 1.7L	*(ACI Code-00)*	(5-27a)
U = 0.75 (1.4D + 1.7L + 1.7W)	*(ACI Code-00)*	(5-27b)
U = 0.9D + 1.3W	*(ACI Code-00)*	(5-27c)

The equations (5-27b) commonly controls the design

Step 2: Selection of a Trial Column Section

Assume short column behavior. Select a trial column section to carry the factored load = P_u and factored moment M_u using equation (5-27b). To select a trial column section follow any procedure mentioned on section 5.3.

Step 3: Determination of Mode of Analysis Either as Non-Sway or Sway Frame

Check for non-sway frame and sway frame.

Step 4: Checking as Non-sway (Braced) Column Under Gravity Loads Only

All columns in sway frames must first be considered as non-sway columns under gravity loads acting alone, *i.e.* for U = 1.4D + 1.7L. This check should be done in accordance to section 5.4.4. After that redesign the column section for sway effect.

Or, the column section may be designed for sway frame and at the end check should be given as non-sway column.

Step 5 : Determination of Effective Length Factor, K

According to *ACI Code Commentary R10.12.1*, k for sway frame can be calculated by any of the two methods.

- Using *Alignment Chart* given in Table 5.3.b
- Using expressions as follows:

$$\Psi_m < 2;\ k = \frac{20 - \Psi_m}{20}\sqrt{1 - \Psi_m} \qquad (5\text{-}28a)$$

$$\Psi_m \geq ;\ k = 0.9\sqrt{1 - \Psi_m} \qquad (5\text{-}28b)$$

Ψ_m is the average of Ψ values at the two ends of the compression member. To determine the values of Ψ_A and Ψ_B use equation (5-21)

Step 6 : Check for Slenderness Effect

According to *ACI Code 10.13.2*, slenderness effect must be considered when

$$\frac{kl_u}{r} \geq 22 \qquad (5\text{-}29)$$

Step 7 : Separation of Sway Loads And Gravity Loads

For sway frame analysis the loads must be separated into gravity loads as sway loads, and appropriate magnification factor must be computed and applied to the sway moment.

Calculate the following:

- M_{1ns} = Factored end moment at the end at witch M_1 acts, resulting from non-sway loads = 0.75 [1.4 x M_1 (D) + 1.7 M_1 (L)]
- M_{2ns} = Factored end moment at the end at which M_2 acts, resulting from non-sway load = 0.75 [1.4 x M_2 (D) +1.7 M_2 (L)]
- M_{1s} = Factored end moment at the end at which M_1 acts, due to lateral load that cause appreciable sideway = 0.75 [1.7 x M_1 (w)]

- M_{2s} = Factored end moment at the end at which M_2 acts, due to lateral load that cause appreciable sidesway = 0.75 [1.7 x M_2 (w)]

The sway effect will amplify the moments M_{1s}, M_{2s} by a moment magnification factor, δ_s $\delta_s M_{1s}$, $\delta_s M_{2s}$ = magnified sway moment

Step 8 : Determination of Magnified Sway Moments, $\delta_S M_s$

ACI Code 10.13.4 provides there alternate methods to calculate the magnified sway moment two methods are illustrated here:

- **Method 1** *(ACI code 10.13.4.2)*

$$\delta_s M_{1s} = \frac{M_{1s}}{1-Q} \geq M_{1s} \qquad (5\text{-}30a)$$

$$\delta_s M_{2s} = \frac{M_{2s}}{1-Q} \leq M_{2s} \qquad (5\text{-}30b)$$

Where $\delta_s = \dfrac{1}{1-Q} \leq 1.5$. for higher values of δ_s Method 2 must be followed.

Q is the stability index as obtained in Step 3.

- **Method 2** *(ACI code 10.13.4.3)*

$$\delta_s M_s = \frac{M_s}{1-\dfrac{\sum P_u}{0.75\sum P_c}} \geq M_s \qquad (5\text{-}31)$$

Where,

$\sum P_u$ = Total axial load on all the column in a story

$\sum P_c$ = Total critical bucking load for all column in the story

P_u for each column should be calculated separately following the equation (5-25)

For calculation of $\sum P_c$:

- Find K'for each type of column of the story under consideration follow step5. δ_o, K_1, K_2, are obtained

- Compute EI using equation either (5-24a) or (5-24b)

For lateral load $\beta_d = 0$. So the equations become:

$EI = 0.2\ E_c\ I_g + E_s\ I_{se}$
or, $EI = 0.4\ E_c\ I_g$

- Calculate P_c

$$P_{c1} = \frac{\pi^2 (EI)_1}{(k_1 l_u)^2}$$

$$P_{c2} = \frac{\pi^2 (EI)_2}{(k_2 l_u)^2}$$

$\sum P_c = P_{c1} + P_{c2} +$ ----------

Now the magnified sway moments are:

$$\delta_s\ M_{1s} = \frac{M_{1s}}{1 - \sum P_u / 0.75 \sum p_c}$$

$$\delta_s\ M_{2s} = \frac{M_{2s}}{1 - \sum P_u / 0.75 \sum p_c}$$

Step 9 : Determination of Total Magnified Moments

$$M_1 = M_{1ns} + \delta_s\ M_{1s} \qquad (5\text{-}32a)$$

$$M_2 = M_{2ns} + \delta_s\ M_{2s} \qquad (5\text{-}32b)$$

Step 10 : Check for Adequacy of Column Section to Resist Axial Factored Load and Magnified Moment

Now the trial column section (b x h) and steel ratio ρ_g should be checked to carry P_u and M_2 (obtained in step 9). Checking can be done by any of the methods mentioned in section 5.3 or by design chart given in Appendix F. Generally, even if the trial section is adequate, ρ_g need to be increased. Accordingly with increased ρ_g total steel area is obtained.

$$A_{st} = \rho_{g\ (increased)} * A_g$$

Then suitable bar size is chosen.

Comments:

If in step 10, column dimension needs to be revised repeat step 5 (for k), Step 6, step 8, basing on new dimensions. Adequacy of new dimensions should be checked again.

Step 11 : Check for Magnified Moment

According to *ACI Code 10.13.5* when

$$\frac{l_u}{r} < \frac{35}{\sqrt{\frac{P_u}{f_{c'}' A_g}}} \tag{5-33}$$

The design is ok, using the magnified total moment as obtained in step 9.

Otherwise, member should be designed for higher magnified moment as follows:

- Find M_1, M_2 using equations (5-32a) and (5-32b)

  $M_1 = M_{1ns} + \delta_s M_{1s}$
  $M_2 = M_{2ns} + \delta_s M_{2s}$

- Using M_1 and M_2 find magnified non-sway moment (M_c) as follows:

  - $C_m = 0.6 + 0.4 \dfrac{M_1}{M_2}$
  - $P_c = \dfrac{\pi^2 EI}{(Kl_u)^2}$
  - $\delta_{ns} = \dfrac{C_m}{1 - \dfrac{P_u}{0.75 P_c}}$
  - $M_c = \delta_{ns} M_2 M_c$ will be used as design moment

Step 12: Check for Possibility of Side Sway Instability Under Gravity Load

To protect against side sway buckling of on entire story under gravity loads alone, *ACI Code 10.13.6* places additional restrictions on sway frame. The form of the restriction depends on the $\delta_s M_s$ in step 8.

For Method 1: For load combination 1.4D + 1.7L $Q \leq 0.6$This corresponds to $\delta_s = \dfrac{1}{1-Q} = 2.5$.

For Method 2: For load combination 1.4D + 1.7L δ is positive ≤ 2.5

5.5 EXAMPLES FOR DESIGN OF COLUMN

5.5.1 EXAMPLE - DESIGN OF SHORT TIED COLUMN

Problem: Design the column subject to axial load as a tied column with following data:

DL = 150 Kips $f_c^{'}$ = 3 ksi

LL = 100 Kips f_y = 40 ksi

Solution: The problem is solved with reference to section 5.3.1.1.

Step 1: Determination of Factored Load

P_u = 1.4 D + 1.7 L
= (1.4 * 150) + (1.7 * 100)
= 380 kips

Step 2: Steel Ratio Assumption

Since value of axial load is low, take $\rho_g = 0.03$

Step 3: Determination of Concrete Gross Area

Pu = 0.80 A_g $\phi \left[0.85 f_c'(1-\rho_g) + \rho_g f_y\right]$

For tied column $\phi = 0.70$, $p_u = 380$ kips. Now,

380 = 0.80 A_g * 0.70 [0.85 * 3 (1- 0.03) + 0.03 * 40]
or, A_g = 185 in^2

Step 4: Selection of Column Size

Let us choose a square column of size = 14in. x 14 in. so A_g = 196 in^2

Step 5 : Check for Steel Ratio

$P_u = 0.80\ A_g\ \phi\left[0.85 f_c'(1-\rho_g) + \rho_g f_y\right]$
380 = (0.80 * 196 * 0.70) $\left[0.85 * 3(1-\rho_g) + \rho_g * 40\right]$
ρ_g = 0.024 = 2.4%; Limit of ρ_g is ok.

Step 6: Calculation of Reinforcement

$A_{st} = A_g * \rho_g$ = 196 * 0.024 = 4.71 in^2

Let us choose # 8 bar (A_b = 0.79 in^2)

No of bar, $N = \frac{A_{st}}{A_b} = \frac{4.71}{0.79} = 6 nos.$

Step 7 : Selection of Ties

Use # 3 bar for ties

Step 8 : Determination of Vertical Spacing of Ties

16 d_b of longitudinal reinforcement = 16 x 1 = 16in (dia # 8 = 1 in)
48 d_b of tie bar = 48 x 0.375 = 18 in (dia # 3 = 0.375 in)
Least dimension of column section = 14 in

Choose vertical spacing of ties = 14 in

Step 9: Arrangement of Ties

Clear spacing between longitudinal bars in y direction

= [Column dimension – (2 x clear cover) – (2 x dia of ties) – (3 x dia of bar)] ÷ 2
= [14 – (2 x 1.5) – (2* 0.375) – (3* 1)] ÷2
= 3.63 in < 6 in

No additional ties are required

Step 10: Detailing

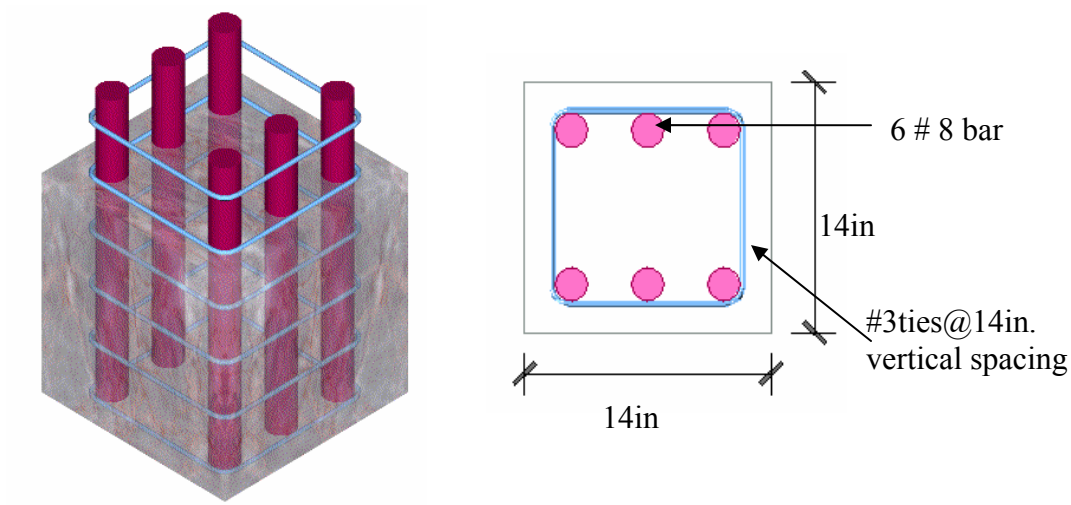


Figure 5.18: Detailing for Example 5.5.1

5.5.2 EXAMPLE - DESIGN OF SPIRALLY REINFORCED COLUMN

Problem: Design the column subject to axial load as spiral column with following data:

DL = 1200 kips $\quad f'_c$ = 3 ksi
LL = 500 kips $\quad f'_y$ = 50 ksi

Solution: The problem is solved with reference to *section 5.3.1.2*

Step 1 : Determination of Factored Load

P_u = 1.4D + 1.7L
= (1.4 * 1200) + (1.7 * 500)
= 2530 kips

Step 2 : Steel Ratio Assumption

Let us choose ρ_g = 0.03

Step 3 : Determination of Concrete Gross Area

$$P_u = 0.85\ A_g \phi \left[0.85 f_c'(1-\rho_g) + \rho_g f_y\right]$$

For spiral column $\phi = 0.75$, $P_u = 2530$ kips

$$2530 = 0.85 * A_g * 0.75\ \left[0.85*3(1-0.3)+(0.3*50)\right]$$

$$A_g = 999 \text{ in}^2$$

Step 4 : Selection of Column Size

Diameter of the column

$$D = \sqrt{\frac{4*A_g}{\pi}} = \sqrt{\frac{4*999}{\pi}} = 35.66 \text{ in}$$

Take D = 36 in

Steps : Check for Steel Ratio

Area of selected column size

$$A_g = \frac{\pi D^2}{4} = \frac{\pi \times 36^2}{4} = 1018 \text{ in}^2$$

Now:

$$P_u = 0.85\ A_g\ \phi \left[0.85 f_c'(1-P_g) + P_g f_y\right]$$

$$2530 = (0.85 * 1018 * 0.75)\ \left[0.85*3(1-P_g)+P_g*50\right]$$

$\rho_g = 0.028$; Steel ratio is within range (OK)

Step 6 : Calculation of Reinforcement

$$A_{st} = A_g * \rho_g = 1018 * 0.028 = 28.50 \text{ in}^2$$

Use # 9 bar ($A_b = 1 \text{ in}^2$)

Total number of bars, $N = \frac{A_{st}}{A_b} = \frac{28.50}{1.0} = 29$ nos

Step 7 : Check for Clear Distance between Longitudinal Reinforcement

Perimeter of spiral core = $\pi D_c = \pi * 33 = 103.67$ in

Clear distance:

$= \pi d_c$ - (N * dia of bar) / (N- 1)
$= [103.67 - (29*1.128)] \div 28$ (dia of # 9 = 1.128 in)
= 2.54 in

Clear distance = 2.54 in > 1.5 d_b = 1.70 in

$> 1\frac{1}{2}$ in

So, choosing of bar is correct.

Step 8 : Selection of Spirals

Use # 3 bar

$A_{sp} = 0.11 \text{ in}^2$

Step 9 : Determination of Spacing of Spiral (Pitch)

$$S_s = \frac{8.90 A_{sp} f_y}{d_c\left(\frac{D^2}{d^2{}_c} - 1\right) f_c'}$$

$$= \frac{8.90*0.11*50}{33\left(\frac{36^2}{33^2} - 1\right)3}$$ A_{sp} = 0.11 in, D = 36 in, d_c = 33 in

= 2.60 in.

Provide S_s = 2.50 in. Here: 1in $\le S_s \le 3in$ (OK)

Step 10 : Detailing

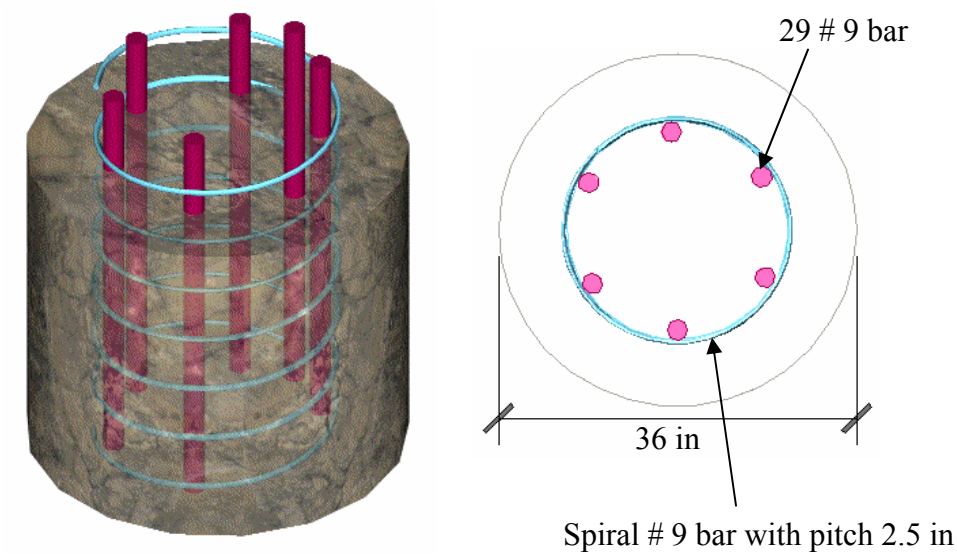


Figure 5.19 : Detailing for Example 5.5.2

For placement of spiral see step 9 of section 5.3.1.2

For splicing of spiral reinforcement see step 10 of section 5.3.1.2

For bar splicing in column see step 11 of section 5.3.1.2

5.5.3 EXAMPLE - DESIGN OF ECCENTRICALLY LOADED COLUMN WITH UNIAXIAL BENDING (MINIMUM ECCENTRICITY)

Problem: Design a short tied interior column with the following data:

DL = 300 kips f'_c = 4 ksi
LL = 250 kips f'_y = 60 ksi
M_{LL}= 500 k-in.

Solution: For minimum eccentricity (e = 0. l0h or $\frac{e}{h} \leq 0.10$) .An eccentrically loaded column, with uniaxial bending shall be designed as concentrically loaded column. At first give a check for minimum eccentricity, then design according to section 5.3.1.1 for tied column, and section 5.3.1.2 for spiral column.

Step 1: Determination of Factored Load and Moment

$$P_u = 1.4D + 1.7L$$
$$= (1.4 * 300) + (1.7 * 250) = 845 \quad \text{kip}$$

$$M_u = 1.7M_{ll} \quad = 1.7 * 500 = 850 \quad \text{k-in.}$$

Step 2: Determination of Eccentricity.

$$e = \frac{M_u}{P_u} = \frac{850}{845} = 1.01 \text{ in}$$

The loading appears to require a column h much greater than 10 in. which means e/h < 0.10. For minimum eccentricity (deign in Region I) design the column as concentrically loaded.

Step 3 : Determinations of Concrete Gross Area.

Assume $p_g = 0.03$

$$p_u = 0.80A_g\varnothing.[0.85 f'_c(1-\rho_g) + \rho_g f_y]$$

$$845 = 0.80 * Ag * 0.70[0.85 * 4(1 - 0.03) + 0.03 * 60]$$

$$Ag = 296in^2$$

Step 4: Seletion of Column Size

Column size = 18 × 18 in. h =180

Hence $\frac{e}{h} = \frac{1.01}{18} = 0.056$ < 0.10; for rest of the design follow section 5.3.1.1

5.5.4 EXAMPLE - DESIGN OF ECCENTRICALLY LOADED COLUMN WITH UNIAXIAL BENDING, COMPRESSION-CONTROLLED SECTION ($e_{min} < e < e_b$)

Problem: Design a short tied interior column with the following data:

DL = 214 kips	f'_c = 4 ksi
LL = 132 kips	f'_y = 60 ksi
M_{DL} = 47 k-ft	M_{LL} = 23 k-ft

Solution: The problem is solved with reference to section 5.3.2.3.

Step 1 : Determination of Factored Load and Moment

$P_u = 1.4D + 1.7L$ = (1.4*214) + (1.7* 1342) = 524 kip

M_u = 1.4 M_{DL} + 1.7 M_{LL} = (1.4* 47) + (1.7* 23) =105 kip-ft

Step 2 : Computation Required Nominal Strength

$$P_n = \frac{p_u}{\varnothing} = \frac{524}{0.70} = 750 \text{ kips}$$

$$M_n = \frac{M_u}{\varnothing} = \frac{105}{0.70} = 150 \text{ k-ft}$$

Step 3 : Computation of Eccentricity

$$e = \frac{M_n}{p_n} = \frac{150*12}{750} = 2\ .40 \text{ in}$$

Step 4 : Check For Minimum Eccentricity

$$e_{\min} = 0.10h \Rightarrow h = \frac{e}{0.10} = \frac{2.40}{0.10} = 24in$$

For h > 24 in, minimum eccentricity controlled
For h < 24 in, the design will be in Region II

Step 5: Determination of Maximum size of Column for Compression-Controlled

Considering $p_n = p_b$

$$p_n = 0.85 f_c^{'} \beta_1 (bd) \frac{\in_u}{\in + \in_y}.$$

$$B_1 = 0.85 \qquad \in_u = 0.003$$

$$\in_y = \frac{f_y}{E_s} = \frac{40}{29.000} = 0.00138$$

$$750 = (0.85*3*0.85)(bd)\left(\frac{0.003}{0.003+0.00138}\right)$$

$$bd = 506in^2$$

$$Ag_b = \frac{bd}{0.9} = \frac{506}{0.9} = 562 \text{ in}^2$$

Which means, that if an area less than 562 sq. in is provided then e $< e_{b,}$ (*i.e.* the section would be compression-controlled)

Step 6 : Selection of the Size of the Column

Select $\rho_g = 0.03$

Use Whitney Formula:

$$p_n = A_g \left[\frac{f_c^{'}}{(3/\xi^2)(e/h)+1.18} + \frac{\rho_g f_y}{(2/\gamma)\left(\frac{e}{h}\right)+1} \right]$$

Assume h = 22 in

$$\frac{e}{h} = \frac{2.4}{22} = 0.109 = 0.12$$

$$\gamma = \frac{d-d'}{h} = \frac{19.5-2.5}{22} = 0.77$$

$$\xi = \frac{d}{h} = \frac{19.5}{22} = 0.89 \Rightarrow \xi^2 = 0.792$$

$$750 = A_g \left[\frac{3}{(3/0.792)(0.12)+1.18} + \frac{0.03*40}{(2/0.77)(0.12)+1} \right]$$

$$\Rightarrow A_g = 272 in^2$$

Select size of column = 17 × 17 in

Step 7 : Estimation of Reinforcement

$$A_{st} = A_g \times \rho_g = 272*0.03 = 8.2 in^2$$

Use # 9 bar ($A_b = 1.0 in^2$)

Number of bar, N = $\frac{A_{st}}{A_b} = \frac{8.2}{1.0} = 9$

Use 10 # 9 bar, with 5 bars in each face

Step 8 : Selection of Ties

Use # 3 tics.

Step 9 : Determination of Vertical spacing of Ties

- 16 bar diameter = 16 * 1.128 = 18 in
- 48 tie diameter = 48 * 0.375 = 18 in
- Least lateral dimension = 17 in

Use # 3 ties at 17 in spacing.

Step 10 : Arrangement of Ties

Clear distance $= [17 - (2*1.5) - (5*1.125)] \div 4$
$= 1.91 \text{ in} \quad > 1.5\, d_b = 1.69 \text{ (OK)}$

Since clear distance $< 6''$, so additional tie should be provided for 3rd bar only. Provide 2 tie per set

Step 11 : Detailing

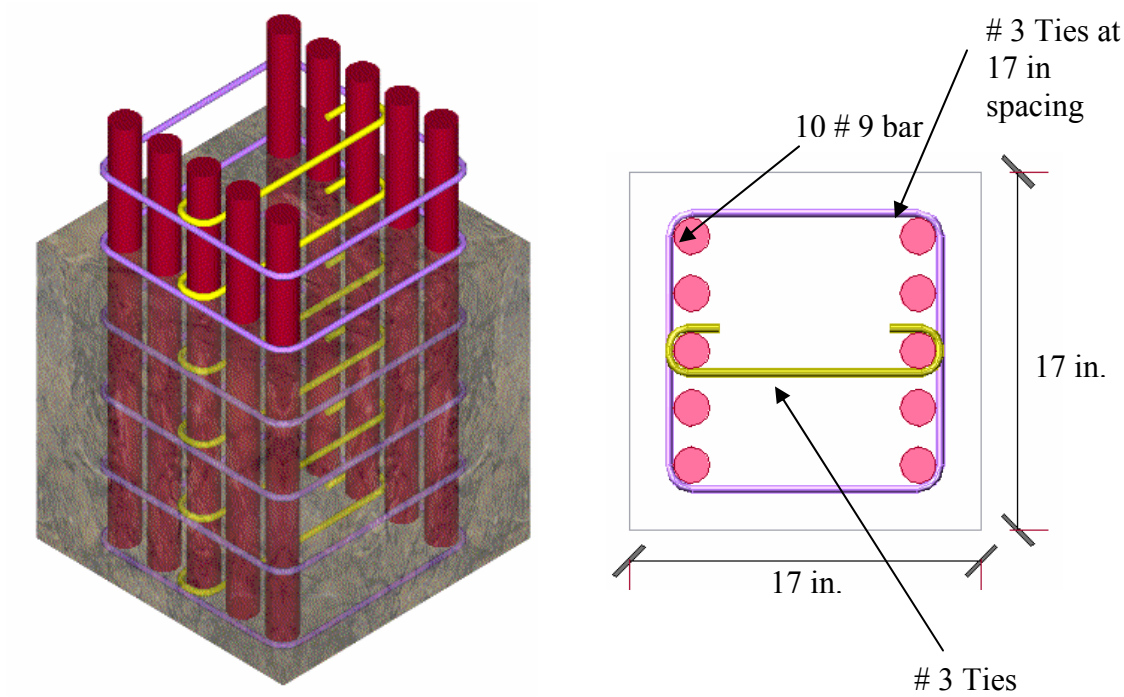


Figure 5.20 : Detailing for Example 5.5.4

5.5.5 EXAMPLE-DESIGN OF ECCENTRICALLY LOADED COLUMN WITH UNIAXIAL BENDING, TRANSITION ZONE TENSION-CONTROLLED SECTION (e > e_b)

Problem: Design a short tied column with the following data:

DL = 43 kips	f'_c = 4.5 ksi
LL = 32 kips	f'_y = 50 ksi
M_{DL} = 96 k-ft	M_{LL} = 85 k-ft
b ≤ 14 in	

Solution: The problem is solved with reference to section 5.3.2.4

Step 1 : Determination of Factored Load and Moment

P_u = 1.4 D + 1.7 L = (1.4 * 43) + (1.7 * 32) = 114.6 kips

M_u = 1.4 M_D + 1.7 M_L = (1.4 * 96) + (1.7 * 85) = 278.9 kip-ft

Step 2 : Computation of Required Nominal Strength

Assume a value for ϕ factor
For tied column, ϕ = 0.70 to 0.90
For spiral column, ϕ = 0.75 to 0.90
Take ϕ = 0.70

$$P_{n,required} = \frac{P_u}{\phi} = \frac{114.6}{0.70} = 164 \text{ kips}$$

$$M_{n,required} = \frac{M_u}{\phi} = \frac{278.9}{0.70} = 398 \text{ kip-ft}$$

Step 3 : Calculation of Eccentricity

$$e = \frac{M_n}{P_n} = \frac{398*12}{164} = 29.1 \text{ in}$$

Step 4: Determination of Minimum Limit of Column Section for Tension Controlled

Considering balanced strain condition $P_n = P_b$

$$P_n = 0.85\ f_c'\ \beta_1\ (bd)\left(\frac{\in_u}{\in_u + \in_y}\right)$$

β_1 =0.85; $\in = 0.003$

$$\in_y = \frac{f_y}{E_s} = \frac{50}{29{,}000} = 0.00172$$

$$164 = (0.85 * 4.5 * 0.85) * (bd)\left(\frac{0.003}{0.003 + 0.00172}\right)$$

bd = 80 in^2

$$Ag_{\text{(balanced)}} = \frac{bd}{0.80} = \frac{80}{0.80} = 100\ \text{in}^2$$

It is reasonably certain that an area larger than this must be used. Therefore $e > e_b$

Step 5 : Determination of Preliminary Size of Column

Select steel ratio, $\rho_g = 0.03$

Use Approximate Formula:

$$p_n = 0.85\, f_c' bd\left[-\rho + 1 - \frac{e'}{d} + \sqrt{\left(1 - \frac{e'}{d}\right)^2 + 2\rho\left[(m-1)\left(1 - d'/d\right) + e'/d\right]}\right]$$

$$\rho = \frac{1}{2}\rho_g = 0.015$$

$$m = \frac{f_y}{0.85\, f_c'} = \frac{50}{0.85 * 4.5} = 13.08$$

Assume h = 20 in , then d = 18.5, $d' = 2.0$

$$\frac{e'}{d} = \frac{\left(d - h/2\right) + e}{d} = \frac{18.5 - 10 + 29.1}{18.5} = 2.03 \approx 2.0$$

$$\frac{d^1}{d} = 0.10$$

$$\Rightarrow 164 = 0.85 * 4.5 * bd\left(\left[-0.015 + 1 - 2 + \sqrt{(-1)^2 + 0.03\{(12.08 * 0.09) + 2\}}\right]\right)$$

$(bd)_{required}$ 263 in^2

$$Ag_{preliminary} = \frac{bd}{0.9} = 292 \text{ in}^2$$

Step 6 : Selection of the Size of Column

Trial-1:

Choose $A_g = (14 \times 18)$ in. column, $A_g = 252 in^2$

$$\frac{e'}{d} = \frac{d - h/2 + e}{d} = \frac{15.5 - 9 + 29.1}{15.5} = 2.30$$

$$\frac{d'}{d} = \frac{2.5}{15.5} = 0.16$$

$$\Rightarrow 164 = 0.85 * 4.5 * (bd)\left(\left[-0.015 + 1 - 2.30 + \sqrt{(-1.3)^2 + 0.03\{(12.08 * 0.84) + 2.30\}}\right]\right)$$

$$\Rightarrow (bd) = 330 in^2 \neq 252 in$$

Trial-2

Choose (14×20) in column, $A_g = 280$ in^2

$$\frac{e'}{d} = \frac{17.5 - 10 + 29.1}{17.5} = 2.09$$

$$\frac{d'}{d} = \frac{2.5}{17.5} = 0.14$$

$$(bd)_{required} = 292 in^2 = 280 in^2$$

Select 14×20 section

Step 7 : Estimation of Reinforcement

$$p_n = 0.85 f_c' ab$$

$$\Rightarrow 164 = 0.85 * 4.5 * a * 14$$

$$\Rightarrow a = 3.06 in$$

$$P_n\left[\left(e-\frac{h}{2}\right)+\frac{a}{2}\right]=A_s f_y\,(d\text{-}d')$$

$$\Rightarrow 164\left[(29.1-10)+\frac{3.06}{2}\right]=A_s 50(17.5-2.5)$$

$$\Rightarrow A_s=4.51in^2$$

Total steel requirement, $A_{st}=2*A_s=2*4.51=9.02in^2$

Check for $\rho_g=\frac{A_{st}}{bh}=\frac{9.02}{14\times 20}=0.0322$

Choose 6 # l 1 bars

Step 8: Detailing

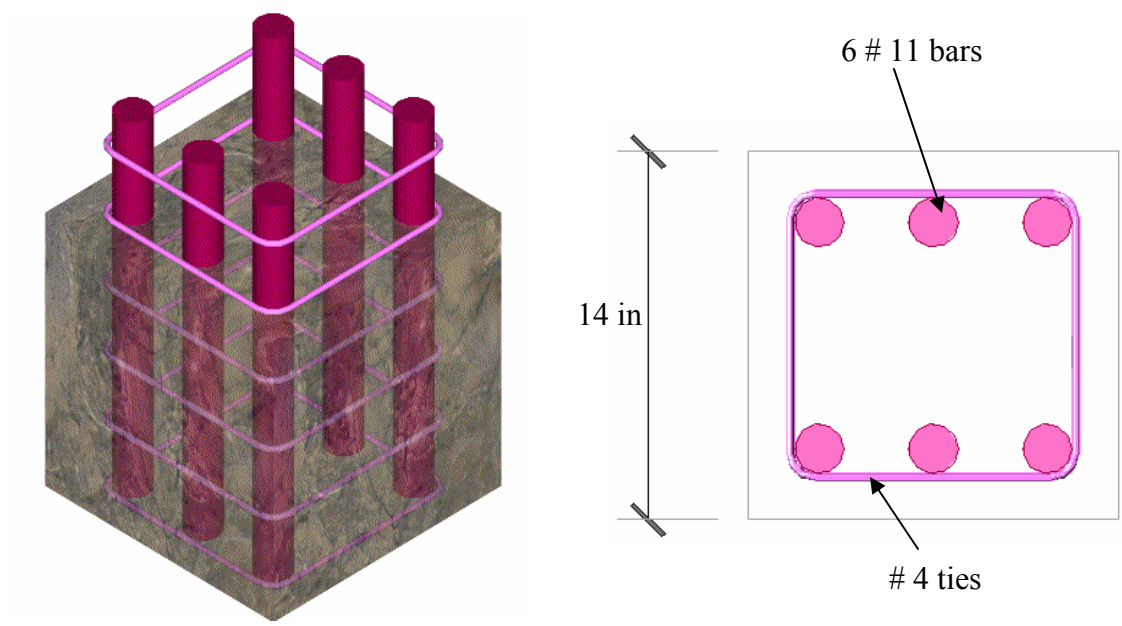


Figure 5.21 : Detailing for Example 5.5.5

5.5.6 EXAMPLE – DESIGN OF A SLENDER COLUMN IN A NONSWAY (BRACED) FRAME

Problem: A multistory concrete frame building is given in Figure 5.21 .The frame is braced against sway by stairs and elevator shafts having concrete walls that are monolithic with the floors.

Beam size = 48in. wide × 12 in. deep

Story height = 14ft

$f_c' = 4000$ psi

$f_y' = 60{,}000$ psi

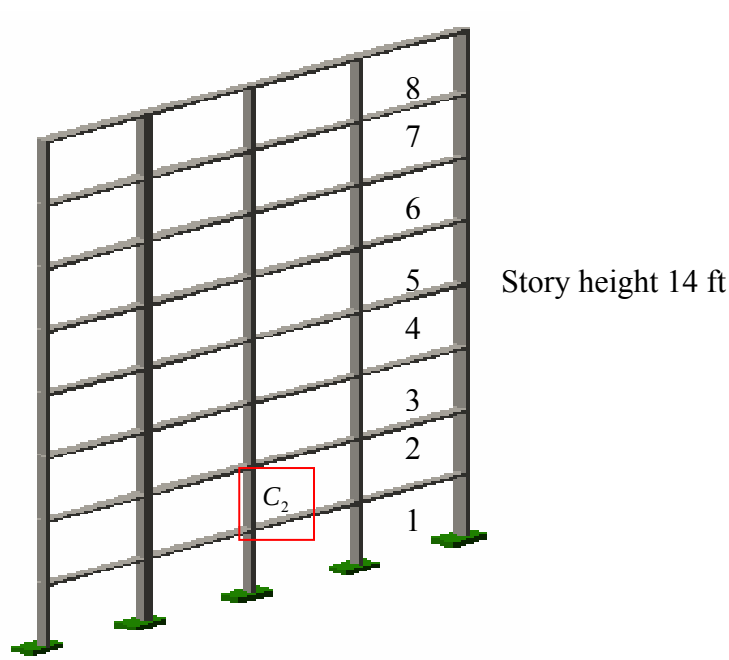


Figure 5.22: Elevation of a building for Example 5.5.6

Loads and moments as obtained by first-degree analysis for column C_2 are:

	Dead 1oad	Live 100 d
P	230k	173 k
M_2	2 k-ft	108 k – f t
M_1	2 k-ft	100 k- t l

Column is subject to double curvature under dead load and single cunvature under live load

Solution: The problem is solved with reference to section 5 .5 .4.

Step 1 : Determination of Factored Load and Moment

P_u = 1.4D + 1.7L = (1.4 * 230) + (1.7 *173) = 616 kips

M_u = 1.4 M_D + 1.7 M_L = (1.4 * 2) + (1.7 * 108) = 186 k-ft

Step 2 : Selection of a Trial Column Section

The column will first be designed as a short column, assuming no slenderness effect. Trial column size can be selected by any of the following methods:

- Follow the procedure mentioned in section 5 .3.2.3 or
- Follow the design aids method mentioned in section 5.3.3.1 or
- From experience choose column section, then using design aids find tentative steel ratio as mentioned in section 5.3.3.2.

Let us choose a column size, A_g = (18 x 18 in)

Follow 3rd approach with 1.5 in. clear distance

3 stirrup (dia = 0.38 in)
10 longitudinal bar (dia = 1.25 in)

$$\gamma = \frac{h - 2d'}{h} = \frac{18 - 2 \times 2.51}{18}$$

= 0.72

Now $\frac{P_u}{A_g} = \frac{616}{324} = 1.90$ ksi

$$\frac{M_u}{A_g h} = \frac{186 \times 12}{324 \times 18} = 0.38 \text{ ksi}$$

Using *Appendix F-7* for γ = 0.75 deter mine $\rho_g = 0.02$

Comment: Since ρ_g = 0.02 is low value. It can be increased, if necessary, to meet slenderness requirements.

Thus, column size = 18×18 in is retained

Step 3 : Determination of Mode of Analysis Either as Non-sway or Sway Frame

Referred to section 5.5.2 due to presence of shear wall, elevator core, by inspection it can be told that the frame is braced or non-sway.

Step 4 : Check for Slenderness Effect

For an initial check assume k = 1.0
Unsupported length of column, $l_u = 13$ ft.
Radius of gyration of rectangular member, $r = 0.30h = 0.30 \times 18 = 5.4$

$$\frac{Kl_u}{r} = \frac{1.0 \times 14 \times 12}{5.4} = 28.9$$

For braced frame allowable short column behavior

$$= \left[34 - \frac{12M_1}{M_2}\right] \leq 40$$

For single curvature, $M_1 = -ve$
For double curvature, $M_1 = +ve$

$$M_1 = 1.4(-2) + (1.7 \times 100) = 167.2$$
$$M_2 = (1.4 \times 2) + (1.7 \times 108) = 186.4$$
$$= \left[34 - 12 * \frac{167.2}{186.4}\right]$$
$$= 23.3$$

$$\frac{kl_u}{r} > \left[34 - \frac{12M_1}{M_2}\right]$$

So, slenderness must be considered

Step 5 : Determination of Refined Effective Length Factor (k)

- **For Column**

  Moment of inertia, $I_g = \frac{bh^3}{12} = \frac{18 \times 18^3}{12} = 8748$

  $I_{column} = 0.70\ I_g = 0.70 * 8748 = 6124\ \text{in}^4$

  $l_c =$ length of column c/c = 14 ft.

  $$\frac{I_{column}}{l_c} = \frac{6124}{14 \times 12} = 36.5 in^3$$

- **For Beam**

  Moment of inertia of "T" beam

  $I_g = 2 \times I_g\ for$ web

  $= 2 \times \frac{48 \times 12^3}{12} = 13{,}824\ \text{in}^4$

  $I_{beam} = 0.35\ I_g = 0.35 * 13{,}824 = 4838\ \text{in}^4$

  $l_n =$ span length of beam c/c = 24 ft.

  $$\frac{I_{beam}}{l_n} = \frac{4838}{24 \times 12} = 16.8\ \text{in}^3$$

- **Rotational Restraint Factor**

  $$\psi_A = \psi_B = \psi = \frac{\sum\left(E_c \frac{I_{column}}{l_c}\right)}{\sum\left(E_b \frac{I_{beam}}{l_n}\right)}$$

  $E_c = E_b$

$$\psi_A = \psi_B = \left(\frac{36.5 + 36.5}{16.8 + +16.8}\right) = 2.17$$

Using alignment chart given in **Table 5.3** determine k =0.87

Step 6 : Recheck for Slenderness Effect

With k = 0.87

$$\frac{Kl_u}{r} = \frac{0.87 \times 13 \times 12}{0.3 \times 18} = 25.1$$

$$\left[34 - 12\frac{M_1}{M_2}\right] = 23.3$$

So, $\frac{Kl_u}{r} > \left[34 - 12\frac{M_1}{M_1}\right]$

Slenderness must be considered

Step 7 : Check for Minimum Design Moment

$M_{2,min} = P_u$ (0.6 + 0.03 h = 616 (0.6 + 0.3×18) $/_{12}$ = 58 ft-kips

It does not control.

Step 8 : Computation of Equivalent Moment Correction Factor, C_m

$$C_m = 0.6 + 0.4\frac{M_1}{M_2} \geq 0.4$$

$$= 0.6 + 0.4\frac{167}{186} = 0.96$$

Step 9 : Computation of Column Flexural Stiffness EI

Find, $\beta_d = \frac{\max imum}{\max imum}\frac{factored}{foctored}\frac{axial}{axial}\frac{dead}{total}\frac{lood}{lood}$

$$= \frac{1.4 \times 230}{1.4 \times 230 + 1.7 \times 173} = 0.52$$

$$\text{EI} = \frac{0.4E_c I_g}{1+\beta_d} = \frac{0.4 * (3.6 \times 10^6) * (\frac{18 \times 18^3}{12})}{(1+0.52)}$$

$$= 8.29 \times 10^9 \ \ in^2 - 1b$$

Step 10 : Determination of Critical Buckling Load P_c

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 * 8.29 * 10^9 I}{(0.87 * 13 * 12)^2} = 4.44 * 10^6 = 4440 \text{ kips}$$

Step 11 : Calculation of Moment Magnification Factor , δ_{ns}

$$\delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 P_c}} = \frac{0.96}{1 - \frac{616}{0.75 * 4440}} = 1018$$

Step 12 : Calculation of Magnified Design Moment ,M_c

$M_c = \delta_{ns} n\ M_2 = 1.18 * 186 = 219$ ft- kips

Step 13 : Check for Steel Ratio Considering Magnified Moment

With reference to column deign chart *Appendix F-7* for

$$\frac{p_u}{A_g} = \frac{616}{324} = 1.90 \quad \text{ksi}$$

$$\frac{M_u}{A_g h} = \frac{219 \times 12}{324 \times 18} = 0.45 \ \text{ksi}$$

Obtain $\rho_g = 0.026$, required steel ratio is increased due to slenderness.
Required steal Area, $A_{st} = 0.026 \times 324 = 8042$ in^2
Use 4 #10 bar and 4 # 9 bar

Step 14 : Selection of Ties

With reference to step 7 of section 5.3.1.1 use # 3 ties.

Step 15 : Vertical Spacing

With reference to step 8 of section 5.3.1.1

$16\ d_b = 16 \times 1.125 = 18$ in (#9bar dia = 1.13 in.)

$48\ d_b$ of tie $= 48 \times 0.38 = 18.24$ in (# 3bar dia = 0.38 in.)

Least dimension of column section = 18 in

So, vertical spacing of ties = 18 in.

Step 16 : Detailing

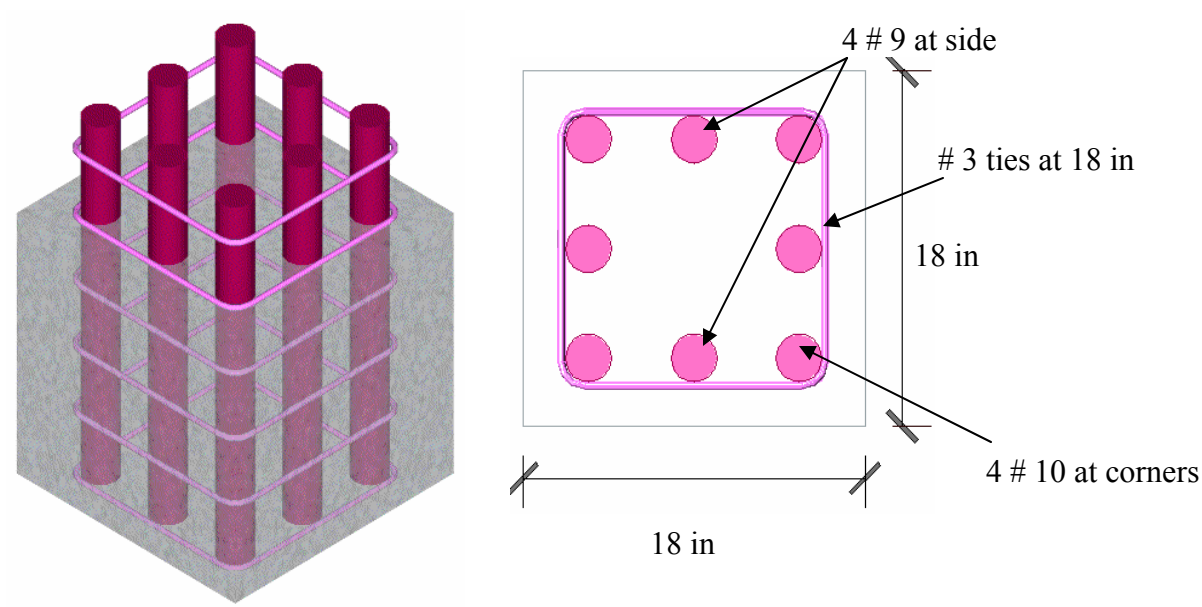


Figure 5.23 : Detailing for Example 5.5.6