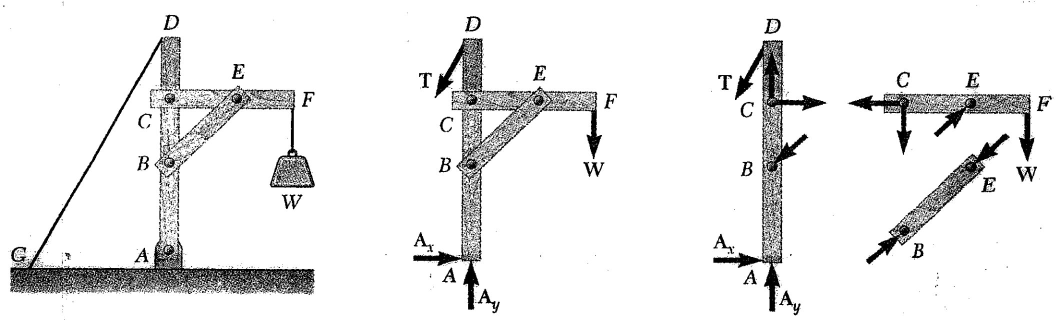
Chapter 6 Analysis of Structures

6.1 Introduction

internal forces : consider structures made of several connected parts, the forces which hold together the various parts of the structures are called internal forces

Newton's third law : the force of action and reaction between bodies in contact have the same magnitude, same line of action and opposite sense



in this chapter, three broad categories of engineering structures will be considered

1. trusses : contain straight members connected at joints located at the end of each member, member of truss are two-force members

2. frames : contain at least one multiforce member (3 or more forces acted)

(1) and (2) are usually stationary fully constrained structures

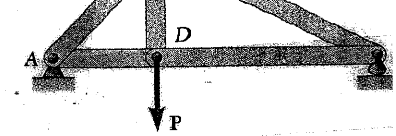
3. machines : which are designed to transmit and modify forces, structures containing moving parts, always contain at least one multiforce member

TRUSS

6.2 Definition of Truss

a truss consists of straight members connected at joints

plane truss : truss of two dimensional configuration

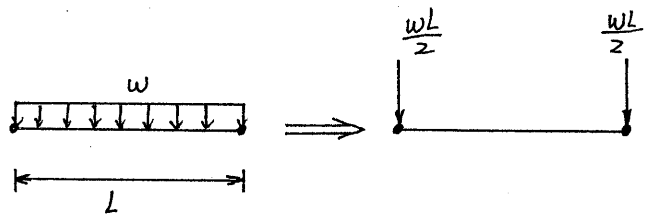


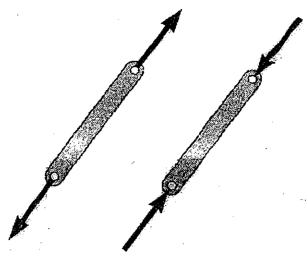
space truss : truss of three dimensional configuration

all members are slender bar, can support only a little lateral load, all loads must be applied to the various joints

when a concentrated load is to be applied between two joints, or when a distributed load is to be supported by the truss, as the case of a bridge truss, a floor system must be provided which, through the use of stringers and floor beams, transmits the load to the joints

the weight of member are also assumed to be applied to the joint, half of weight being applied to each of two joints

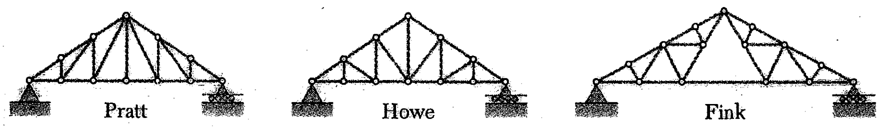




assume the members are pinned together, the force acting at each end of member reduce to a single force and no couple

each member may be treated as a two-force member

several typical trusses are shown (more trusses are shown in figure 6.5)

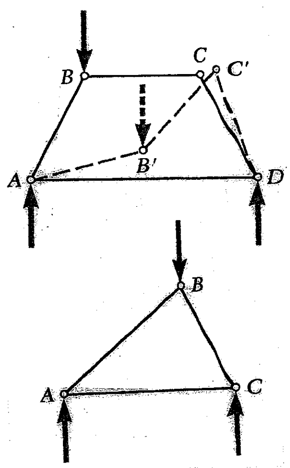


Typical Roof Trusses

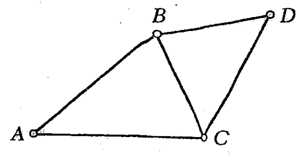
6.3 Simple Truss

consider a four member truss pinned at four joints, if a load is applied at B, the truss will collapse

for a triangular truss, it will not collapse when load is applied, it is called rigid truss

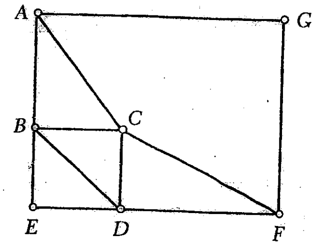


a large rigid truss may be obtained by adding two members to the basic triangular truss, adding two new members, attach them to separate existing joints and connect them at connect them at a new joint (three joints must not be in a straight line), a truss which may be constructed in this manner is called simple truss



simple truss is no necessarily made only of triangles

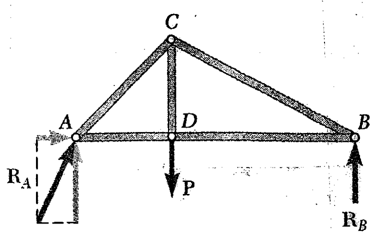
in simple truss, the total number of members is



$$m = 2n - 3$$

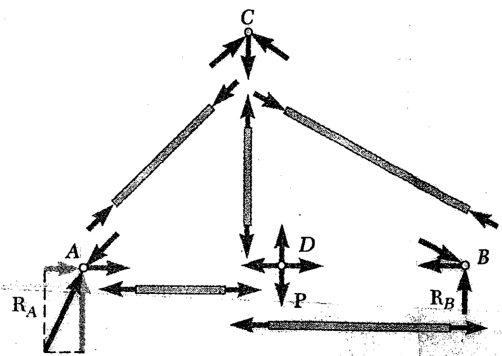
where n is the total number of joints

6.4 Analysis of Trusses by the Method of Joints



consider a simple truss with simply support, take the whole truss as the free body, we have three unknown reactions A_x, A_y and B

choose the free bodies for each member and joint, since the entire truss is in equilibrium, each pin must be in equilibrium



truss contains n pins, we have $2n$ equations

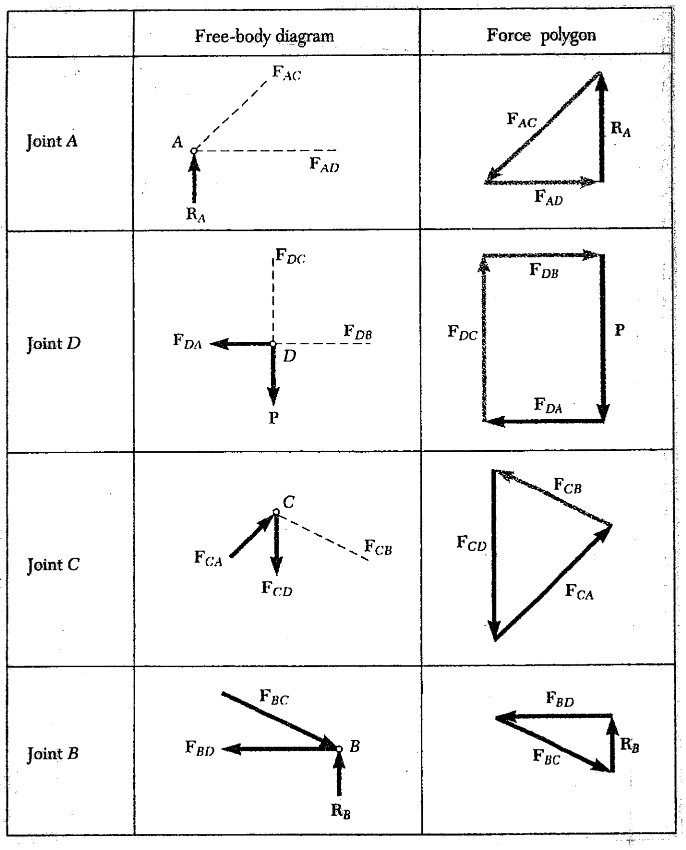
m members and 3 reactions, we have m + 3 unknowns

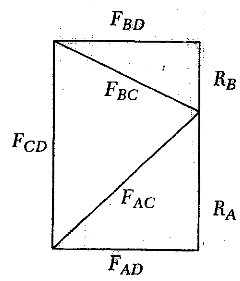
for a simple truss $m = 2n - 3$ i.e. $2n = m + 3$

thus number of equations = number or unknowns

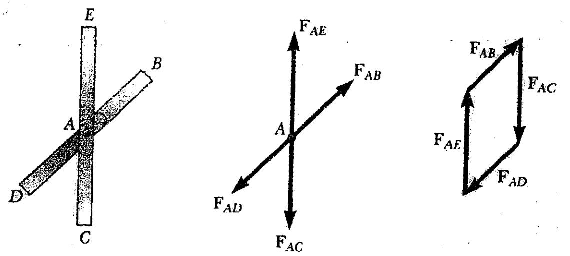
the reactions R_A and R_B can be obtained first

at joint A, F_{AC} and F_{AD} can be obtained, similarly for points D and C, then all member forces can be found, finally use point B to check the result, for the whole truss, the magnitude of each member force can be represented by the Maxwell's diagram



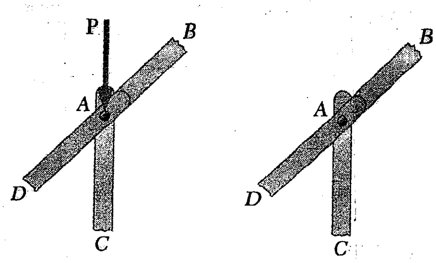


6.5 Joints Under Special Loading Conditions



for a joint connects four members lying in two intersecting straight lines, the forces in opposite members must be equal

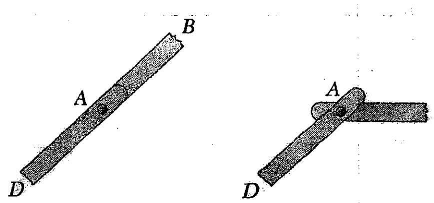
$$F_{AC} = F_{AE} \qquad F_{AB} = F_{AD}$$



for a joint connects three members and supports a load P with two members lie in the same line, and the load P acts along the third member, then

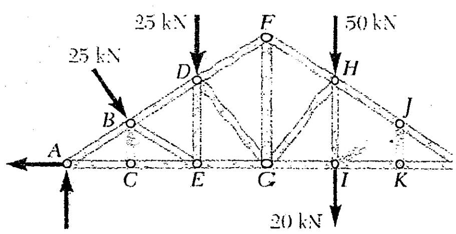
$$F_{AB} = F_{AD} \qquad F_{AC} = P$$

if $P = 0$ (no loading applied), then $F_{AC} = 0$, AC is said to be a *zero-force member*



for a joint connects two member only, if two members lying in the same line, the force in two members must be equal, it two members lying not in the same line, the forces of two members are equal to zero

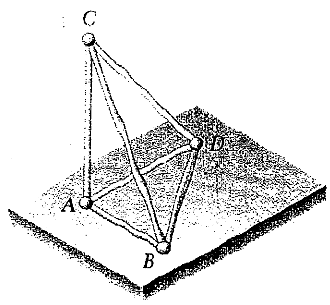
zero-force members are not useless, they do not carry any load under particular loading condition, but they may carry loads if the loading condition are changed



6.6 Space Truss

basic tetrahedron truss has 6 members and four joints, every three members are added, connected at a new joint

if truss is constructed in this manner, it is called simple space truss

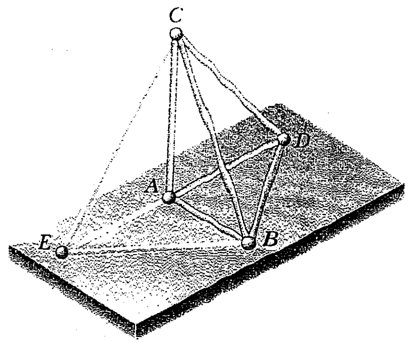


total number of member $m = 3n - 6$

support has 6 unknown reactions

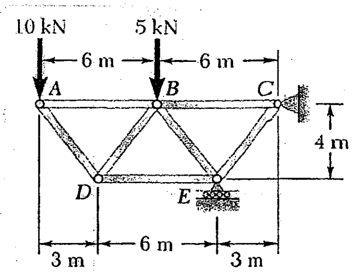
the condition of equilibrium for each joint will be expressed by the three equations

$$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$$



Sample Problem 6.1

determine the force in each member



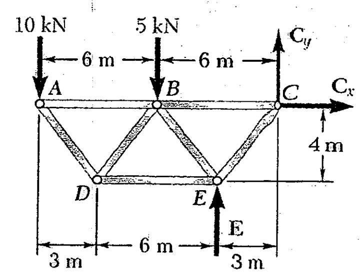
$\Sigma M_C = 0 \quad 10 \times 12 + 5 \times 6 - E \times 3 = 0$

$E = 50 \text{ kN} \uparrow$

$\Sigma F_y = 0 \quad -10 - 5 + 50 + C_y = 0$

$C_y = -35 \text{ kN} \downarrow$

$\Sigma F_x = 0 \quad C_x = 0$

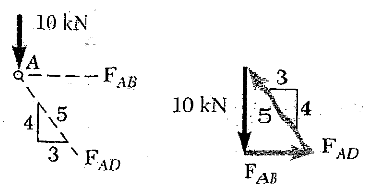


at joint A

$F_{AB} / 3 = F_{AD} / 5 = 10 / 4$

$F_{AB} = 7.5 \text{ kN (T)}$

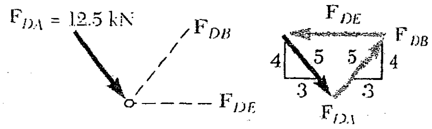
$F_{AD} = 12.5 \text{ kN (C)}$



at joint D

$F_{BD} = F_{AD} = 12.5$ kN (T)

$F_{DE} = F_{AD} \times (3/5) \times 2 = 15$ kN (C)



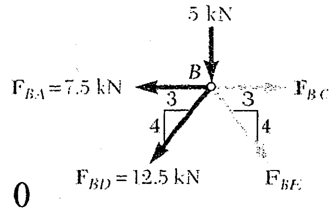
at joint B

$\Sigma F_y = 0 \quad -5 - 12.5\,(4/5) - F_{BE}\,(4/5) = 0$

$F_{BE} = -18.75$ kN (C)

$\Sigma F_x = 0 \quad F_{BC} - 7.5 - 12.5\,(3/5) - 18.75\,(3/5) = 0$

$F_{BC} = 26.25$ kN (T)



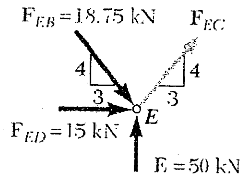
at joint E

$\Sigma F_x = 0 \quad F_{EC}\,(3/5) + 15 + 18.75\,(3/5) = 0$

$F_{EC} = -43.75$ kN (C)

$\Sigma F_y = 0 \quad 50 - 18.75\,(4/5) - 43.75\,(4/5) = 0$

O. K. (check)

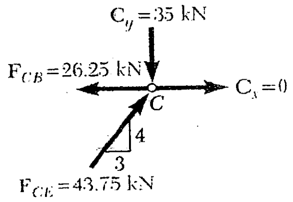


at joint C

$\Sigma F_x = 0 \quad -26.25 + 43.75\,(3/5) = 0$

$\Sigma F_y = 0 \quad -35 + 43.75\,(4/5) = 0$

O. K. (check)

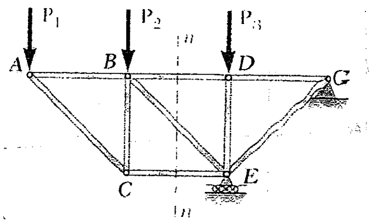


6.7 Analysis of Trusses by the Method of Section

method of joints : determine the forces in all members

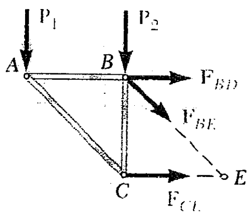
method of sections : determine few members which are desired

choose a larger portion of truss as a free body, composed of several joints and members, provided that the desired force in one of the external forces acting on that portion



$\Sigma M_E = 0 \quad \Rightarrow \quad F_{BD}$

$\Sigma M_B = 0 \quad \Rightarrow \quad F_{CE}$



$\Sigma F_y = 0 \quad \Rightarrow \quad F_{BE}$

one equation to get one unknown, it is more efficient

the computation can be checked by using $\Sigma F_x = 0$

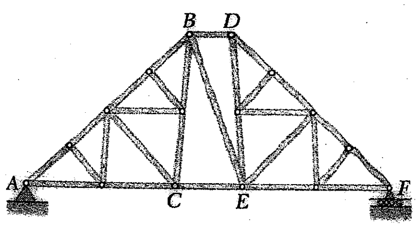
6.8 Trusses Made of Several Simple Trusses

trusses made of several simple trusses rigidly connected are known as compound trusses

consider two simple trusses ABC and DEF

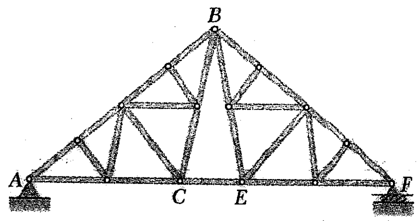
1. connected by three members

$$n = n_1 + n_2$$
$$m = m_1 + m_2 + 3$$
$$= (2n_1 - 3) + (2n_2 - 3) + 3$$
$$= 2n - 3$$



2. combined at pin B and connected by CE

$$n = n_1 + n_2 - 1$$
$$m = m_1 + m_2 + 1$$
$$= (2n_1 - 3) + (2n_2 - 3) + 1$$
$$= 2n - 3$$



number of members are still related by $m = 2n - 3$

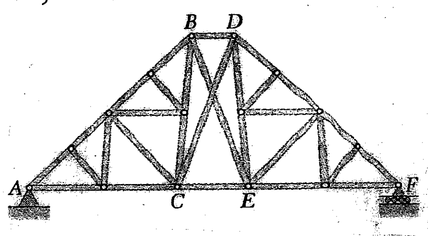
it is statically determinate, rigid and completely constrained if the compound truss supported on simply support

if two simple trusses are connected by four members, then

$$m = 2n - 2$$

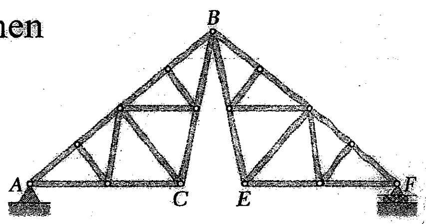
unknowns $m + 3 > 2n$

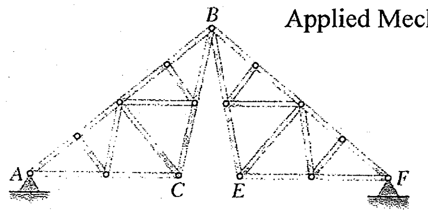
this truss is statically indeterminate



it two simple trusses connected only by one pin, then

$$m = 2n - 4$$



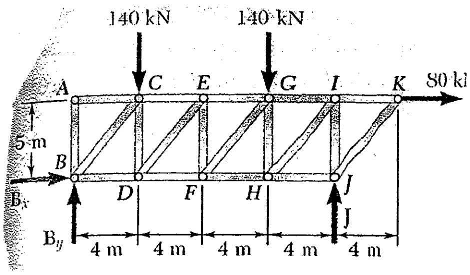


unknowns $m + 3 < 2n$

this truss is nonrigid

but if it is supported at two hinge supports, it has four reactions

then unknowns $= m + 4 = 2n$, it is still rigid



Sample Problem 6.2

determine the force in EF and GI

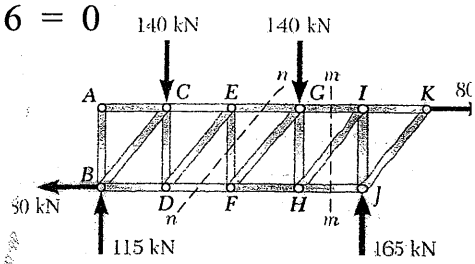
$\Sigma M_B = 0 \quad -140 \times 4 - 140 \times 12 - 80 \times 5 + J \times 16 = 0$

$J = 165 \text{ kN} \uparrow$

$\Sigma F_x = 0 \quad B_x = -80 \text{ kN} \leftarrow$

$\Sigma F_y = 0 \quad B_y + 165 - 140 - 140 = 0$

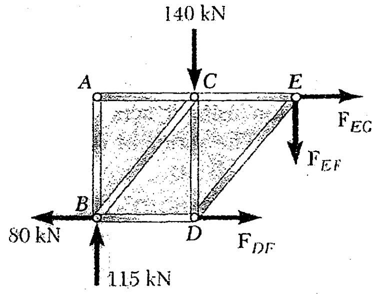
$B_y = 115 \text{ kN} \uparrow$



take the left part of section n-n

$\Sigma F_y = 0 \quad 115 - 140 - F_{EF} = 0$

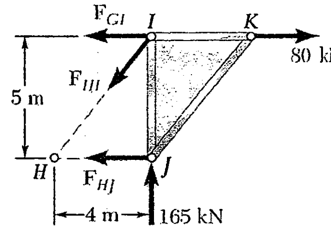
$F_{EF} = -25 \text{ kN (C)}$



take the right part of section m-m

$\Sigma M_H = 0 \quad 165 \times 4 - 80 \times 5 + F_{GI} \times 5 = 0$

$F_{GI} = -52 \text{ kN (C)}$



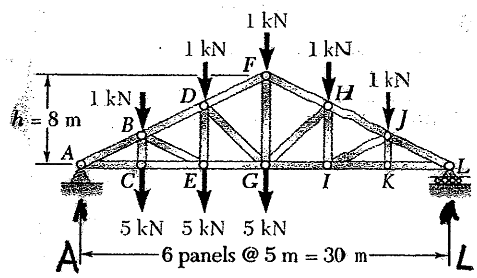
Sample Problem 6.3

determine the force in FH, GH and GI

take entire truss as free body, the reactions are

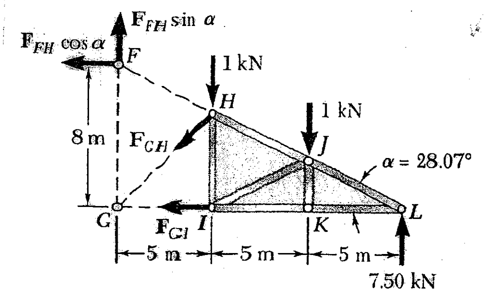
$A = 12.5 \text{ kN} \uparrow \qquad L = 7.5 \text{ kN} \uparrow$

$\tan\alpha = 8/15 = 0.533 \quad \Rightarrow \quad \alpha = 28.07^\circ$



take free for the right part of section n-n

$\Sigma M_H = 0 \quad 7.5 \times 10 - 1 \times 5 - F_{GI} \times 5.33 = 0$



$$F_{GI} = 13.13 \text{ kN (T)}$$

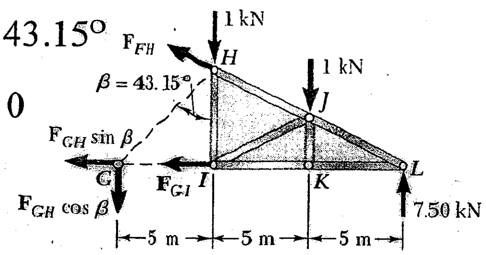
$$\Sigma M_G = 0 \quad 7.5 \times 15 - 1 \times 10 - 1 \times 5 - F_{FH} \cos\alpha \times 8 = 0$$

$$F_{FH} = -13.81 \text{ kN (C)}$$

$$\tan\beta = 5 / (\tfrac{2}{3} \times 8) = 0.9375 \quad \Rightarrow \quad \beta = 43.15°$$

$$\Sigma M_L = 0 \quad 1 \times 10 + 1 \times 5 + F_{GH} \cos\beta \times 15 = 0$$

$$F_{GH} = -1.371 \text{ kN (C)}$$



FRAMES AND MACHINES

6.9 Structures Containing Multiforce Members

consider structures in which at least one of the members is a multiforce members (not only two force member), e.g. frame and machines

frame are designed to support loads and are usually stationary, fully constrained structures

machines are designed to transmit and modify forces, may or may not be stationary and will always contain moving parts

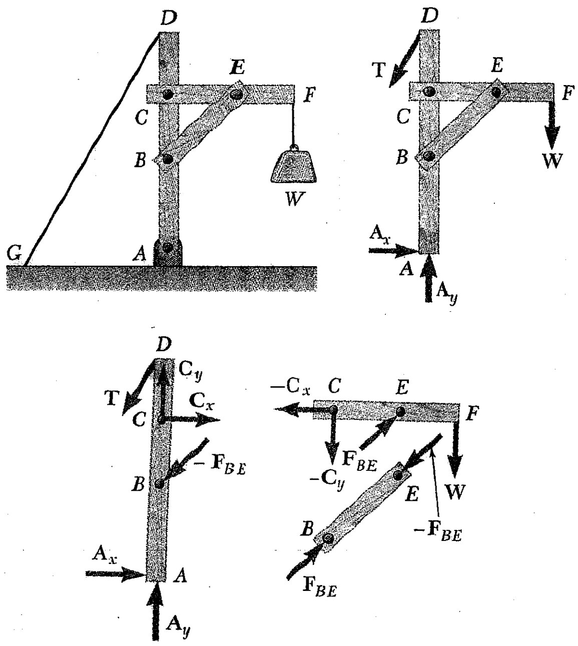
6.10 Analysis of a Frame

consider a frame carries a given load W

the cable force T and the reactions A_x and A_y can be determined

on each free body, three equilibrium equation may be used, $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$

in this frame, BE is a two force member, the force F_{BE} is acting along BE



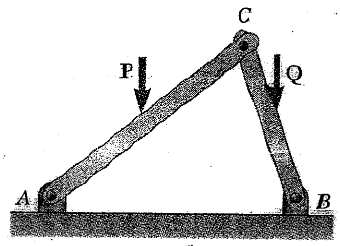
now consider the multiforce members *ABCD*, the internal forces C_x, C_y applied at *C* will be arbitrarily directed to the right and upward (and directed to the left and downward on member *CEF*), three components of internal force can be determined

the pins were assumed to form an integral part of one of the two members they connected and so it was not necessary to shown their free-body diagram, when a pin connects three or more members, or when a pin connects a support and tow or more members, or when a load is applied to a pin, a clear decision must be made is choosing the member to which the pin will be assumed to belong

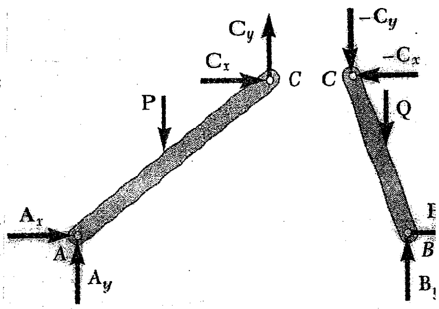
6.11 Frames Which Cease to be Rigid When Detached From Their Supports

for the frame considered in section 6.10, it is rigid without the help of its support (3 members : 9 equations, 6 internal unknown forces, 3 unknowns reactions, equations = unknowns)

for the frame *ABC*, it detached from its supports, this frame will not maintain its shape, if it is supported on a hinge and a roller, it will be nonrigid (2 members ; 6 equations, 2 internal unknown forces, 3 unknowns reactions, equations > unknowns)



if it is supported on hinges *A* and *B*, it will become rigid (4 unknown reactions, equations = unknowns), but the reactions cannot be completely determined from the free body diagram of the whole frame



the equilibrium equations obtained for free body *ABC* are necessary condition for the equilibrium of a nonrigid structure, but not sufficient condition statically determinate and rigid : unknowns = equations, all unknowns may be determined and all the equations satisfied under general loading condition

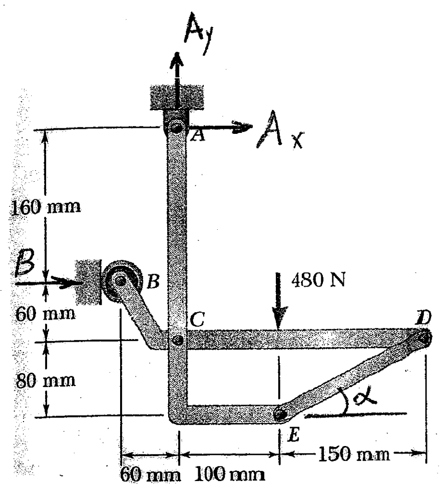
statically indeterminate : unknowns > equations

nonrigid : unknowns < equations

statically indeterminate and nonrigid : unknowns ≧ equations, improper arrangement of members and supports

Sample Problem 6.4

determine F_{DE} and the internal forces at C on member BCD



$\Sigma F_y = 0 \quad A_y = 480\text{ N} \uparrow$

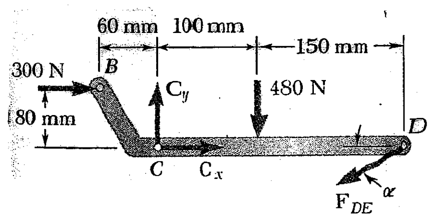
$\Sigma M_A = 0 \quad 480 \times 100 = B \times 160$

$B = 300\text{ N} \rightarrow$

$\Sigma F_x = 0 \quad A_x = 300\text{ N} \leftarrow$

take free body on member BCD

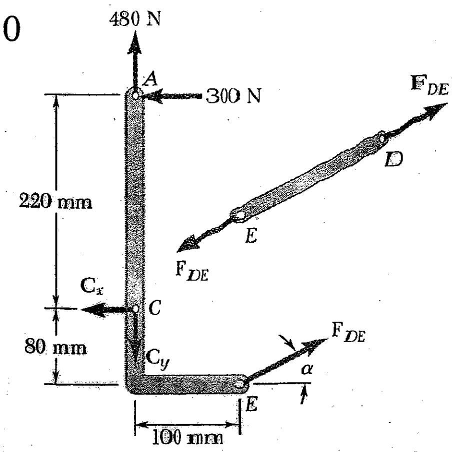
(DE is a two-force member)



$\Sigma M_C = 0 \qquad (\alpha = 28.07°)$

$300 \times 60 + 480 \times 100 + F_{DE} \sin \alpha \times 250 = 0$

$F_{DE} = -561\text{ N} = 561\text{ N (C)}$



$\Sigma F_x = 0 \quad C_x - F_{DE} \cos \alpha + 300 = 0$

$C_x = -795\text{ N} = 795\text{ N} \leftarrow$

$\Sigma F_y = 0 \quad C_y - F_{DE} \sin a - 480 = 0$

$C_y = 216\text{ N} = 216\text{ N} \uparrow$

take free body on member ACE to check the result

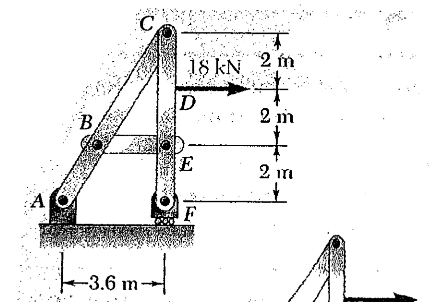
$\Sigma M_A = 0 \quad 795 \times 220 - 561 \times \cos a \times 300 - 561 \times \sin a \times 100 = 0$

(O. K.)

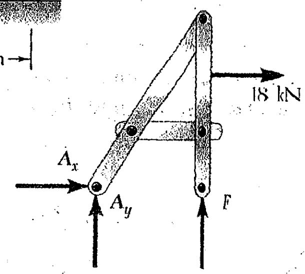
$\Sigma F_x = 0 \quad \Sigma F_y = 0$ (O. K.)

Sample Problem 6.5

determine all the reaction at the pins and supports



free body for entire frame



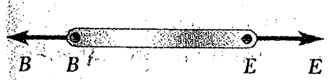
$\Sigma F_x = 0 \quad A_x + 18 = 0 \qquad A_x = -18 \text{ kN} \leftarrow$

$\Sigma M_F = 0 \quad -A_y \times 3.6 - 18 \times 4 = 0$

$A_y = -20 \text{ kN} \downarrow$

$\Sigma F_y = 0 \quad A_y + F = 0 \qquad A_y = 20 \text{ kN} \uparrow$

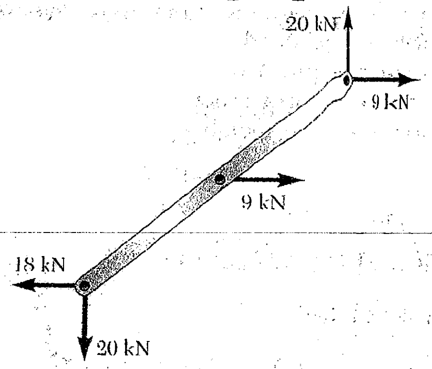
free body for member *BE*



$\Sigma F_x = 0 \quad -B + E = 0 \qquad B = E$

$C_y = 3600 \text{ N}$

free body for member *ABC*



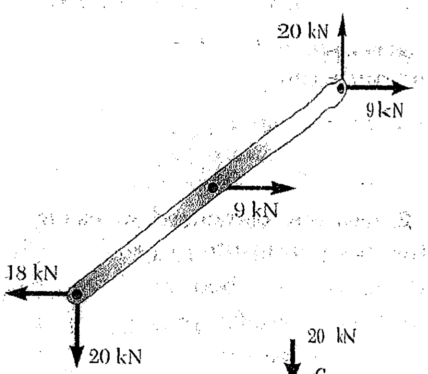
$\Sigma F_y = 0 \quad -20 + C_y = 0 \qquad C_y = 20 \text{ kN} \uparrow$

$\Sigma M_C = 0 \quad B \times 4 + 20 \times 3.6 - 18 \times 6 = 0$

$B = 9 \text{ kN} \uparrow$

$\Sigma F_x = 0 \quad -18 + 9 + C_x = 0$

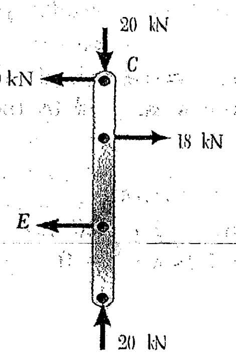
$C_x = 9 \text{ kN} \rightarrow$



from free body of *BE*

$B = E \qquad E = 9 \text{ kN}$

free body for member *CEF*

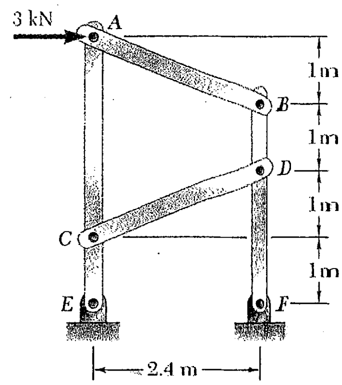


$\Sigma F_x = 0 \quad -9 - 9 + 18 = 0$ (OK)

$\Sigma F_y = 0 \quad -20 + 20 = 0$ (OK)

Sample Problem 6.6

determine the forces acting on two vertical members



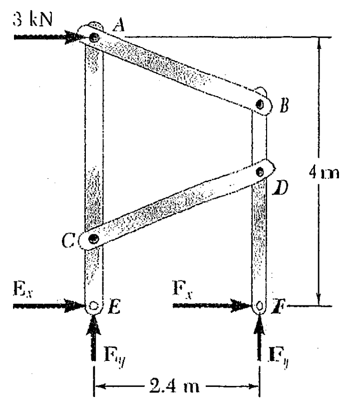
free body for entire frame

$\Sigma M_E = 0 \quad F_y \times 2.4 = 3 \times$

$F_y = 5 \text{ kN} \uparrow$

$\Sigma F_y = 0 \quad E_y = 5 \text{ kN} \downarrow$

$\Sigma F_x = 0 \quad E_x + F_x + 3 = 0$



AD and *CD* are two force members, assume that the 600 N force is applied at *A* belong to member *ACE*

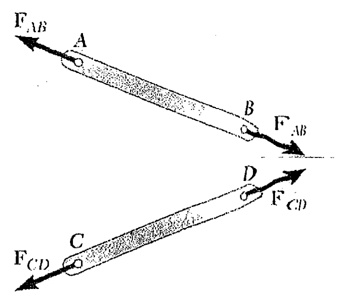
free body for member *ACE*

$\Sigma F_y = 0 \quad -F_{AB}(5/13) + F_{CD}(5/13) - 5 = 0$

$\Sigma M_E = 0 \quad -3 \times 4 - F_{AB}(12/13) \times 4$

$- F_{CD}(12/13) \times 1 = 0$

$F_{AB} = -5.2 \text{ kN} \qquad F_{CD} = 7.8 \text{ kN}$



$\Sigma F_x = 0 \quad 3 + (-5.2)(12/13) + 7.8(12/13)$

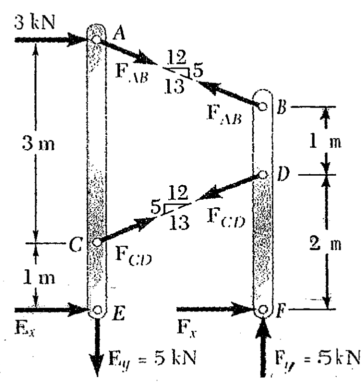
$+ E_x = 0$

$E_x = -5.4 \text{ kN} \leftarrow$

return to the entire frame

$\Sigma F_x = 0 \quad -5.4 + F_x + 3 = 0$

$F_x = 2.4 \text{ kN} \rightarrow$

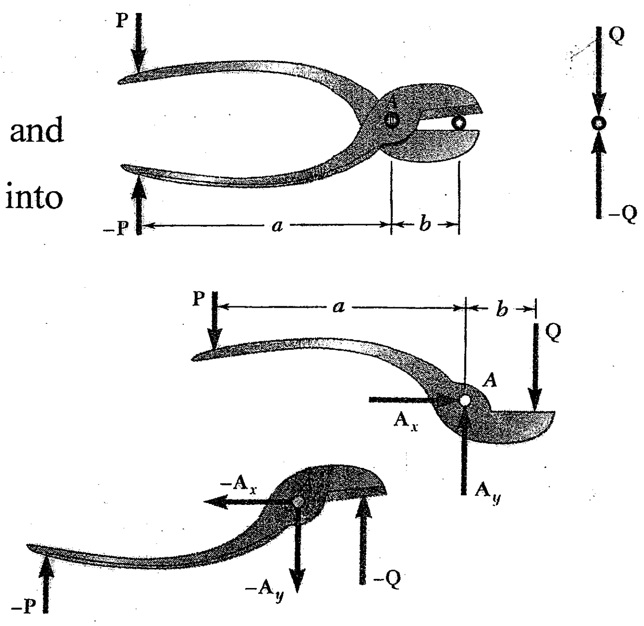


use free body for member *BDF* to check

$\Sigma M_B = -F_{CD}(12/13) \times 1 + F_x \times 3$

$= -7.8 \times (12/13) \times 1 + 2.4 \times 3 = 0$ (O. K.)

6.12 Machines



machines are designed to transmit and modify forces, transform input forces into output forces

consider a pair of cutting pliers

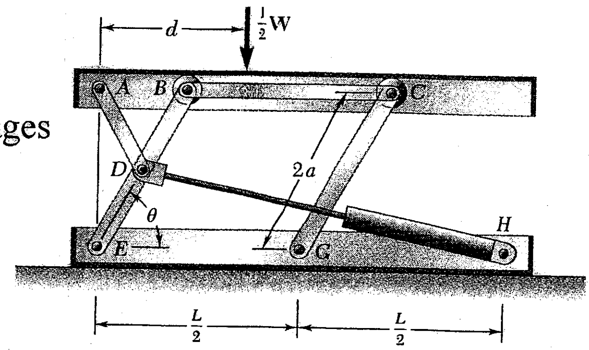
$\Sigma M_A = 0 \qquad P\,a = Q\,b$

$Q = P\,a\,/\,b$

also $A_x = 0$

$A_y = P + Q$

Sample Problem 6.7

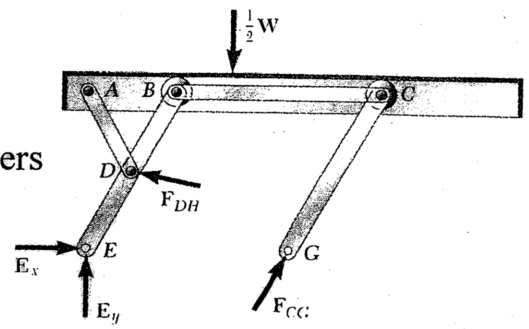


a crate is supported by two identical linkages

$W = 1000 \times 9.81 = 9.81$ kN

$a = 0.7$ m $\qquad L = 3.2$ m $\qquad \theta = 60^o$

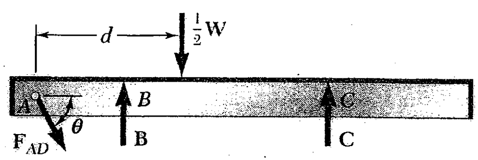
determine the force exerted by each cylinder



for the free body considered

AD, *BC*, *CG* and *DH* are two force members

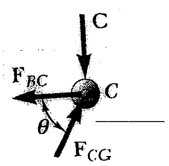
on platform *ABC*



$\Sigma F_x = 0 \quad F_{AD} = 0$

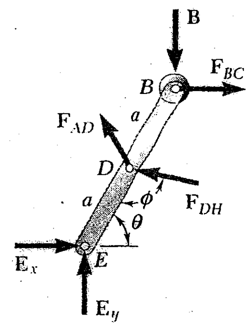
$\Sigma F_y = 0 \quad B + C = \frac{1}{2} W$

free body on roller *C*



$F_{BC} = C \cot\theta$

free body on member *BDE*



$$\Sigma M_E = 0 \quad F_{DH} \cos(\phi - 90^\circ)\, a \; - \; B \times 2a \cos\theta$$

$$- \; F_{BC} \times 2a \sin\theta = 0$$

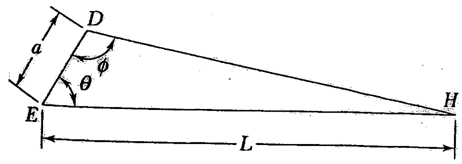
$$F_{DH}\, a \sin\phi \; - \; B \times 2a \cos\theta \; - \; C \cot\theta \times 2a \sin\theta = 0$$

$$F_{DH} \sin\phi \; - \; 2\,(B + C) \cos\theta = 0$$

$$F_{DH} = W \cos\theta / \sin\phi$$

it is shown that F_{DH} is independent of d

by the law of sines for triangle *EDH*



$$\sin\phi / EH = \sin\theta / DH$$

$$\sin\phi = \sin\theta\,(EH / DH)$$

$$(DH)^2 = a^2 + L^2 - 2\,a\,L \cos\theta$$

$$= 0.7^2 + 3.2^2 - 2 \times 0.7 \times 3.2 \cos 60'' = 8.49$$

$$DH = 2.91 \text{ m}$$

$$F_{DH} = W \cos\theta / \sin\phi = W \cot\theta\,(DH / EH)$$

$$= 9.81 \cos 60^\circ\,(2.91 / 3.2) = 5.15 \text{ kN}$$