Simple Stresses

Simple stresses are expressed as the ratio of the applied force divided by the resisting area or

$$\sigma = \text{Force / Area.}$$

It is the expression of force per unit area to structural members that are subjected

to external forces and/or induced forces. Stress is the lead to accurately describe and predict the elastic deformation of a body.

Simple stress can be classified as **normal stress, shear stress, and bearing stress**. Normal stress develops when a force is applied perpendicular to the cross-sectional area of the material. If the force is going to pull the material, the stress is said to be tensile stress and compressive stress develops when the material is being compressed by two opposing forces. Shear stress is developed if the applied force is parallel to the resisting area. Example is the bolt that holds the tension rod in its anchor. Another condition of shearing is when we twist a bar along its longitudinal axis. This type of shearing is called torsion and covered in Chapter 3. Another type of simple stress is the bearing stress, it is the contact pressure between two bodies.

Stress

Stress is the expression of force applied to a unit area of surface. It is measured in psi (English unit) or in MPa (SI unit). Stress is the ratio of force over area.

stress = force / area

Simple Stresses

There are three types of simple stress namely; normal stress, shearing stress, and

bearing stress

Normal Stress

The resisting area is perpendicular to the applied force, thus normal. There are two types of normal stresses; tensile stress and compressive stress. Tensile stress applied to bar tends the bar to elongate while compressive stress tend to shorten the bar.

$$\sigma = \frac{P}{A}$$

Where; P is the applied normal load in Newton and A is the area in mm^2

The maximum stress in tension or compression occurs over a section normal to the load.

Bar in Tension   Bar in Compression

Problem .1

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m^2

Solution 1

$P = \sigma A$

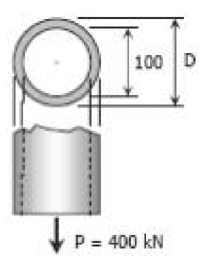
where:

$P = 400\text{ kN} = 400\,000\text{ N}$

$\sigma = 120\text{ MPa}$

$A = \frac{1}{4}\pi D^2 - \frac{1}{4}\pi(100^2)$

$= \frac{1}{4}\pi(D^2 - 10\,000)$



thus,

$$400\ 000 = 120[\tfrac{1}{4}\pi(D^2 - 10\ 000$$

$$400\ 000 = 30\pi D^2 - 300\ 000\pi$$

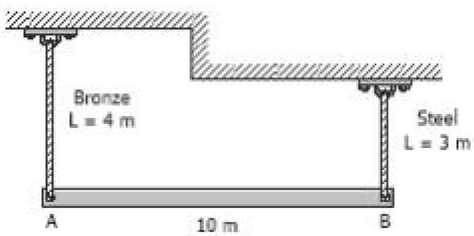
$$D^2 = \frac{400\ 000 + 300\ 000\pi}{30\pi}$$

$$D = \mathbf{119.35\ mm}$$

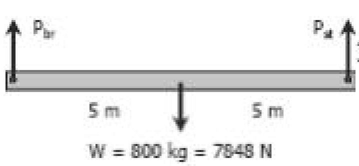
Problem 2

A homogeneous 800 kg bar AB is supported at either end by a cable as shown in Fig. P-105. Calculate the smallest area of each cable if the stress is not to exceed

90 MPa in bronze and 120 MPa in steel.



Solution



By symmetry:

$$P_{br} = P_{st} = \tfrac{1}{2}(7848)$$
$$= 3924\ \text{N}$$

For bronze cable:

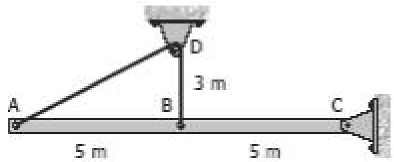
$$P_{br} = \sigma_{br} A_{br}$$
$$3924 = 90A_{br}$$
$$A_{br} = \mathbf{43.6\ mm^2}$$

For steel cable:

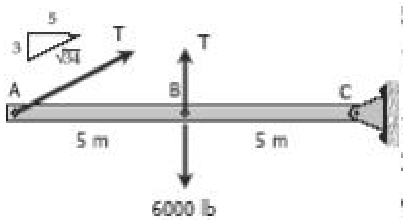
$$P_{st} = \sigma_{st} A_{st}$$
$$3924 = 120A_{st}$$
$$A_{st} = \mathbf{32.7\ mm^2}$$

Problem 3

The homogeneous bar shown in Fig. 2 is supported by a smooth pin at C and a cable that runs from A to B around the smooth peg at D. Find the stress in the cable if its diameter is 0.6 inch and the bar weighs 6000 lb.



Solution



$\Sigma M_C = 0$

$5T + 10(\frac{3}{\sqrt{34}}T) = 5(6000)$

$T = 2957.13$ lb

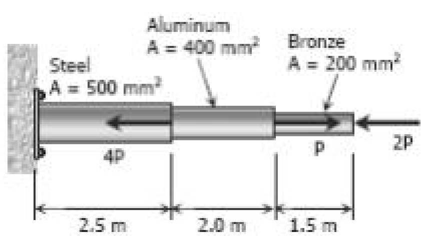
$T = \sigma A$

$2957.13 = \sigma[\frac{1}{4}\pi(0.6^2)]$

$\sigma = 10\,458.72$ **psi**

Problem 4

An aluminum rod is rigidly attached between a steel rod and a bronze rod as shown in Fig.3. Axial loads are applied at the positions indicated. Find the maximum value of P that will not exceed a stress in steel of 140 MPa, in aluminum of 90 MPa, or in bronze of 100 MPa.

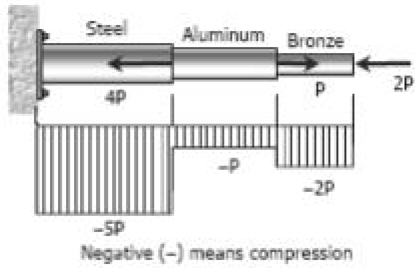


Solution

For bronze:
$\sigma_{br} A_{br} = 2P$
$100(200) = 2P$
$P = 10\,000$ N

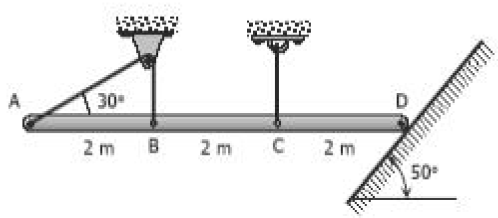
For aluminum:
$\sigma_{al} A_{al} = P$
$90(400) = P$
$P = 36\,000$ N
For Steel:
$\sigma_{st} A_{st} = 5P$
$P = 14\,000$ N

For safe P, use $P = 10\,000$ N = **10 kN**

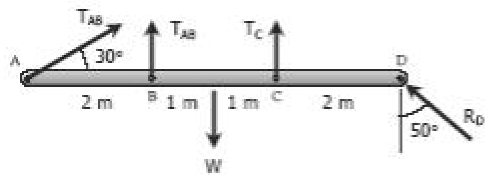


Problem 5

The homogeneous bar ABCD shown in Fig. 5 is supported by a cable that runs from A to B around the smooth peg at E, a vertical cable at C, and a smooth inclined surface at D. Determine the mass of the heaviest bar that can be supported if the stress in each cable is limited to 100 MPa. The area of the cable AB is 250 mm2 and that of the cable at C is 300 mm2



Solution



$\Sigma F_H = 0$
$T_{AB} \cos 30° = R_D \sin 50°$
$R_D = 1.1305T_{AB}$

$\Sigma F_V = 0$
$T_{AB} \sin 30° + T_{AB} + T_C + R_D \cos 50° = W$
$T_{AB} \sin 30° + T_{AB} + T_C + (1.1305T_{AB}) \cos 50° = W$
$2.2267T_{AB} + T_C = W$
$T_C = W - 2.2267T_{AB}$

$\Sigma M_D = 0$
$6(T_{AB} \sin 30°) + 4T_{AB} + 2T_C = 3W$
$7T_{AB} + 2(W - 2.2267T_{AB}) = 3W$
$2.5466T_{AB} = W$
$T_{AB} = 0.3927W$

$T_C = W - 2.2267T_{AB}$
$= W - 2.2267(0.3927W)$
$= 0.1256W$

Based on cable AB:
$T_{AB} = \sigma_{AB} A_{AB}$
$0.3927W = 100(250)$
$W = 63\ 661.83$ N

Based on cable at C:
$T_2 = \sigma_C A_C$
$0.1256W = 100(300)$
$W = 238\ 853.50$ N

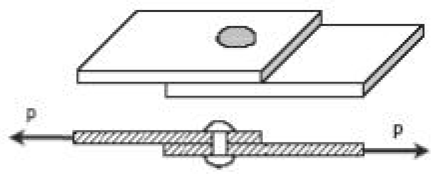
Safe weight $W = 63\ 669.92$ N
$W = mg$
$63\ 669.92 = m(9.81)$
$m = 6\ 490$ kg
= 6.49 Mg

Shearing Stress

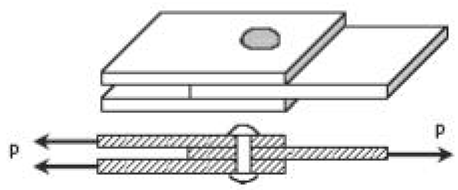
Forces parallel to the area resisting the force cause shearing stress. It differs to tensile and compressive stresses, which are caused by forces perpendicular to the area on which they act. Shearing stress is also known as tangential stress.

$$\tau = \frac{V}{A}$$

Where ;V is the resultant shearing force which passes which passes through the centroid of the area A being sheared.



Single Shear



Double Shear

Problem.1

Find the smallest diameter bolt that can be used in the clevis shown in Fig. 1.if P = 400 kN. The shearing strength of the bolt is 300 MPa.

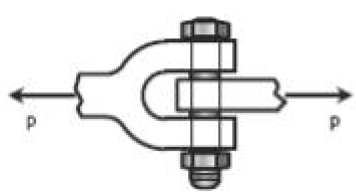
Solution

The bolt is subject to double shear.

$V = \tau A$

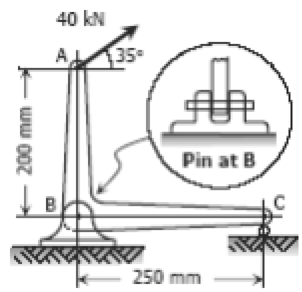
$400(1000) = 300[2(\frac{1}{4}\pi d^2)]$

$d = 29.13$ mm

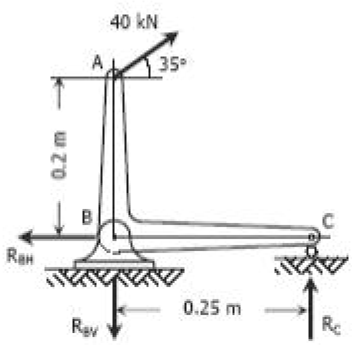


Problem 2

Compute the shearing stress in the pin at B for the member supported as shown in Fig. 2. The pin diameter is 20 mm.



Solution



Free Body Diagram

From the FBD:

$\Sigma M_C = 0$

$0.25 R_{BV} = 0.25(40 \sin 35°) + 0.2(40 \cos 35°)$

$R_{BV} = 49.156$ kN

$\Sigma F_H = 0$

$R_{BH} = 40 \cos 35°$

$= 32.766$ kN

$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$

$= \sqrt{32.766^2 + 49.156^2}$

$= 59.076$ kN → shear force of pin at B

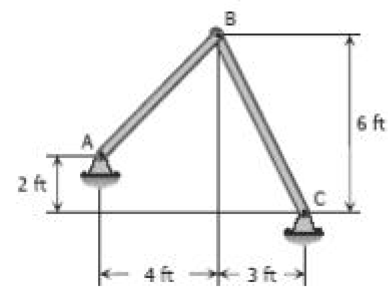
$V_B = \tau_B A$ → double shear

$59.076\,(1000) = \tau_B \{2[\frac{1}{4}\pi(20^2)]\}$

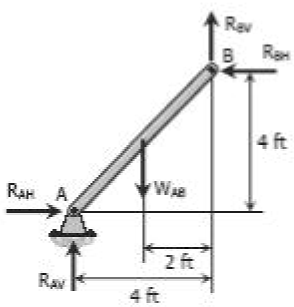
$\tau_B =$ **94.02 MPa**

Problem 3

The members of the structure in Fig. 3 weigh 200 lb/ft. Determine the smallest diameter pin that can be used at A if the shearing stress is limited to 5000 psi. Assume single shear.



Solution



FBD of member

For member *AB*:

Length, $L_{AB} = \sqrt{4^2 + 4^2}$
$= 5.66$ ft

Weight, $W_{AB} = 5.66(200)$
$= 1132$ lb

$\Sigma M_A = 0$

$4R_{BH} + 4R_{BV} = 2W_{AB}$

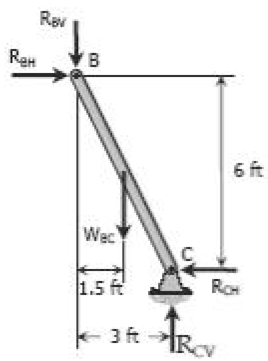
$4R_{BH} + 4R_{BV} = 2(1132)$

$R_{BH} + R_{BV} = 566 \qquad \rightarrow (1)$

For member *BC*:

Length, $L_{BC} = \sqrt{3^2 + 6^2}$
$= 6.71$ ft

Weight, $W_{BC} = 6.71(200)$
$= 1342$ lb



FBD of member BC

$\Sigma M_C = 0$
$6R_{BH} = 1.5W_{BC} + 3R_{BV}$
$6R_{BH} - 3R_{BV} = 1.5(1342)$
$2R_{BH} - R_{BV} = 671 \qquad \rightarrow (2)$

Add equations (1) and (2)
$R_{BH} + R_{BV} = 566 \qquad \rightarrow (1)$
$\underline{2R_{BH} - R_{BV} = 671} \qquad \rightarrow (2)$
$3R_{BH} = 1237$
$R_{BH} = 412.33$ lb

From equation (1):
$412.33 + R_{BV} = 566$
$R_{BV} = 153.67$ lb

From the FBD of member AB
$\Sigma F_H = 0$
$R_{AH} = R_{BH} = 412.33$ lb

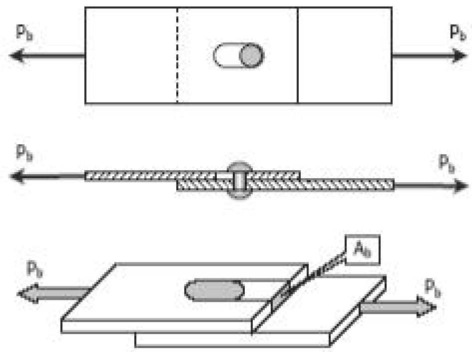
$\Sigma F_V = 0$
$R_{AV} + R_{BV} = W_{AB}$
$R_{AV} + 153.67 = 1132$
$R_{AV} = 978.33$ lb

$R_A = \sqrt{R_{AH}^2 + R_{AV}^2}$
$= \sqrt{412.33^2 + 978.33^2}$
$= 1061.67$ lb $\rightarrow$ shear force of pin at A

$V = \tau A$
$1061.67 = 5000(\frac{1}{4}\pi d^2)$
$d =$ **0.520 in**

Bearing Stress

Bearing stress is the contact pressure between the separate bodies. It differs from compressive stress, as it is an internal stress caused by compressive forces.



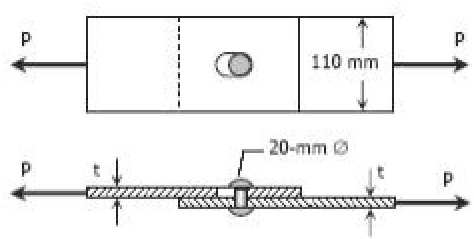
$$\sigma_b = \frac{P_b}{A_b}$$

Problem 1

In Fig. 1, assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine

(a) the minimum thickness of each plate; and

(b) the largest average tensile stress in the plates.



Solution

(a) From shearing of rivet:

$P = \tau A_{rivets}$

$= 60[\frac{1}{4}\pi(20^2)]$

$= 6000\pi$ N

From bearing of plate material:

$P = \sigma_b A_b$

$6000\pi = 120(20t)$

$t = 7.85$ **mm**

(b) Largest average tensile stress in the plate:

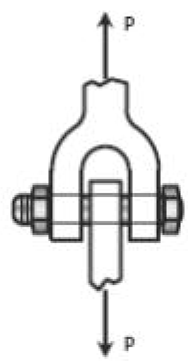
$P = \sigma A$

$6000\pi = \sigma[7.85(110 - 20)]$

$\sigma = 26.67$ **MPa**

Problem 2

In the clevis shown in Fig. 2 find the minimum bolt diameter and the minimum thickness of each yoke that will support a load P = 14 kips without exceeding a shearing stress of 12 ksi and a bearing stress of 20 ksi.



Solution

For shearing of rivets (double shear)

$P = \tau A$

$14 = 12[2(\frac{1}{4}\pi d^2)]$

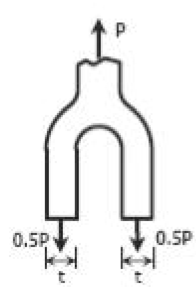
$d = 0.8618$ **in** → diameter of bolt

For bearing of yoke:

$P = \sigma_b A_b$

$14 = 20[2(0.8618t)]$

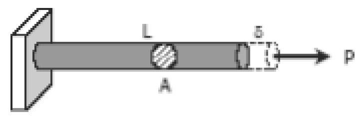
$t = 0.4061$ **in** → thickness of yoke



STRAIN

Simple Strain

Also known as unit deformation, strain is the ratio of the change in length caused by the applied force, to the original length.



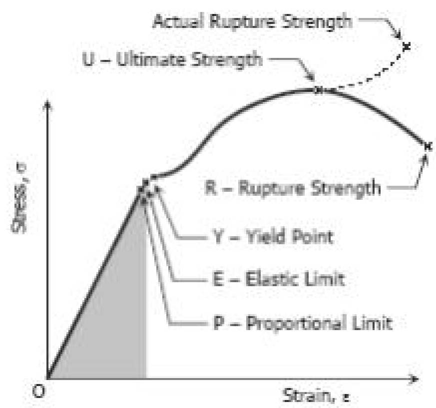
$$\varepsilon = \frac{\delta}{L}$$

Where ; δ is the deformation and L is the original length, thus ε is dimensionless.

Stress-Strain Diagram

Suppose that a metal specimen be placed in tension-compression testing machine. As the axial load is gradually increased in increments, the total elongation over the gage length is measured at each increment of the load and this is continued until failure of the specimen takes place. Knowing the original cross-sectional area and length of the specimen, the normal stress σ and the strain ε can be obtained. The graph of these quantities with the stress σ along the y-axis and the strain ε along the x-axis is called the stress-strain diagram. The stress-strain diagram differs in form for various materials. The diagram shown below is that for a medium carbon structural steel. Metallic engineering materials are classified as either ductile or brittle materials.

A ductile material is one having relatively large tensile strains up to the point of rupture like structural steel and aluminum, whereas brittle materials has a relatively small strain up to the point of rupture like cast iron and concrete. An arbitrary strain of 0.05 mm/mm is frequently taken as the dividing line between these two classes.



PROPORTIONAL LIMIT (HOOKE'S LAW)

From the origin O to the point called proportional limit, the stress-strain curve is a straight line. This linear relation between elongation and the axial force causing was first noticed by Sir Robert Hooke in 1678 and is called Hooke's Law that within the proportional limit, the stress is directly proportional to strain or

$$\sigma \propto \epsilon \ \ or\ \sigma = k\ \epsilon$$

The constant of proportionality k is called the Modulus of Elasticity E or Young's Modulus and is equal to the slope of the stress-strain diagram from O to P. Then

$$\sigma = E\ \epsilon$$

ELASTIC LIMIT

The elastic limit is the limit beyond which the material will no longer go back to its original shape when the load is removed, or it is the maximum stress that may E developed such that there is no permanent or residual deformation when the load is entirely removed.

ELASTIC AND PLASTIC RANGES

The region in stress-strain diagram from O to P is called the elastic range. The region from P to R is called the plastic range.

YIELD POINT

Yield point is the point at which the material will have an appreciable elongation or yielding without any increase in load.

ULTIMATE STRENGTH

The maximum ordinate in the stress-strain diagram is the ultimate strength or tensile strength.

RAPTURE STRENGTH

Rapture strength is the strength of the material at rupture. This is also known as the breaking strength.

MODULUS OF RESILIENCE

Modulus of resilience is the work done on a unit volume of material as the force is gradually increased from O to P, in Nm/m3. This may be calculated as the area under the stress-strain curve from the origin O to up to the elastic limit E (the shaded area in the figure). The resilience of the material is its ability to absorb energy without creating a permanent distortion.

MODULUS OF TOUGHNESS

Modulus of toughness is the work done on a unit volume of material as the force is gradually increased from O to R, in Nm/m3. This may be calculated as the area under the entire stress-strain curve (from O to R). The toughness of a material is its ability to absorb energy without causing it to break.

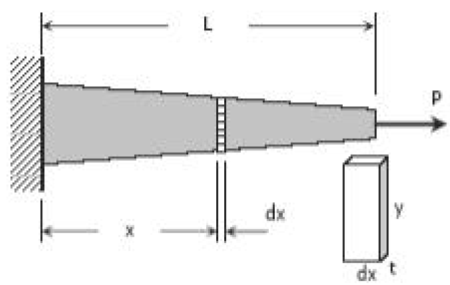
AXIAL DEFORMATION

In the linear portion of the stress-strain diagram, the tress is proportional to strain and is given by

$$\sigma = E \in$$

$$\delta = \frac{P.L}{A.E} = \frac{\sigma.L}{E}$$

To use this formula, the load must be axial, the bar must have a uniform cross-sectional area, and the stress must not exceed the proportional limit. If however, the cross- sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration. If however, the cross-sectional area is not uniform, the axial deformation can be determined by considering a differential length and applying integration.



$$\delta = \frac{P}{E}\int_0^L \frac{dx}{A}$$

where A = ty and y and t, if variable, must be expressed in terms of x. For a rod of unit mass ρ suspended vertically from one end, the total elongation due to its own weight is

$$\delta = \frac{\rho g L^2}{2E} = \frac{MgL}{2AE}$$

where ρ is in kg/m3, L is the length of the rod in mm, M is the total mass of the rod in kg, A is the cross-sectional area of the rod in mm2 , and g = 9.81 m/s2

Problem 1

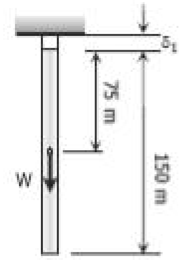
A steel rod having a cross-sectional area of 300 mm2 and a length of 150 m is suspended vertically from one end. It supports a tensile load of 20 kN at the lower end. If the unit mass of steel is 7850 kg/m3 and E = 200 × 103 MN/m2 , find the total elongation of the rod.

Solution

Let δ = total elongation

δ_1 = elongation due to its own weight

δ_2 = elongation due to applied load

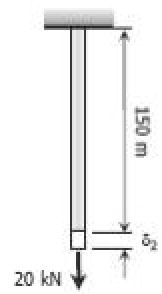


$$\delta = \delta_1 + \delta_2$$

$$\delta_1 = \frac{PL}{AE}$$

Where: $P = W = 7850(1/1000)3(9.81)[300(150)(1000)]$
$P = 3465.3825$ N
$L = 75(1000) = 75\ 000$ mm
$A = 300\ mm^2$
$E = 200\ 000$ MPa

$$\delta_1 = \frac{3465.3825\,(75000)}{300\,(200\,000)} = 4.33\ \text{mm}$$



$$\delta_2 = \frac{PL}{AE}$$

Where: $P = 20$ kN $= 20\ 000$ N
$L = 150$ m $= 150\ 000$ mm
$A = 300\ mm^2$
$E = 200\ 000$ MPa

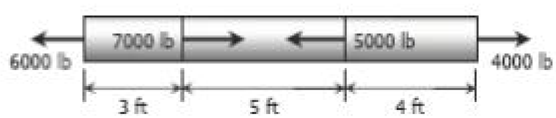
$$\delta_2 = \frac{20000(150000)}{300(200000)} = 50\ \text{mm}$$

Total elongation:

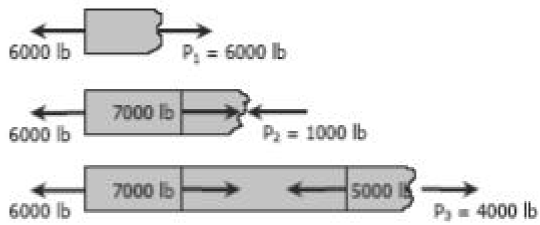
$\delta = 4.33 + 50 = 54.33$ mm

Problem 2

An aluminum bar having a cross-sectional area of 0.5 in2 carries the axial loads applied at the positions shown in Fig. 2. Compute the total change in length of the bar if E = 10 × 106 psi. Assume the bar is suitably braced to prevent lateral buckling.



Solution 209



$P_1 = 6000$ lb tension
$P_2 = 1000$ lb compression
$P_3 = 4000$ lb tension

$$\delta = \frac{PL}{AE}$$

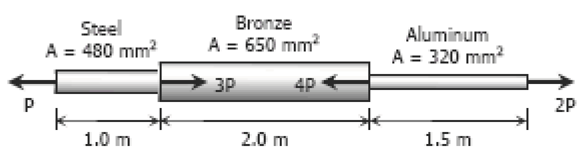
$$\delta = \delta_1 - \delta_2 + \delta_3$$

$$\delta = \frac{6000(3\times12)}{0.5(10\times10^6)} - \frac{1000(5\times12)}{0.5(10\times10^6)} + \frac{4000(4\times12)}{0.5(10\times10^6)}$$

$\delta = 0.0696$ **in (lengthening)**

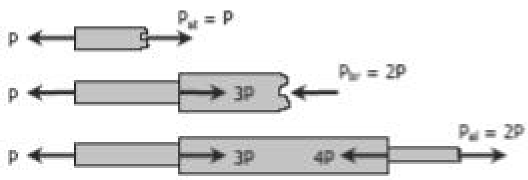
Problem 3

A bronze bar is fastened between a steel bar and an aluminum bar as shown in Fig.3. Axial loads are applied at the positions indicated. Find the largest value of P that will not exceed an overall deformation of 3.0 mm, or the following stresses: 140 MPa in the steel, 120 MPa in the bronze, and 80 MPa in the aluminum. Assume that the assembly is suitably braced to prevent buckling. Use Est = 200 GPa, Eal = 70 GPa, and Ebr = 83 GPa.



Solution

Based on allowable stresses:



Steel:

$P_{st} = \sigma_{st}A_{st}$

$P = 140(480) = 67\ 200$ N

$P = 67.2$ kN

Bronze:

$P_{br} = \sigma_{br}A_{br}$

$2P = 120(650) = 78\ 000$

$P = 39\ 000$ N $= 39$ kN

Aluminum:

$P_{al} = \sigma_{al}A_{al}$

$2P = 80(320) = 25\ 600$ N

$P = 12\ 800$ N $= 12.8$ kN

Based on allowable deformation:
(steel and aluminum lengthens, bronze shortens)

$$\delta = \delta_{st} - \delta_{br} + \delta_{al}$$

$$3 = \frac{P(1000)}{480(200\,000)} - \frac{2P(2000)}{650(70\,000)} + \frac{2P(1500)}{320(83\,000)}$$

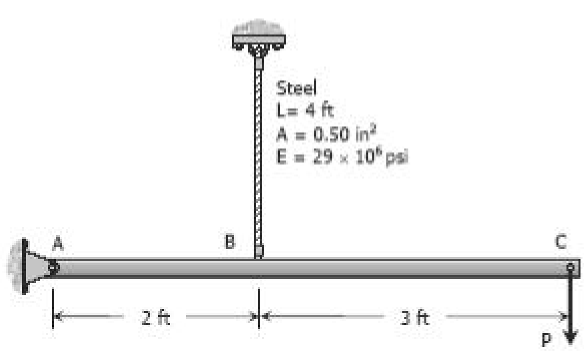
$$3 = \left(\tfrac{1}{96000} - \tfrac{1}{11375} + \tfrac{3}{26560}\right)P$$

$$P = 84\ 610.99 \text{ N} = 84.61 \text{ kN}$$

Use the smallest value of P, $P =$ **12.8 kN**

Problem 4

The rigid bar ABC shown in Fig. 4 is hinged at A and supported by a steel rod at B. Determine the largest load P that can be applied at C if the stress in the steel rod is limited to 30 ksi and the vertical movement of end C must not exceed 0.10 in.



Solution

Based on maximum stress of steel rod:

$\Sigma M_A = 0$

$5P = 2P_{st}$

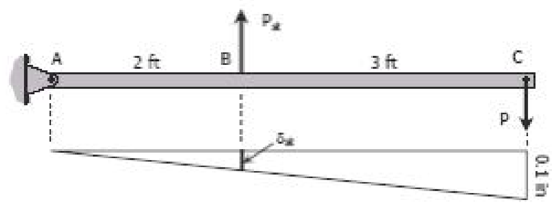
$P = 0.4P_{st}$

$P = 0.4\sigma_{st}A_{st}$

$P = 0.4[30(0.50)]$

$P = 6$ kips

Free body and deformation diagrams:



Based on movement at C:

$$\frac{\delta_{st}}{2} = \frac{0.1}{5}$$

$\delta_{st} = 0.04$ in

$$\frac{P_{st}L}{AE} = 0.04$$

$$\frac{P_{st}(4\times12)}{0.50(29\times10^6)} = 0.04$$

$P_{st} = 12\,083.33$ lb

$\Sigma M_A = 0$

$5P = 2P_{st}$

$P = 0.4P_{st}$

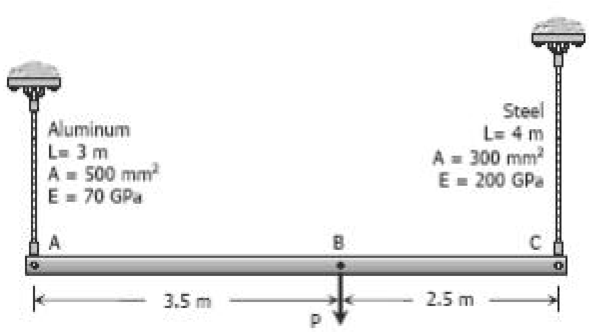
$P = 0.4(12\,083.33)$

$P = 4833.33$ lb $= 4.83$ kips

Use the smaller value, P = **4.83 kips**

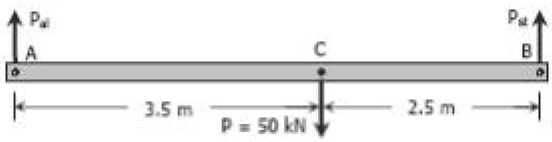
Problem 5

The rigid bar AB, attached to two vertical rods as shown in Fig. 5, is horizontal before the load P is applied. Determine the vertical movement of P if its magnitude is 50 kN.



Solution

Free body diagram:



For aluminum:

$[\Sigma M_B = 0]$ $\quad 6P_{al} = 2.5(50)$

$P_{al} = 20.83$ kN

$$\left[\delta = \frac{PL}{AE}\right]_{al} \qquad \delta_{al} = \frac{20.83(3)1000^2}{500(70000)}$$

$\delta_{al} = 1.78$ mm

For steel:

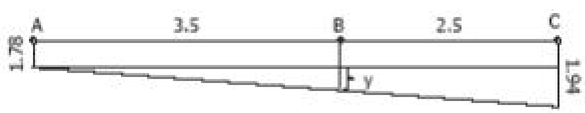
$[\Sigma M_A = 0]$ $\quad 6P_{st} = 3.5(50)$

$P_{st} = 29.17$ kN

$$\left[\delta = \frac{PL}{AE}\right]_{st} \qquad \delta_{st} = \frac{29.17(4)1000^2}{300(200000)}$$

$\delta_{st} = 1.94$ mm

Movement diagram:



$$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$$

$y = 0.09$ mm

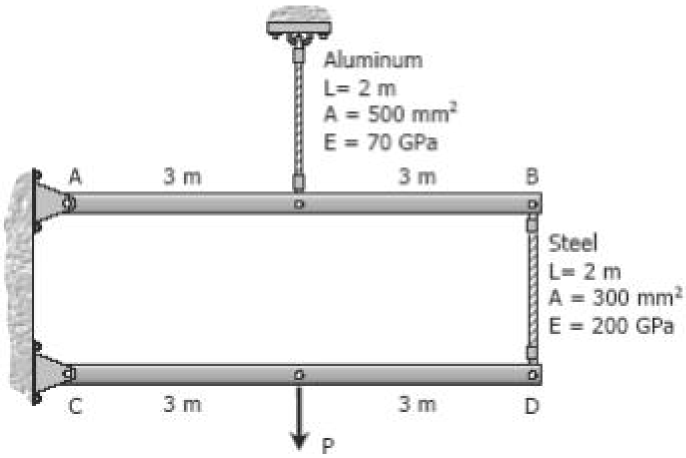
δ_B = vertical movement of P

$\delta_B = 1.78 + y = 1.78 + 0.09$

$\delta_B =$ **1.87 mm**

Problem 6

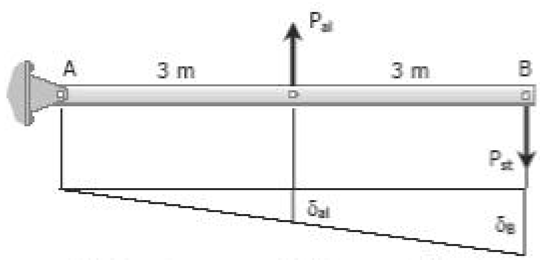
The rigid bars AB and CD shown in Fig. 6 are supported by pins at A and C and the two rods. Determine the maximum force P that can be applied as shown if its vertical movement is limited to 5 mm. Neglect the weights of all members.



Solution

$$[\Sigma M_A = 0] \qquad 3P_{al} = 6P_{st}$$

$$P_{al} = 2P_{st}$$



FBD and movement diagram of bar AB

By ratio and proportion:

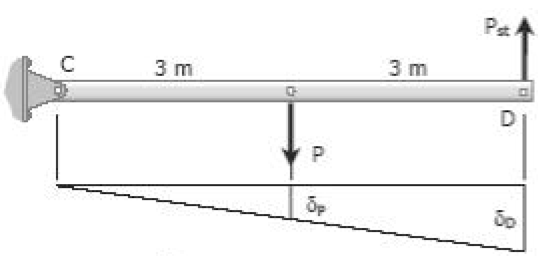
$$\frac{\delta_B}{6} = \frac{\delta_{al}}{3}$$

$$\delta_B = 2\delta_{al} = 2\left[\frac{PL}{AE}\right]_{al}$$

$$\delta_B = 2\left[\frac{P_{al}(2000)}{500(70\,000)}\right]$$

$$\delta_B = \frac{1}{8750}\,P_{al} = \frac{1}{8750}\,(2P_{st})$$

$$\delta_B = \frac{1}{4375}\,P_{st} \rightarrow \text{movement of B}$$



FBD and movement diagram of bar CD

Movement of D:

$$\delta_D = \delta_{st} + \delta_B = \left[\frac{PL}{AE}\right]_{st} + \frac{1}{4375}\,P_{st}$$

$$\delta_D = \frac{P_{st}(2000)}{300(200\,000)} + \frac{1}{4375}\,P_{st}$$

$$\delta_D = \frac{11}{42\,000}\,P_{st}$$

$[\Sigma M_C = 0]$ $\qquad 6P_{st} = 3P$

$$P_{st} = \tfrac{1}{2} P$$

By ratio and proportion:

$$\frac{\delta_P}{3} = \frac{\delta_D}{6}$$

$$\delta_P = \tfrac{1}{2}\,\delta_D = \tfrac{1}{2}\left(\tfrac{11}{42000} P_{st}\right)$$

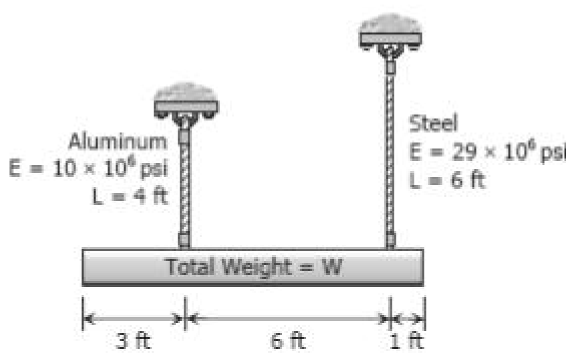
$$\delta_P = \tfrac{11}{84000} P_{st}$$

$$5 = \tfrac{11}{84000}\left(\tfrac{1}{2} P\right)$$

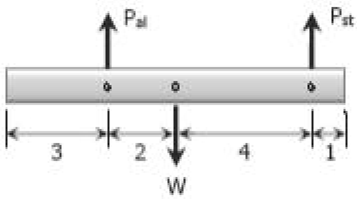
$P = 76\,363.64$ N = **76.4 kN**

Problem 7

A uniform concrete slab of total weight W is to be attached, as shown in Fig. 7, to two rods whose lower ends are on the same level. Determine the ratio of the areas of the rods so that the slab will remain level



Solution



$$[\Sigma M_{al} = 0] \qquad 6P_{st} = 2W$$

$$P_{st} = \tfrac{1}{3}W$$

$$[\Sigma M_{st} = 0] \qquad 6P_{al} = 4W$$

$$P_{al} = \tfrac{2}{3}W$$

$$\delta_{st} = \delta_{al}$$

$$\left[\frac{PL}{AE}\right]_{st} = \left[\frac{PL}{AE}\right]_{al}$$

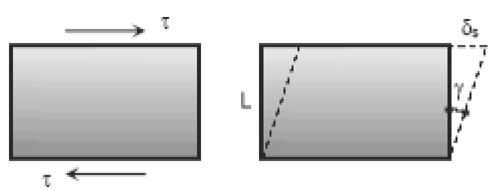
$$\frac{\frac{1}{3}W(6\times 12)}{A_{st}(29\times 10^6)} = \frac{\frac{2}{3}W(4\times 12)}{A_{al}(10\times 10^6)}$$

$$\frac{A_{al}}{A_{st}} = \frac{\frac{2}{3}W(4\times 12)(29\times 10^6)}{\frac{1}{3}W(6\times 12)(10\times 10^6)}$$

$$A_{al}/A_{st} = 3.867$$

Shearing Deformation

Shearing forces cause shearing deformation. An element subject to shear does not change in length but undergoes a change in shape.



The change in angle at the corner of an original rectangular element is called the shear strain and is expressed as

$$\gamma = \frac{\delta_s}{L}$$

The ratio of the shear stress τ and the shear strain γ is called the modulus of elasticity in shear or modulus of rigidity and is denoted as G, in MPa.

$$\gamma = \frac{\tau}{\gamma}$$

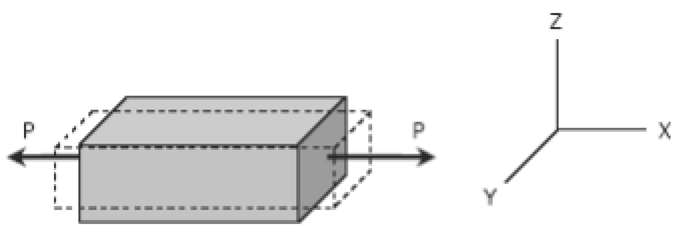
The relationship between the shearing deformation and the applied shearing force is

$$\delta_s = \frac{VL}{AG} = \frac{\tau L}{G}$$

Where; V is the shearing force acting over an area A.

Poisson's Ratio

When a bar is subjected to a tensile loading there is an increase in length of the bar in the direction of the applied load, but there is also a decrease in a lateral dimension perpendicular to the load. The ratio of the sidewise deformation (or strain) to the longitudinal deformation (or strain) is called the Poisson's ratio and is denoted by ν. For most steel, it lies in the range of 0.25 to 0.3, and 0.20 for concrete.



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

where ε_x is strain in the x-direction and ε_y and ε_z are the strains in the perpendicular direction. The negative sign indicates a decrease in the transverse dimension when ε_x is positive.

BIAXIAL DEFORMATION

If an element is subjected simultaneously by ensile stresses, σx and σy, in the x and y directions, the strain in the x-direction is σx / E and the strain in the y direction is σy / E. Simultaneously, the stress in the y direction will produce a lateral contraction on the xx direction of the amount -ν εy or -ν σy/E. The resulting strain in the x direction will be

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \text{ or } \sigma_x = \frac{(\varepsilon_x + \nu\varepsilon_y)E}{1-\nu^2}$$

and

$$\varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} \text{ or } \sigma_y = \frac{(\varepsilon_y + \nu\varepsilon_x)E}{1-\nu^2}$$

TRIAXIAL DEFORMATION

If an element is subjected simultaneously by three mutually perpendicular normal stresses σ_x, σ_y, and σ_z, which are accompanied by strains ε_x, ε_y, and ε_z, respectively,

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Tensile stresses and elongation are taken as positive. Compressive stresses and contraction are taken as negative.

Relationship Between E, G, and ν

The relationship between modulus of elasticity E, shear modulus G and Poisson's ratio ν is:

$$G = \frac{E}{2(1+\nu)}$$

Bulk Modulus of Elasticity or Modulus of Volume Expansion, K

The bulk modulus of elasticity K is a measure of a resistance of a material to change in volume without change in shape or form. It is given as

$$K = \frac{E}{3(1-2\nu)} = \frac{\sigma}{\Delta V / V}$$

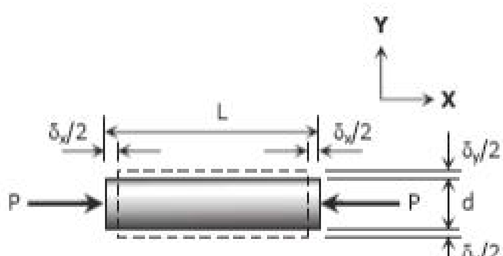
$$\frac{\Delta V}{V} = \frac{\sigma}{K} = \frac{3(1-2\nu)}{E}$$

Where V is the volume and ΔV is change in volume. The ratio ΔV / V is called volumetric strain and can be expressed as

Problem 1

A solid cylinder of diameter d carries an axial load P. Show that its change in diameter is 4Pν / πEd.

Solution



The load P can be compressive or tensile

$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$

$$\varepsilon_y = -\nu\varepsilon_x$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E}$$

$$\frac{\delta_y}{d} = -\nu\frac{-P}{AE}$$

$$\delta_y = \nu\frac{Pd}{\frac{1}{4}\pi d^2 E}$$

$$\delta_y = \frac{4P\nu}{\pi E d} \qquad ok!$$

Problem 2

A welded steel cylindrical drum made of a 10-mm plate has an internal diameter of 1.20 m. Compute the change in diameter that would be caused by an internal pressure of 1.5 MPa. Assume that Poisson's ratio is 0.30 and E = 200 GPa.

Solution

σ_y = longitudinal stress

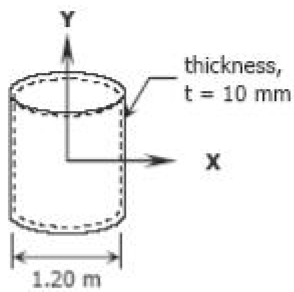
$$\sigma_y = \frac{pD}{4t} = \frac{1.5(1200)}{4(10)}$$

$\sigma_y = 45$ MPa

σ_x = tangential stress

$$\sigma_x = \frac{pD}{2t} = \frac{1.5(1200)}{2(10)}$$

$\sigma_x = 90$ MPa



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

$$\varepsilon_x = \frac{90}{200000} - 0.3\left(\frac{45}{200000}\right)$$

$\varepsilon_x = 3.825 \times 10^{-4}$

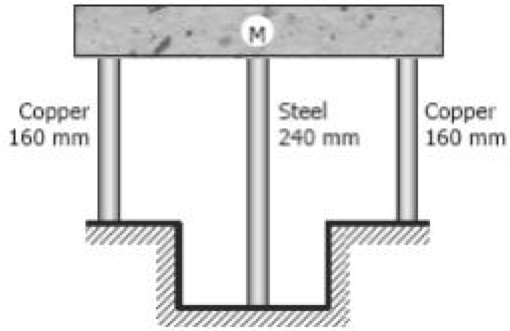
$$\varepsilon_x = \frac{\Delta D}{D}$$

$\Delta D = \varepsilon_x D = (3.825 \times 10^{-4})(1200)$

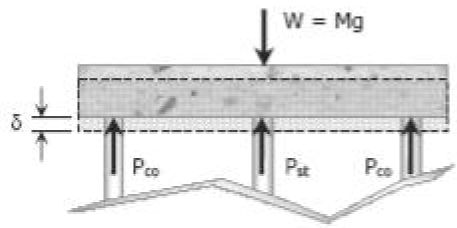
$\Delta D =$ **0.459 mm**

Problem 3

A rigid block of mass M is supported by three symmetrically spaced rods as shown in figure. Each copper rod has an area of 900 mm2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm2 ; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass M which can be supported.



Solution



$$\delta_{co} = \delta_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

$$\frac{\sigma_{co}(160)}{120000} = \frac{\sigma_{st}(240)}{200000}$$

$$10\sigma_{co} = 9\sigma_{st}$$

When $\sigma_{st} = 140$ MPa

$$\sigma_{co} = \tfrac{9}{10}(140)$$

$$\sigma_{co} = 126 \text{ MPa} > 70 \text{ MPa (not ok!)}$$

When $\sigma_{co} = 70$ MPa

$$\sigma_{st} = \tfrac{10}{9}(70)$$

$$\sigma_{st} = 77.78 \text{ MPa} < 140 \text{ MPa (ok!)}$$

Use $\sigma_{co} = 70$ MPa and $\sigma_{st} = 77.78$ MPa

$$\Sigma Fv = 0$$

$$2P_{co} + P_{st} = W$$

$$2(\sigma_{co}A_{co}) + \sigma_{st}A_{st} = Mg$$

$$2[70(900)] + 77.78(1200) = M(9.81)$$

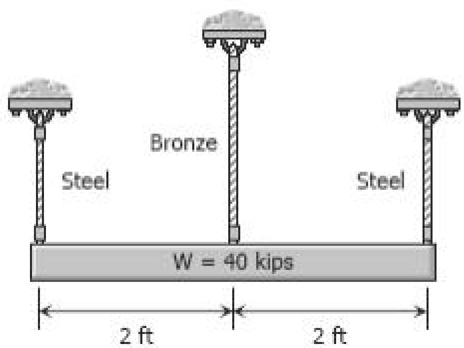
$$M = 22\,358.4 \text{ kg}$$

Problem 4

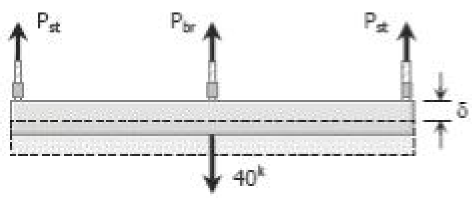
The lower ends of the three bars in Figure are at the same level before the uniform rigid block weighing 40 kips is attached. Each steel bar has a length of 3 ft, and area of 1.0 in.2, and E = 29 × 106 psi. For the bronze bar, the area is 1.5 in.2 and E = 12 × 106 psi. Determine

(a) the length of the bronze bar so that the load on each steel bar is twice the load on the bronze bar, and

(b) the length of the bronze that will make the steel stress twice the bronze stress.



Solution



(a) Condition: $P_{st} = 2P_{br}$

$\sum F_V = 0$

$2P_{st} + P_{br} = 40$

$2(2P_{br}) + P_{br} = 40$

$P_{br} = 8$ kips

$P_{st} = 2(8) = 16$ kips

$\delta_{br} = \delta_{st}$

$$\left(\frac{PL}{AE}\right)_{br} = \left(\frac{PL}{AE}\right)_{st}$$

$$\frac{8000L_{br}}{1.5(12\times10^6)} = \frac{16\,000(3\times12)}{1.0(29\times10^6)}$$

$L_{br} = 44.69$ in

$L_{br} = \mathbf{3.72\ ft}$

(b) Condition: $\sigma_{st} = 2\sigma_{br}$

$\sum F_V = 0$

$2P_{st} + P_{br} = 40$

$2(\sigma_{st}A_{st}) + \sigma_{br}A_{br} = 40$

$2[(2\sigma_{br})A_{st}] + \sigma_{br}A_{br} = 40$

$4\sigma_{br}\,(1.0) + \sigma_{br}\,(1.5) = 40$

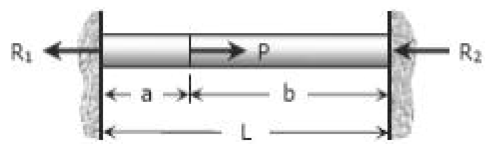
$\sigma_{br} = 7.27$ ksi

$\sigma_{st} = 2(7.27) = 14.54$ ksi

$$\delta_{br} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{7.27(1000)L_{br}}{12\times10^6} = \frac{14.54(1000)(3\times12)}{29\times10^6}$$
$$L_{br} = 29.79 \text{ in}$$
$$L_{br} = \mathbf{2.48 \text{ ft}}$$

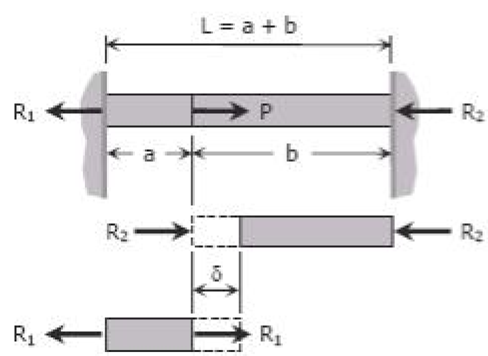
Problem 5

A homogeneous rod of constant cross section is attached to unyielding supports. It carries an axial load P applied as shown in Figure. Prove that the reactions are given by R1 = Pb/L and R2 = Pa/L.



Solution

$$\sum F_H = 0$$
$$R_1 + R_2 = P$$
$$R_2 = P - R_1$$

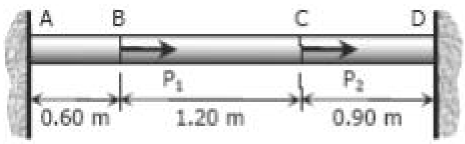


$$\delta_1 = \delta_2 = \delta$$
$$\left(\frac{PL}{AE}\right)_1 = \left(\frac{PL}{AE}\right)_2$$
$$\frac{R_1 a}{AE} = \frac{R_2 b}{AE}$$
$$R_1 a = R_2 b$$
$$R_1 a = (P - R_1)b$$
$$R_1 a = Pb - R_1 b$$
$$R_1(a + b) = Pb$$
$$R_1 L = Pb$$
$$R_1 = Pb/L \qquad \text{ok!}$$

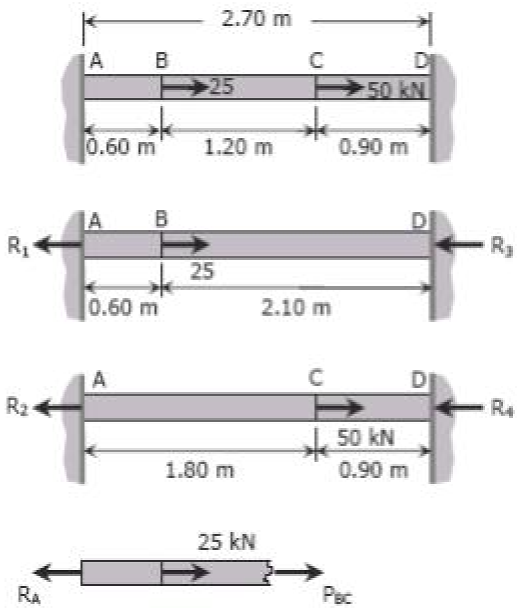
$$R_2 = P - Pb/L$$
$$R_2 = \frac{P(L-b)}{L}$$
$$R_2 = Pa/L \qquad \textit{ok!}$$

Problem 6

A homogeneous bar with a cross sectional area of 500 mm2 is attached to rigid supports. It carries the axial loads P1 = 25 kN and P2 = 50 kN, applied as shown in Figure. Determine the stress in segment BC, and compute the reactions caused by P1 and P2 acting separately. Then use the principle of superposition to compute the reactions when both loads are applied.)



Solution



$R_1 = 25(2.10)/2.70$
$R_1 = 19.44$ kN

$R_2 = 50(0.90)/2.70$
$R_2 = 16.67$ kN

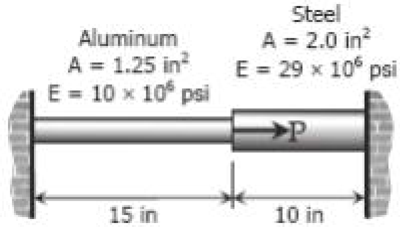
$R_A = R_1 + R_2$
$R_A = 19.44 + 16.67$
$R_A = 36.11$ kN

For segment BC
$P_{BC} + 25 = R_A$
$P_{BC} + 25 = 36.11$
$P_{BC} = 11.11$ kN

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{11.11(1000)}{500}$$
$\sigma_{BC} = 22.22$ MPa

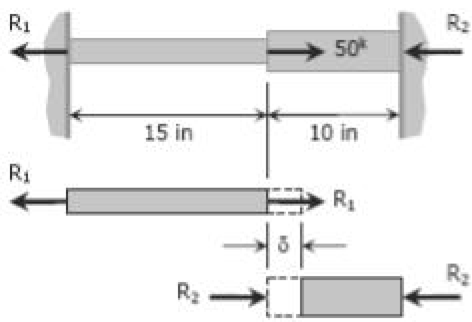
Problem 7

The composite bar in Figure is firmly attached to unyielding supports. Compute the stress in each material caused by the application of the axial load P = 50 kips.



Solution

$$\sum F_H = 0$$
$$R_1 + R_2 = 50\,000$$
$$R_1 = 50\,000 - R_2$$



$$\delta_{al} = \delta_{st} = \delta$$
$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st}$$
$$\frac{R_1(15)}{1.25(10\times10^6)} = \frac{R_2(10)}{2.0(29\times10^6)}$$
$$R_2 = 6.96R_1$$
$$R_2 = 6.96(50\,000 - R_2)$$
$$7.96R_2 = 348\,000$$
$$R_2 = 43\,718.59 \text{ lb}$$

$$\sigma_{st} = \frac{R_2}{A_{st}} = \frac{43718.59}{2.0}$$
$$\sigma_{st} = \mathbf{21\,859.30 \text{ psi}}$$

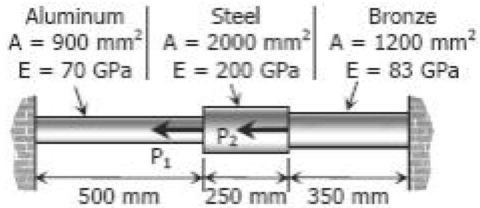
$$R_1 = 50\,000 - 43\,718.59$$
$$R_1 = 6281.41 \text{ lb}$$

$$\sigma_{al} = \frac{R_1}{A_{al}} = \frac{6281.41}{1.25}$$

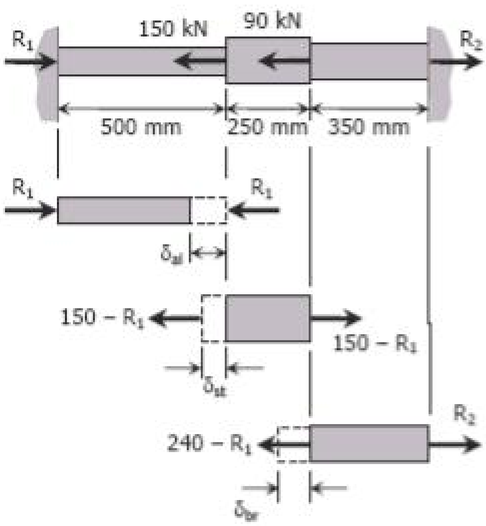
$$\sigma_{al} = \mathbf{5025.12 \text{ psi}}$$

Problem 8

The composite bar in Figure is stress-free before the axial loads P1 and P2 are applied. Assuming that the walls are rigid, calculate the stress in each material if P1 =150 kN and P2 = 90 kN.



Solution



From the *FBD* of each material shown:

δ_{al} is shortening

δ_{st} and δ_{br} are lengthening

$R_2 = 240 - R_1$

$P_{al} = R_1$

$P_{st} = 150 - R_1$

$P_{br} = R_2 = 240 - R_1$

$$\delta_{al} = \delta_{st} + \delta_{br}$$

$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br}$$

$$\frac{R_1(500)}{900(70\,000)} = \frac{(150 - R_1)(250)}{2000(200\,000)} + \frac{(240 - R_1)(350)}{1200(83\,000)}$$

$$\frac{R_1}{126000} = \frac{150 - R_1}{1600000} + \frac{(240 - R_1)7}{1992000}$$

$$\tfrac{1}{63} R_1 = \tfrac{1}{800}(150 - R_1) + \tfrac{7}{996}(240 - R_1)$$

$$\left(\tfrac{1}{63} + \tfrac{1}{800} + \tfrac{7}{996}\right)R_1 = \tfrac{1}{800}(150) + \tfrac{7}{996}(240)$$

$$R_1 = 77.60 \text{ kN}$$

$$P_{al} = R_1 = 77.60 \text{ kN}$$
$$P_{st} = 150 - 77.60 = 72.40 \text{ kN}$$
$$P_{br} = 240 - 77.60 = 162.40 \text{ kN}$$

$$\sigma = P/A$$

$$\sigma_{al} = 77.60(1000)/900$$
$$= \mathbf{86.22 \text{ MPa}}$$

$$\sigma_{st} = 72.40(1000)/2000$$
$$= \mathbf{36.20 \text{ MPa}}$$

$$\sigma_{br} = 162.40(1000)/1200$$
$$= \mathbf{135.33 \text{ MPa}}$$

THERMAL STRESS

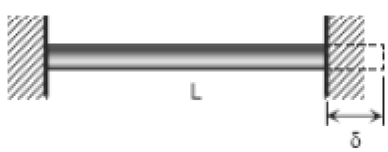
Temperature changes cause the body to expand or contract. The amount δT, is given by

$$\delta_T = \alpha L(T_f - T_i) = \alpha L \, \Delta T$$

where α is the coefficient of thermal expansion in m/m°C, L is the length in meter, and Ti and Tf are the initial and final temperatures, respectively in °C. For steel, α = 11.25 × 10–6 / °C.

If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress.

For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as:



deformation due to temperature changes;

$$\delta_T = \alpha L \,\Delta T$$

deformation due to equivalent axial stress;

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$\delta_T = \delta_P$$

$$\alpha L \,\Delta T = \frac{\sigma L}{E}$$

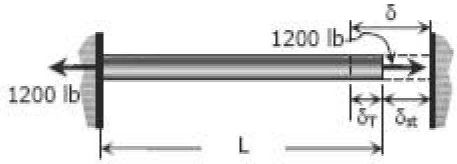
$$\sigma = E\,\alpha\,\Delta T$$

where σ is the thermal stress in MPa and E is the modulus of elasticity of the rod in MPa.

Problem.1

A steel rod with a cross-sectional area of 0.25 in2 is stretched between two fixed points. The tensile load at 70°F is 1200 lb. What will be the stress at 0°F? At what temperature will the stress be zero? Assume α = 6.5 × 10-6 in / (in·°F) and E = 29 × 106 psi.

Solution



For the stress at 0°C:

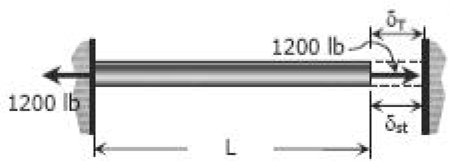
$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

$$\sigma = (6.5 \times 10^{-6})(29 \times 10^{6})(70) + \frac{1200}{0.25}$$

$$\sigma = 17\,995 \text{ psi} = \mathbf{18 \text{ ksi}}$$



For the temperature that causes zero stress:

$$\delta_T = \delta_{st}$$

$$\alpha \cancel{L}(\Delta T) = \frac{P\cancel{L}}{AE}$$

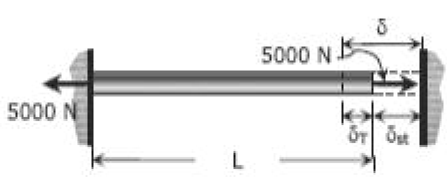
$$(6.5 \times 10^{-6})(T - 70) = \frac{1200}{0.25(29 \times 10^{6})}$$

$$T = \mathbf{95.46°C}$$

Problem 2

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma \cancel{L}}{E} = \alpha \cancel{L}(\Delta T) + \frac{P\cancel{L}}{AE}$$

$$\sigma = \alpha E(\Delta T) + \frac{P}{A}$$

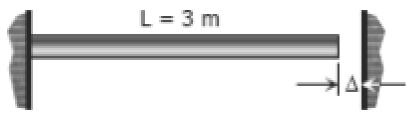
$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

$$A = \frac{5000}{36.4} = 137.36 \text{ mm}^2$$

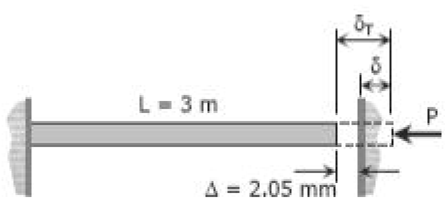
$$\tfrac{1}{4}\pi d^2 = 137.36; \quad d = \mathbf{13.22 \text{ mm}}$$

Problem 3

A bronze bar 3 m long with a cross sectional area of 320 mm2 is placed between two rigid walls as shown in Figure. At a temperature of -20°C, the gap Δ = 25 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use α = 18.0 × 10-6 m/(m·°C) and E = 80 GPa.



Solution



$$\delta_T = \delta + \Delta$$

$$\alpha L(\Delta T) = \frac{\sigma L}{E} + 2.5$$

$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80000} + 2.5$$

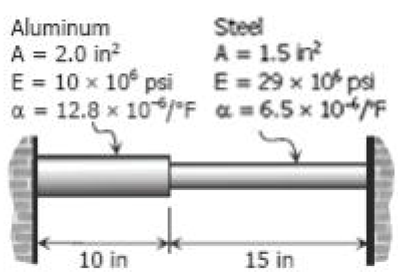
$$\Delta T = 70.6°C$$

$$T = 70.6 - 20$$

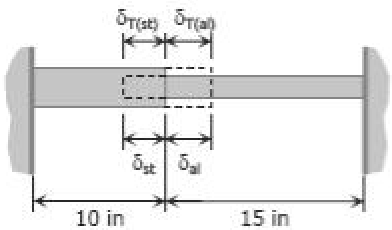
$$T = 50.6°C$$

Problem 4

Calculate the increase in stress for each segment of the compound bar shown in Figure if the temperature increases by 100°F. Assume that the supports are unyielding and that the bar is suitably braced against buckling.



Solution



$$\delta_T = \alpha L \, \Delta T$$

$$\delta_{T(st)} = (6.5 \times 10^{-6})(15)(100)$$
$$\delta_{T(st)} = 0.00975$$

$$\delta_{T(al)} = (12.8 \times 10^{-6})(10)(100)$$
$$\delta_{T(al)} = 0.0128 \text{ in}$$

$$\delta_{st} + \delta_{al} = \delta_{T(st)} + \delta_{T(al)}$$
$$\left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{al} = 0.00975 + 0.0128$$

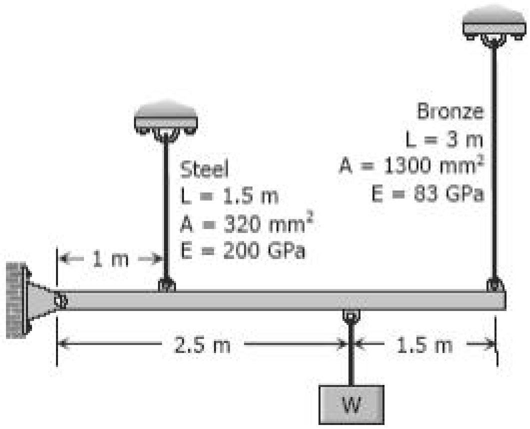
where $P = P_{st} = P_{al}$

$$\frac{P(15)}{1.5(29 \times 10^6)} + \frac{P(10)}{2(10 \times 10^6)} = 0.02255$$
$$P = 26\,691.84 \text{ psi}$$

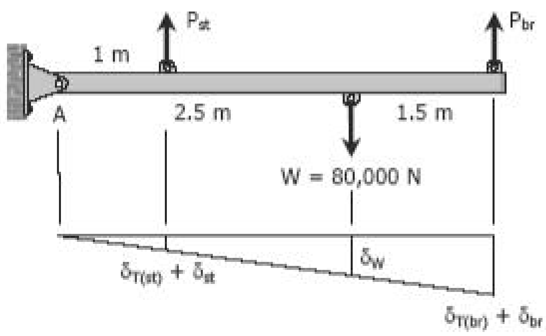
$$\sigma = \frac{P}{A}$$
$$\sigma_{st} = \frac{26691.84}{1.5} = 17\,794.56 \text{ psi}$$
$$\sigma_{al} = \frac{26691.84}{2.0} = 13\,345.92 \text{ psi}$$

Problem.8

A rigid bar of negligible weight is supported as shown in Figure. If W = 80 kN, compute the temperature change that will cause the stress in the steel rod to be 55 MPa. Assume the coefficients of linear expansion are 11.7 µm/(m·°C) for steel and 18.9 µm / (m·°C) for bronze.



Solution



Stress in bronze when $\sigma_{st} = 55$ MPa

$$\Sigma M_A = 0$$

$$4P_{br} + P_{st} = 2.5(80000)$$

$$4\sigma_{br}(1300) + 55(320) = 2.5(80000)$$

$$\sigma_{br} = 35.08 \text{ MPa}$$

By ratio and proportion:

$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$

$$\delta_{T(st)} + \delta_{st} = 0.25[\delta_{T(br)} + \delta_{br}]$$

$$(\alpha L\ \Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} = 0.25\left[(\alpha L\ \Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br}\right]$$

$$(11.7 \times 10^{-6})(1500)\ \Delta T + \frac{55(1500)}{2000} = 0.25\left[(18.9 \times 10^{-6})(3000)\ \Delta T + \frac{35.08(3000)}{83000}\right]$$

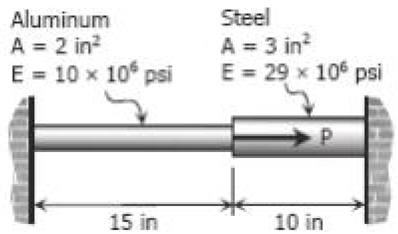
$$0.017\,55\ \Delta T + 0.4125 = 0.014\,175\ \Delta T + 0.317$$

$$\Delta T = -28.3\ °\text{C}$$

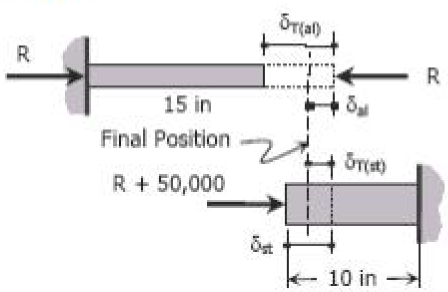
A temperature drop of 28.3 °C is needed to stress the steel to 55 MPa.

Problem 8

The composite bar shown in Figure is firmly attached to unyielding supports. An axial force P = 50 kips is applied at 60°F. Compute the stress in each material at 120°F. Assume α = 6.5 × 10–6 in/(in·°F) for steel and 12.8 × 10–6 in/(in·°F) for aluminum.



Solution



$$\delta_{T(al)} = (\alpha L\ \Delta T)_{al}$$

$$\delta_{T(al)} = (12.8 \times 10^{-6})(15)(120 - 60)$$

$$\delta_{T(al)} = 0.011\,52 \text{ inch}$$

$$\delta_{T(st)} = (\alpha L\ \Delta T)_{st}$$

$$\delta_{T(st)} = (6.5 \times 10^{-6})(10)(120 - 60)$$

$$\delta_{T(st)} = 0.0039 \text{ inch}$$

$$\delta_{T(al)} - \delta_{al} = \delta_{st} - \delta_{T(st)}$$

$$0.011\,52 - \left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} - 0.0039$$

$$0.011\,52 - \frac{R(15)}{2(10\times 10^6)} = \frac{(R+50\,000)(10)}{3(29\times 10^6)} - 0.0039$$

$$100\,224 - 6.525R = R + 50\,000 - 33\,930$$

$$84\,154 = 7.525R$$

$$R = 11\,183.25 \text{ lbs}$$

$$P_{al} = R = 11\,183.25 \text{ lbs}$$

$$P_{st} = R + 50\,000 = 61\,183.25 \text{ lbs}$$

$$\sigma = \frac{P}{A}$$

$$\sigma_{al} = \frac{11\,183.25}{2}$$

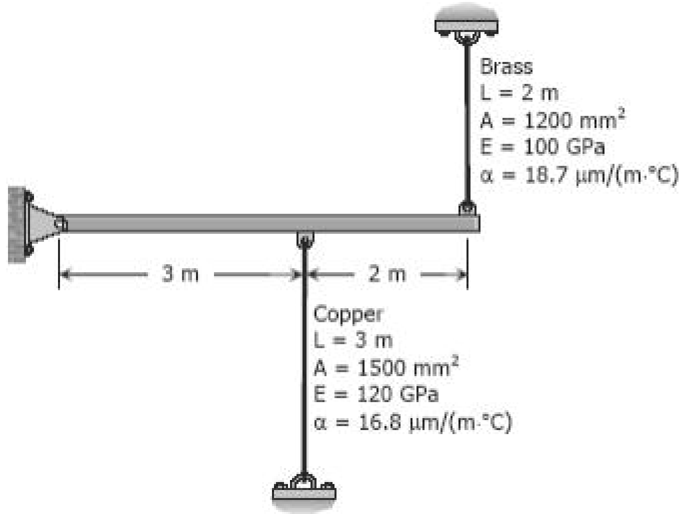
$$= \mathbf{5\,591.62\ psi}$$

$$\sigma_{st} = \frac{61\,183.25}{3}$$

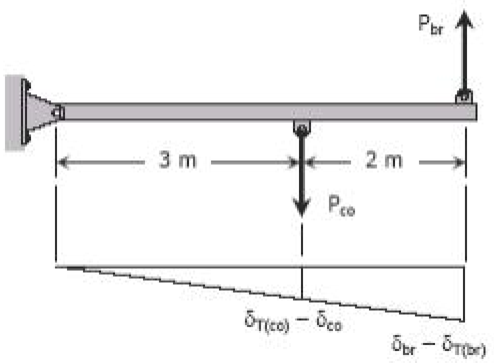
$$= \mathbf{20\,394.42\ psi}$$

Problem 9

A rigid horizontal bar of negligible mass is connected to two rods as shown in Figure. If the system is initially stress-free. Calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both rods are subjected to the change in temperature.



Solution



$\Sigma M_{\text{hinge support}} = 0$

$5P_{br} - 3P_{co} = 0$

$5\sigma_{br} A_{br} - 3\sigma_{co} A_{co} = 0$

$5(90)(1200) - 3\sigma_{co}(1500) = 0$

$\sigma_{co} = 120 \text{ MPa}$

$\delta = \sigma L / E$

$\delta_{br} = 90(2000) / 100\,000 = 1.8 \text{ mm}$

$\delta_{co} = 120(3000) / 120\,000 = 3 \text{ mm}$

$$\frac{\delta_{T(co)} - \delta_{co}}{3} = \frac{\delta_{br} - \delta_{T(br)}}{5}$$

$5\delta_{T(co)} - 5\delta_{co} = 3\delta_{br} - 3\delta_{T(br)}$

$5(16.8\times10^{-6})(3000)\,\Delta T - 5(3)$

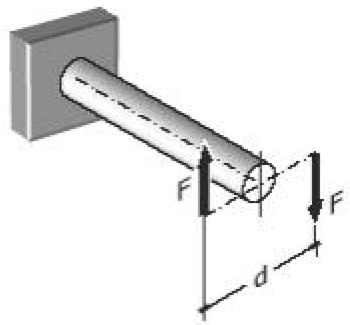
$\quad = 3(1.8) - 3(18.7\times10^{-6})(2000)\,\Delta T$

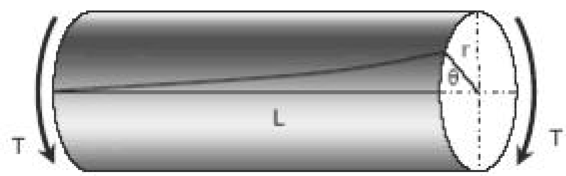
$0.3642\,\Delta T = 20.4$

$\Delta T = 56.01°\text{C}$ drop in temperature

Torsion

Consider a bar to be rigidly attached at one end and twisted at the other end by a torque or twisting moment T equivalent to F × d, which is applied perpendicular to the axis of the bar, as shown in the figure. Such a bar is said to be in torsion.





TORSIONAL SHEARING STRESS, τ

For a solid or hollow circular shaft subject to a twisting moment T, the torsional shearing stress τ at a distance ρ from the center of the shaft is

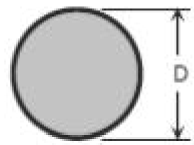
$$\tau = \frac{T\rho}{J} \text{ and } \tau_{max} = \frac{Tr}{J}$$

Where; J is the polar moment of inertia of the section and r is the outer radius.

For solid cylindrical shaft

$$J = \frac{\pi}{32} D^4$$

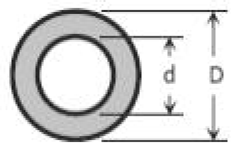
$$\tau_{max} = \frac{16T}{\pi D^3}$$



For hollow cylindrical shaft:

$$J = \frac{\pi}{32} (D^4 - d^4)$$

$$\tau_{max} = \frac{16TD}{\pi (D^4 - d^4)}$$



ANGLE OF TWIST

The angle θ through which the bar length L will twist is

$$\theta = \frac{TL}{JG} \text{ in radians}$$

where ;

T -is the torque in N·mm,

L- is the length of shaft in mm,

G- is shear modulus in MPa,

J- is the polar moment of inertia in mm4

D and d are diameter in mm, and

r- is theradius in mm.

POWER TRANSMITTED BY THE SHAFT

A shaft rotating with a constant angular velocity ω (in radians per second) is being acted by a twisting moment T. The power transmitted by the shaft is

$$P = T\omega = 2\pi Tf$$

Where;

T- is the torque in N·m,

f- is the number of revolutions per second,

P- is the power in watts.

Problem 1

A solid steel shaft 5 m long is stressed at 80 MPa when twisted through 4°. Using G = 83 GPa, compute the shaft diameter. What power can be transmitted by the shaft at 20 Hz?

Solution

$$\theta = \frac{TL}{JG}$$

$$4^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{T(5)(1000)}{\frac{1}{32}\pi d^4(83000)}$$

$$T = 0.1138d^4$$

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$80 = \frac{16(0.1138d^4)}{\pi d^3}$$

$$d = 138\ \text{mm}$$

$$T = \frac{P}{2\pi f}$$

$$0.1138d^4 = \frac{P}{2\pi(20)}$$

$$P = 14.3d^4 = 14.3(138^4)$$

$$P = 5\,186\,237\,285\ \text{N·mm/sec}$$

$$P = 5\,186\,237.28\ \text{W}$$

$$P = 5.19\ \text{MW}$$

Problem 2

A steel propeller shaft is to transmit 4.5 MW at 3 Hz without exceeding a shearing stress of 50 MPa or twisting through more than 1° in a length of 26 diameters. Compute the proper diameter if G = 83 GPa.

Solution 309

$$T = \frac{P}{2\pi f} = \frac{4.5(1\,000\,000)}{2\pi(3)}$$

$$T = 238\,732.41 \text{ N}\cdot\text{m}$$

Based on maximum allowable shearing stress:

$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$50 = \frac{16(238732.41)(1000)}{\pi d^3}$$

$$d = 289.71 \text{ mm}$$

Based on maximum allowable angle of twist:

$$\theta = \frac{TL}{JG}$$

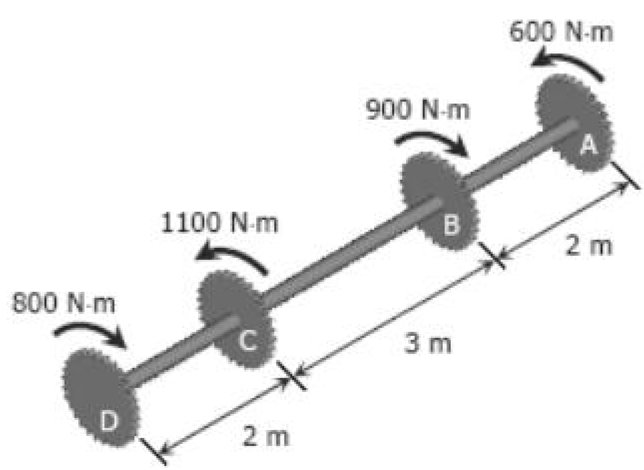
$$1°\left(\frac{\pi}{180°}\right) = \frac{238732.41(26d)(1000)}{\frac{1}{32}\pi d^4(83000)}$$

$$d = 352.08 \text{ mm}$$

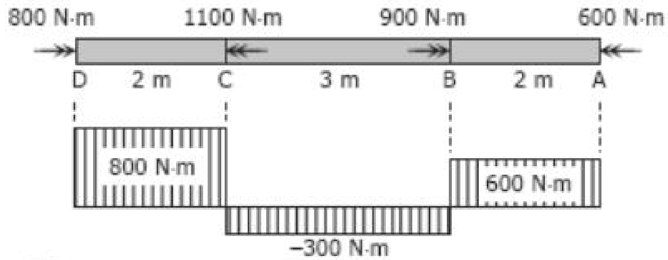
Use the bigger diameter, d = **352 mm**

Problem 3

An aluminum shaft with a constant diameter of 50 mm is loaded by torques applied to gears attached to it as shown in Figure. Using G = 28 GPa, determine the relative angle of twist of gear D relative to gear A.



Solution



$$\theta = \frac{TL}{JG}$$

Rotation of D relative to A:

$$\theta_{D/A} = \frac{1}{JG}\sum TL$$

$$\theta_{D/A} = \frac{1}{\frac{1}{32}\pi(50^4)(28000)}[800(2) - 300(3) + 600(2)]\ (100^2)$$

$$\theta_{D/A} = 0.1106\ \text{rad}$$

$$\theta_{D/A} = 6.34°$$

Problem 4

Determine the maximum torque that can be applied to a hollow circular steel shaft of 100-mm outside diameter and an 80-mm inside diameter without exceeding a shearing stress of 60 MPa or a twist of 0.5 deg/m. Use G = 83 GPa.

Solution

Based on maximum allowable shearing stress:

$$\tau_{max} = \frac{16TD}{\pi(D^4 - d^4)}$$

$$60 = \frac{16T(100)}{\pi(100^4 - 80^4)}$$

$$T = 6\,955\,486.14 \text{ N·mm}$$

$$T = 6\,955.5 \text{ N·m}$$

Based on maximum allowable angle of twist:

$$\theta = \frac{TL}{JG}$$

$$0.5°\left(\frac{\pi}{180°}\right) = \frac{T(1000)}{\frac{1}{32}\pi(100^4 - 80^4)(83000)}$$

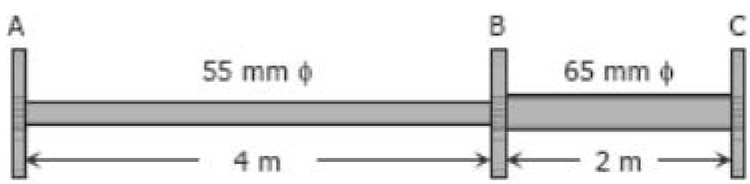
$$T = 4\,198\,282.97 \text{ N·mm}$$

$$T = 4\,198.28 \text{ N·m}$$

Use the smaller torque, T = **4 198.28 N·m**

Problem 5

The steel shaft shown in Figure rotates at 4 Hz with 35 kW taken off at A, 20 kW removed at B, and 55 kW applied at C. Using G = 83 GPa, find the maximum shearing stress and the angle of rotation of gear A relative to gear C.



Solution

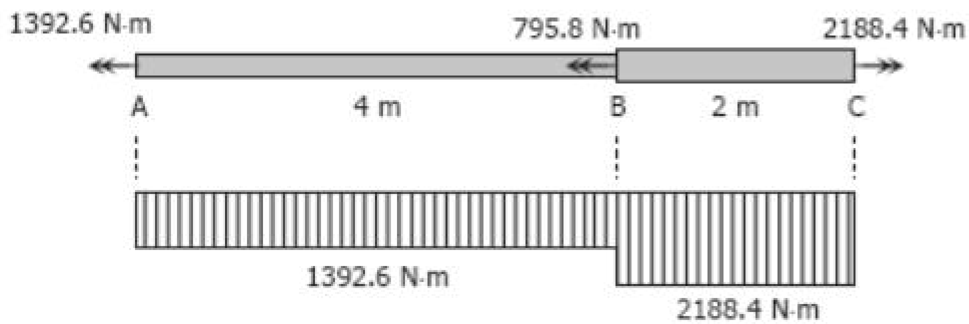
$$T = \frac{P}{2\pi f}$$

$$T_A = \frac{-35(1000)}{2\pi(4)} = -1392.6 \text{ N}\cdot\text{m}$$

$$T_B = \frac{-20(1000)}{2\pi(4)} = -795.8 \text{ N}\cdot\text{m}$$

$$T_C = \frac{55(1000)}{2\pi(4)} = 2188.4 \text{ N}\cdot\text{m}$$

Relative to C:



$$\tau_{max} = \frac{16T}{\pi d^3}$$

$$\tau_{AB} = \frac{16(1392.6)(1000)}{\pi(55^3)} = 42.63\text{ MPa}$$

$$\tau_{BC} = \frac{16(2188.4)(1000)}{\pi(65^3)} = 40.58\text{ MPa}$$

$$\therefore \quad \tau_{max} = \tau_{AB} = 42.63\text{ MPa}$$

$$\theta = \frac{TL}{JG}$$

$$\theta_{A/C} = \frac{1}{G}\sum\frac{TL}{J}$$

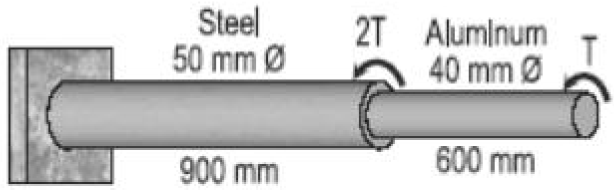
$$\theta_{A/C} = \frac{1}{83\,000}\left[\frac{1392.6(4)}{\frac{1}{32}\pi(55^4)} + \frac{2188.4(2)}{\frac{1}{32}\pi(65^4)}\right](1000^2)$$

$$\theta_{A/C} = 0.104\,796\,585\text{ rad}$$

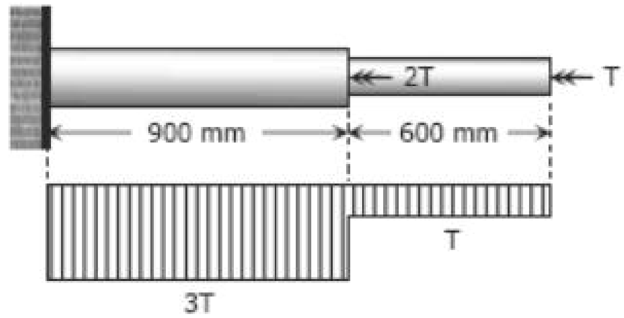
$$\theta_{A/C} = 6.004°$$

Problem 6

A compound shaft consisting of a steel segment and an aluminum segment is acted upon by two torques as shown in Fig. P-316. Determine the maximum permissible value of T subject to the following conditions: τ_{st} = 83 MPa, τ_{al} = 55 MPa, and the angle of rotation of the free end is limited to 6°. For steel, G = 83 GPa and for aluminum, G = 28 GPa.



Solution



Based on maximum shearing stress $\tau_{max} = 16T \;/\; \pi d^3$:

$$\tau_{st} = \frac{16(3T)}{\pi(50^3)} = 83$$

$T = 679\ 042.16$ N·mm

$T = 679.04$ N·m

$$\tau_{al} = \frac{16T}{\pi(40^3)} = 55$$

$T = 691\ 150.38$ N·mm

$T = 691.15$ N·m

Based on maximum angle of twist:

$$\theta = \left(\frac{TL}{JG}\right)_{st} + \left(\frac{TL}{JG}\right)_{al}$$

$$6^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{3T(900)}{\frac{1}{32}\pi(50^4)(83000)} + \frac{T(600)}{\frac{1}{32}\pi(40^4)(28000)}$$

$T = 757\ 316.32$ N·mm

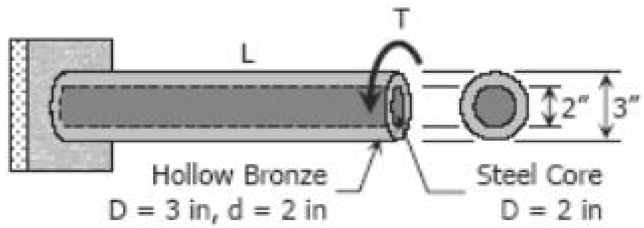
$T = 757.32$ N·m

Use T = **679.04 N·m**

Problem 7

A hollow bronze shaft of 3 in. outer diameter and 2 in. inner diameter is slipped over a solid steel shaft 2 in. in diameter and of the same length as the hollow shaft. The two shafts are then fastened rigidly together at their ends. For bronze, G = 6 × 106 psi, and for steel, G = 12 × 106 psi. What torque can be applied to the composite shaft without exceeding a shearing stress of 8000 psi in the bronze or 12 ksi in the steel?

Solution



$$\theta_{st} = \theta_{br}$$

$$\left(\frac{TL}{JG}\right)_{st} = \left(\frac{TL}{JG}\right)_{br}$$

$$\frac{T_{st}L}{\frac{1}{32}\pi(2^4)(12\times10^6)} = \frac{T_{br}L}{\frac{1}{32}\pi(3^4-2^4)(6\times10^6)}$$

$$\frac{T_{st}}{192\times10^6} = \frac{T_{br}}{390\times10^6} \qquad \rightarrow \text{Equation (1)}$$

Applied Torque = Resisting Torque

$$T = T_{st} + T_{br} \qquad \rightarrow \text{Equation (2)}$$

Equation (1) with T_{st} in terms of T_{br} and Equation (2)

Applied Torque = Resisting Torque

$$T = T_{st} + T_{br} \quad \rightarrow \text{Equation (2)}$$

Equation (1) with T_{st} in terms of T_{br} and Equation (2)

$$T = \frac{192 \times 10^6}{390 \times 10^6} T_{br} + T_{br}$$

$$T_{br} = 0.6701T$$

Equation (1) with T_{br} in terms of T_{st} and Equation (2)

$$T = T_{st} + \frac{390 \times 10^6}{192 \times 10^6} T_{st}$$

$$T_{st} = 0.3299T$$

Based on hollow bronze ($T_{br} = 0.6701T$)

$$\tau_{max} = \left[\frac{16TD}{\pi(D^4 - d^4)} \right]_{br}$$

$$8000 = \frac{16(0.6701T)(3)}{\pi(3^4 - 2^4)}$$

$$8000 = \frac{16(0.6701T)(3)}{\pi(3^4 - 2^4)}$$

$T = 50\ 789.32$ lb·in

$T = 4232.44$ lb·ft

Based on steel core ($T_{st} = 0.3299T$):

$$\tau_{max} = \left[\frac{16T}{\pi D^3}\right]_{st}$$

$$12\ 000 = \frac{16(0.3299T)}{\pi(2^3)}$$

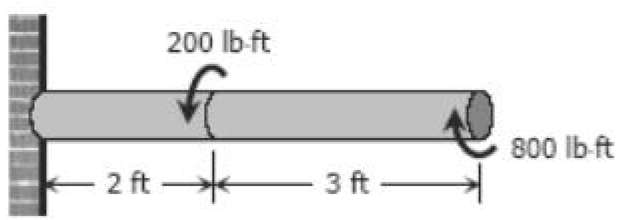
$T = 57\ 137.18$ lb·in

$T = 4761.43$ lb·ft

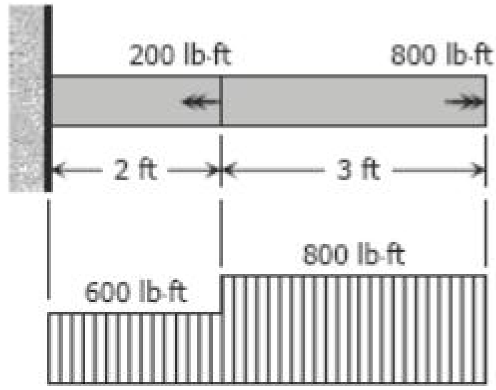
Use $T =$ **4232.44 lb·ft**

Problem 8

A solid aluminum shaft 2 in. in diameter is subjected to two torques as shown in Figure. Determine the maximum shearing stress in each segment and the angle of rotation of the free end. Use G = 4 × 106 psi.



Solution



$$\tau_{max} = \frac{16T}{\pi D^3}$$

For 2-ft segment:

$$\tau_{max2} = \frac{16(600)(12)}{\pi(2^3)} = \mathbf{4583.66\ psi}$$

For 3-ft segment:

$$\tau_{max3} = \frac{16(800)(12)}{\pi(2^3)} = \mathbf{6111.55\ psi}$$

$$\theta = \frac{TL}{JG}$$

$$\theta = \frac{1}{JG}\sum TL$$

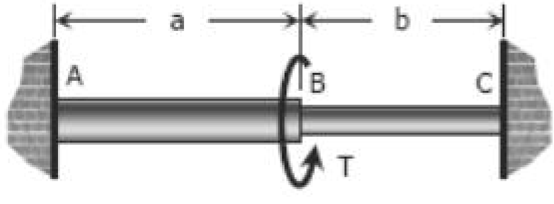
$$\theta = \frac{1}{\frac{1}{32}\pi(2^4)(4\times10^6)}\ [600(2) + 800(3)]\ (12^2)$$

$$\theta = 0.0825\ \text{rad}$$

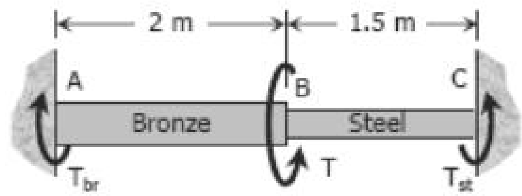
$$\theta = \mathbf{4.73°}$$

Problem 9

The compound shaft shown in Figure is attached to rigid supports. For the bronze segment AB, the diameter is 75 mm, $\tau \leq 60$ MPa, and G = 35 GPa. For the steel segment BC, the diameter is 50 mm, $\tau \leq 80$ MPa, and G = 83 GPa. If a = 2 m and b = 1.5 m, compute the maximum torque T that can be applied.



Solution



$$\Sigma M = 0$$

$$T = T_{br} + T_{st} \quad \rightarrow \text{Equation (1)}$$

$$\theta_{br} = \theta_{st}$$

$$\left(\frac{TL}{JG}\right)_{br} = \left(\frac{TL}{JG}\right)_{st}$$

$$\frac{T_{br}(2)(1000)}{\frac{1}{32}\pi(75^4)(35000)} = \frac{T_{st}(1.5)(1000)}{\frac{1}{32}\pi(50^4)(83000)}$$

$$\left.\begin{aligned} T_{br} &= 1.6011 T_{st} \\ T_{st} &= 0.6246 T_{br} \end{aligned}\right\} \text{Equations (2)}$$

$$\tau_{max} = \frac{16T}{\pi D^3}$$

Based on $\tau_{br} \leq 60$ MPa

$$60 = \frac{16T_{br}}{\pi(75^3)}$$

T_{br} = 4 970 097.75 N·mm

T_{br} = 4.970 kN·m → Maximum allowable torque for bronze

T_{st} = 0.6246(4.970) → From one of Equations (2)

T_{st} = 3.104 kN·m

Based on $\tau_{st} \leq 80$ MPa

$$80 = \frac{16T_{st}}{\pi(50^3)}$$

T_{st} = 1 963 495.41 N·mm

T_{st} = 1.963 kN·m → maximum allowable torque for steel

T_{br} = 1.6011(1.963) → From Equations (2)

T_{br} = 3.142 kN·m

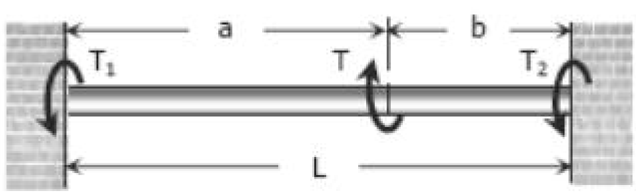
Use T_{br} = 3.142 kN·m and T_{st} = 1.963 kN·m

T = 3.142 + 1.963 → From Equation (1)

T = 5.105 kN·m

Problem 10

A torque T is applied, as shown in Figure to a solid shaft with built-in ends. Prove that the resisting torques at the walls are T1 = Tb/L and T2 = Ta/L. How would these values be changed if the shaft were hollow?



Solution

$$\Sigma M = 0$$

$$T = T_1 + T_2 \qquad \rightarrow \text{Equation (1)}$$

$$\theta_1 = \theta_2$$

$$\left(\frac{TL}{JG}\right)_1 = \left(\frac{TL}{JG}\right)_2$$

$$\frac{T_1 a}{JG} = \frac{T_2 b}{JG}$$

$$\left.\begin{aligned} T_1 &= \frac{b}{a} T_2 \\ T_2 &= \frac{a}{b} T_1 \end{aligned}\right\} \text{Equations (2)}$$

Equations (1) and (2) with T_2 in terms of T_1:

Equations (1) and (2) with T_2 in terms of T_1:

$$T = T_1 + \frac{a}{b} T_1$$

$$T = \frac{T_1 b + T_1 a}{b}$$

$$T = \frac{(b + a)T_1}{b}$$

$$T = \frac{LT_1}{b}$$

$$T_1 = \boldsymbol{Tb/L}$$

Equations (1) and (2) with T_1 in terms of T_2:

$$T = \frac{b}{a} T_2 + T_2$$

$$T = \frac{T_2 b + T_2 a}{a}$$

$$T = \frac{(b + a)T_2}{a}$$

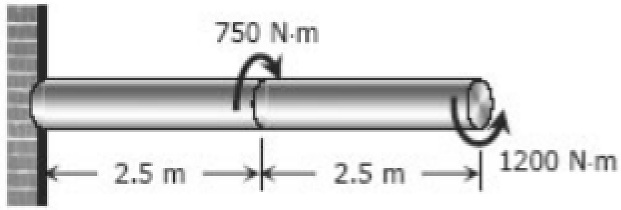
$$T = \frac{LT_2}{a}$$

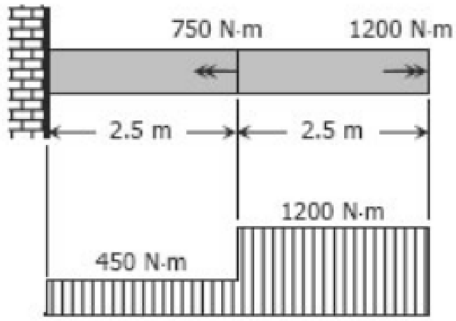
$$T_2 = \boldsymbol{Ta/L}$$

If the shaft were hollow, Equation (1) would be the same and the equality $\theta_1 = \theta_2$, by direct investigation, would yield the same result in Equations (2). Therefore, the values of T_1 and T_2 are the same (**no change**) if the shaft were hollow.

Problem 12

A solid steel shaft is loaded as shown in Figure. Using G = 83 GPa, determine the required diameter of the shaft if the shearing stress is limited to 60 MPa and the angle of rotation at the free end is not to exceed 4 deg.





Based on maximum allowable shear:

$$\tau_{max} = \frac{16T}{\pi D^3}$$

For the 1st segment:

$$60 = \frac{450(2.5)(1000^2)}{\pi D^3}$$

$$D = 181.39 \text{ mm}$$

For the 2nd segment:

$$60 = \frac{1200(2.5)(1000^2)}{\pi D^3}$$

$$D = 251.54 \text{ mm}$$

Based on maximum angle of twist:

$$\theta = \frac{TL}{JG}$$

$$\theta = \frac{1}{JG}\sum TL$$

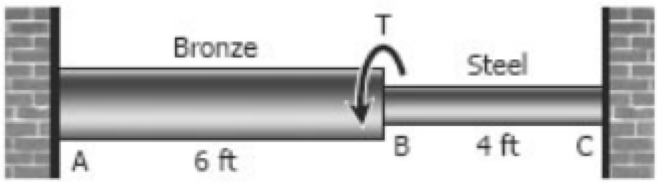
$$4°\left(\frac{\pi}{180°}\right) = \frac{1}{\frac{1}{32}\pi D^4(83000)}[450(2.5) + 1200(2.5)](1000^2)$$

$$D = 51.89 \text{ mm}$$

Use $D =$ **251.54 mm**

Problem 13

The compound shaft shown in Figure is attached to rigid supports. For the bronze segment AB, the maximum shearing stress is limited to 8000 psi and for the steel segment BC, it is limited to 12 kpsi. Determine the diameters of each segment so that each material will be simultaneously stressed to its permissible limit when a torque T = 12 kip·ft is applied. For bronze, G = 6 × 106 psi and for steel, G = 12 × 106 psi.



Solution

$$\tau_{max} = \frac{16T}{\pi D^3}$$

For bronze:

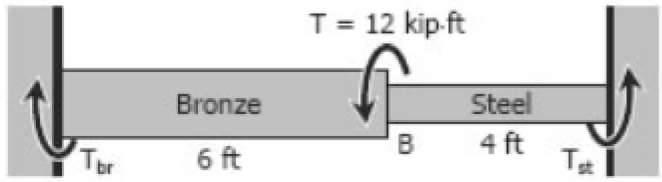
$$8000 = \frac{16T_{br}}{\pi D_{br}^3}$$

$$T_{br} = 500\pi D_{br}^3 \text{ lb·in}$$

For steel:

$$12\,000 = \frac{16T_{st}}{\pi D_{st}^3}$$

$T_{st} = 750\pi\, D_{st}^{\ 3}$ lb·in



$\Sigma M = 0$

$T_{br} + T_{st} = T$

$T_{br} + T_{st} = 12(1000)(12)$

$T_{br} + T_{st} = 144\ 000$ lb·in

$500\pi\, D_{br}^{\ 3} + 750\pi\, D_{st}^{\ 3} = 144\ 000$

$D_{br}^{\ 3} = 288/\pi - 1.5\, D_{st}^{\ 3}$ → equation (1)

$$\theta_{br} = \theta_{st}$$

$$\left(\frac{TL}{JG}\right)_{br} = \left(\frac{TL}{JG}\right)_{st}$$

$$\frac{T_{br}(6)}{\frac{1}{32}\pi D_{br}^{\ 4}(6\times 10^6)} = \frac{T_{st}(4)}{\frac{1}{32}\pi D_{st}^{\ 4}(12\times 10^6)}$$

$$\frac{T_{br}}{D_{br}^{\ 4}} = \frac{T_{st}}{3D_{st}^{\ 4}}$$

$$\frac{500\pi D_{br}^{\ 3}}{D_{br}^{\ 4}} = \frac{750\pi D_{st}^{\ 3}}{3D_{st}^{\ 4}}$$

$$D_{st} = 0.5D_{br}$$

From Equation (1)

$$D_{br}^{\ 3} = 288/\pi - 1.5\,(0.5D_{br})^3$$

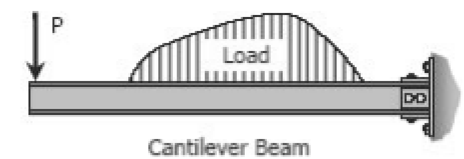
$$1.1875\,D_{br}^{\ 3} = 288/\pi$$

$$D_{br} = \mathbf{4.26\ in.}$$

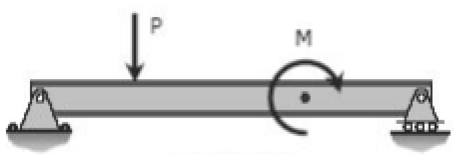
$$D_{st} = 0.5(4.26) = \mathbf{2.13\ in.}$$

DEFINITION OF A BEAM
A beam is a bar subject to forces or couples that lie in a plane containing the longitudinal of the bar. According to determinacy, a beam may be determinate or indeterminate.

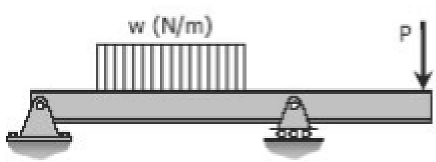
STATICALLY DETERMINATE BEAMS
Statically determinate beams are those beams in which the reactions of the supports may be determined by the use of the equations of static equilibrium. The beams shown below are examples of statically determinate beams.



Cantilever Beam

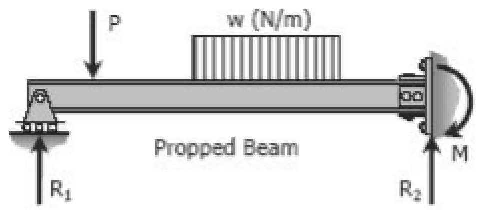


Simple Beam

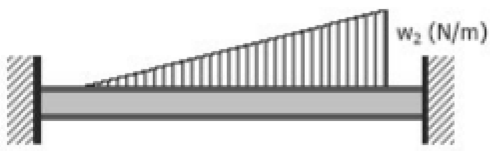


Overhanging Beam

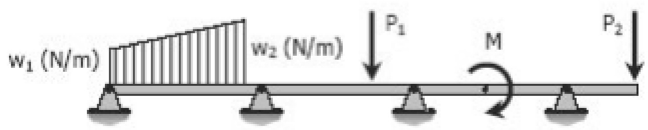
STATICALLY INDETERMINATE BEAMS
If the number of reactions exerted upon a beam exceeds the number of equations in static equilibrium, the beam is said to be statically indeterminate. In order to solve the reactions of the beam, the static equations must be supplemented by equations based upon the elastic deformations of the beam.
The degree of indeterminacy is taken as the difference between the umber of reactions to the number of equations in static equilibrium that can be applied. In the case of the propped beam shown, there are three reactions R1, R2, and M and only two equations ($\sum M = 0$ and sum;Fv = 0) can be applied, thus the beam is indeterminate to the first degree ($3 – 2 = 1$).



Propped Beam



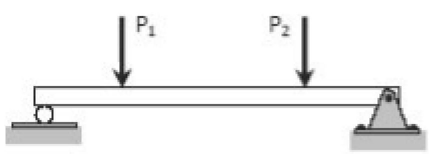
Fixed or Restrained Beam



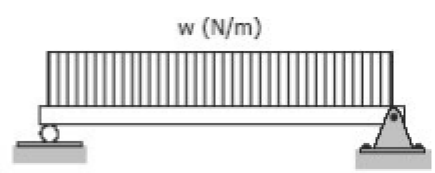
Continuous Beam

TYPES OF LOADING

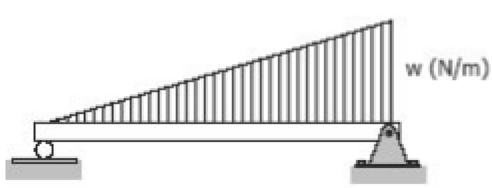
Loads applied to the beam may consist of a concentrated load (load applied at a point), uniform load, uniformly varying load, or an applied couple or moment. These loads are shown in the following figures.



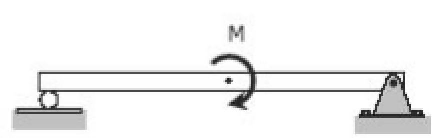
Concentrated Loads



Uniform Load



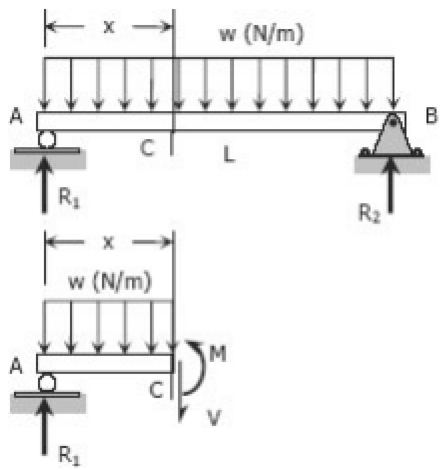
Uniformly Varying Load



Applied Couple

Shear and Moment Diagrams

Consider a simple beam shown of length L that carries a uniform load of w (N/m) throughout its length and is held in equilibrium by reactions R1 and R2. Assume that the beam is cut at point distance of x from he left support and the portion o the beam to the right of C be removed. The portion removed must then be replaced by vertical shearing force V together with a couple M to hold the left portion of the bar in equilibrium under the C action of R1 and wx. The couple M is called the resisting moment or moment and the force V is called the resisting shear or shear. The sign of V and M are taken to be positive if they have the senses indicated above.



Problem 403

Beam loaded as shown in Fig. P-403.

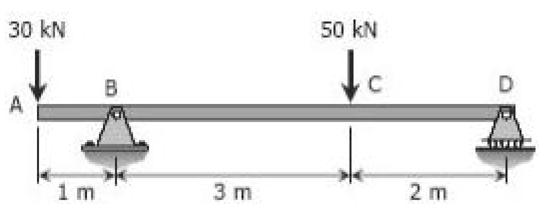


Figure P-403

Solution 403

From the load diagram:

$\sum M_B = 0$

$5R_D + 1(30) = 3(50)$

$R_D = 24$ kN

$\sum M_D = 0$

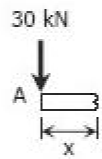
$5R_B = 2(50) + 6(30)$

$R_B = 56$ kN

Segment AB:

$V_{AB} = -30$ kN

$M_{AB} = -30x$ kN·m



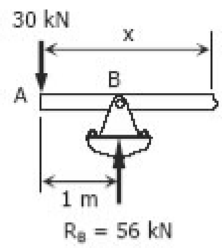
Segment BC:

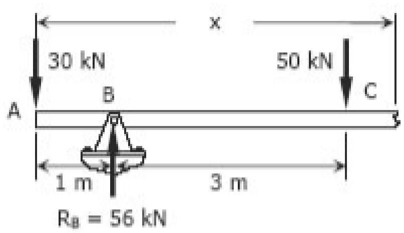
$V_{BC} = -30 + 56$

$= 26$ kN

$M_{BC} = -30x + 56(x - 1)$

$= 26x - 56$ kN·m





Segment CD:

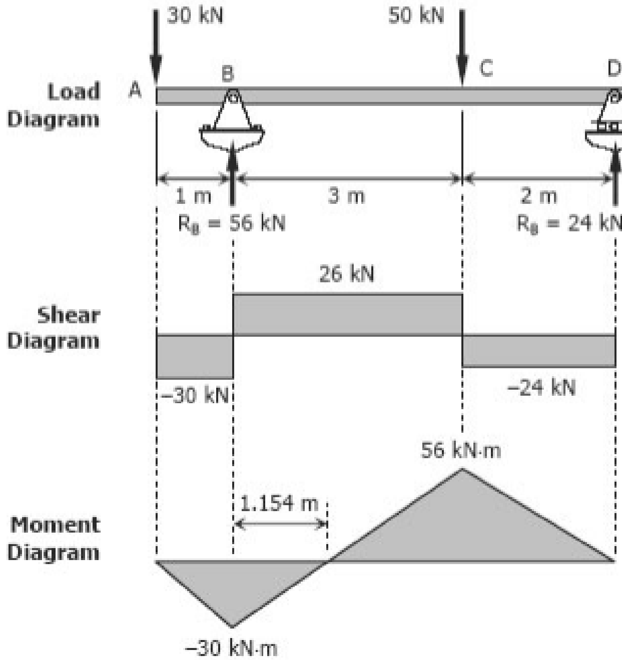
$V_{CD} = -30 + 56 - 50$

$= -24$ kN

$M_{CD} = -30x + 56(x - 1) - 50(x - 4)$

$= -30x + 56x - 56 - 50x + 200$

$= -24x + 144$



To draw the Shear Diagram:

(1) In segment AB, the shear is uniformly distributed over the segment at a magnitude of –30 kN.

(2) In segment BC, the shear is uniformly distributed at a magnitude of 26 kN.

(3) In segment CD, the shear is uniformly distributed at a magnitude of –24 kN.

To draw the Moment Diagram:

(1) The equation $M_{AB} = -30x$ is linear, at $x = 0$, $M_{AB} = 0$ and at $x = 1$ m, $M_{AB} = -30$ kN·m.

(2) $M_{BC} = 26x - 56$ is also linear. At $x = 1$ m, $M_{BC} = -30$ kN·m; at $x = 4$ m, $M_{BC} = 48$ kN·m. When $M_{BC} = 0$, $x = 2.154$ m, thus the moment is zero at 1.154 m from B.

(3) $M_{CD} = -24x + 144$ is again linear. At $x = 4$ m, $M_{CD} = 48$ kN·m; at $x = 6$ m, $M_{CD} = 0$.

Problem 404

Beam loaded as shown in Fig. P-404.

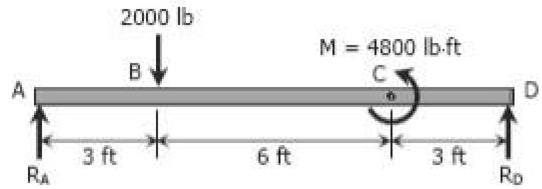


Figure P-404

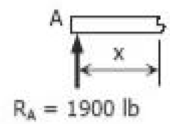
Solution 404

$$\sum M_A = 0$$
$$12R_D + 4800 = 3(2000)$$
$$R_D = 100 \text{ lb}$$

$$\sum M_D = 0$$
$$12R_A = 9(2000) + 4800$$
$$R_A = 1900 \text{ lb}$$

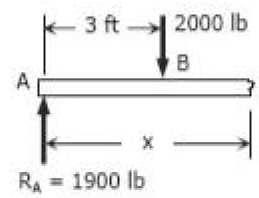
Segment AB:

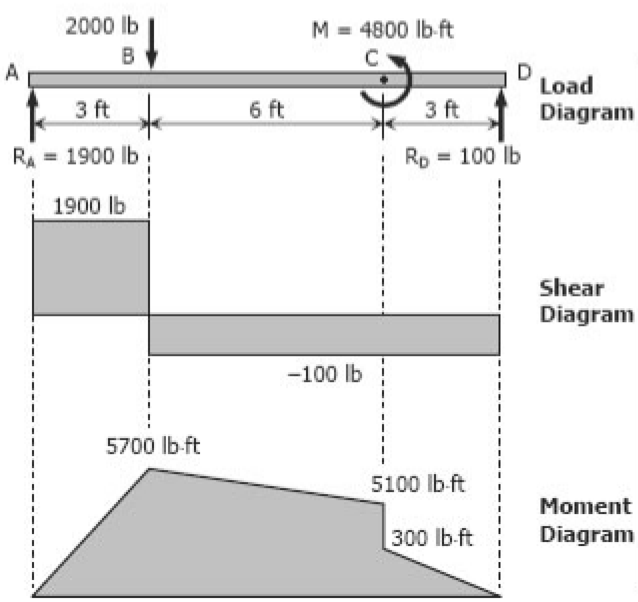
$$V_{AB} = 1900 \text{ lb}$$
$$M_{AB} = 1900x \text{ lb·ft}$$



Segment BC:

$$V_{BC} = 1900 - 2000$$
$$= -100 \text{ lb}$$
$$M_{BC} = 1900x - 2000(x - 3)$$
$$= 1900x - 2000x + 6000$$
$$= -100x + 6000$$





To draw the Shear Diagram:

(1) At segment AB, the shear is uniformly distributed at 1900 lb.

(2) A shear of −100 lb is uniformly distributed over segments BC and CD.

To draw the Moment Diagram:

(1) $M_{AB} = 1900x$ is linear; at $x = 0$, $M_{AB} = 0$; at $x = 3$ ft, $M_{AB} = 5700$ lb·ft.

(2) For segment BC, $M_{BC} = -100x + 6000$ is linear; at $x = 3$ ft, $M_{BC} = 5700$ lb·ft; at $x = 9$ ft, $M_{BC} = 5100$ lb·ft.

(3) $M_{CD} = -100x + 1200$ is again linear; at $x = 9$ ft, $M_{CD} = 300$ lb·ft; at $x = 12$ ft, $M_{CD} = 0$.

Problem 405

Beam loaded as shown in Fig. P-405.

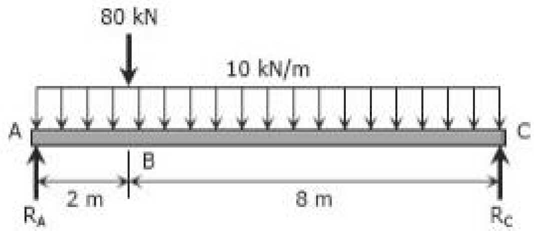
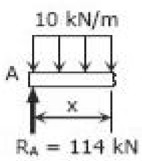


Figure P-405

Solution 405

$$\Sigma M_A = 0$$
$$10R_C = 2(80) + 5[10(10)]$$
$$R_C = 66 \text{ kN}$$

$$\Sigma M_C = 0$$
$$10R_A = 8(80) + 5[10(10)]$$
$$R_A = 114 \text{ kN}$$

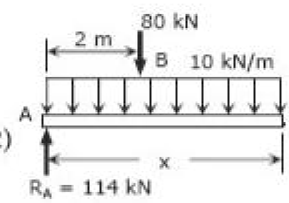


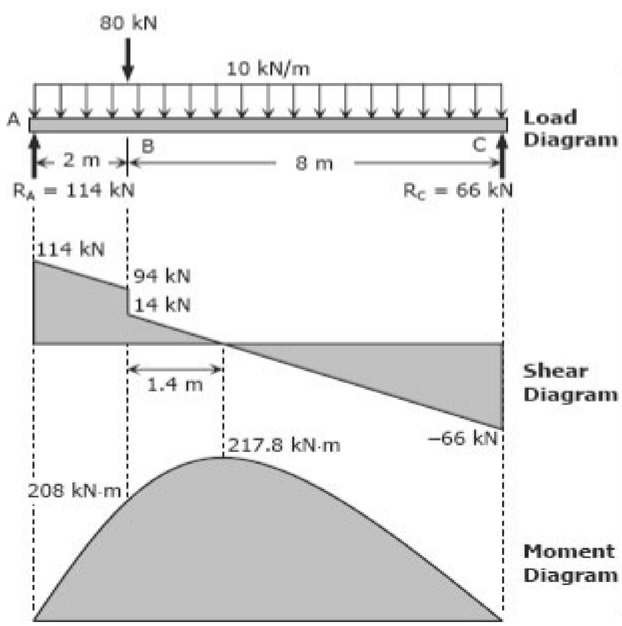
Segment *AB*:

$$V_{AB} = 114 - 10x \text{ kN}$$
$$M_{AB} = 114x - 10x(x/2)$$
$$= 114x - 5x^2 \text{ kN·m}$$

Segment *BC*:

$$V_{BC} = 114 - 80 - 10x$$
$$= 34 - 10x \text{ kN}$$
$$M_{BC} = 114x - 80(x - 2) - 10x(x/2)$$
$$= 160 + 34x - 5x^2$$





To draw the Shear Diagram:

(1) For segment AB, $V_{AB} = 114 - 10x$ is linear; at x = 0, V_{AB} = 14 kN; at x = 2 m, V_{AB} = 94 kN.

(2) $V_{BC} = 34 - 10x$ for segment BC is linear; at x = 2 m, V_{BC} = 14 kN; at x = 10 m, V_{BC} = −66 kN. When V_{BC} = 0, x = 3.4 m thus V_{BC} = 0 at 1.4 m from B.

To draw the Moment Diagram:

(1) $M_{AB} = 114x - 5x^2$ is a second degree curve for segment AB; at x = 0, M_{AB} = 0; at x = 2 m, M_{AB} = 208 kN·m.

(2) The moment diagram is also a second degree curve for segment BC given by $M_{BC} = 160 + 34x - 5x^2$; at x = 2 m, M_{BC} = 208 kN·m; at x = 10 m, M_{BC} = 0.

(3) Note that the maximum moment occurs at point of zero shear. Thus, at x = 3.4 m, M_{BC} = 217.8 kN·m.

Problem 406

Beam loaded as shown in Fig. P-406.

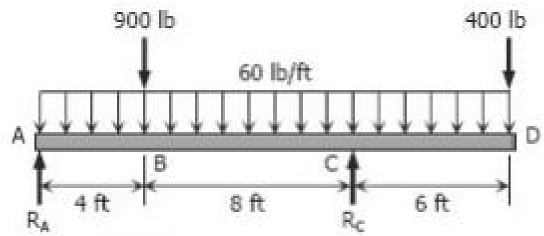


Figure P-406

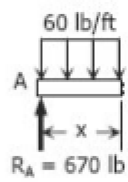
Solution 406

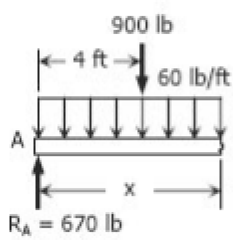
$$\sum M_A = 0$$
$$12R_C = 4(900) + 18(400) + 9[(60)(18)]$$
$$R_C = 1710 \text{ lb}$$

$$\sum M_C = 0$$
$$12R_A + 6(400) = 8(900) + 3[60(18)]$$
$$R_A = 670 \text{ lb}$$

Segment AB:

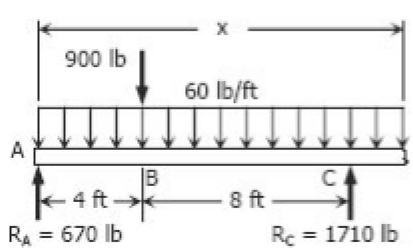
$$V_{AB} = 670 - 60x \text{ lb}$$
$$M_{AB} = 670x - 60x(x/2)$$
$$= 670x - 30x^2 \text{ lb·ft}$$





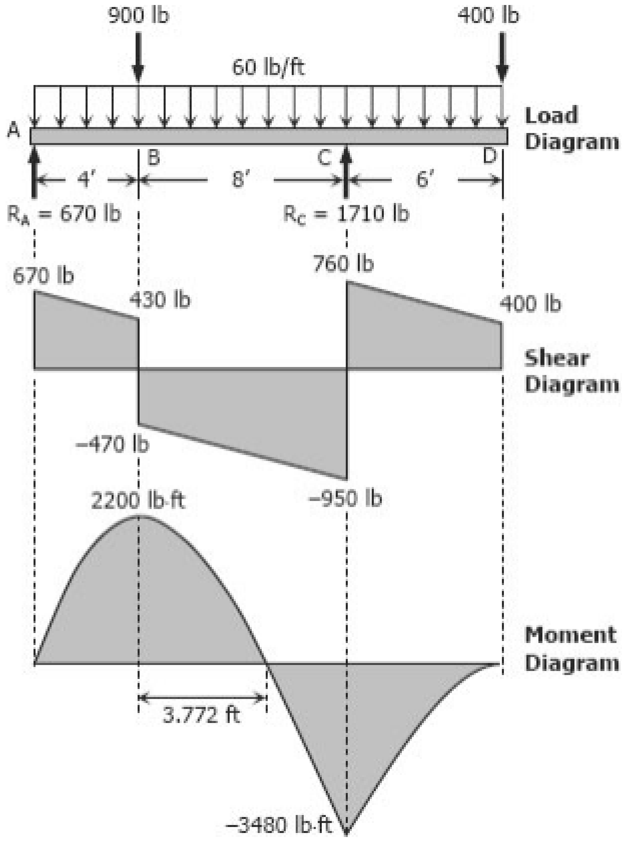
Segment BC:

$$V_{BC} = 670 - 900 - 60x$$
$$= -230 - 60x \text{ lb}$$
$$M_{BC} = 670x - 900(x - 4) - 60x(x/2)$$
$$= 3600 - 230x - 30x^2 \text{ lb·ft}$$



Segment CD:

$$V_{CD} = 670 + 1710 - 900 - 60x$$
$$= 1480 - 60x \text{ lb}$$
$$M_{CD} = 670x + 1710(x - 12)$$
$$- 900(x - 4) - 60x(x/2)$$
$$= -16920 + 1480x - 30x^2 \text{ lb·ft}$$



To draw the Shear Diagram:

(1) $V_{AB} = 670 - 60x$ for segment AB is linear; at $x = 0$, $V_{AB} = 670$ lb; at $x = 4$ ft, $V_{AB} = 430$ lb.

(2) For segment BC, $V_{BC} = -230 - 60x$ is also linear; at $x = 4$ ft, $V_{BC} = -470$ lb, at $x = 12$ ft, $V_{BC} = -950$ lb.

(3) $V_{CD} = 1480 - 60x$ for segment CD is again linear; at $x = 12$, $V_{CD} = 760$ lb; at $x = 18$ ft, $V_{CD} = 400$ lb.

To draw the Moment Diagram:

(1) $M_{AB} = 670x - 30x^2$ for segment AB is a second degree curve; at $x = 0$, $M_{AB} = 0$; at $x = 4$ ft, $M_{AB} = 2200$ lb·ft.

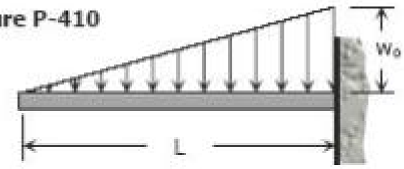
(2) For BC, $M_{BC} = 3600 - 230x - 30x^2$, is a second degree curve; at $x = 4$ ft, $M_{BC} = 2200$ lb·ft, at $x = 12$ ft, $M_{BC} = -3480$ lb·ft; When $M_{BC} = 0$, $3600 - 230x - 30x^2 = 0$, $x = -15.439$ ft and 7.772 ft. Take $x = 7.772$ ft, thus, the moment is zero at 3.772 ft from B.

(3) For segment CD, $M_{CD} = -16920 + 1480x - 30x^2$ is a second degree curve; at $x = 12$ ft, $M_{CD} = -3480$ lb·ft; at $x = 18$ ft, $M_{CD} = 0$.

Problem 410

Cantilever beam carrying the uniformly varying load shown in Fig. P-410.

Figure P-410



Solution 410

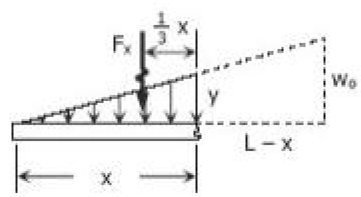
$$\frac{y}{x} = \frac{w_o}{L}$$

$$y = \frac{w_o}{L}x$$

$$F_x = \tfrac{1}{2}xy$$

$$= \frac{1}{2}x\left(\frac{w_o}{L}x\right)$$

$$= \frac{w_o}{2L}x^2$$



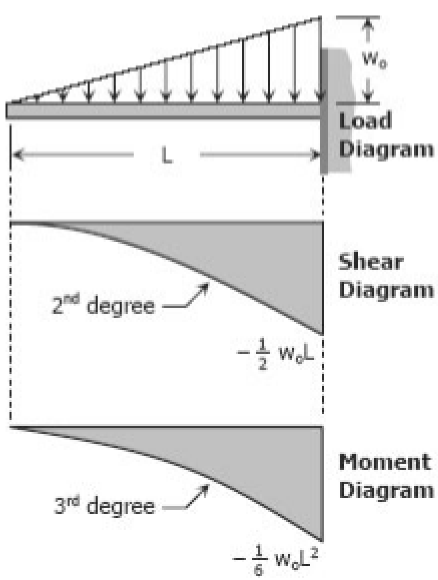
Shear equation:

$$V = -\frac{w_o}{2L}x^2$$

Moment equation:

$$M = -\tfrac{1}{3}xF_x = -\frac{1}{3}x\left(\frac{w_o}{2L}x^2\right)$$

$$= -\frac{w_o}{6L}x^3$$



To draw the Shear Diagram:

$V = -\frac{w_o}{2L}x^2$ is a second degree curve;

at $x = 0$, $V = 0$; at $x = L$, $V = -\frac{1}{2}w_oL$.

To draw the Moment Diagram:

$M = -\frac{w_o}{6L}x^3$ is a third degree curve; at

$x = 0$, $M = 0$; at $x = L$, $M = -\frac{1}{6}w_oL^2$.

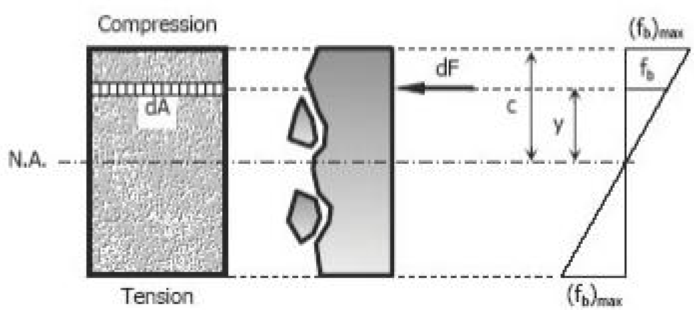
Stresses in Beams

Forces and couples acting on the beam cause bending (flexural stresses) and shearing stresses on any cross section of the beam and deflection perpendicular to the longitudinal axis of the beam. If couples are applied to the ends of the beam and no forces act on it, the bending is said to be pure bending. If forces produce the bending the bending is called ordinary bending.

ASSUMPTIONS

In using the following formulas for flexural and shearing stresses, it is assumed that a plane section of the beam normal to its longitudinal axis prior to loading remains plan after the forces and couples have been applied, and that the beam is initially straight and of uniform cross section and that the modulis of elasticity in tension and compression are equal.

For this chapter, the notation f_b will be used instead of σ, to denote flexural stresses.



$$M = \frac{EI}{\rho} \text{ or } \rho = \frac{EI}{M}$$

SECTION MODULUS

In the formula

$$(f_b)_{max} = \frac{Mc}{I} = \frac{M}{I/c},$$

the ratio I/c is called the section modulus and is usually denoted by S with units of mm^3 (in^3). The maximum bending stress may then be written as

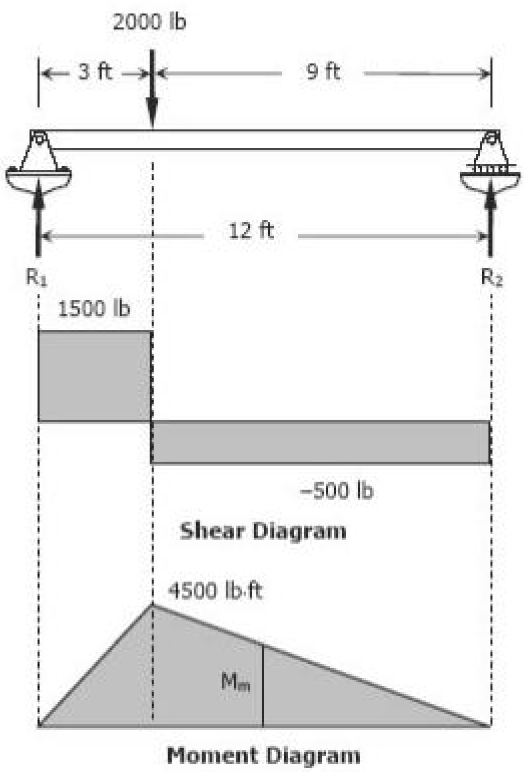
$$(f_b)_{max} = \frac{M}{S}$$

This form is convenient because the values of S are available in handbooks for a wide range of standard structural shapes.

Problem 504

A simply supported beam, 2 in wide by 4 in high and 12 ft long is subjected to a concentrated load of 2000 lb at a point 3 ft from one of the supports. Determine the maximum fiber stress and the stress in a fiber located 0.5 in from the top of the beam at midspan.

Solution 504



$\Sigma M_{R2} = 0$

$12R_1 = 9(2000)$

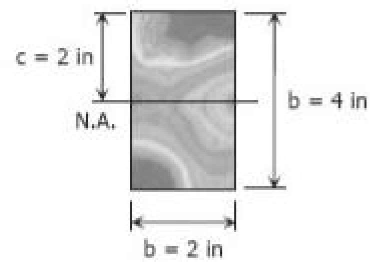
$R_1 = 1500 \text{ lb}$

$\Sigma M_{R1} = 0$

$12R_2 = 3(2000)$

$R_2 = 500 \text{ lb}$

Maximum fiber stress:



$$(f_b)_{max} = \frac{Mc}{I} = \frac{4500(12)(2)}{\dfrac{2(4)^3}{12}}$$

$(f_b)_{max} =$ **10,125 psi**

Stress in a fiber located 0.5 in from the top of the beam at midspan:

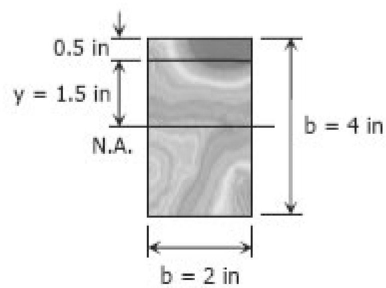
$$\frac{M_m}{6} = \frac{4500}{9}$$

$M_m = 3000 \text{ lb·ft}$

$$f_b = \frac{My}{I}$$

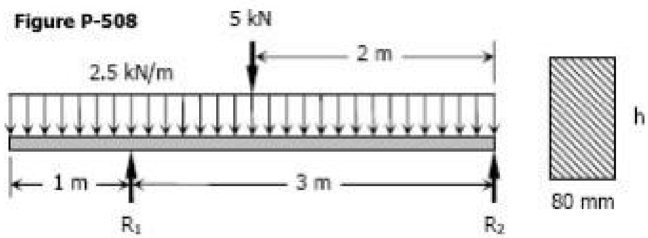
$$f_b = \frac{3000(12)(1.5)}{\dfrac{2(4^3)}{12}}$$

$f_b =$ **5,062.5 psi**

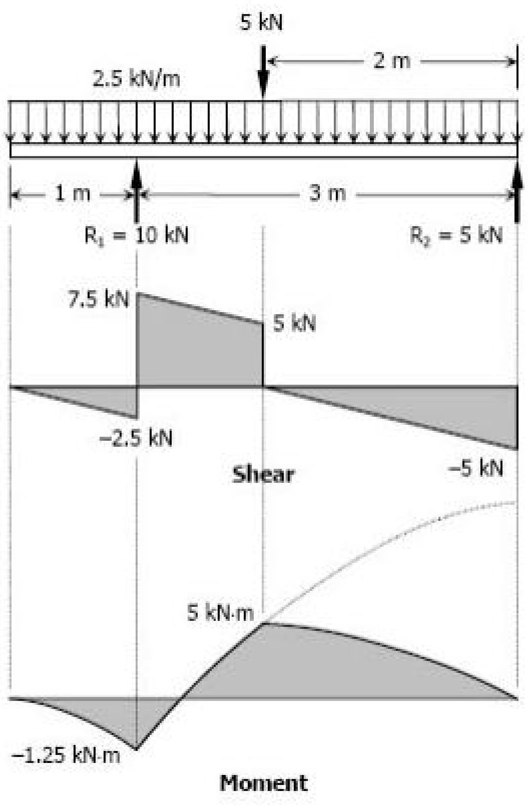


Problem 508

Determine the minimum height h of the beam shown in Fig. P-508 if the flexural stress is not to exceed 20 MPa.



Solution 508



$\Sigma M_{R2} = 0$

$3R_1 = 2(5) + 2(2.5)(4)$

$R_1 = 10 \text{ kN}$

$\Sigma M_{R1} = 0$

$3R_2 = 1(5) + 1(2.5)(4)$

$R_2 = 5 \text{ kN}$

$$f_b = \frac{Mc}{I}$$

Where:

$f_b = 20 \text{ MPa}$

$M = 5 \text{ kN·m}$

$= 5(1000)^2 \text{ N·mm}$

$c = \frac{1}{2}h$

$$I = \frac{bh^3}{12} = \frac{80h^3}{12}$$

$$= \frac{20}{3}h^3$$

Thus,

$$20 = \frac{5(1000)^2(\frac{1}{2}h)}{\frac{20}{3}h^3}$$

$h^2 = 18\,750$

$h = 137 \text{ mm}$

Problem 513

A rectangular steel beam, 2 in wide by 3 in deep, is loaded as shown in Fig. P-513. Determine the magnitude and the location of the maximum flexural stress.

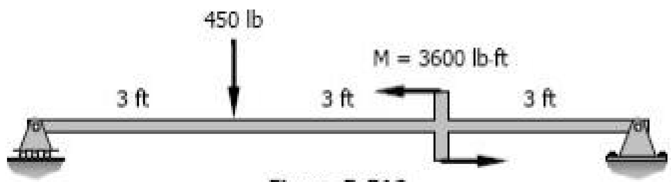
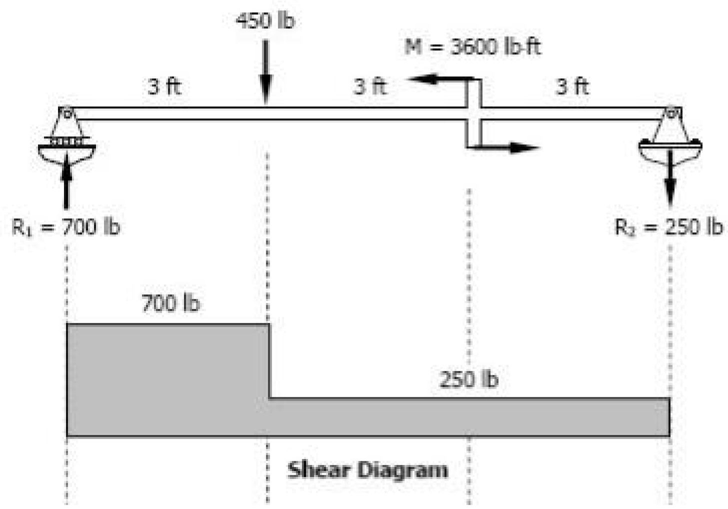


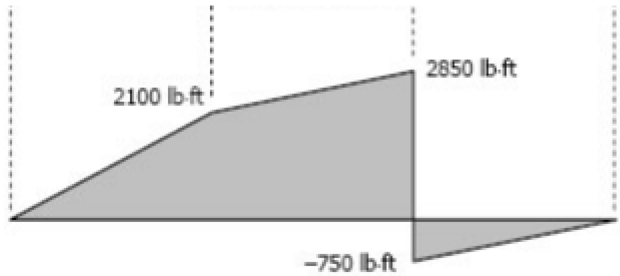
Figure P-513

Solution 513

$$\Sigma M_{R2} = 0$$
$$9R_1 = 6(450) + 3600$$
$$R_1 = 700 \text{ lb}$$

$$\Sigma M_{R1} = 0$$
$$9R_2 + 3(450) = 3600$$
$$R_2 = 250 \text{ lb}$$





Moment Diagram

$$(f_b)_{max} = \frac{Mc}{I}$$

where $M = 2850$ lb·ft
$c = h/2 = 3/2$
$= 1.5$ in
$I = \dfrac{bh^3}{12} = \dfrac{2(3^3)}{12}$
$= 4.5$ in^4

$$(f_b)_{max} = \frac{2850(12)(1.5)}{4.5}$$

$(f_b)_{max}$ **= 11400 psi @ 3 ft from right support**

Problem 524

A beam with an S380 &times 74 section carries a total uniformly distributed load of 3W and a concentrated load W, as shown in Fig. P-524. Determine W if the flexural stress is limited to 120 MPa.

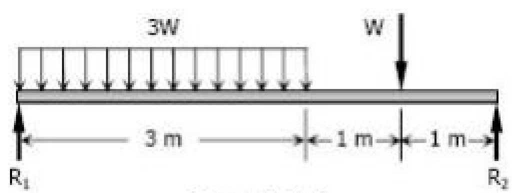
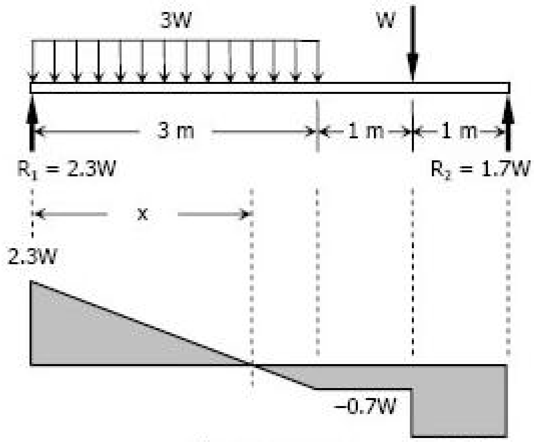


Figure P-524

Solution 524



Shear Diagram

$$\Sigma M_{R2} = 0$$
$$5R_1 = 3W(3.5) + W(1)$$
$$R_1 = 2.3W$$

$$\Sigma M_{R1} = 0$$
$$5R_2 = 3W(1.5) + W(4)$$
$$R_2 = 1.7W$$

$$2.3W - Wx = 0$$
$$x = 2.3\text{ m}$$

$$M_{max} = \tfrac{1}{2}x\,(2.3W)$$
$$M_{max} = \tfrac{1}{2}(2.3)(2.3W)$$
$$M_{max} = 2.645W$$

From Appendix B, Table B-3 Properties of I-Beam Sections (S-Shapes): SI Units, of text book.

Designation............... S380 × 74

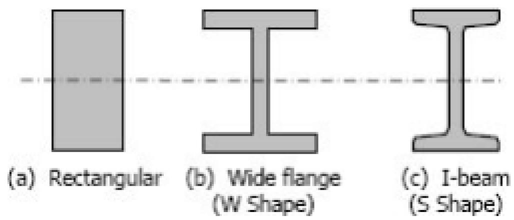
S.............................. 1 060 × 10^3 mm^3

$$(f_b)_{max} = \frac{M}{S}$$
$$120 = \frac{2.645W(1000)}{1060 \times 10^3}$$
$$W = 48\ 090.74\text{ N}$$

Economic Sections

From the flexure formula fb = My / I, it can be seen that the bending stress at the neutral axis, where y = 0, is zero and increases linearly outwards. This means that for a rectangular or circular section a large portion of the cross section near the middle section is under stressed.

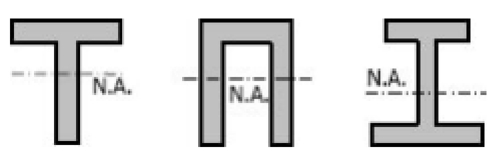
For steel beams or composite beams, instead of adopting the rectangular shape, the area may be arranged so as to give more area on the outer fiber and maintaining the same overall depth, and saving a lot of weight**.**



When using a wide flange or I-beam section for long beams, the compression flanges tend to buckle horizontally sidewise. This buckling is a column effect, which may be prevented by providing lateral support such as a floor system so that the full allowable stresses may be used, otherwise the stress should be reduced. The reduction of stresses for these beams will be discussed in steel design. In selecting a structural section to be used as a beam, the resisting moment must be equal or greater than the applied bending moment.

Unsymmetrical Beams

Flexural Stress varies directly linearly with distance from the neutral axis. Thus for a symmetrical section such as wide flange, the compressive and tensile stresses will be the same. This will be desirable if the material is both equally strong in tension and compression. However, there are materials, such as cast iron, which are strong in compression than in tension. It is therefore desirable to use a beam with unsymmetrical cross section giving more area in the compression part making the stronger fiber located at a greater distance from the neutral axis than the weaker fiber. Some of these sections are shown below.



The proportioning of these sections is such that the ratio of the distance of the neutral axis from the outermost fibers in tension and in compression is the same as the ratio of the allowable stresses in tension and in compression. Thus, the allowable stresses are reached simultaneously. In this section, the following notation will be use:

f_{bt} = flexure stress of fiber in tension

f_{bc} = flexure stress of fiber in compression

N.A. = neutral axis

y_t = distance of fiber in tension from N.A.

y_c = distance of fiber in compression from N.A.

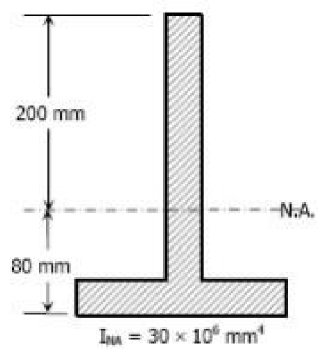
M_r = resisting moment

M_c = resisting moment in compression

M_t = resisting moment in tension

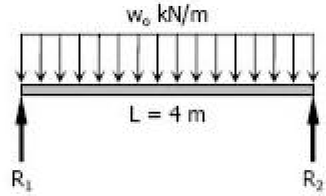
Problem 548

The inverted T section of a 4-m simply supported beam has the properties shown in Fig. P-548. The beam carries a uniformly distributed load of intensity w_o over its entire length. Determine wo if $f_{bt} \leq 40$ MPa and $f_{bc} \leq 80$ MPa.



Solution 548

$$M_{max} = \frac{1}{8} w_o L^2$$
$$= \frac{1}{8} w_o (4^2)$$
$$= 2w_o$$



Member B – 1
M_{max} = 1/8 w_o L^2

$$M_r = \frac{f_b I}{y}$$

$$M_t = \frac{40(30 \times 10^6)}{80}$$
$$= 15\,000\,000 \text{ N·mm}$$
$$= 15 \text{ kN·mm}$$

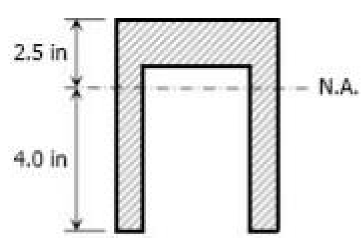
$$M_c = \frac{80(30 \times 10^6)}{200}$$
$$= 12\,000\,000 \text{ N·mm}$$
$$= 12 \text{ kN·mm}$$

The section is stronger in tension and weaker in compression, so compression governs in selecting the maximum moment.

$$M_{max} = M_r$$
$$2w_o = 12$$
$$w_o = 6 \text{ kN/m}$$

Problem 549

A beam with cross-section shown in Fig. P-549 is loaded in such a way that the maximum moments are +1.0P lb·ft and -1.5P lb·ft, where P is the applied load in pounds. Determine the maximum safe value of P if the working stresses are 4 ksi in tension and 10 ksi in compression.



I_{NA} = 192 in^4

Solution 549

At $M = +1.0P$ lb·ft the upper fiber is in compression while the lower fiber is in tension.

$$M = M_r$$

$$M = \frac{f_b I}{y}$$

For fibers in compression (upper fiber):

$$M_c = \frac{10(192)(1000)}{2.5}$$

$$1.0P = 768\,000 \text{ lb·in}$$

$$1.0P = 64\,000 \text{ lb·ft}$$

$$P = 64\,000 \text{ lb}$$

For fibers in tension (lower fiber):

$$M_c = \frac{4(192)(1000)}{4}$$

$$1.0P = 192\,000 \text{ lb·in}$$

$$1.0P = 16\,000 \text{ lb·ft}$$

$$P = 16\,000 \text{ lb}$$

At $M = -1.5P$ lb·ft, the upper fiber is in tension while the lower fiber is in compression.

$$M = M_r$$

$$M = \frac{f_b I}{y}$$

For fibers in compression (lower fiber):

$$M_c = \frac{10(192)(1000)}{4}$$

$$1.5P = 480\,000 \text{ lb·in}$$

$$1.5P = 40\,000 \text{ lb·ft}$$

$$P = 26\,666.67 \text{ lb}$$

For fibers in tension (upper fiber):

$$M_c = \frac{4(192)(1000)}{2.5}$$

$$1.5P = 307\,200 \text{ lb·in}$$

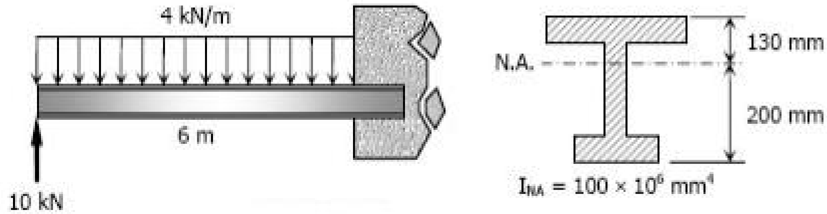
$$1.5P = 25\,600 \text{ lb·ft}$$

$$P = 17\,066.67 \text{ lb}$$

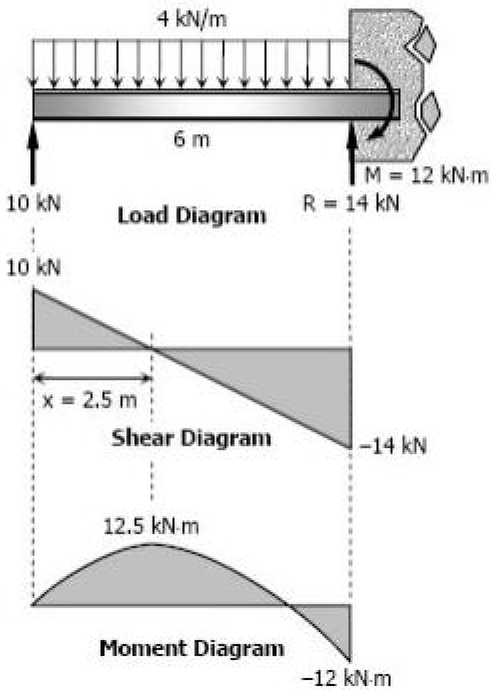
The safe load P = **16 000 lb**

Problem 551

Find the maximum tensile and compressive flexure stresses for the cantilever beam shown in Fig. P-551.



Solution 551



$$M = 4(6)(3) - 10(6)$$
$$= 12\text{ kN}\cdot\text{m}$$

$$R = 4(6) - 10$$
$$= 14\text{ kN}$$

$$\frac{x}{10} = \frac{6-x}{14}$$
$$14x = 60 - 10x$$
$$x = 2.5\text{ m}$$

$$f_b = \frac{My}{I}$$

At $M = +12.5$ kN·m

$$f_{bc} = \frac{12.5(130)(1000^2)}{100 \times 10^6}$$
$$= 16.25\text{ MPa} \quad \rightarrow \text{upper fiber}$$

$$f_{bt} = \frac{12.5(200)(1000^2)}{100 \times 10^6}$$
$$= 25\text{ MPa} \quad \rightarrow \text{lower fiber}$$

At $M = -12$ kN·m

$$f_{bc} = \frac{12(200)(1000^2)}{100 \times 10^6}$$

$$= 24 \text{ MPa} \quad \rightarrow \text{lower fiber}$$

$$f_{bt} = \frac{12(130)(1000^2)}{100 \times 10^6}$$

$$= 15.6 \text{ MPa} \quad \rightarrow \text{upper fiber}$$

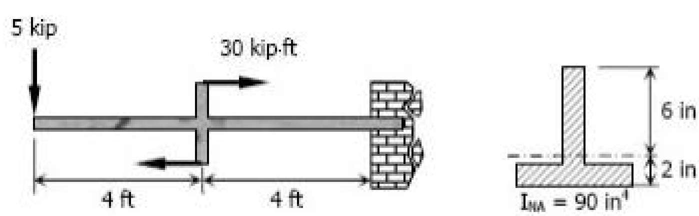
Maximum flexure stresses:

f_{bc} = **24 MPa** at the fixed end

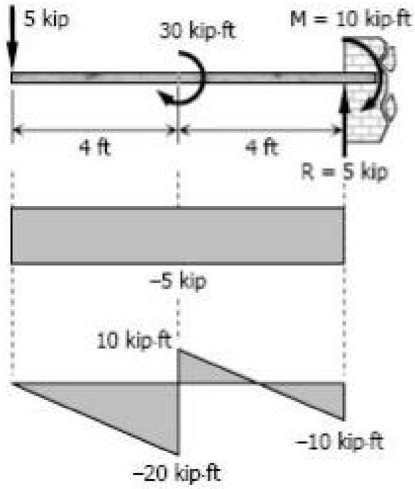
f_{bt} = **25 MPa** at 2.5 m from the free end

Problem 552

A cantilever beam carries the force and couple shown in Fig. P-552. Determine the maximum tensile and compressive bending stresses developed in the beam.



Solution 552



$R = 5\text{ kip}$

$M = 5(8) - 30$

$\quad = 10\text{ kip·ft}$

$$f_b = \frac{My}{I}$$

At $M = +10$ kip·ft of moment diagram

$$f_{bc} = \frac{10(6)(12)}{90}$$

$= 8\text{ ksi}$ → upper fiber

$$f_{bt} = \frac{10(2)(12)}{90}$$

$= 2.67\text{ ksi}$ → lower fiber

At $M = -20$ kip·ft of moment diagram

$$f_{bc} = \frac{20(2)(12)}{90}$$

$= 5.33\text{ ksi}$ → lower fiber

$$f_{bt} = \frac{20(6)(12)}{90}$$

$= 16\text{ ksi}$ → upper fiber

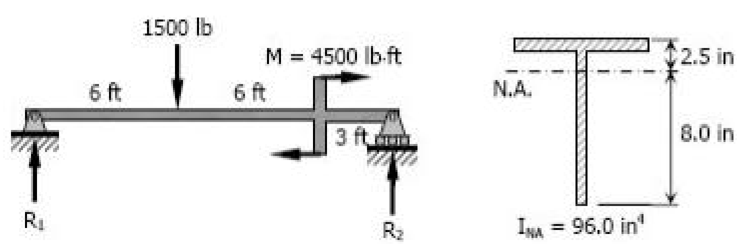
Maximum bending stresses:

$\boldsymbol{f_{bc} = 8\text{ ksi}}$

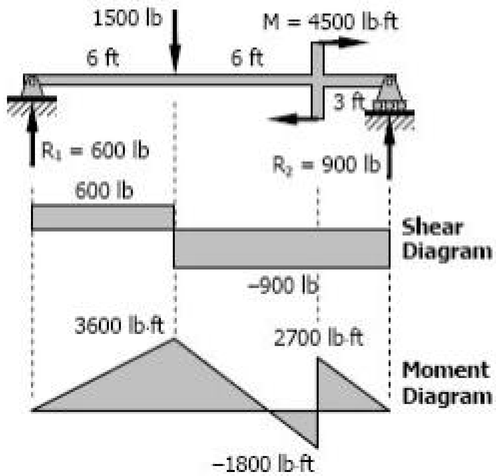
$\boldsymbol{f_{bt} = 16\text{ ksi}}$

Problem 553

Determine the maximum tensile and compressive bending stresses developed in the beam as shown in Fig. P-553.



Solution 553



$\Sigma M_{R2} = 0$
$15R_1 + 4500 = 1500(9)$
$R_1 = 600 \text{ lb}$

$\Sigma M_{R1} = 0$
$15R_2 = 1500(6) + 4500$
$R_2 = 900 \text{ lb}$

$$f_b = \frac{My}{I}$$

At $M = +3600$ lb·ft

$$f_{bc} = \frac{3600(2.5)(12)}{96.0}$$
$= 1125$ psi → upper fiber

$$f_{bt} = \frac{3600(8.0)(12)}{96.0}$$
$= 3600$ psi → lower fiber

At $M = -1800$ lb·ft

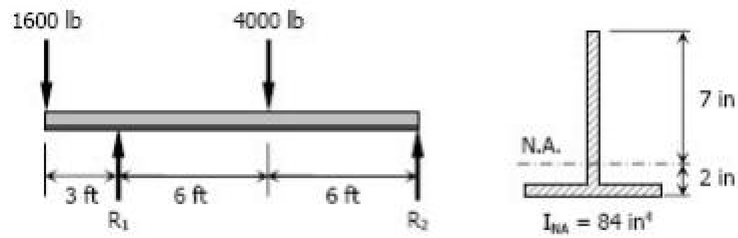
$$f_{bc} = \frac{1800(8.0)(12)}{96.0}$$
$= 1800$ psi → lower fiber

$$f_{bt} = \frac{1800(2.5)(12)}{96.0}$$
$= 562.5$ psi → upper fiber

Maximum flexure stresses
f_{bc} = **1800 psi**
f_{bt} = **3600 psi**

Problem 554

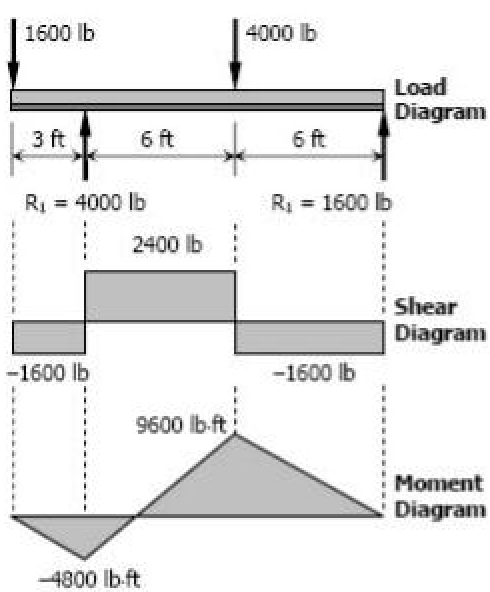
Determine the maximum tensile and compressive stresses developed in the overhanging beam shown in Fig. P-554. The cross-section is an inverted T with the given properties.



Solution 554

$$\Sigma M_{R2} = 0$$
$$12R_1 = 1600(15) + 4000(6)$$
$$R_1 = 4000 \text{ lb}$$

$$\Sigma M_{R1} = 0$$
$$12R_2 + 1600(3) = 4000(6)$$
$$R_2 = 1600 \text{ lb}$$



$$f_b = \frac{My}{I}$$

At $M = -4800$ lb-ft

$$f_{bc} = \frac{4800(2)(12)}{84}$$
$$= 1371.43 \text{ psi} \quad \rightarrow \text{lower fiber}$$

$$f_{bt} = \frac{4800(7)(12)}{84}$$
$$= 4800 \text{ psi} \quad \rightarrow \text{upper fiber}$$

At $M = +9600$ lb-ft

$$f_{bc} = \frac{9600(7)(12)}{84}$$
$$= 9600 \text{ psi} \quad \rightarrow \text{upper fiber}$$

$$f_{bt} = \frac{9600(2)(12)}{84}$$
$$= 2742.86 \text{ psi} \quad \rightarrow \text{lower fiber}$$

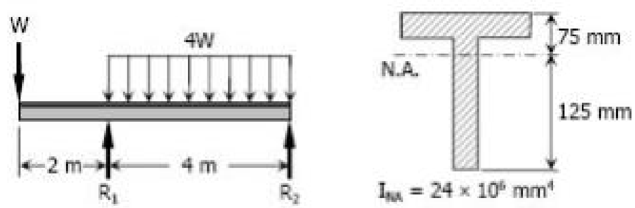
Maximum flexure stress:

$f_{bc} = \mathbf{9600\ psi}$

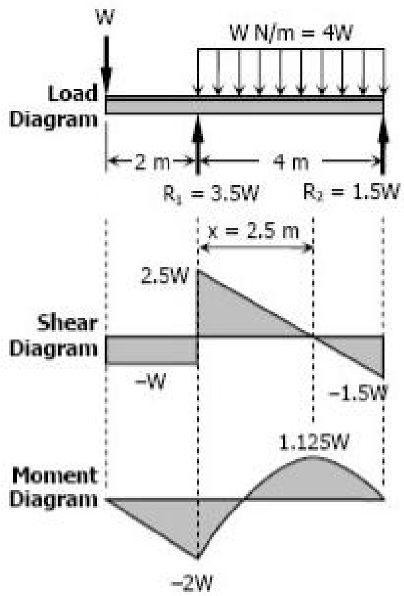
$f_{bt} = \mathbf{4800\ psi}$

Problem 555

A beam carries a concentrated load W and a total uniformly distributed load of 4W as shown in Fig. P-555. What safe value of W can be applied if $f_{bc} \le 100$ MPa and $f_{bt} \le 60$ MPa? Can a greater load be applied if the section is inverted? Explain.



Solution 555



$[\,\Sigma M_{R2} = 0\,]$ $\quad 4R_1 = 6W + 4W(2)$

$R_1 = 3.5W$

$[\,\Sigma M_{R1} = 0\,]$ $\quad 4R_2 + 2W = 4W(2)$

$R_2 = 1.5W$

$$\frac{x}{2.5W} = \frac{4-x}{1.5W}$$

$$1.5Wx = 10W - 2.5Wx$$

$$x = 2.5\text{ m}$$

$$f_b = \frac{My}{I}$$

At $M = -2W$

For lower fiber, $f_{bc} \le 100$ MPa

$$100 = \frac{2W(125)(1000)}{24\times10^6}$$

$$W = 9600\text{ N}$$

For upper fiber, $f_{bt} \le 60$ MPa

$$60 = \frac{2W(75)(1000)}{24\times10^6}$$

$$W = 9600\text{ N}$$

At $M = 1.125W$

For upper fiber, $f_{bc} \le 100$ MPa

$$100 = \frac{1.125W(75)(1000)}{24\times10^6}$$

$$W = 28{,}444.44\text{ N}$$

For lower fiber, $f_{bt} \le 60$ MPa

$$60 = \frac{1.125W(125)(1000)}{24\times10^6}$$

$$W = 10{,}240\text{ N}$$

For safe load W, use $W =$ **9600 N**

Discussion:

At W = 9600 N, the allowable f_b in tension and compression are reached simultaneously when M = $-2W$. This is the same even if the section is inverted. Therefore, no load can be applied greater than W = 9600 N.

Principal Stresses and Principal Planes

Machine design involves, the proper sizing of a machine member to safely withstand the max. stress which is induced within the member when it is subjected separately or to any combination of bending, torsional,axial or transverse loads.
In general ductile materials are weaker in shear are designed on the basis of the max. shear stress, while brittle materials are usually designed on the basis of the max. stress in either tension or compression.

The max. and mim. Normal stresses

The max. and min. stresses are tensile or compressive stresses can be determined for the general case of 2-D on particle by

∴ Maximum principal (or normal) stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

and minimum principal (or normal) stress,

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

Where,

σ_x – is a stress at critical point in tension or compression normal to the cross section
under consideration and may be due to either bending or axial loads or to a combination of two
σ_y – is a stress at the same critical point and in a direction normal to the σ_x
τ_{xy} – is the shear stress at the same critical point acting in the plane normal to the y-
axis (which is the xz plane) and in the plane normal to the x-axis (which is the yz
plane). This shear stress may be due to torsion; moment, a transverse load or to
a combination of the two.

σ_1, σ_2- are called principal stresses and occur on planes that are at 90° to each other called principal planes, These are also planes of zero shear. For 2-D loading, the third principal stress is zero.

The maximum shear stress at the critical point is equal to half of the greatest difference of any two of the three principal stresses

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

The planes of max. shear are inclined at 45° with the principal planes.

Application of Principal Stresses

There are many cases in practice, in which machine members are subjected to combined stresses due to simultaneous action of either tensile or compressive stresses combined with shear stresses. In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses

with shear stresses may be obtained.

Example 1. A hollow shaft of 40 mm outer diameter and 25 mm inner diameter is subjected to a twisting moment of 120 N-m, simultaneously, it is subjected to an axial thrust of 10 kN and a bending moment of 80 N-m. Calculate the maximum compressive and shear stresses

Example 2. A shaft, as shown in Fig.(3.7), is subjected to a bending load of 3 kN, pure torque of 1000 N-m and an axial pulling force of 15 kN.

Calculate the stresses at A and B.

Fig.(3.7)

Example 3. An overhang crank with pin and shaft is shown in Fig(3.8). A tangential load of 15 kN acts on the crank pin. Determine the maximum principal stress and the maximum shear stress at the centre of the crankshaft bearing.

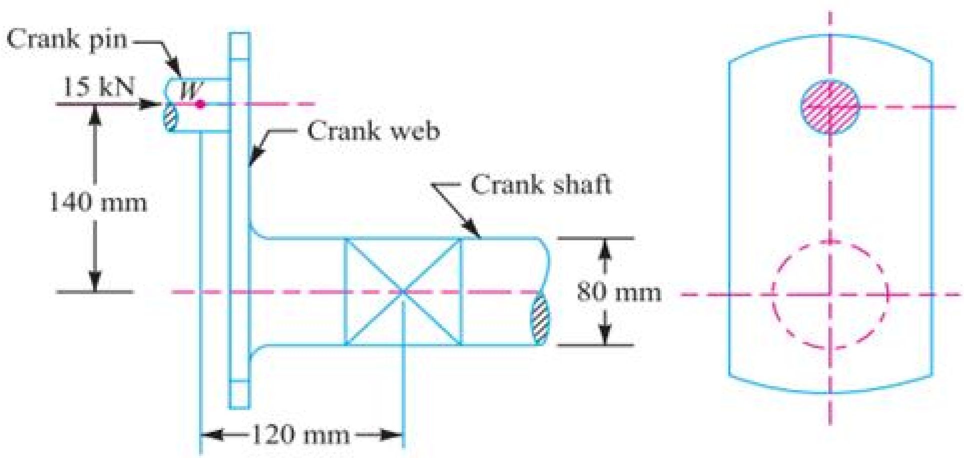
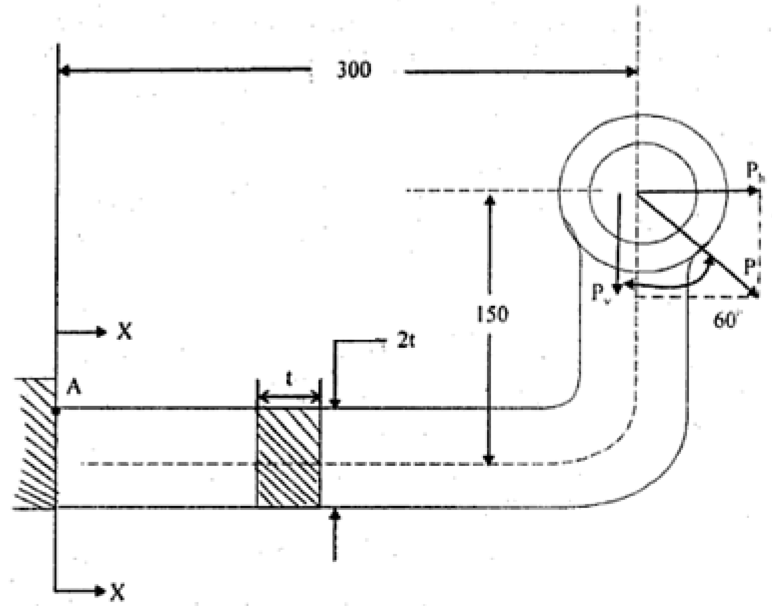


Fig.(3.8)

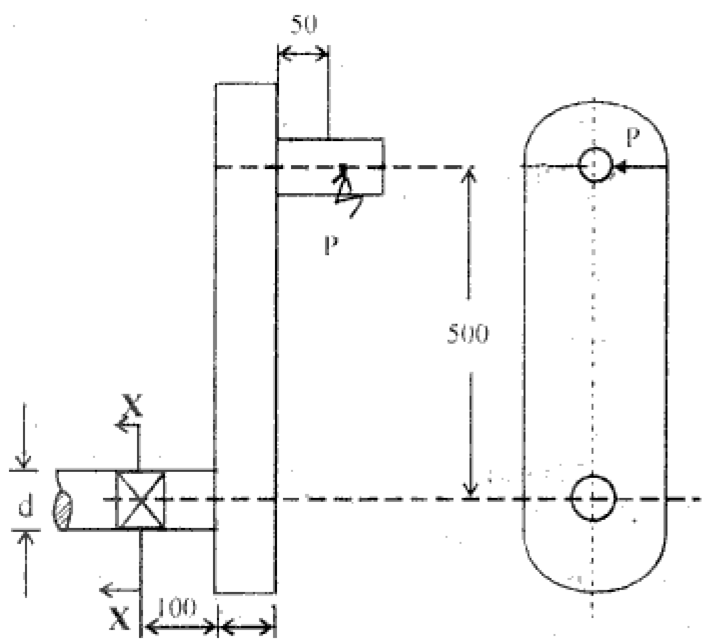
Ex2.

A wall bracket with a rectangular cross-section is shown in Figure 2.12.6. The depth of the cross section is twice the width. The force acting on the bracket at 60^0 to the vertical is 10 kN. The material of the bracket is grey cost iron FG 200 and the factor of safety is 3.5. Determine the dimensions of the cross-section of the bracket using the maximum-normal stress theory of failure.



Ex3.

The dimensions of an overhang crank are given in Figure 2.12.7. The force acting at the crank pin is 2 kN. The crank is made of steel 30 C8 ($\sigma_{yt} = 400\ \text{N/mm}^2$) and the factor of safety is 2. Using the maximur shear stress theory of failure, determine the diameter d at section XX.

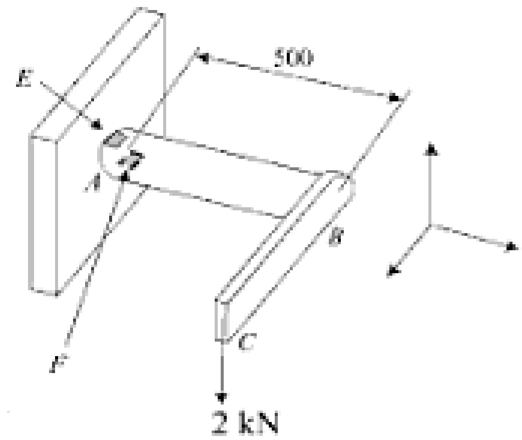


Ex4.

A rod of 50 mm diameter is subjected to a load $F = 2$ kN at the end of a lever (300 mm long) fixed to the bar as shown. Calculate σ_1, σ_2 and τ_{max} at

(a) point E, located at the top surface near fixed end

(b) point F, located at bottom surface near fixed end.

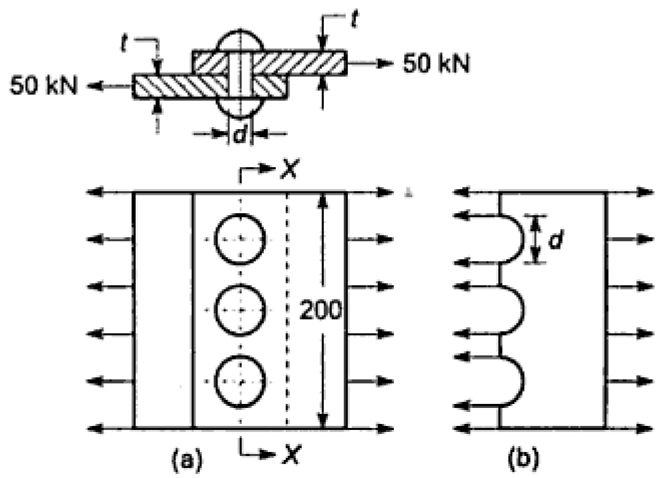


EX.5

Two plates, subjected to a tensile force of 50 kN, are fixed together by means of three rivets as shown in Fig. 3.9 (a). The plates and rivets are made of plain carbon steel 10C4 with a tensile yield strength of 250 N/mm². The yield strength in shear is 50% of the tensile yield strength, and the factor of safety is 2.5. Neglecting stress concentration, determine:

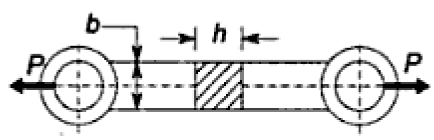
(i) the diameter of the rivets, and

(ii) the thickness of the plates.



Ex.6

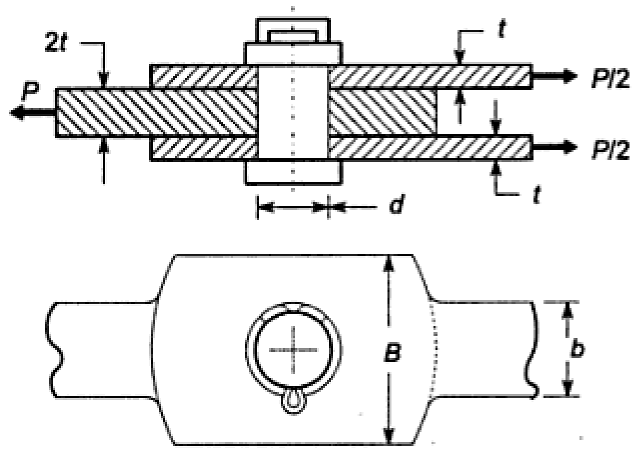
A link, shown in Fig. 3.10, is made of grey cast iron FG150. It transmits a pull P of 10 kN. Assuming that the link has square cross-section ($b = h$) and using a factor of safety of 5, determine the dimensions of the cross-section of the link.



Ex.7

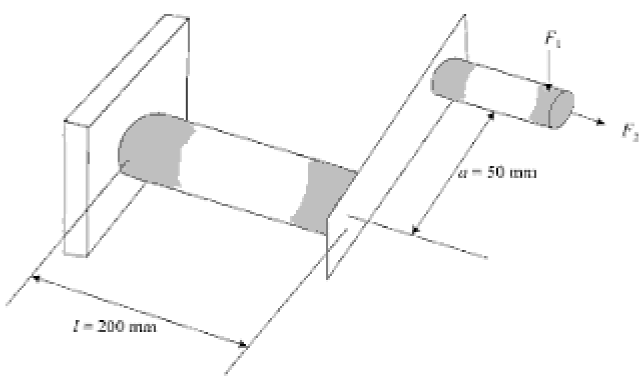
A suspension link, used in the bridge, is shown in Fig. 3.12. The plates and the pin are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 5. The maximum load (P) on the link is 100kN. The ratio of width of the link plate to its thickness (b/t) can be taken as 5. Calculate:

(i) Thickness and width of the link plates,
(ii) Diameter of the knuckle pin,
(iii) Width of the link plate at the centre line of the pin, and
(iv) Crushing stress on the pin.



Ex.9

A circular bar is attached to a wall as shown and is subjected to vertical and horizontal loads of 0.5 and 0.25 kN respectively. Design the bar if it is made of C40 steel (S_{ut} = 500 MPa and S_{yt} = 278 MPa) using maximum shear stress theory. Neglect the effect of direct shear.



Solution

Effect of F_1

(i) to bend the bar (due to bending moment in x–y plane (M_{xy})

$$M_{xy} = F_1 l = (500)(200) = 50\text{kN.mm}$$

(ii) to twist the bar in y–z plane due to twisting moment

$$M_{yz} = F_1\ (500)(50)\text{kN.mm}$$

Effect of F_2

(i) to elongate the bar along x axis due to axial force of F_2

(ii) to bend the bar (due to bending moment in x–z plane (M_{xz})

$$M_{xz} = F_2 a = (250)(50)\text{kN.mm}$$

Resultant loading

Tensile load $F_2 = 0.25$ kN

Bending moment $M = \sqrt{M_{xy}^2 + M_{xz}^2}$

Twisting moment $M_t = M_{yz} = 250$ N.mm

Tensile stress $\sigma_x = \sigma_{x2} + \sigma_{x2} = \dfrac{F_2}{A} + \dfrac{M}{Z}$

$$\sigma_x = \frac{4F}{\pi d^2} + \frac{32M}{\pi d^3}$$

$$\sigma_y = 0$$

$$\tau_{xy} = \frac{16M_t}{\pi d^3}$$

Maximum shear stress theory

$$\sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = \tau_d$$

Substitute and solve for diameter (left for readers as an exercise)

EX.

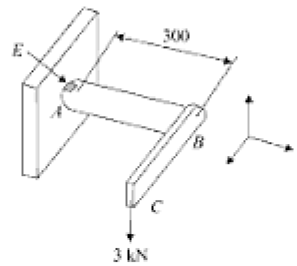
Determine the factor of safety for the rod shown based on both maximum distortion energy theory and maximum shear stress theory. The diameter of rod is 35 mm. The yield strength of the rod material is 150 N/mm^2. The length of the lever is 50 mm.

Solution

Point E will the most stressed element. The tensile stress at E

$$\sigma_x = \frac{M}{Z} = \frac{(3)(10)^3\,(300)}{\frac{\pi}{32}(50)^3} = 73.35\ \text{N/mm}^2$$

Maximum shear stress



$$\tau_{xy} = \frac{M_t}{J} r_{\max} = \frac{(3)(10)^3(300)}{\frac{\pi}{32}(50)^4}(25)\ \text{N/mm}^2$$

Principal stresses $\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$

$$\sigma_{1,2} = \frac{73.35}{2} \pm \sqrt{\left(\frac{73.35}{2}\right)^2 + (40.5)^2}$$

$$\sigma_1 = 95.7 \text{ MPa} \quad \sigma_2 = -17.3 \text{ MPa}$$

Maximum shear stress

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = 56.53 \text{ MPa}$$

Using maximum shear stress theory

$$n = \frac{S_{yt}}{2n} = \frac{150}{(2)(56.53)} = 1.33$$

Using maximum distortion energy theory

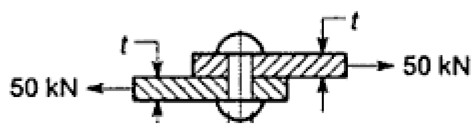
$$n = \frac{S_{yt}}{\sqrt{\sigma_x^2 + 3\tau_{xy}^2}} = \frac{150}{\sqrt{73.35^2 + (56.53)^2}} = 1.23$$

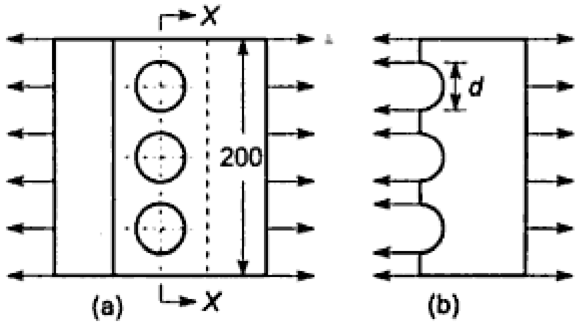
EX.10

Two plates, subjected to a tensile force of 50 kN, are fixed together by means of three rivets as shown in Fig. 3.9 (a). The plates and rivets are made of plain carbon steel 10C4 with a tensile yield strength of 250 N/mm^2. The yield strength in shear is 50% of the tensile yield strength, and the factor of safety is 2.5. Neglecting stress concentration, determine:

(i) the diameter of the rivets, and

(ii) the thickness of the plates.





Solution

For rivets,

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.50\left(S_{yt}\right)}{(fs)} = \frac{0.50\,(250)}{(2.5)} = 50\,\mathrm{N/mm^2}$$

Since there are three rivets,

$$3\left[\frac{\pi}{4}d^2\right]\tau = P$$

or $$3\left[\frac{\pi}{4}d^2\right]50 = 50 \times 10^3$$

or $$d = 20.60 \text{ or } 22 \text{ mm} \qquad \text{(i)}$$

For plates,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{250}{2.5} = 100\ \mathrm{N/mm^2}$$

As shown in Fig. 3.9 (b),

$$\sigma_t\,(200 - 3d)\,t = P$$

or $$100(200 - 3 \times 22)\,t = 50 \times 10^3$$

or $$t = 3.73 \quad \text{or} \quad 4 \text{ mm} \qquad \text{(ii)}$$

Example 3.4

A suspension link, used in the bridge, is shown in Fig. 3.12. The plates and the pin are made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 5. The maximum load (P) on the link is 100kN. The ratio of width of the link plate to its thickness (b/t) can be taken as 5. Calculate:

(i) Thickness and width of the link plates,
(ii) Diameter of the knuckle pin,
(iii) Width of the link plate at the centre line of the pin, and
(iv) Crushing stress on the pin.

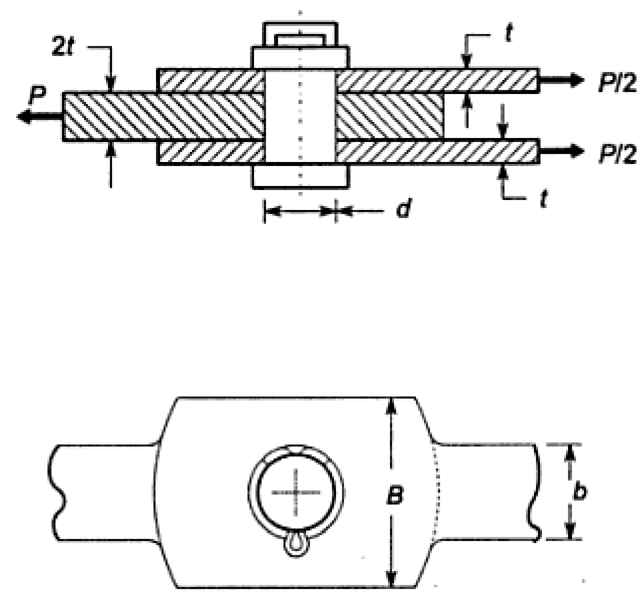


Fig. 3.12 Suspension Link

Solution Permissible stresses :

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{0.5\, S_{yt}}{(fs)} = \frac{0.5\,(400)}{5} = 40 \text{ N/mm}^2$$

Considering tensile failure of link plates,

$$P = (2t \times b)\ \sigma_t$$

$$(100 \times 10^3) = (2t \times 5t)\ (80)$$

$$\therefore \quad t = 11.18 \text{ or } 12 \text{ mm}$$

$$b = 5t = 5(12) = 60 \text{ mm} \qquad \text{(i)}$$

Considering shear failure of the knuckle pin,

$$P = 2\left[\frac{\pi}{4} d^2\right]\tau$$

Substituting values,

$$(100 \times 10^3) = 2\left[\frac{\pi}{4} d^2\right](40)$$

$$\therefore \quad d = 39.89 \text{ or } 40 \text{ mm} \qquad \text{(ii)}$$

The width of the link plate at the centre line of the knuckle pin is given by,

$$B = b + d = 60 + 40 = 100 \text{ mm} \qquad \text{(iii)}$$

The projected area of pin in each side plate is $(d \times t)$. Therefore,

$$\sigma_c = \frac{(P/2)}{(d \times t)}$$

$$= \frac{(50 \times 10^3)}{(40 \times 12)}$$

$$= 104.17 \text{ N/mm}^2$$

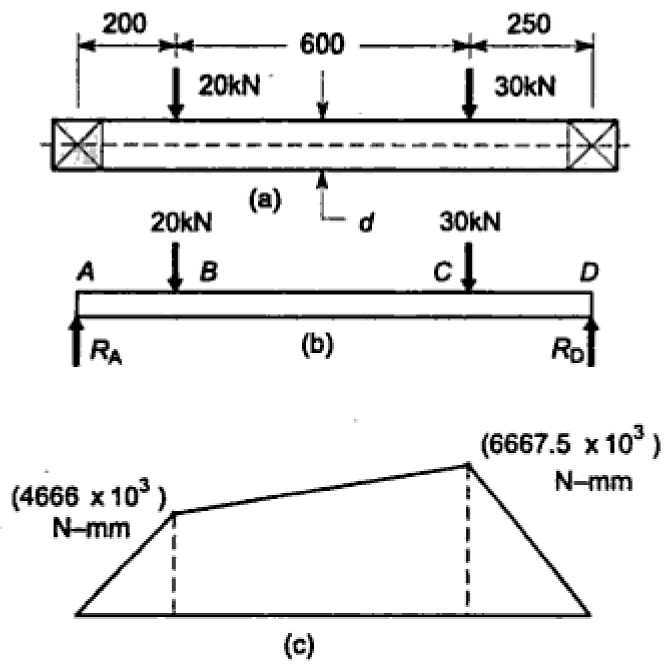
Example 3.5

The forces exerted by the levers of the pump on a rocking shaft are shown in Fig. 3.13(a). The rocking shaft does not transmit torque. It is made of plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 5. Calculate the diameter of the shaft.

Solution The forces acting on the rocking shaft are shown in Fig. 3.13 (b). Taking moment of forces about bearing A,

$$20 \times 200 + 30 \times 800 = R_D \times 1050$$

$$\therefore \qquad R_D = 26.67 \text{ kN}$$



Considering equilibrium of vertical forces,

$$R_A + R_D = 20 + 30$$

$$R_A + 26.67 = 20 + 30$$

$$\therefore \quad R_A = 23.33 \text{ kN}$$

The bending moment diagram is shown in Fig. 3.13(c). The maximum bending moment at C is (6667.5×10^3) N-mm.
From Eq. (3.12),

$$\sigma_b = \frac{M_b y}{I}$$

From Eq. (3.14),

$$I = \frac{\pi d^4}{64} \quad \text{and} \quad y = \frac{d}{2}$$

Substituting Eq. (3.14) in Eq. (3.12),

$$\sigma_b = \frac{32 M_b}{\pi d^3} \tag{a}$$

The permissible tensile stress is given by,

$$\sigma_t = \frac{S_{yt}}{(fs)} = \frac{400}{5} = 80 \text{ N/mm}^2$$

Substituting above value in Eq. (a),

$$d^3 = \frac{32 M_b}{\pi \sigma_b} = \frac{32\,(6667.5 \times 10^3)}{\pi(80)}$$

$$\therefore \quad d = 94.69 \quad \text{or} \quad 95 \text{ mm}$$

Example 3.7

A shaft transmits 20kW power and rotates at 500 r.p.m. The material of shaft is 50C4 (S_{yt} = 460 N/mm^2) and the factor of safety is 2.

(a) Determine the diameter of shaft on the basis of its shear strength.

(b) Determine the diameter of shaft on the basis of its torsional rigidity, if the permissible angle of twist is 3° per metre length and modulus of rigidity of shaft material is 79 300 N/mm^2.

Solution

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n}$$

$$= \frac{60 \times 10^6 (20)}{2\pi(500)}$$

$$= 381\,971.86 \text{ N-mm}$$

From Eq. (3.24),

$$S_{sy} = 0.5S_{yt} = 0.5(460) = 230 \text{ N/mm}^2$$

and,

$$\tau = \frac{S_{sy}}{(fs)} = \frac{230}{2} = 115 \text{ N/mm}^2$$

From Eq. (3.17),

$$\tau = \frac{M_t r}{J}$$

From Eq. (3.19),

$$J = \frac{\pi d^4}{32}$$

Substituting Eq. (3.19) in Eq. (3.17) and rearranging the terms,

$$d^3 = \frac{16\,M_t}{\pi\tau}$$

Substituting numerical values in above equation,

$$d^3 = \frac{16(381\,971.86)}{\pi(115)}$$

$$\therefore \qquad d = 25.67 \text{ mm} \qquad \text{(a)}$$

From Eq. (3.21),

$$\theta = \frac{584\,M_t l}{Gd^4}$$

$$3 = \frac{584(381\,971.86)(1000)}{79300\,d^4}$$

$$\therefore \qquad d = 31.12 \text{ mm} \qquad \text{(b)}$$

Example 3.8

A hollow shaft is required to transmit 500kW power at 120 r.p.m. The maximum torque is 25% greater than the mean torque. The shaft is made of plain carbon steel 45C8 (S_{yt} = 380 N/mm^2) and the factor of safety is 3.5. The shaft should not twist more than 1.5 degrees in a length of 3 metres. The internal diameter of shaft is (3/8) times of external diameter. The modulus of rigidity of shaft material is 80 kN/mm^2. Determine the external diameter of shaft on the basis of its shear strength and on the basis of permissible angle of twist.

Solution

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n}$$

$$M_t = \frac{60 \times 10^6 (500)}{2\pi (120)}$$

$$= 39\ 788.74 \times 10^3 \text{ N-mm}$$

The maximum torque is 25% greater than the mean torque. Therefore,

$$M_t = 1.25\ (39\ 788.74 \times 10^3)$$

$$= 49\ 735.92 \times 10^3 \text{ N-mm}$$

From Eq. (3.24),

$$S_{sy} = 0.5\ S_{yt} = 0.5(380) = 190 \text{ N/mm}^2$$

$$\tau = \frac{S_{sy}}{(fs)} = \frac{190}{3.5} = 54.29 \text{ N/mm}^2$$

From Eq. (3.20),

$$J = \frac{\pi (d_o^4 - d_i^4)}{32}$$

$$= \frac{\pi}{32}\left[\left(\frac{8}{3}d_i\right)^4 - d_i^4\right]$$

$$= 4.8663\, d_i^4 \text{ mm}^4$$

From Eq. (3.17),

$$\tau = \frac{M_t\, r}{J}$$

Substituting values,

$$54.29 = \frac{49\,735.92 \times 10^3 \left(\frac{1}{2} \times \frac{8d_i}{3}\right)}{4.8663\, d_i^4}$$

$\therefore$

$$d_i = 63.08 \text{ mm}$$

$$d_o = \frac{8d_i}{3}\ \frac{8(63.08)}{3} = 168.22 \text{ mm} \qquad \text{(i)}$$

From Eq. (3.18),

$$\theta = \frac{M_t l}{JG}$$

$$\frac{1.5 \times \pi}{1.80} = \frac{49\,735.62 \times 10^3 (3000)}{(4.8663\, d_i^4)(80\,000)}$$

$\therefore$

$$d_i = 61.86 \text{ mm}$$

$$d_o = \frac{8d_i}{3} = \frac{8(61.86)}{3} = 164.95 \text{ mm} \qquad \text{(ii)}$$