STABILITY & DETERMINACY OF STRUCTURES

1.1. STABILITY OF STRUCTURES:

Before deciding the determinacy or indeterminacy of a structure we should first of all have a structure which is stable. The question of determinacy or indeterminacy comes next. We shall now discuss 2-D or single plane structures. (Defined and accommodated in a single plane).

1.1.1. STABLE STRUCTURE:

A stable structure is the one, which remains stable for any conceivable (imaginable) system of loads. Therefore, we do not consider the types of loads, their number and their points of application for deciding the stability or determinacy of the structure. Normally internal and external stability of a structure should be checked separately and if its overall stable then total degree of indeterminacy should be checked.

1.2. ARTICULATED STRUCTURES:

This may be defined as "A truss, or an articulated structure, composed of links or bars, assumed to be connected by frictionless pins at the joints, and arranged so that the area enclosed within the boundaries of the structure is subdivided by the bars into geometrical figures which are usually triangles."

1.3. CONTINUOUS FRAME:

"A continuous frame is a structure which is dependent, in part, for its stability and load carrying capacity upon the ability of one or more of its joints to resist moment." In other words, one or more joints are more or less rigid.

1.4. DETERMINACY:

A statically indeterminate structure is the one in which all the reactive components plus the internal forces cannot be calculated only from the equations of equilibrium available for a given force system. These equations, of course, are

$$\sum H = 0, \quad \sum V = 0 \quad and \quad \sum M = 0$$

The degree of indeterminacy for a given structure is, in fact, the excess of total number of reactive components or excess of members over the equations of equilibrium available.

It is convenient to consider stability and determinacy as follows.

a) With respect to reactions, i.e. external stability and determinacy.

b) With respect to members, i.e. internal stability and determinacy.

c) A combination of external and internal conditions, i.e. total stability and determinacy.

1.4.1. EXTERNAL INDETERMINACY:

A stable structure should have at least three reactive components, (which may not always be sufficient) for external stability of a 2-D structure, which are non-concurrent and non-parallel.

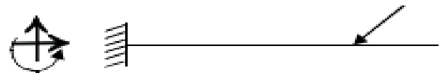


Fig. 1.1. Stable & determinate.

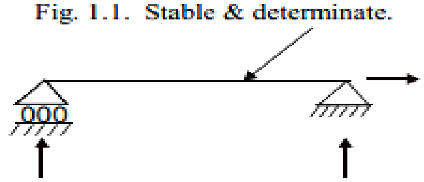


Fig. 1.2. Stable & determinate.

External indeterminacy is, in fact, the excess of total number of reactive components over the equations of equilibrium available.

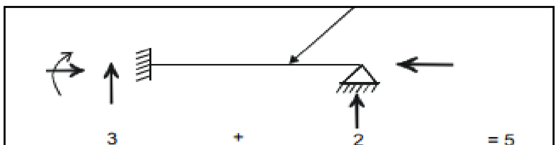


Fig. 1.3.

No. of reactions possible = 5
No. of Equations of equilibrium available = 3
Degree of External indeterminacy = 5 – 3 = 2

Stable & Indeterminate to 2nd degree.

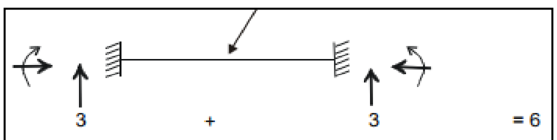


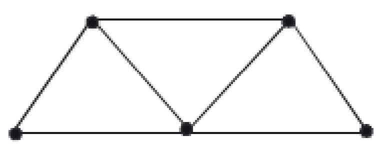
Fig. 1.4

Stable & Indeterminate to 3rd degree.

1.4.2. INTERNAL INDETERMINACY:

This question can be decided only if the minimum number of reactive components necessary for external stability and determinacy are known and are acting on the structure. This type of indeterminacy is normally associated with articulated structures like trusses. We assume that the structure whose internal indeterminacy is being checked is under the action of minimum reactive components required for external stability at the supports.

The basic form of the truss is a triangle. To make the truss, add two members and one joint and repeat.



Let us assume that

j = Total number of joints.

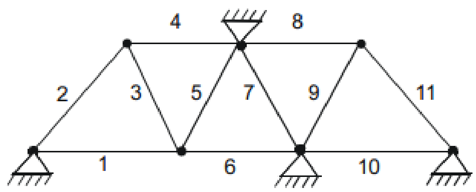
b = Total number of bars.

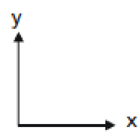
r = Minimum number of reactive components required for external stability/determinacy.

b + r ***(total number of unknowns)*** **= 2j** ***(total number of equations available (at joints))***

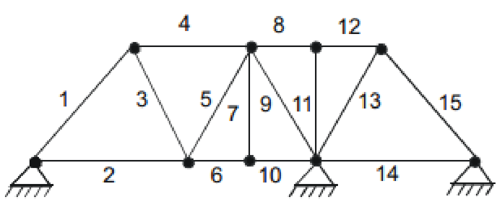
Case	Decisions
If b + r = 2 j	Stable & internally determinate. Check the arrangement of members also.
If b + r > 2 j	Stable & internally indeterminate. (degree of indeterminacy would be decided by the difference of these two quantities).
If b + r < 2 j	Unstable.

A structure is said to have determinacy or indeterminacy only if it is stable. Now we consider some examples.



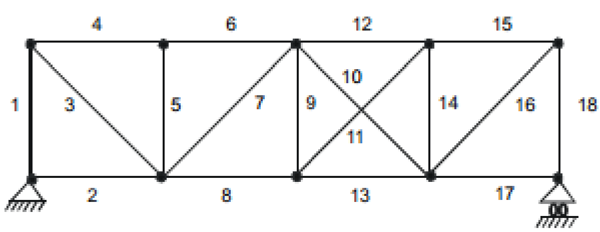


b = 11
r = 3
j = 7
b + r = 2 j
11 + 3 = 2 × 7
14 = 14



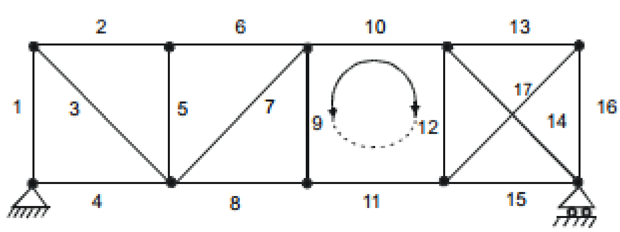
This truss is stable and internally determinate.

b = 15
r = 3
j = 9
b + r = 2 j
15 + 3 = 2 × 9
18 = 18



b = 18
r = 3
j = 10
b + r = 2 j
18 + 3 = 2 × 10
21 > 20

The truss is stable and internally determinate.



b = 16
r = 3
j = 10
b + r = 2 j
17 + 3 = 2 × 10
20 = 20

This truss is stable & internally indeterminate to 1st degree.

Special Case:

This truss is Unstable by inspection although the criterion equation is satisfied. The members in indicated square may get displaced and rotated due to gravity loads. Always inspect member positions. Insert one member in the encircled box or manage prevention of sliding by external supports to make it stable.

NOTE: *- The difference between the internal and the external indeterminacy is only in the definition of 'r'*

1.4.3. TOTAL INDETERMINACY

The question of total indeterminacy is of little interest and we have got different equations for different types of structures. For example, the previous equation, i.e., $b + r = 2j$ can be used to check the total degree of indeterminacy of an articulated structure like truss by slightly modifying the definition of "r" which should now be considered as the "total number of reactive components available".

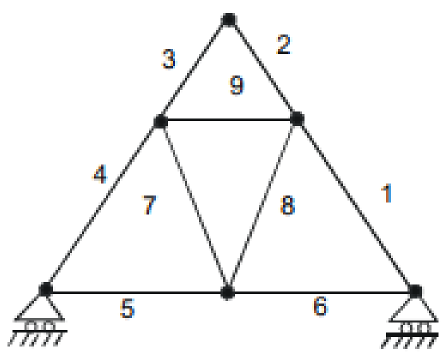
b + r = 2 j

Where **b** = Total number of bars.

r = Total number of reactive components available.

j = Total number of joints

Example No. 1: Determine the external and internal conditions of stability and determinateness for the following structures:-



(i) External Stability And Determinacy:-

Number of reactive components available = 2

Number of equations of equilibrium available = 3

$\therefore$ Unstable. (Visible also)

(ii) Internal Stability And Determinacy

$b = 9$
$r = 3$
$j = 6$
$b + r = 2j$
$9 + 3 = 2 \times 6$
$12 = 12$
Degree of Indeterminacy = $D = 12 - 12 = 0$

$\therefore$ Stable and Internally Determinate, if arrangement is improved to have $\Sigma = 3$.