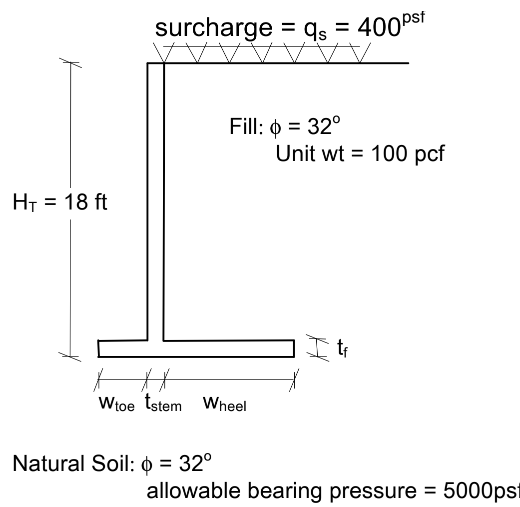
Design a reinforced concrete retaining wall for the following conditions.

f'c = 3000 psi
fy = 60 ksi



Development of Structural Design Equations. In this example, the structural design of the three retaining wall components is performed by hand. Two equations are developed in this section for determining the thickness & reinforcement required to resist the bending moment in the retaining wall components (stem, toe and heel).

Equation to calculate effective depth, d: Three basic equations will be used to develop an equation for d.

$$M_u = \phi M_n$$

$$M_n = A_s f_y \left(d - \frac{a}{2} \right)$$

$$M_u = \phi A_s f_y \left(d - \frac{a}{2} \right) \quad [Eqn\,1]$$

$$C = T, \quad 0.85 f_c^{'} a\, b = A_s f_y$$

$$A_s = 0.85 \frac{f_c^{'}}{f_y} ab \qquad [Eqn\,2]$$

$$strain\ compatibility: \ \frac{0.003}{a/\beta_1} = \frac{\varepsilon_s + 0.003}{d}, \quad \frac{a}{d} = \frac{0.003}{\varepsilon_s + 0.003}\beta_1$$

Assuming $\beta_1 = 0.85$,

ε_s	a/d
0.005	0.319
0.00785	0.235
0.010	0.196

and choosing a value for ε_s in about the middle of the practical design range,

$$\frac{a}{d} = 0.235, \quad a = 0.235d \qquad [Eqn\ 3]$$

Substituting Eqn. 2 into Eqn. 1:

$$M_u = \phi\left(0.85\frac{f_c'}{f_y}ab\right)f_y\left(d-\frac{a}{2}\right)$$

And substituting Eqn. 3 into the above:

$$M_u = \phi\ 0.85\frac{f_c'}{f_y}0.235d\,b\,f_y\underbrace{\left(d-\frac{0.235d}{2}\right)}_{0.883d}$$

Inserting the material properties: f'c = 3 ksi and fy = 60 ksi, and b = 12^{in} (1-foot-wide strip of wall, in the direction out of the paper).

$$M_u = 0.90(0.85)3^{ksi}(12^{in})(0.235)(0.883)d^2$$

$$\boxed{M_u = 5.71^{\frac{k}{in}}d^2}$$

<u>Equation for area of reinforcement, A_s</u>. The area of reinforcement required is calculated from Eqn. 1:

$$M_u = \phi A_s\,f_y\,0.883d = 0.90\,A_s\,60^{ksi}\,0.883d$$

$$\boxed{M_u = 47.7^{ksi}\,A_s\,d}$$

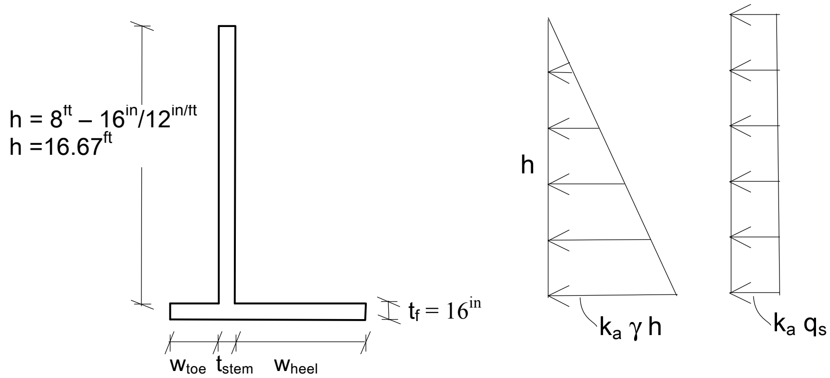
Design Procedure (after Phil Ferguson, Univ. Texas)

<u>1. Determine H_T.</u> Usually, the top-of-wall elevation is determined by the client. The bottom-of-wall elevation is determined by foundation conditions. <u>H_T = 18 feet.</u>

<u>2. Estimate thickness of base.</u> $t_f \approx 7\%$ to 10% H_T (12" minimum)

T_f = 0.07 (18' x 12"/') = 15.1" <u>use t_f = 16"</u>

3. <u>Design stem</u> (t_{stem}, As_{stem}). The stem is a vertical cantilever beam, acted on by the horizontal earth pressure.



<u>calc. d:</u>

$$P_{fill} = \frac{1}{2}(k_a \gamma h)\, h \;\; (1^{ft} \; out \; of \;\; page)$$

$$k_a = \frac{1-\sin\phi}{1+\sin\phi} = \frac{1-\sin(32^o)}{1+\sin(32^o)} = 0.31$$

$$P_{fill} = \frac{1}{2}(0.31)(100pcf)(16.67^{ft})^2 \;\; (1^{ft}) = 4310^{lb}$$

$$P_{sur} = k_a \; q_{sur} h \;\; (1^{ft}) = 0.31(400psf)(16.67^{ft})(1^{ft}) = 2070^{lb}$$

$$M_u = (\text{Earth Pressure LoadFactor})(P_{fill})(\frac{h}{3}) + (\text{Live LoadFactor})(\text{P}_{\text{sur}})(\frac{h}{2})$$

$$M_u = (1.6)(4310^{lb})(\frac{16.67^{ft}}{3}) + (1.6)(2070^{lb})(\frac{16.67^{ft}}{2}) = 65.9^{k-ft}$$

$$M_u = 5.71^{\frac{k}{in}} d^2$$

$$65.9^{k-ft}(12\frac{in}{ft}) = 5.71^{\frac{k}{in}} d^2, \;\; d = 11.8^{in}$$

$$t_{stem} = 11.8^{in} + 2^{in}\,\text{cover} + \frac{1}{2}(1.0^{in}) = 14.3^{in}, \;\; (assume \; \#8 \; bars)$$

<u>*use* $t_{stem} = 15^{in}$</u>

$$d = 15^{in} - 2^{in} - 0.5^{in} = 12.5^{in}$$

calc. A_S:

$$M_u = 47.7^{ksi} A_s d$$

$$65.9^{k-ft}(12\frac{in}{ft}) = 47.7^{ksi} A_s (12.5^{in}),\ \ A_s = 1.33 in^2$$

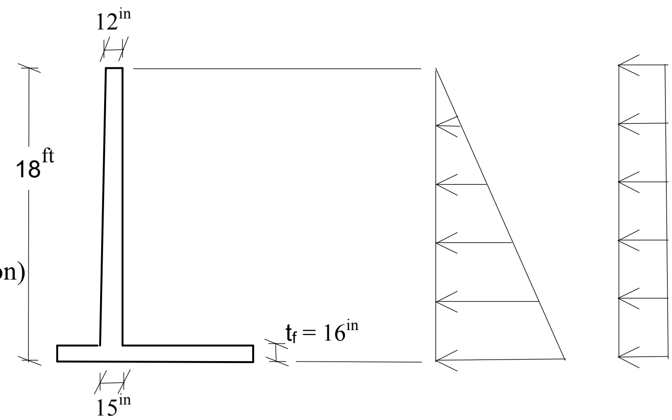
$$A_s\ of\ one\ \#8\ bar = 0.79\ in^2$$

$$\frac{0.79\frac{in^2}{bar}}{1.33\frac{in^2}{ft\ of\ wall}} 12\frac{in}{ft} = 7.13\frac{in}{bar},$$

use #8 @ 6in

4. Choose Heel Width, w_{heel} Select w_{heel} to prevent sliding. Use a key to force sliding failure to occur in the soil (soil-to-soil has higher friction angle than soil-to-concrete).

Neglect soil resistance in front of the wall.



$$set\frac{F_{resist}}{FS} = F_{sliding}$$

$$\text{FS} = \text{Factor of Safety} = 1.5 \text{ for sliding}$$

$$F_{resist} = (\text{Vertical Force})(\text{coefficient of friction})$$

$$F_{resist} = W_T(\tan\phi_{natural\ soil})$$

$$\tan\phi_{natural\ soil} = \tan(32^o) = 0.62$$

$$W_T = W_{fill} + W_{stem} + W_{found}$$

$$W_{fill} = (100pcf)(16.67^{ft})(w_{heel})(1^{ft}) = 1670\frac{lb}{ft} w_{heel}$$

$$W_{stem} = (150pcf)(16.67^{ft})(\frac{12^{in}+15^{in}}{2}\frac{1^{ft}}{12^{in}})(1^{ft}) = 2810^{lb}$$

$$W_{found} = (150pcf)(\frac{16}{12}ft)(w_{heel}+\frac{15}{12}ft+3^{ft})(1^{ft}) = 200plf\ w_{heel} + 850$$

$$F_{sliding} = P_{fill} + P_{sur}$$

$$P_{fill} = \frac{1}{2}(0.31\times 100pcf)(18^{ft})^2(1^{ft}) = 5020^{lb}$$

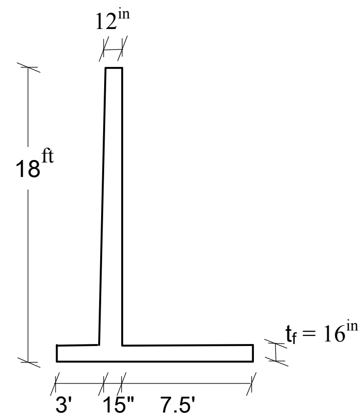
$$P_{sur} = (0.31\times 400psf)(18^{ft})(1^{ft}) = 2230^{lb}$$

$$F_{sliding} = 5020^{lb} + 2230^{lb} = 7250^{lb}$$

$$7250^{lb} = \frac{\left[1670\frac{lb}{ft}w_{heel} + 2810^{lb} + 200\frac{lb}{ft}w_{heel} + 850^{lb}\right](0.62)}{1.5}$$

$$7250^{lb}\frac{1.5}{0.62} = 3660^{lb} + 1870\frac{lb}{ft}w_{heel}, \quad w_{heel} = 7.42^{ft}, \qquad \underline{use\ w_{heel} = 7.5^{ft}}$$

5. Check Overturning.



$$M_{over} = P_{fill}(\frac{18^{ft}}{3}) + P_{sur}(\frac{18^{ft}}{2})$$

$$M_{over} = 5.02^{k}(6^{ft}) + 2.23^{k}(9^{ft}) = 50.2^{k-ft}$$

$$M_{resist} = W_{fill}(\frac{7.5^{ft}}{2} + \frac{15}{12}ft + 3^{ft}), \ assume\ w_{toe} = 3^{ft}$$

$$+W_{stem}(3^{ft} + \frac{1.25^{ft}}{2})$$

$$+W_{found}(\frac{11.75^{ft}}{2})$$

$$M_{resist} = \underbrace{(1.67^{klf} \times 7.5^{ft})}_{12.53^{k}}(8^{ft}) + (2.81^{k})(3.625^{ft}) + \underbrace{(0.20^{klf} \times 7.5^{ft} + 0.85^{k})}_{2.35^{k}}(5.875^{ft})$$

$$M_{resist} = 124.2^{k-ft}$$

$$\frac{M_{resist}}{M_{over}} = \frac{124.2^{k-ft}}{50.2^{k-ft}} = 2.47 > 2.0 = FS_{over}, OK$$

6. Check Bearing.

$$\sigma_v\ at\ end\ of\ toe\ = \frac{W_T}{bL} + \frac{M}{\frac{bL^2}{6}}, \quad \text{equation is valid only if } e < \frac{L}{6}$$

$$W_T = W_{fill} + W_{stem} + W_{found}$$

$$W_T = 12.45^{k} + 2.81^{k} + 2.35^{k} = 17.69^{k}$$

$$M = M_{over} - W_{fill}(5.875^{ft} - \frac{7.5^{ft}}{2}) + W_{stem}(7.5^{ft} + \frac{1.25^{ft}}{2} - 5.875^{ft}) + W_{found}(0)$$

$$M = 50.2^{k-ft} - 12.53^{k}(2.125^{ft}) + 2.81^{k}(2.25^{ft}) = 29.9^{k-ft}$$

<u>Check that e < L/6:</u>

$$e = \frac{m}{W_T} = \frac{29.9^{k-ft}}{17.69^{k}} = 1.68^{ft}, \quad \frac{L}{6} = \frac{11.75^{ft}}{6} = 1.96^{ft}, \quad \therefore e < \frac{L}{6}, \ OK$$

$$\sigma_v = \frac{17.69^k}{(1^{ft})(11.75^{ft})} + \frac{29.9^{k-ft}}{\frac{1}{6}(1^{ft})(11.75^{ft})^2} = 2.80^{ksf} < 5.0^{ksf} = \text{allowable bearing capacity, OK}$$

7. Heel Design.

Max. load on heel is due to the weight of heel + fill + surcharge as the wall tries to tip over.

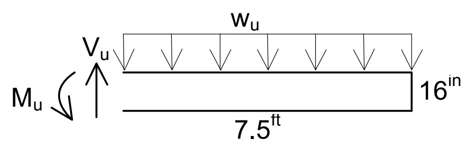
Flexure:

$$W = W_{heel} + W_{fill} + W_{sur}$$

$$W = 1.2(150pcf)(\frac{16}{12}ft)(1^{ft}) + 1.2(100pcf)(16.67^{ft})(1^{ft}) + 1.6(400plf)$$

$$W = 2.88^{klf}$$

$$M_u = \frac{w_u L^2}{2} = \frac{2.88^{klf}(7.5^{ft})^2}{2} = 81.0^{k-ft}$$



$$M_u = 5.71\frac{k}{in} d^2$$

$$81.0^{k-ft}(12\frac{in}{ft}) = 5.71\frac{k}{in}d^2, \quad d = 13.0^{in} \quad \textit{for flexure}$$

Shear:

$$V_u = w_u(7.5^{ft}) = 2.88^{klf}(7.5^{ft}) = 21.6^k$$

$$\phi V_c = (0.75)2\sqrt{f_c'}\, b_w d = (0.75)2\sqrt{3000psi}(12^{in})d$$

$$set\, V_u = \phi V_c, \quad 21{,}600^{lb} = (0.75)2\sqrt{3000psi}(12^{in})d, \quad d = 21.9^{in} \quad \textit{for shear, controls}$$

Shear controls the thickness of the heel.

$$t_{heel} = 21.9^{in} + 2^{in}\text{cover} + \frac{1}{2}in = 24.4^{in} \quad (\textit{assume \#8 bar}), \qquad \underline{use\, t_{heel} = 21.5^{in}}$$

Reinforcement in heel:

$$M_u = 47.7^{ksi} A_s d$$

$$81.0^{k-ft}(12\frac{in}{ft}) = 47.7^{ksi} A_s(21.9^{in}), \quad A_s = 1.07in^2$$

$$\frac{0.79\frac{in^2}{bar}}{1.07\frac{in^2}{ft}}(12\frac{in}{ft}) = 8.83^{in}, \qquad \underline{use\ \#8\ @8''}$$

8. Toe Design.

Earth Pressure at Tip of Toe:

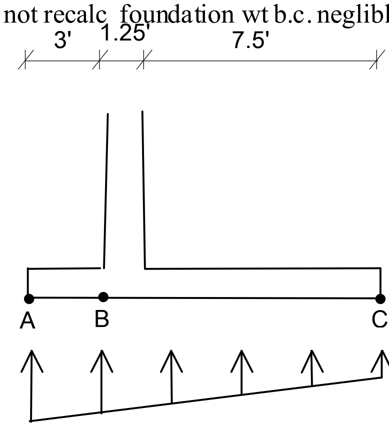
$$\sigma_v = \frac{W_u}{bL} \pm \frac{M_u}{\frac{1}{6}bL^2}$$

$$W_u = 1.2(W_{fill} + W_{stem} + W_{found}) + 1.6(W_{sur})$$

$$W_u = 1.2(12.53^k + 2.81^k + 2.35^k) + 1.6(0.4^{ksf})(18^{ft})(1^{ft}) = 32.7^k,$$ (did not recalc foundation wt b.c. neglible change)

$$M_u = 1.6 M_{over} - 1.2(W_{soil} \times 2.125^{ft} + W_{stem} \times 1.0^{ft})$$

$$M_u = 1.6(50.2^{k-ft}) - 1.2\left[12.53^k(2.125^{ft}) + 2.81^k(1^{ft})\right] = 45.0^{k-ft}$$



$$\sigma_v = \frac{32.7^k}{(1^{ft})(11.75^{ft})} + \frac{45.0^{k-ft}}{\frac{1}{6}(1^{ft})(11.75^{ft})^2}$$

$$\sigma_{v_A} = 2.78^{ksf} + 1.96^{ksf} = 4.74^{ksf}$$

$$\sigma_{v_C} = 2.78^{ksf} - 1.96^{ksf} = 0.82^{ksf}$$

$$\sigma_{v_B} = 0.82^{ksf} + \frac{4.74^{ksf} - 0.82^{ksf}}{11.75^{ft}}(8.75^{ft}) = 3.74^{ksf}$$

d for flexure:

$$M_u = (3.74^{ksf})(3^{ft})(1^{ft})(\frac{3^{ft}}{2}) + \frac{1}{2}(1.00^{ksf})(3^{ft})(1^{ft})(\frac{2}{3}3^{ft}) = 19.8^{k-ft}$$

$$M_u = 5.71\frac{k}{in}\,d^2$$

$$19.8^{k-ft}(12\frac{in}{ft}) = 5.71\frac{k}{in}d^2, \quad d = 6.5^{in} \ \ for\ \ flexure$$

d for shear:

Assume $t_{heel} = t_{toe} = 21.5^{in}$

Critical section for shear occurs at "d" from face of stem, d = 21.5" – 3"cover-1/2"=18"

$$\sigma_{v_{critical\,section}} = 0.82^{ksf} + \frac{4.74^{ksf} - 0.82^{ksf}}{11.75^{ft}}(8.75^{ft} + \frac{18}{12}ft) = 4.24^{ksf}$$

$$V_u = \frac{1}{2}(4.74^{ksf} + 4.24^{ksf})(3^{ft} - \frac{18}{12}ft)(1^{ft}) = 6.74^k$$

$$\phi V_c = (.75)2\sqrt{3000psi}\ (12^{in})(18^{in}) = 17{,}750^{lb} > V_u, \ \ OK, \ \ d\ \ for\ \ flexure\ controls$$

Reinforcement in toe:

$$M_u = 47.7^{ksi} A_s d$$

$$19.8^{k-ft}(12\frac{in}{ft}) = 47.7^{ksi} A_s (18^{in}), \quad A_s = 0.28 in^2$$

$$\frac{0.79\frac{in^2}{bar}}{0.28\frac{in^2}{ft}}(12\frac{in}{ft}) = 33^{in}, \textit{ try smaller bars, say \#4}$$

$$\frac{0.20\frac{in^2}{bar}}{0.28\frac{in^2}{ft}}(12\frac{in}{ft}) = 8.6^{in}$$

use #4 @8"