

One-way and Two-way Slabs

One-way slabs transfer the imposed loads in one direction only. They may be supported on two opposite sides only [Fig. 1.1(a)], in which the structural action is essentially one-way, the loads being carried in direction perpendicular to the supporting beams or walls.

But rectangular slabs often have such proportions and supports (e.g., relatively deep, stiff monolithic concrete beams) that result in two-way action [Fig. 1.1(b)]. At any point, such slabs are curved in both directions resulting in biaxial bending moments. It is convenient to think of such slabs as consisting of two sets of parallel strips, in each direction and intersecting each other. So part of the load is carried by one set and the remainder by the other.

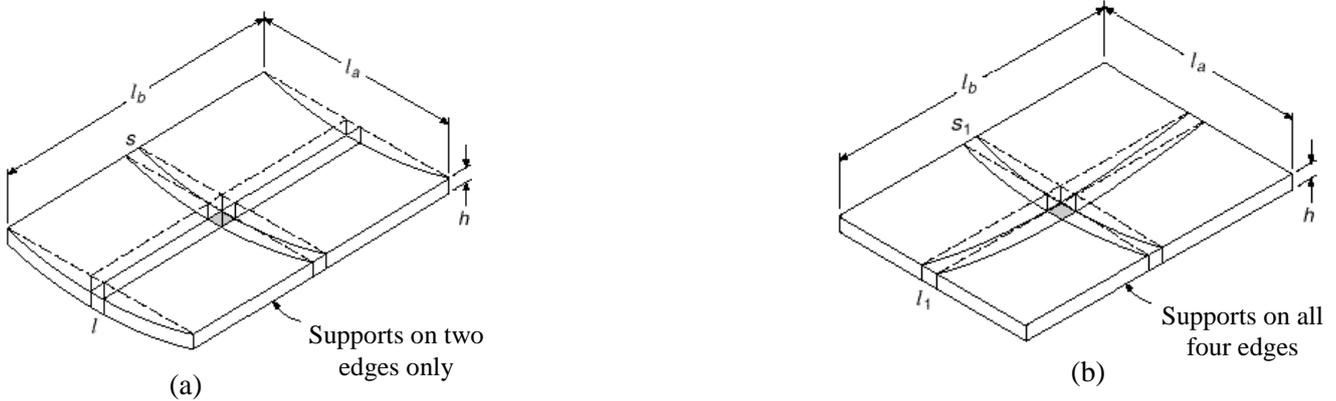


Fig. 1.1: Load transfer in (a) One-way slab, (b) Two-way Slab (Nilson)

Fig. 1.1(b) shows two center strips of a rectangular plate with spans l_a and l_b . For uniformly distributed loads of w per square foot of the slab, each strip acts approximately like a simple beam uniformly loaded by its share of w ; i.e., w_a and w_b . Since they are part of the same slab, their midspan deflections must be the same. Equating the center deflections of the short and long strips

$$5w_a l_a^4 / 384EI = 5w_b l_b^4 / 384EI \Rightarrow w_a / w_b = (l_b / l_a)^4 \dots\dots\dots(1.1)$$

Therefore, larger share of the load is carried in the shorter direction, the ratio of the two portions of the load being inversely proportional to the fourth power of the ratio of spans. For example, if $l_b / l_a = 2$, $w_a / w_b = 16$; i.e., about 94% of the load is carried in the shorter direction and only 6% in the longer direction.

However, this proportion also depends on the support conditions in each direction, because the maximum midspan deflection is $(wl^4/192EI)$ for hinged-fixed ends and $(wl^4/384EI)$ for fixed-fixed ends. Therefore, if $l_b / l_a = 2$ and span l_a is simply supported, about 14% of w is carried by hinged-fixed span l_b and 24% by fixed-fixed span l_b . On the other hand, if l_b is simply supported, it carries only 2.4% of w if l_a is hinged-fixed and 1.2% of w if l_a is fixed-fixed.

For a simply supported square slab, $l_b / l_a = 1$; therefore Eq. (1.1) $\Rightarrow w_a = w_b = w/2$ (1.2)

So if only bending was present, the maximum bending moment in each slab would be

$$M_{max} = (w/2) l^2 / 8 = 0.0625 w l^2 \dots\dots\dots(1.3)$$

However, the actual behavior of a slab is more complex than that of two intersecting strips. As shown in Fig. 1.2, slab can be modeled as grid, some strips of which (particularly the outer strips) are not only bent but also twisted. Consequently, the total load on the slab is carried not only by bending moments in two directions, but also twisting moments. For this reason, bending moments in elastic slabs would be smaller than that would be computed for sets of unconnected strips loaded by w_a and w_b .

The theory of bending of elastic plates shows that the exact maximum bending moment in such a square slab is only $0.048wl^2$. However, a slab does not behave like a beam and would not collapse if only one hypothetical central strip reaches ultimate load. Therefore, in reality this moment can be further reduced to $0.036wl^2$ due to inelastic redistribution of the load among various strips of the slab.

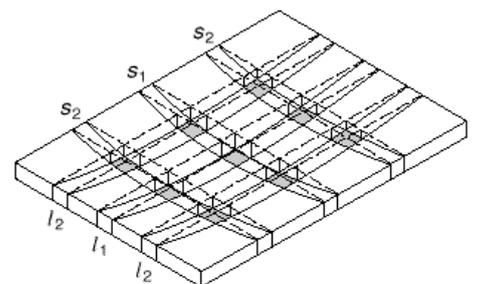


Fig. 1.2: Grid model of two-way slab (Nilson)

Analysis of Two-way Slabs by Coefficient Method

The determination of exact moments in two-way slabs with various support conditions is mathematically formidable and not suited to design practice. Various simplified methods are therefore adopted for determining moments, shears and reactions in such slabs. Quite popular and widely used among these methods is one using ‘Moment Coefficient’ based on the 1963 ACI Code, for the special case of two-way slabs supported on four sides by relatively stiff beams.

The method uses tables of moment coefficients for a variety of support conditions. These coefficients are based on elastic analysis but also accounts for inelastic redistribution. Moments in the middle strips in the two directions are

$$M_{a,max} = C_a w l_a^2 \quad \text{and} \quad M_{b,max} = C_b w l_b^2 \quad \dots\dots\dots(1.4)$$

where C_a, C_b = Tabulated moment coefficients, w = Uniform load per unit area (e.g., psf or kPa)

l_a, l_b = Length of clear span in short and long directions respectively

This method provides the values of $M_{a,max}$ and $M_{b,max}$ along the central strip of the slab, as demonstrated in Fig. 1.3 for a slab simply supported on all sides. As shown, the maximum moments are less elsewhere. Therefore, other design values can be reduced according to the variation shown. These variations in maximum moment across the width and length of a rectangular slab are accounted for approximately by designing the outer quarters of the slab span in each direction for a reduced moment.

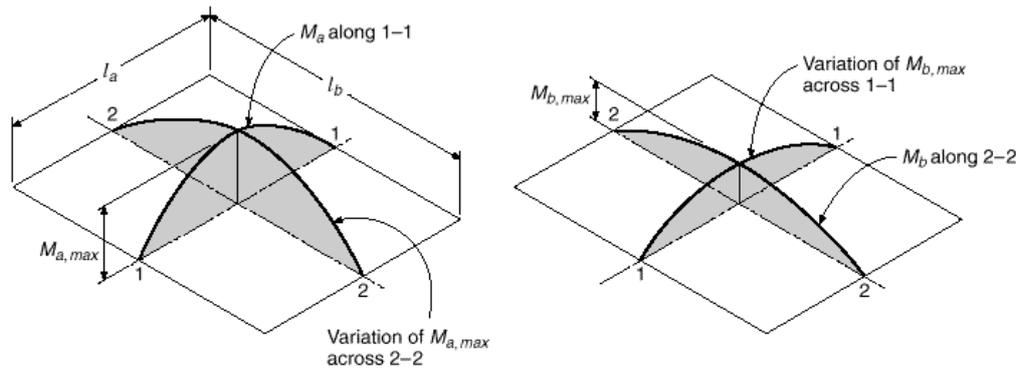


Fig. 1.3: Variation of moments in a uniformly loaded slab simply supported on all sides

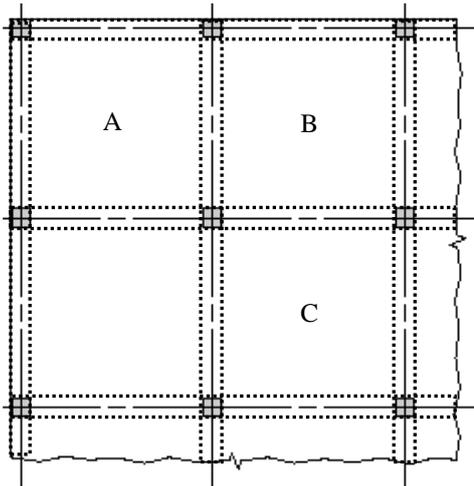


Fig. 1.4: Plan of typical slab

Compared to the idealized ‘simply supported’ slab, Fig. 1.4 shows a more ‘realistic’ scenario where a system of beams supports a two-way slab. For this slab, panel A has two discontinuous exterior edges and two continuous interior edges, panel B has one discontinuous and three continuous edges, while the interior panel C has all edges continuous.

The design bending moments are zero at discontinuous ends, negative at continuous ends and positive at midspans. Fig. 1.5 shows the moment coefficients for positive dead load [$C_{a(D)+}, C_{b(D)+}$] and live load [$C_{a(L)+}, C_{b(L)+}$] as well as negative dead or live load [C_{a-}, C_{b-}].

Maximum positive moments are obtained by putting live loads on alternate spans, while maximum negative moments are obtained by loading adjacent spans, similar to the loading condition for dead loads. So, the live load coefficients are different from dead load coefficients for positive moments, but they are equal for negative moments.

In this figure, the comparatively thin/light lines along perimeters represent discontinuous edges and thick/dark lines represent continuous edges. Therefore, ‘Case 1’ has all discontinuous edges (similar to the slab in Fig. 1.3) and ‘Case 2’ has all continuous edges (similar to the panel C in Fig. 1.4). Moreover, panel A in Fig. 1.4 represents ‘Case 4’ while panel B represents ‘Case 8’ or ‘Case 9’ (depending on the relative slab dimension in both directions).

Example 1.1

Use Fig. 1.5 to determine the moment coefficients for square panels A, B and C in Fig. 1.4.

For square panels, the span ratio = $l_a/l_b = 1.0$, for all the panels. As mentioned, panel A, B and C represents Case 4, 8 (or 9) and 2 respectively. Therefore, Fig. 1.5 \Rightarrow

In Panel A, $C_{a(D)+} = C_{b(D)+} = 0.027$, $C_{a(L)+} = C_{b(L)+} = 0.032$, $C_{a-} = C_{b-} = 0.050$

In Panel B, $C_{a(D)+} = 0.020$, $C_{b(D)+} = 0.023$, $C_{a(L)+} = 0.028$, $C_{b(L)+} = 0.030$, $C_{a-} = 0.033$, $C_{b-} = 0.061$

In Panel C, $C_{a(D)+} = C_{b(D)+} = 0.018$, $C_{a(L)+} = C_{b(L)+} = 0.027$, $C_{a-} = C_{b-} = 0.045$

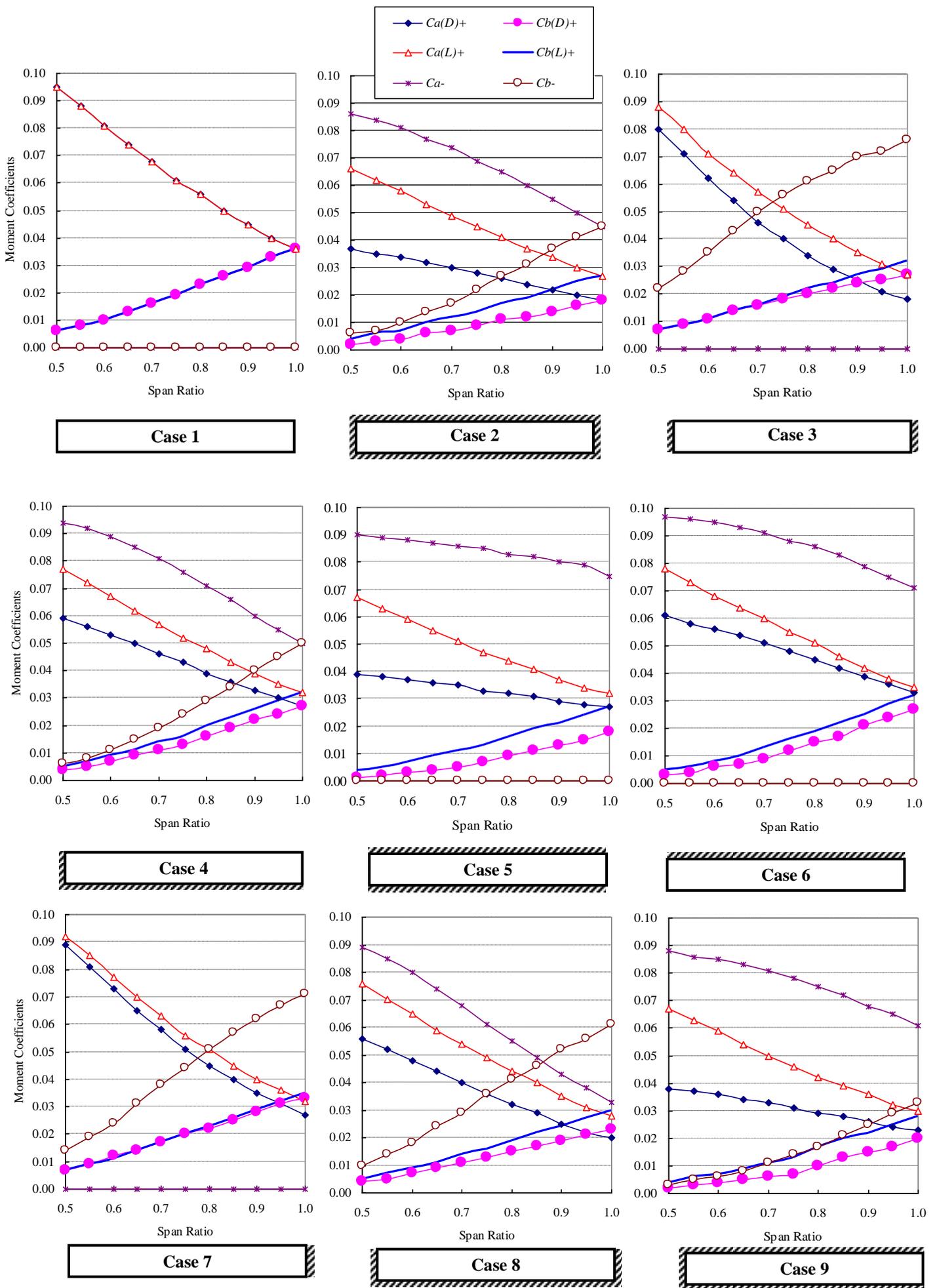
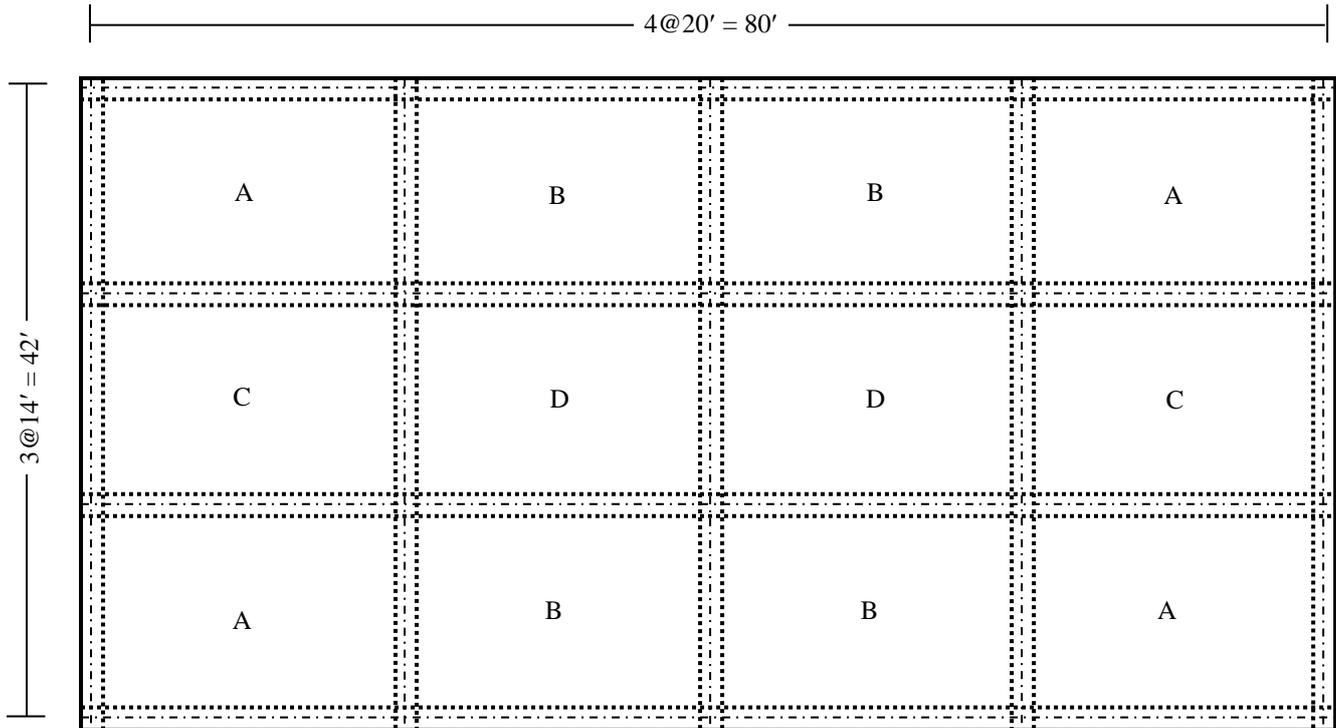


Fig. 1.5: Moment coefficients for different support conditions

Design of Two-way Slabs

Example 1.2

Use (i) WSD, (ii) USD to design the two-way slab shown below, carrying floor finish = 30 psf, random wall = 50 psf and live load = 60 psf [Given: $f_c' = 3$ ksi, $f_y = 50$ ksi].



Clear area for each slab is $(13' \times 19')$; therefore Span Ratio $m = 13/19 = 0.68$.

Slab	l_a	l_b	m	Case	$C_{a(D)+}$	$C_{a(L)+}$	C_{a-}	$C_{b(D)+}$	$C_{b(L)+}$	C_{b-}
A	13	19	0.68	4	0.047	0.058	0.082	0.010	0.013	0.020
B	13	19	0.68	8	0.042	0.057	0.070	0.010	0.013	0.028
C	13	19	0.68	9	0.034	0.052	0.082	0.006	0.010	0.010
D	13	19	0.68	2	0.031	0.050	0.076	0.007	0.011	0.016

\therefore Assumed slab thickness, $t = (13' + 19') \times 2/180 = 4.33''$; i.e., $4.5'' \Rightarrow d = 3.5''$ (or $3''$ for M_{min})

\therefore Self Wt. = 56.25 psf \Rightarrow DL = 56.25 + 30 + 50 = 136.25 psf = 0.136 ksf

LL = 60 psf = 0.06 ksf \Rightarrow Total Load per slab area = 0.136 + 0.06 = 0.196 ksf

Factored DL = $1.4 \times 136.25 = 190.75$ psf = 0.191 ksf, LL = $1.7 \times 60 = 102$ psf = 0.102 ksf

\Rightarrow Total factored load per slab area = 0.191 + 0.102 = 0.293 ksf

For WSD, $n = 9$, $k = 9/(9 + 20/1.35) = 0.378$, $j = 1 - k/3 = 0.874$, $R = \frac{1}{2} \times 1.35 \times 0.378 \times 0.874 = 0.223$ ksi

$$A_s = M/f_j d = M \times 12/(20 \times 0.874 \times 3.5) = M/5.10 \text{ (or } M/4.37 \text{ for } M_{min})$$

$$A_{s(Temp)} = 0.0030 bt = 0.0030 \times 12 \times 4.5 = 0.162 \text{ in}^2/'$$

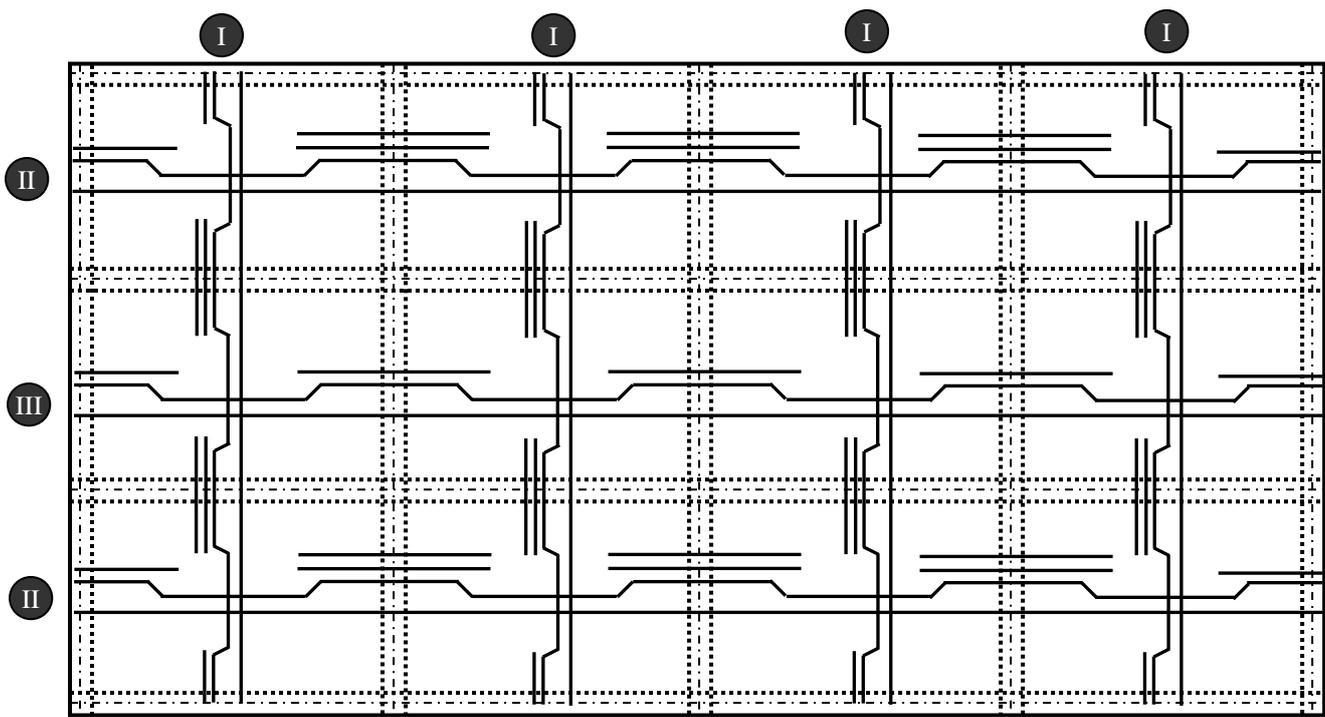
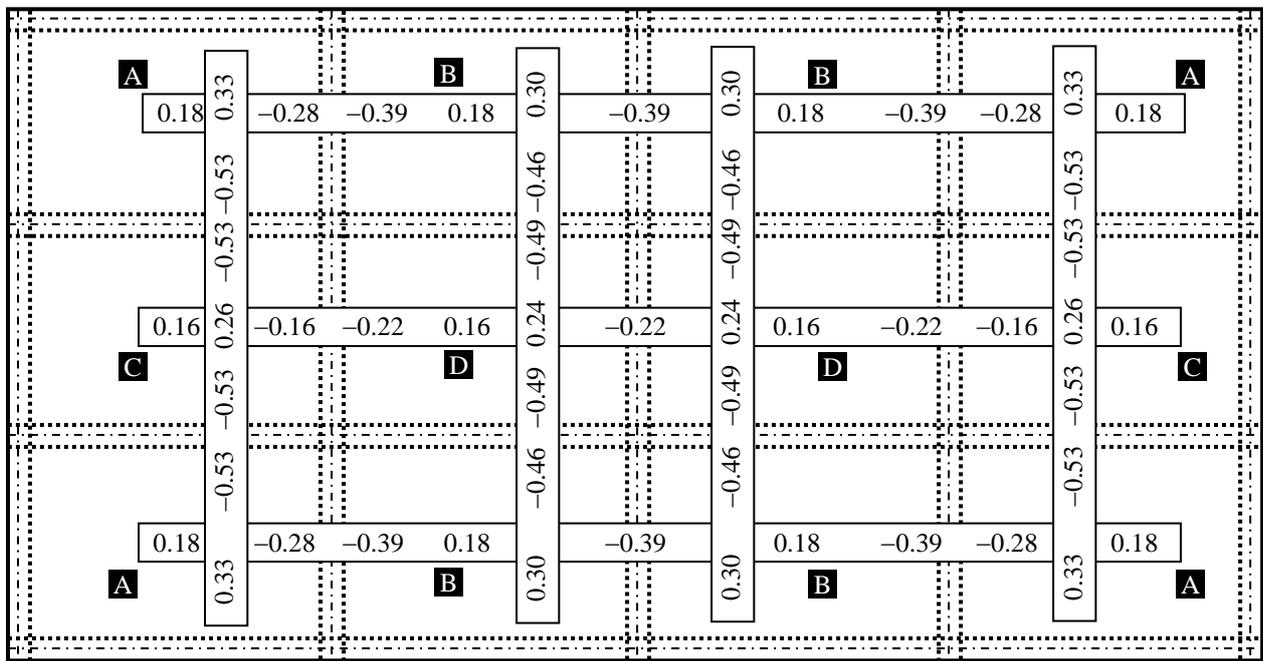
Using moments in k''
and reinforcements in in²/'

Slab	M_{a+}	A_{sa+}	M_{a-}	A_{sa-}	M_{b+}	A_{sb+}	M_{b-}	A_{sb-}
A	1.67	0.33	2.72	0.53	0.77	0.18	1.42	0.28
B	1.55	0.30	2.32	0.46	0.77	0.18	1.98	0.39
C	1.31	0.26	2.72	0.53	0.51	0.16	0.71	0.16
D	1.22	0.24	2.52	0.49	0.58	0.16	1.13	0.22

$$d_{req} = \sqrt{(M_{max}/R)}$$

$$= \sqrt{(2.72/0.223)}$$

$$= 3.49'' < d, \text{ OK}$$



I #3 @4''c/c, alt ckd + 2#3 extra top

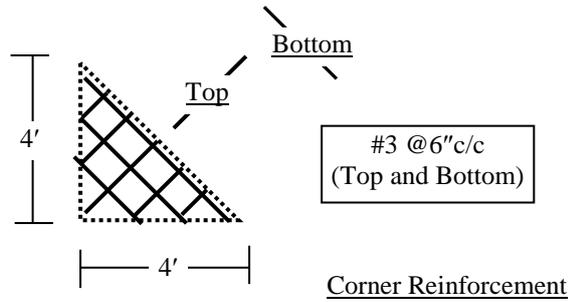
II #3@7''c/c, alt ckd + 1#3, 1#4 extra top

III #3 @8''c/c, alt ckd + 1#4 extra top

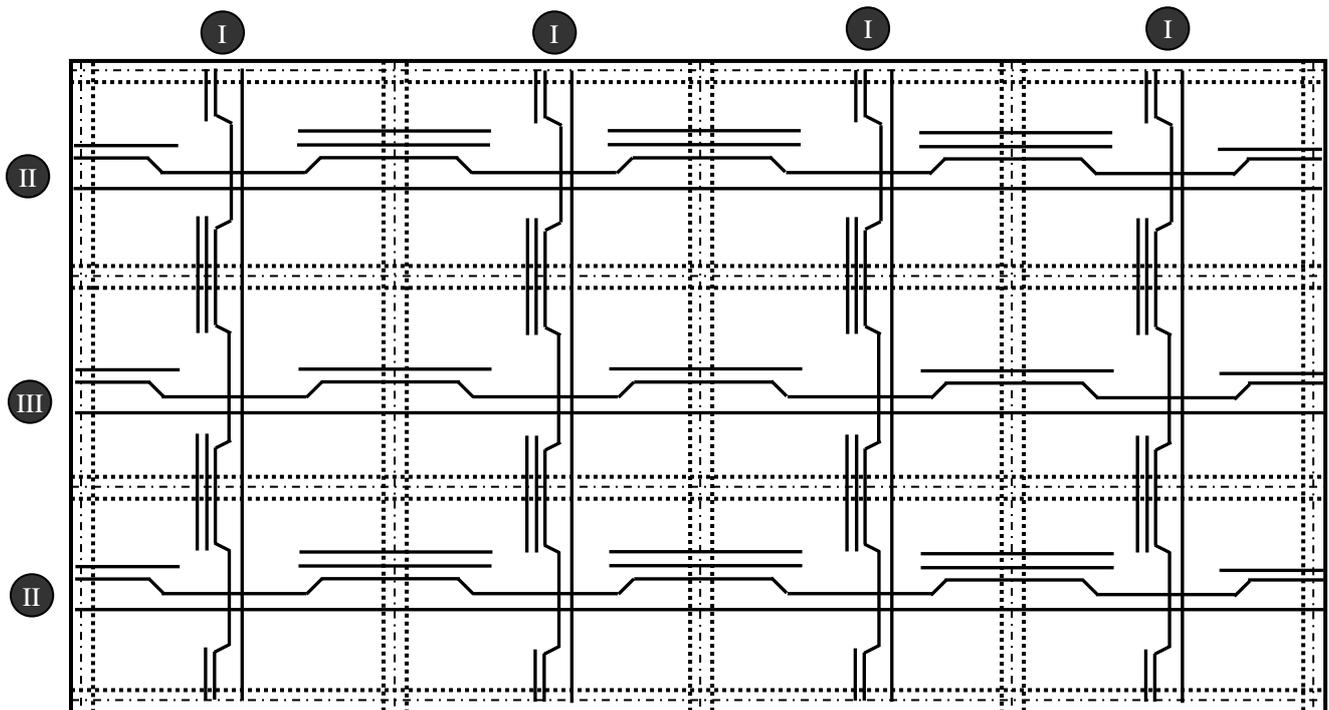
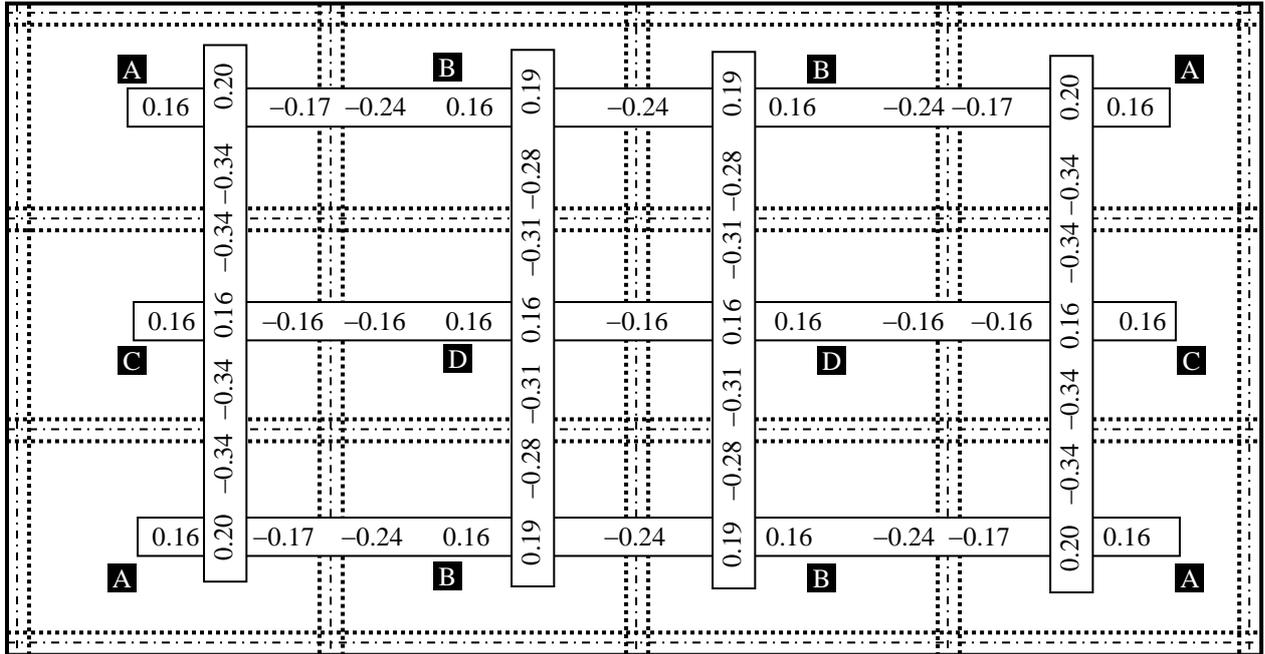
For USD, using $f_c = 0.85f'_c$, $A_s = (f_c/f_y) [1 - \sqrt{1 - 2M_u/(\phi f_c b d^2)}] b d = (2.55/50) [1 - \sqrt{1 - 2M_u/(2.55 \times 3.5^2)}] (12 \times 3.5)$
 $= 2.14 [1 - \sqrt{1 - M_u/15.62}]$ (or $= 1.84 [1 - \sqrt{1 - M_u/11.48}]$ for M_{min})

Also $A_{s(Temp)} = 0.0030 b t = 0.0030 \times 12 \times 4.5 = 0.162 \text{ in}^2/'$

Slab	M_{a+}	A_{sa+}	M_{a-}	A_{sa-}	M_{b+}	A_{sb+}	M_{b-}	A_{sb-}
A	2.51	0.20	4.06	0.34	1.17	0.16	2.11	0.17
B	2.34	0.19	3.46	0.28	1.17	0.16	2.96	0.24
C	1.99	0.16	4.06	0.34	0.78	0.16	1.06	0.16
D	1.86	0.16	3.76	0.31	0.89	0.16	1.69	0.16



Using moments in $k/'$ reinforcements in $\text{in}^2/'$



I #3 @ 6" c/c, alt ckd + 2#3 extra top

II #3 @ 8" c/c, alt ckd + 2#3 extra top

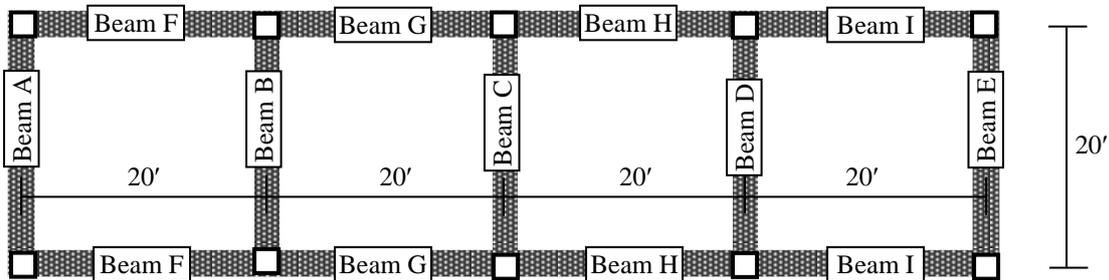
III #3 @ 8" c/c, alt ckd + 1#3 extra top

Questions and Problems (1)

1. (i) What is two-way slab? Mention the differences between one-way and two-way slabs.
 (ii) What is the Coefficient Method of two-way slab design?
 Mention the parameters governing slab moment coefficients and explain their effects on moment coefficients.
 (iii) Specify the moment coefficient for a simply supported square slab and explain how it is derived.
 (iv) Explain why the slab coefficients (C_a) for short direction moments decrease with span ratio, while the slab coefficients (C_b) for long direction moments increase with span ratio.
 (v) Explain why slab coefficients for DL and LL are different for M+ but equal for M-.
 (vi) What are corner reinforcements in two-way slabs? Mention the ACI provisions for corner reinforcements.

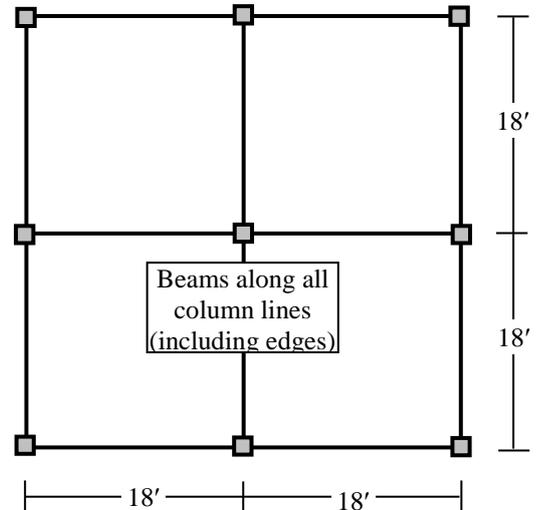
[Use $f'_c = 3 \text{ ksi}$, $f_y = 60 \text{ ksi}$ for all questions]

2. Calculate the required slab thickness and indicate qualitative slab reinforcements of the floor system shown below, if it is supported (i) as shown, (ii) after removing beams (a) B, D, (b) B, C, D, (c) A, E, (d) F, G, H, I.



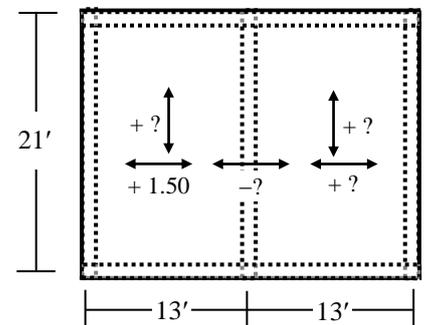
3. Figure below shows floor plan of a RC beam-column slab, with 12" x 18" beams and 5" thick partition walls along column lines (column size 18" x 18"). Floor loads also include working FF = 25 psf, RW = 50 psf.

- (i) Calculate the required slab thickness from deflection considerations, the allowable bending moment (using WSD) for this slab thickness and corresponding allowable LL.
- (ii) Design the slab (with neat sketches of reinforcements) using the loads calculated in (i).

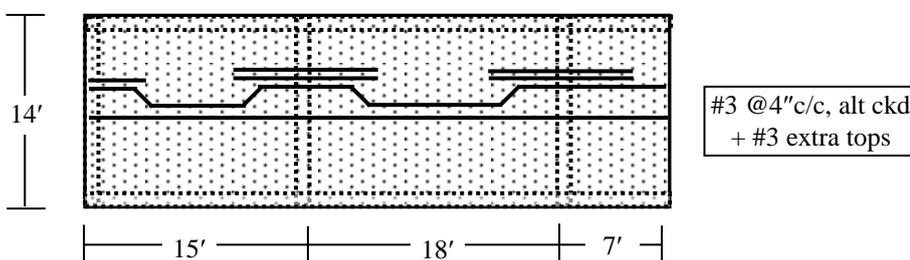


4. Shown below is a bending moment (in k'') of slab system, supported on all sides by beams.

- (i) Calculate the missing moments (denoted by ?) and the distributed load on the slab [assuming LL = 40 psf]
- (ii) Use the WSD to calculate the slab reinforcements and show them in a plan and cross-sectional view.

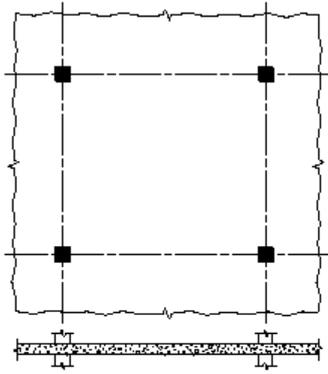


5. Calculate the live load that can be carried by the slab shown below, if it is reinforced (longitudinally) as shown below. Also show its reinforcements in the transverse direction.



Column-Supported Slabs: Flat Plates and Flat Slabs

Concrete slabs are often carried directly by columns without the use of beams or girders.



Such slabs are described as *Flat Plates* [Fig. 2.1(a)] and are commonly used where spans are not large and not particularly heavy.

A very similar construction *Flat Slab* [Fig. 2.1(b)] is also beamless but incorporates a thickened slab region in the vicinity of columns (called *Drop Panels*) and often employs flared up column tops (*Column Capitals*). Both are devices to reduce stresses due to shear and negative bending around the columns.

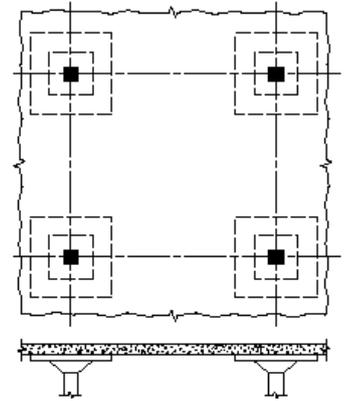


Fig. 2.1: Typical flat slab structural forms (a) Flat Plate, (b) Flat Slab with Drop Panel and Column Capital [Nilson]

Flat plates have assumed wide acceptance because of their aesthetic beauty, reduced story height, simpler formwork, quick construction, adequacy for placing utilities of air conditioning and plumbing as well as flexibility of using the spaces without obstacle. Much brighter appearance due to little obstruction to light, better ventilation, better fire-resistance due to fewer sharp corners and better scope of structural inspection are among some other advantages of flat plate system. Figs. 2.2 and 2.3 show some of these advantages in typical flat plate and flat slab constructions in office building, parking lot and residential facilities.



Fig. 2.2: (a) Flat plate in an office building, (b) Flat slab in a parking structure

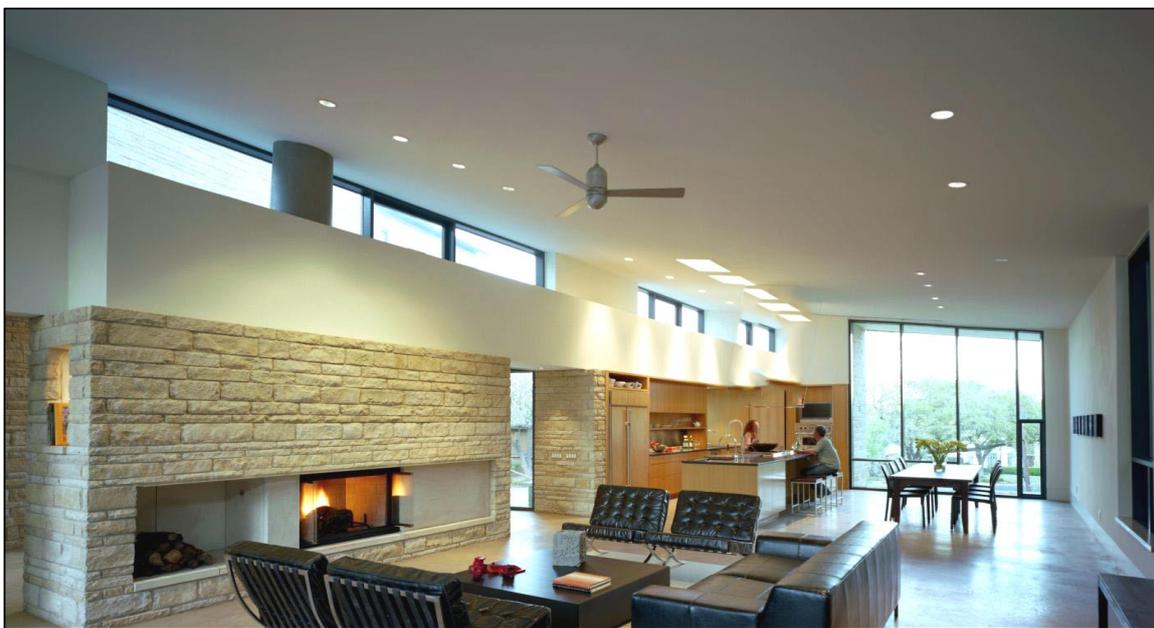


Fig. 2.3: No beam, free layout and large windows create bright and open spaces in flat slab floors

Elements of Flat Slab

Drop Panel

As mentioned, Drop Panels reduce the stresses due to shear and negative bending moment around the columns. They are square or rectangular, with prescribed sides at least one-third the span. ACI Code specifies that its total effective thickness must be not more than 1.5 times the slab thickness.

Column Capital

The columns in practically all cases flare out toward the top, forming a capital of a shape somewhat similar to an inverted truncated cone. The effective diameter of the capital should be taken as the diameter of the circle at the point at which a 45° line from the base of the capital intersects the bottom of the slab or dropped panel.

Column Strip and Middle Strip

For the purposes of design, a typical flat slab panel is divided into *Column Strips* and *Middle Strips*.

A *Column Strip* is defined as a strip of slab having a width on each side of the column centerline equal to one-fourth the smaller of the panel dimensions L_1 and L_2 . Such a strip includes column-line beams, if present.

A *Middle Strip* is a design strip bounded by two *Column Strips*.

Fig. 2.4 shows the distribution of *Column Strips* and *Middle Strips* in a typical slab, where $L_1 \geq L_2$.

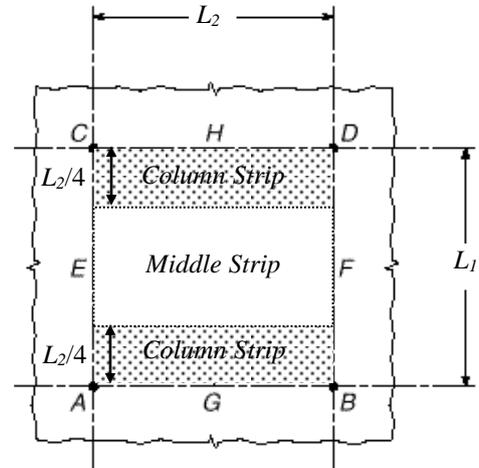


Fig. 2.4: Column Strip and Middle Strip

Slab Thickness

The required thickness of flat slabs is governed by deflection criteria. The minimum thickness of flat slabs (i.e., slabs without interior beams) according to ACI 9.5.3.2 must not be less than those mentioned in Table 2.1.

Table 2.1: Minimum Thickness of Flat Plates [$f_y = 40$ ksi]
(L_n is clear span in long direction)

Exterior Panels without Edge Beams	Exterior Panels with Edge Beams	Interior Panels
$L_n/33$	$L_n/36$	$L_n/36$

- * For reinforcements with $f_y \neq 40$ ksi, the tabulated values are to be multiplied by $(0.8 + f_y/200)$.
- * Drop panels permit a further reduction in minimum thickness of about 10%.
- * In all cases, the minimum thickness of flat plates should not be less than 5 in, and those of flat slabs (with drop panels) not less than 4 in.

Analysis of Flat Slabs for Vertical Loads

In flat-slab analysis, the full load is assumed to be carried by the slab in each direction. This is in apparent contrast to the analysis of two-way beam-supported slabs, in which the load is divided. In two-way slabs, as in flat slabs, equilibrium conditions require that the entire load is carried in each of two principal directions.

Though the structural analysis of flat slabs can be carried out using computer based structural modeling, the two widely used methods for this purpose are the semi-empirical *Direct Design Method* and *Equivalent Frame Method*.

Direct Design Method

Moments in two-way slabs can be found by the Direct Design Method, subject to the following restrictions:

- * There must be a minimum of three continuous spans in each direction.
- * The panels must be rectangular, with the ratio of the longer to shorter spans within a panel not greater than 2.
- * The successive span lengths in each direction must not differ by more than one-third of the longer span.
- * Columns may be offset a maximum of 10% of the span from either axis between column centerlines.
- * Loads must be due to gravity only and the live load must not exceed twice the dead load.
- * The ratio of stiffness of the beams (given by α/L^2) in the two perpendicular directions must not exceed 5.

Total Static Moment at Factored Loads

For purposes of calculating the total static moment M_0 in a panel, the clear span L_n in the direction of moment is used. The clear span is defined to extend from face to face of the column, capitals brackets, or walls but is not to be less than $0.65L_l$. The total factored moment in a span, for a strip bounded laterally by the centerline of the panel on each side, is

$$M_0 = w_n L_2 L_n^2 / 8 \quad \dots\dots\dots(2.1)$$

Assignment of Moment to Critical Sections

For interior spans, the total static moment is apportioned between the critical positive and negative bending sections according to the following ratios:

$$M_u^{(-)} = 0.65 M_0 \quad \quad \quad M_u^{(+)} = 0.35 M_0 \quad \dots\dots\dots(2.2)$$

Depending on the conditions of edge restraint, the ACI Code specifies five alternative sets of moment distribution coefficients for end spans, as shown in Table 2.2.

Table 2.2: Distribution Factors applied to Static Moment M_0 for Positive and Negative Moments

Position of Moment	Ext Edge unrestrained (a)	Slab with beams between all supports (b)	No beam between interior supports		Exterior Edge fully restrained (e)
			Without edge beam (c)	With edge beam (d)	
Exterior $M^{(-)}$	0.00	0.16	0.26	0.30	0.65
Interior $M^{(-)}$	0.75	0.70	0.70	0.70	0.65
$M^{(+)}$	0.63	0.57	0.52	0.50	0.35

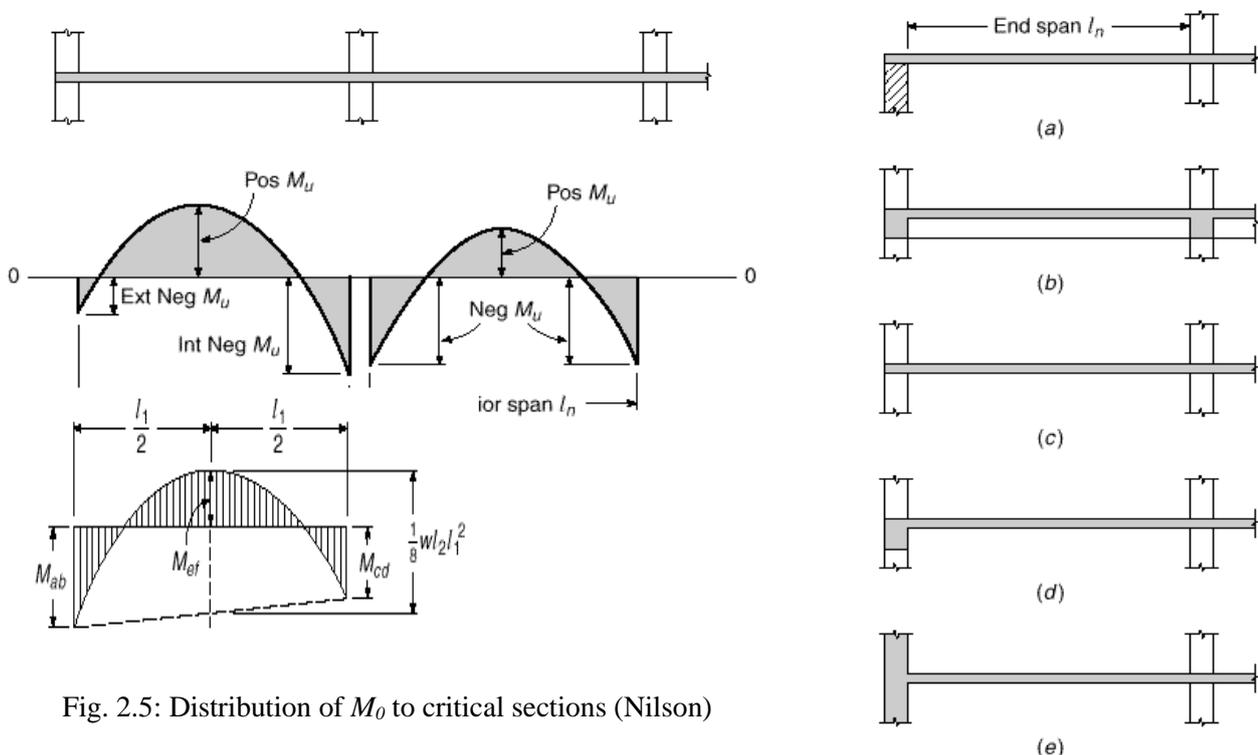


Fig. 2.5: Distribution of M_0 to critical sections (Nilson)

Transverse Distribution of Moment

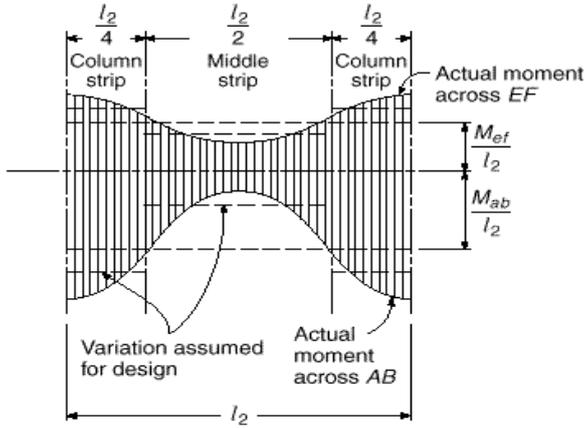


Fig. 2.6: Moment variation across width of sections

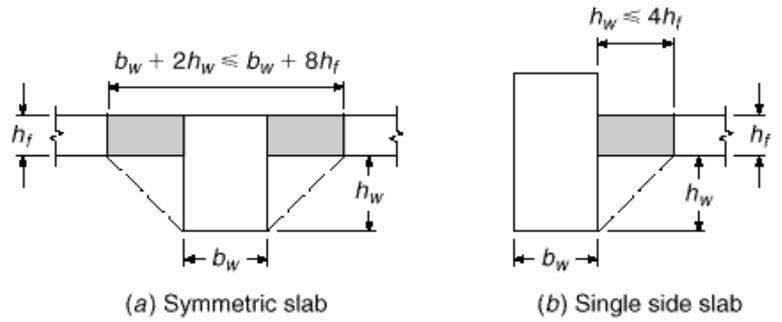


Fig. 2.7: Portion of slab to be included with beam

Having distributed the moment M_0 to the positive and negative moment section as described, one must distribute these design moments across the width of the critical sections. For design purposes, it is convenient to consider the moments constant within the bounds of a middle or column strip (Fig. 2.6) unless there is a beam present on the column line.

The distribution of total negative or positive moment between slab middle strips, slab column strips, and beams depends upon the ratio L_2/L_1 , the relative stiffness of the beam and slab, and degree of torsional restraint provided by the edge beam. A convenient parameter defining the relative stiffness of beam and slab spanning in either direction is

$$\alpha = E_{cb}I_b/E_{cs}I_s \tag{2.3}$$

where E_{cb} and E_{cs} are the moduli of elasticity of the beam and slab concrete and I_b and I_s are the moments of inertia of the effective beam and the slab respectively. Subscripted parameters α_1 and α_2 are used to identify α computed for the directions of L_1 and L_2 respectively.

Beams include that part of the slab on each side of the beam extending a distance equal to its projection above or below the slab h_w (whichever is greater) but not greater than 4 times the slab thickness h_f (Fig. 2.7).

The relative restraint provided by the torsional resistance of the effective transverse edge beam is reflected by the parameter β_t defined as

$$\beta_t = E_{cb}C/2E_{cs}I_s \tag{2.4}$$

where I_s is calculated for the slab spanning in direction L_1 and having width bounded by panel centerlines in the L_2 direction. The constant C pertains to the torsional rigidity of the effective transverse beam.

The constant C is calculated by dividing the section into its components rectangles, each having smaller dimension x and larger dimension y , and summing the contributions of all the parts by the equation

$$C = \sum (1-0.63 x/y) x^3y/3 \tag{2.5}$$

The subdivision can be done in such a way as to maximize C .

With the parameters defined, ACI Code 13.6.4 distributes the negative and positive moments between column strips and middle strips, assigning to column strips a percentages of positive and negative moments, depending on the panel width-to-length ratio and beam-to-slab stiffness ratio in the direction of the analysis. Linear interpolations are to be made between the values shown. As an alternative, the following equations are suggested in ACI Code 318-95.

$$\% \text{ of Exterior } M^{(-)} \text{ supported by Column Strip} = 100 - 10\beta_t + 12 \beta_t (\alpha_1 L_2/L_1) (1-L_2/L_1) \tag{2.6a}$$

$$\% \text{ of } M^{(+)} \text{ supported by Column Strip} = 60 + 30 (\alpha_1 L_2/L_1) (1.5-L_2/L_1) \tag{2.6b}$$

$$\% \text{ of Interior } M^{(-)} \text{ supported by Column Strip} = 75 + 30 (\alpha_1 L_2/L_1) (1-L_2/L_1) \tag{2.6c}$$

In Eqs. (2.6), $\alpha_1 L_2/L_1$ is to be taken = 1, when it is > 1 and β_t is to be taken = 2.5, when it is > 2.5

For slabs without beams between supports ($\alpha_1 = 0$) and without edge beams ($\beta_t = 0$), the portion of negative moments in column strip is simply 100% and 75% for exterior and interior supports, respectively, and portion of positive moment in column strip is simply 60%.

Example 2.1

Design the panels of a flat slab of size 20' × 14' c/c (without any edge beam) as shown in Fig. 2.8, if it carries FF = 30 psf, RW = 50 psf, LL = 60 psf [Given: $f_c' = 3$ ksi, $f_y = 50$ ksi, $f_{c,all} = 1.35$ ksi, $f_{s,all} = 20$ ksi].

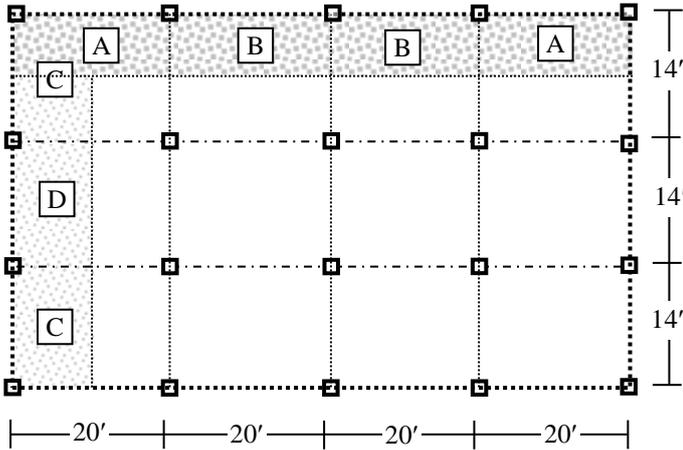


Fig. 2.8: Plan of sample flat plate

Clear slab size = 19' × 13'; i.e., Maximum Clear Span = 19'
Slab without edge beam \Rightarrow Thickness = $L_n(0.8 + f_y/200)/33$
= $19 \times (0.8 + 50/200) \times 12/33 = 7.25''$; i.e., assume 7.5" slab

Using $k = 0.378$, $j = 0.874$, $d = 6.5''$, $6.0''$;
i.e., $A_s = M/9.47$, $M/8.74$

No edge beam along panel length $\Rightarrow \alpha_1 = 0$,
and no transverse beam $\Rightarrow \beta_t = 0$

Self weight = $7.5 \times 150/12 = 93.75$ psf
 \Rightarrow Total load on slab $w = 93.75 + 30 + 50 + 60$
= 233.75 psf = 0.234 ksf

Column Strip width = $14'/4 = 3.5'$
 \therefore Middle Strip width = $7' - 3.5' = 3.5'$ (for A, B)
and = $10' - 3.5' = 6.5'$ (for C, D)

Slab A

$$M_0 = wL_2L_n^2/8 = 0.234 \times 7 \times 19^2/8 = 73.84 \text{ k'}$$

$$\text{Support (c)} \Rightarrow M_{Ext}^- = 0.26 M_0 = 19.20 \text{ k'}, M^+ = 0.52 M_0 = 38.39 \text{ k'}, M_{Int}^- = 0.70 M_0 = 51.69 \text{ k'}$$

$$\therefore \text{Column strip moments are } M_{CExt}^- = 100\% M_{Ext}^- = 19.20 \text{ k'}, M_C^+ = 60\% M^+ = 23.04 \text{ k'}, M_{CInt}^- = 75\% M_{Int}^- = 38.76 \text{ k'}$$

\therefore Column strip moments and reinforcements are

$$M_{CExt}^- = 19.20 \text{ k'} = 5.48 \text{ k''} \Rightarrow A_{sCExt}^- = 0.58 \text{ in}^2/''$$

$$M_C^+ = 23.04 \text{ k'} = 6.58 \text{ k''} \Rightarrow A_{sC}^+ = 0.70 \text{ in}^2/''$$

$$M_{CInt}^- = 38.76 \text{ k'} = 11.08 \text{ k''} \Rightarrow A_{sCInt}^- = 1.17 \text{ in}^2/''$$

\therefore Middle strip moments and reinforcements are

$$M_{MExt}^- = 0 \text{ k'} = 0 \text{ k''} \Rightarrow A_{sMExt}^- = 0.00 \text{ in}^2/''$$

$$M_M^+ = 15.36 \text{ k'} = 4.39 \text{ k''} \Rightarrow A_{sM}^+ = 0.46 \text{ in}^2/''$$

$$M_{MInt}^- = 12.92 \text{ k'} = 3.69 \text{ k''} \Rightarrow A_{sMInt}^- = 0.39 \text{ in}^2/''$$

Slab B

$$M_0 = wL_2L_n^2/8 = 0.234 \times 7 \times 19^2/8 = 73.84 \text{ k'}$$

$$\text{Interior Span} \Rightarrow M_{Int}^- = 0.65 M_0 = 47.99 \text{ k'}, M^+ = 0.35 M_0 = 25.84 \text{ k'}$$

$$\therefore \text{Column strip moments are } M_{CInt}^- = 75\% M_{Int}^- = 35.99 \text{ k'}, \text{ and } M_C^+ = 60\% M^+ = 15.51 \text{ k'}$$

\therefore Column strip moments and reinforcements are

$$M_{CInt}^- = 35.99 \text{ k'} = 10.28 \text{ k''} \Rightarrow A_{sCInt}^- = 1.09 \text{ in}^2/''$$

$$M_C^+ = 15.51 \text{ k'} = 4.43 \text{ k''} \Rightarrow A_{sC}^+ = 0.47 \text{ in}^2/''$$

\therefore Middle strip moments and reinforcements are

$$M_{MInt}^- = 12.00 \text{ k'} = 3.43 \text{ k''} \Rightarrow A_{sMInt}^- = 0.36 \text{ in}^2/''$$

$$M_M^+ = 10.34 \text{ k'} = 2.95 \text{ k''} \Rightarrow A_{sM}^+ = 0.31 \text{ in}^2/''$$

Slab C

$$M_0 = wL_2L_n^2/8 = 0.234 \times 10 \times 13^2/8 = 49.38 \text{ k'}$$

$$\text{Support (c)} \Rightarrow M_{Ext}^- = 0.26 M_0 = 12.84 \text{ k'}, M^+ = 0.52 M_0 = 25.68 \text{ k'}, M_{Int}^- = 0.70 M_0 = 34.57 \text{ k'}$$

$$\therefore \text{Column strip moments are } M_{CExt}^- = 100\% M_{Ext}^- = 12.84 \text{ k'}, M_C^+ = 60\% M^+ = 15.41 \text{ k'}, M_{CInt}^- = 75\% M_{Int}^- = 25.92 \text{ k'}$$

\therefore Column strip moments and reinforcements are

$$M_{CExt}^- = 12.84 \text{ k'} = 3.67 \text{ k''} \Rightarrow A_{sCExt}^- = 0.39 \text{ in}^2/''$$

$$M_C^+ = 15.41 \text{ k'} = 4.40 \text{ k''} \Rightarrow A_{sC}^+ = 0.50 \text{ in}^2/''$$

$$M_{CInt}^- = 25.92 \text{ k'} = 7.41 \text{ k''} \Rightarrow A_{sCInt}^- = 0.78 \text{ in}^2/''$$

\therefore Middle strip moments and reinforcements are

$$M_{MExt}^- = 0 \text{ k'} = 0 \text{ k''} \Rightarrow A_{sMExt}^- = 0.00 \text{ in}^2/''$$

$$M_M^+ = 10.27 \text{ k'} = 1.58 \text{ k''} \Rightarrow A_{sM}^+ = 0.18 \text{ in}^2/''$$

$$M_{MInt}^- = 8.64 \text{ k'} = 1.33 \text{ k''} \Rightarrow A_{sMInt}^- = 0.14 \text{ in}^2/''$$

Slab D

$$M_0 = wL_2L_n^2/8 = 0.234 \times 10 \times 13^2/8 = 49.38 \text{ k'}$$

$$\text{Interior Span} \Rightarrow M_{Int}^- = 0.65 M_0 = 32.10 \text{ k'}, M^+ = 0.35 M_0 = 17.28 \text{ k'}$$

$$\therefore \text{Column strip moments are } M_{CInt}^- = 75\% M_{Int}^- = 24.07 \text{ k'}, M_C^+ = 60\% M^+ = 10.37 \text{ k'}$$

\therefore Column strip moments and reinforcements are

$$M_{CInt}^- = 24.07 \text{ k'} = 6.88 \text{ k''} \Rightarrow A_{sCInt}^- = 0.73 \text{ in}^2/''$$

$$M_C^+ = 10.37 \text{ k'} = 2.96 \text{ k''} \Rightarrow A_{sC}^+ = 0.34 \text{ in}^2/''$$

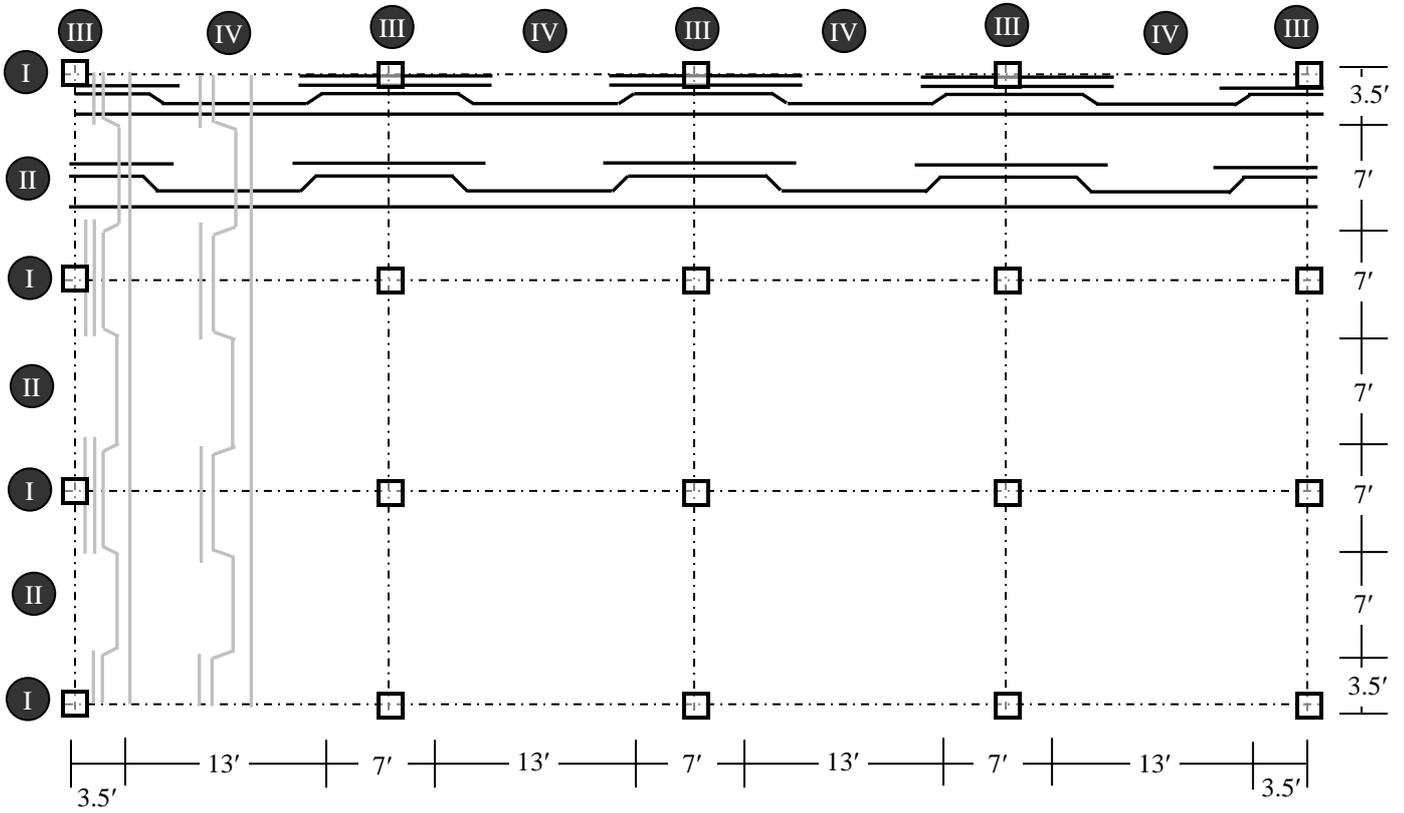
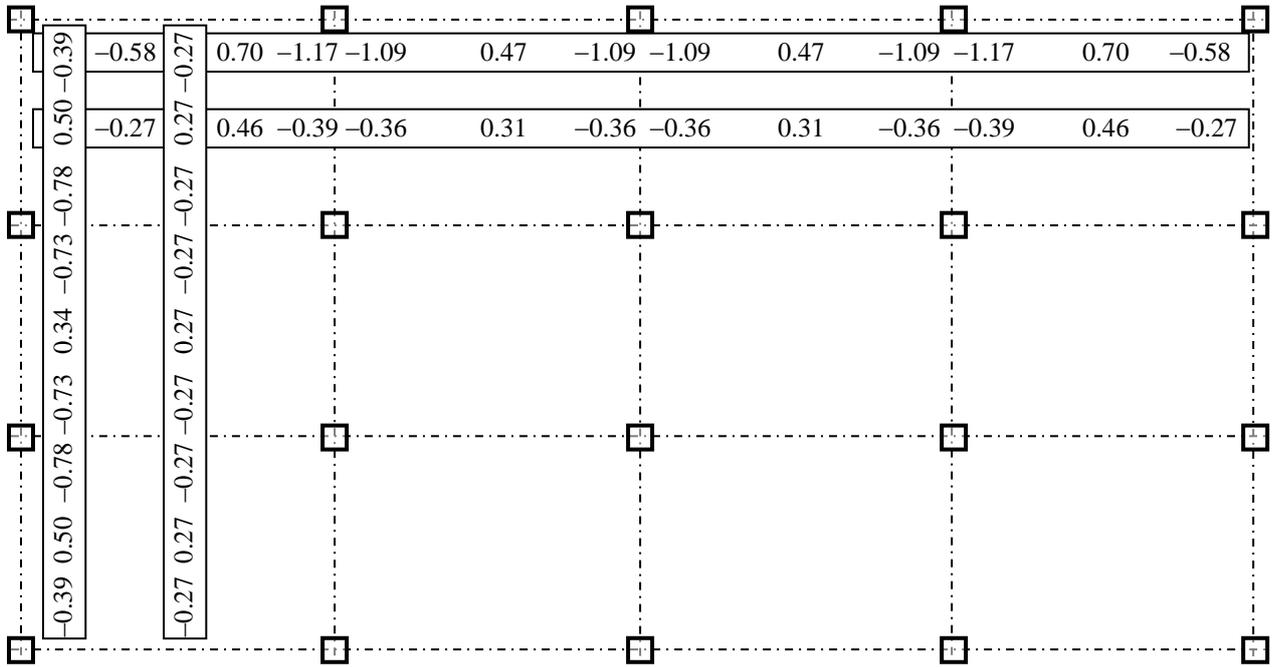
\therefore Middle strip moments and reinforcements are

$$M_{MInt}^- = 8.02 \text{ k'} = 1.23 \text{ k''} \Rightarrow A_{sMInt}^- = 0.13 \text{ in}^2/''$$

$$M_M^+ = 6.91 \text{ k'} = 1.06 \text{ k''} \Rightarrow A_{sM}^+ = 0.12 \text{ in}^2/''$$

Flexural reinforcements should be checked against $A_{sTemp} = 0.036t = 0.27 \text{ in}^2/''$, which may govern in some cases.

However, $d_{req} = \sqrt{(M_{max}/R)} = \sqrt{(11.08/0.223)} = 7.05'' \Rightarrow$ Thicker slab, or stronger concrete, or USD required.



- I #4@3.5"/c/c, alt ckd + 1#4, 1#5 extra top
- II #4@5"/c/c, alt ckd + 1#4 extra top

- III #4@5"/c/c, alt ckd + 2#4 extra top
- IV #4 @8"/c/c, alt ckd + 1#4 extra top

Example 2.2

Design the panels of a flat slab of size 20' × 14' c/c (supported on 12" × 12" edge beams), if it carries FF = 30 psf, RW = 50 psf, LL = 60 psf [Given: $f_c' = 3$ ksi, $f_y = 50$ ksi, $f_{c,all} = 1.35$ ksi, $f_{s,all} = 20$ ksi].

Slab size (= 20' × 14' c/c) = 19' × 13'; i.e., Maximum Clear Span = 19'

Slab with edge beam ⇒ Thickness = $L_n(0.8 + f_y/200)/36 = 19 \times (0.8 + 50/200) \times 12/36 = 6.65''$; i.e., assume 7" slab

Using $k = 0.378$, $j = 0.874$, $d = 6.0''$, or $5.5''$; i.e., $A_s = M/8.74$ or $M/8.01$

The edge beam is made of two rectangular sections (12" × 19") and (12" × 7")

$$\therefore \bar{y} = (12 \times 19 \times 9.5 + 12 \times 7 \times 3.5)/(12 \times 19 + 12 \times 7) = 7.88''$$

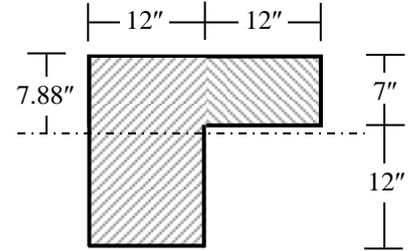
$$\text{Moment of Inertia of external beam-slab, } I_b = 12 \times 19^3/3 + 12 \times 7^3/3 - (228 + 84) \times 7.88^2 = 9412 \text{ in}^4$$

$$\text{Torsional rigidity of edge beam, } C = (1 - 0.63 \times 12/19) 12^3 \times 19/3 + (1 - 0.63 \times 7/12) 7^3 \times 12/3 = 7457 \text{ in}^4$$

$$\text{Width} = 7.5'; \text{ Moment of Inertia of edge slab, } I_s \cong 7.5 \times 12 \times 7^3/12 = 2573 \text{ in}^4$$

$$\text{For the edge beam along panel length; } \therefore \alpha_1 = E_{cb}I_b/E_{cs}I_s = 9412/2573 = 3.66$$

$$\beta_t = E_{cb}C/2E_{cs}I_s = 7457/(2 \times 2573) = 1.45 < 2.5$$



Effective beam with slab

Design for Distributed Moments in Slab

$$\text{Self weight} = 7.0 \times 150/12 = 87.5 \text{ psf}$$

$$\Rightarrow \text{Total load on slab } w = 87.5 + 30 + 50 + 60 = 227.5 \text{ psf} = 0.228 \text{ ksf}$$

Slab A

$$M_0 = wL_2L_n^2/8 = 0.228 \times 7.5 \times 19^2/8 = 76.99 \text{ k'}$$

$$\text{Support (d)} \Rightarrow M_{Ext}^- = 0.30 M_0 = 23.10 \text{ k'}, M^+ = 0.50 M_0 = 38.50 \text{ k'}, M_{Int}^- = 0.70 M_0 = 53.90 \text{ k'}$$

$$L_2/L_1 = 14/20 = 0.70, \alpha_1 L_2/L_1 = 2.56 > 1.0, \beta_t = 1.45$$

∴ Total column strip moment percentages are

$$\text{For } M_{Ext}^- = 100 - 10 \times 1.45 + 12 \times 1.45 \times 1.0 \times (1 - 0.70) = 91\%$$

$$\text{For } M^{(+)} = 60 + 30 \times 1.0 \times (1.5 - 0.70) = 84\%, \text{ For } M_{Int}^- = 75 + 30 \times 1.0 \times (1 - 0.70) = 84\%$$

∴ Total column strip moments and reinforcements are

$$M_{CExt}^- = 0.91 M_{Ext}^- = 20.96 \text{ k'}; \text{ i.e., } 20.96 \times 0.85 = 17.81 \text{ k' in beam, } 3.15/3.0 = 1.05 \text{ k'' in slab, } A_{sCExt}^- = 0.12 \text{ in}^2/''$$

$$M_C^+ = 0.84 M^+ = 32.34 \text{ k'}; \text{ i.e., } 32.34 \times 0.85 = 27.49 \text{ k' in beam, } 4.85/3.0 = 1.62 \text{ k'' in slab, } A_{sC}^+ = 0.18 \text{ in}^2/''$$

$$M_{CInt}^- = 0.84 M_{Int}^- = 45.27 \text{ k'}; \text{ i.e., } 45.27 \times 0.85 = 38.48 \text{ k' in beam, } 6.79/3.0 = 2.26 \text{ k'' in slab, } A_{sCInt}^- = 0.26 \text{ in}^2/''$$

∴ Total middle strip moments and reinforcements are

$$M_{MExt}^- = 23.10 - 20.96 = 2.14 \text{ k'}; \text{ i.e., } 2.14/3.5 = 0.61 \text{ k'' in slab, } A_{sMExt}^- = 0.07 \text{ in}^2/''$$

$$M_M^+ = 38.50 - 32.34 = 6.16 \text{ k'}; \text{ i.e., } 6.16/3.5 = 1.76 \text{ k'' in slab, } A_{sM}^+ = 0.20 \text{ in}^2/''$$

$$M_{MInt}^- = 53.90 - 45.27 = 8.62 \text{ k'}; \text{ i.e., } 8.62/3.5 = 2.45 \text{ k'' in slab, } A_{sMInt}^- = 0.28 \text{ in}^2/''$$

Slab D

$$M_0 = wL_2L_n^2/8 = 0.228 \times 14 \times 19^2/8 = 143.72 \text{ k'}$$

$$\text{Interior Span} \Rightarrow M_{Int}^- = 0.65 M_0 = 93.42 \text{ k'}, M^+ = 0.35 M_0 = 50.30 \text{ k'}$$

$$L_2/L_1 = 14/20 = 0.70, \text{ No longitudinal beam} \Rightarrow \alpha_1 L_2/L_1 = 0, \text{ and no transverse beam} \Rightarrow \beta_t = 0$$

∴ Total column strip moment percentages are

$$\text{For } M_{Int}^- = 75 + 30 \times 0.0 \times (1 - 0.70) = 75\%, \text{ For } M^{(+)} = 60 + 30 \times 0.0 \times (1.5 - 0.70) = 60\%$$

∴ Total column strip moments and reinforcements are

$$M_{CInt}^- = 0.75 M_{Int}^- = 70.07 \text{ k'}; \text{ i.e., } 70.07/7.0 = 10.01 \text{ k'' in slab, } A_{sCInt}^- = 1.15 \text{ in}^2/''$$

$$M_C^+ = 0.60 M^+ = 30.18 \text{ k'}; \text{ i.e., } 30.18/7.0 = 4.31 \text{ k'' in slab, } A_{sC}^+ = 0.49 \text{ in}^2/''$$

∴ Total middle strip moments and reinforcements are

$$M_{MInt}^- = 93.42 - 70.07 = 23.36 \text{ k'}; \text{ i.e., } 23.36/7.0 = 3.34 \text{ k'' in slab, } A_{sMInt}^- = 0.38 \text{ in}^2/''$$

$$M_M^+ = 50.30 - 30.18 = 20.12 \text{ k'}; \text{ i.e., } 20.12/7.0 = 2.87 \text{ k'' in slab, } A_{sM}^+ = 0.33 \text{ in}^2/''$$

Flexural reinforcements should be checked against $A_{sTemp} = 0.036t = 0.25 \text{ in}^2/''$, which may govern in some cases.

Punching Shear Failure of Flat Slabs

Most collapses related to flat slabs can be attributed to punching shear failure. It is a type of failure of RC slabs subjected to high localized forces. This occurs around column support in flat slab structures. Fig. 2.10 shows the basic mechanism of punching shear failure in laboratory tests and at a public place.

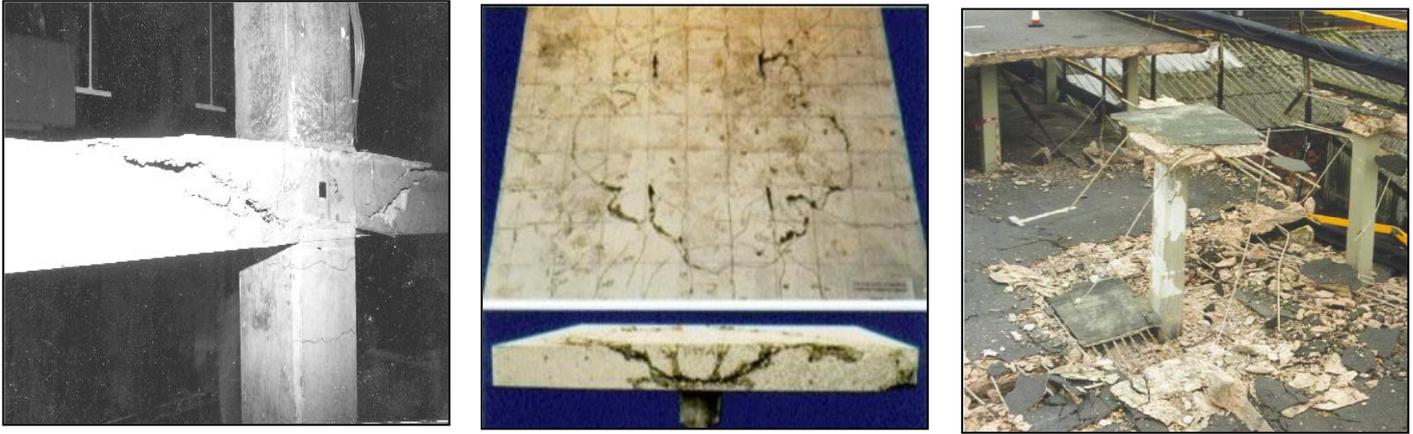


Fig. 2.10: Punching Shear failure in (a), (b) Laboratory Tests, (c) Public place at UK

Punching is an extremely brittle failure mode. A column can suddenly punch through the slab without nearly any warning sign that could cause the slab to collapse on the slab underneath. The sudden drop of the top slab causes a large impact load on the slab below, which it is unable to withstand and collapses as well. This creates a series of slab failures heaped on one another like a pack of cards (called a ‘pancake’ failure). Figs. 2.11(a)~(c) show pancake failures of overloaded slabs in Bangladesh, USA and earthquake action in Haiti.



Fig. 2.11: Pancake failure of slabs in (a) USA (1981), (b) Bangladesh (2005), (c) Haiti earthquake (2010)

Punching Shear Strength

Failure due to punching shear occurs with the potential diagonal crack following the surface of a truncated cone or pyramid around the column, capital or drop panel. The failure surface extends from the bottom of the slab, at the support, diagonally upward to the top surface. The critical section for shear is taken perpendicular to the plan of the slab and a distance $d/2$ from the periphery of the support. At such a section, in addition to the shearing stresses and horizontal compressive stresses due to negative bending moment, vertical or somewhat inclined compressive stress is present, owing to the reaction of the column. The simultaneous presence of vertical and horizontal compression increases the shear strength of the concrete. Tests indicate that the nominal shear strength (in psi) may be taken as

$$V_c = 4\sqrt{f'_c} b_o d \quad \dots \dots \dots (2.3a)$$

according to ACI Code 11.12.2 [where b_o = Perimeter along the critical section]. However ACI states further that V_c in punching shear must not be taken greater than

$$V_c = (2 + 4/\beta_c) \sqrt{f'_c} b_o d \quad \dots \dots \dots (2.3b)$$

$$V_c = (2 + \alpha_s d/b_o) \sqrt{f'_c} b_o d \quad \dots \dots \dots (2.3c)$$

with β_c (= Long/Short side of column), and $\alpha_s = 40, 30, 20$ for interior, edge and corner columns respectively. The punching shear strength of flat slabs is to be taken as the smallest of the values of V_c given by Eqs. (2.3a), (2.3b) and (2.3c). The basic requirement is then $V_u \leq \phi V_c$ [$\phi = 0.75$] $\dots \dots \dots (2.4)$

In WSD, allowable punching shear stresses can be taken as half the values given by Eqs. (2.3a)~(2.3c), with $\phi = 1$.

Shear Reinforcements in Flat Slabs

Shear reinforcements are provided in flat slabs mainly to protect them from the destructive shear failures mentioned. Among various such options used in practice, only two are mentioned here (and shown in Fig. 2.12).

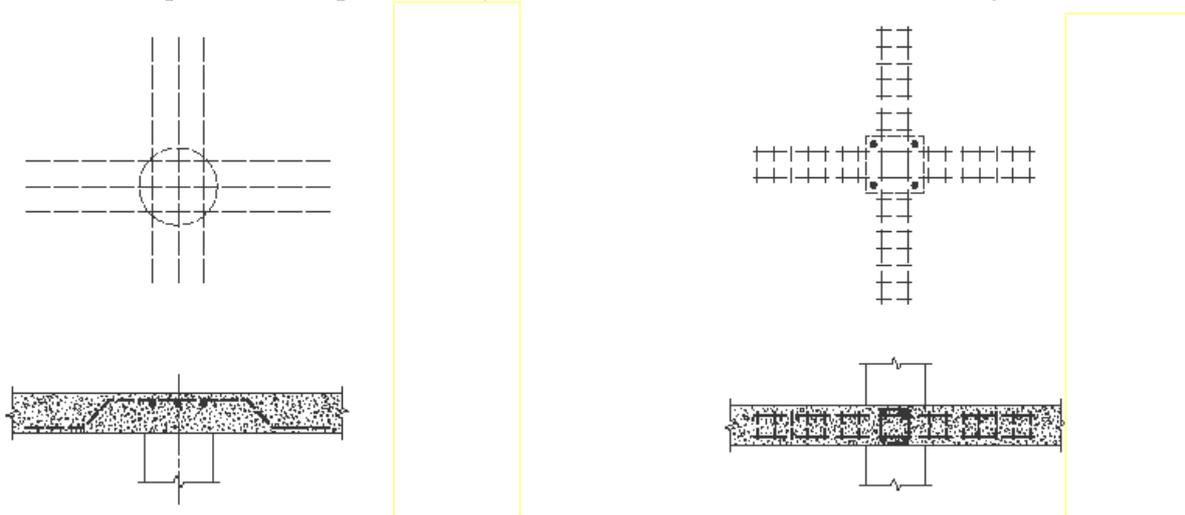


Fig. 2.12: Shear Reinforcements for Flat Slabs (a) Bent Bars, (b) Integral Beams with Vertical Stirrups

Bent Bar Reinforcement

If shear reinforcement in the form of bars is used, the limit value of nominal shear strength V_n , calculated at the critical section $d/2$ from the support face, may be increased to $6\sqrt{f_c'}b_0d$ ($3\sqrt{f_c'}b_0d$ for WSD) or according to ACI Code 11.12.3. The shear resistance of the concrete, V_c is reduced to $2\sqrt{f_c'}b_0d$ ($1.1\sqrt{f_c'}b_0d$ for WSD), and reinforcement must be provided for the excess shear above ϕV_c . The total bar area A_v crossing the critical section at slope angle α is easily obtained by equating the vertical component of the steel force to the excess shear force to be accommodated

$$\phi A_v f_y \sin \alpha = V_u - \phi V_c \quad \dots\dots\dots(2.5)$$

Therefore $A_v = (V_n - V_c)/(f_y \sin \alpha) \quad \dots\dots\dots(2.6)$

In WSD, the allowable steel stress f_v should replace f_y in these equations.

Integral Beams with Vertical Stirrups

The bent bar shear reinforcement is not expensive, but may lead to troublesome congestion of reinforcement in the column-slab joint region. Shear reinforcement using vertical stirrups in integral beams avoids much of this difficulty.

The first critical section for shear design in the slab is taken at $d/2$ from the column face, as usual, and the stirrups if needed are extended outward from the column in four direction for the typical interior case, until the concrete alone can carry the shear, with $V_c = 4\sqrt{f_c'}b_0d$ (or $2\sqrt{f_c'}b_0d$ in WSD) at the second critical section.

Within the region adjacent to the column, where shear resistance is provided by a combination of concrete and steel, the nominal shear strength V_n must not exceed $6\sqrt{f_c'}b_0d$ according to ACI Code (or $3\sqrt{f_c'}b_0d$ in WSD). In this region, the concrete contribution is reduced to $V_c = 2\sqrt{f_c'}b_0d$ (or $1.1\sqrt{f_c'}b_0d$ in WSD).

The second critical section crosses each integral beam at a distance $d/2$ measured outward from the last stirrup and is located so that its perimeter b_0 is a minimum for the typical case, defined by 45° lines between the integral beams.

Example 2.3

Check central columns of the slab (in Example 2.1) for punching shear and calculate shear reinforcements if required.

Tributary area for central columns is $= 20' \times 14' \Rightarrow$ The maximum axial force in column $= 0.234 \times 20 \times 14 = 65.45$ k
 For slab thickness $= 7.5''$, and $d = 6.5''$, punching perimeter $b_0 = 4 \times 18.5 = 74''$ and punching area $= 74 \times 6.5 = 481$ in²
 \Rightarrow Punching shear force $= 65.45 - 0.234 \times (18.5/12)^2 = 64.89$ k

Using WSD version of Eqs. (2.3), [using $\beta_c = 1.0$, $\alpha_s = 40$, $d = 6.5''$, $b_0 = 74''$]

Maximum allowable punching shear force $= 2\sqrt{f_c'}b_0d = 2\sqrt{(3/1000)} \times 481 = 52.69$ k, which is < 64.89 k; i.e. Not safe

If shear reinforcements are provided (as may be even more necessary for seismic design)

Maximum allowable punching shear force $= 1.1\sqrt{f_c'}b_0d = 1.1\sqrt{(3/1000)} \times 481 = 28.98$ k

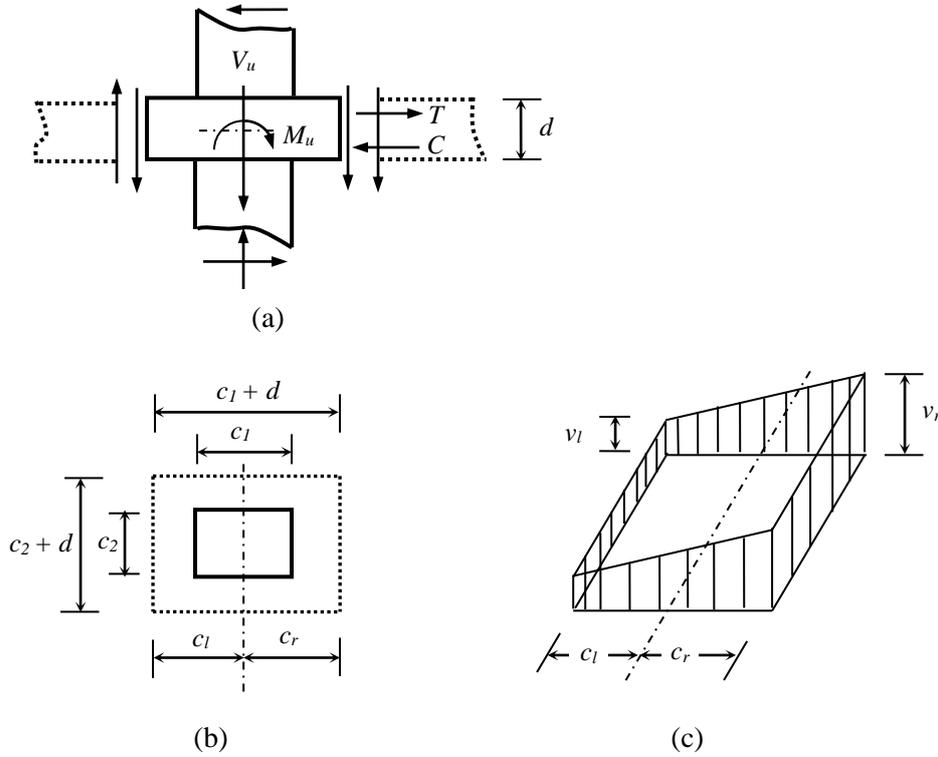
\therefore Shear reinforcements are to be provided for the additional force $= 64.89 - 28.98 = 35.91$ k

\therefore Spacing of #4 stirrups $S = A_v f_y d / (V - V_c) = 1.60 \times 20 \times 6.5 / 35.91 = 5.79'' \Rightarrow d/2 = 3.25''$ around four sides of column

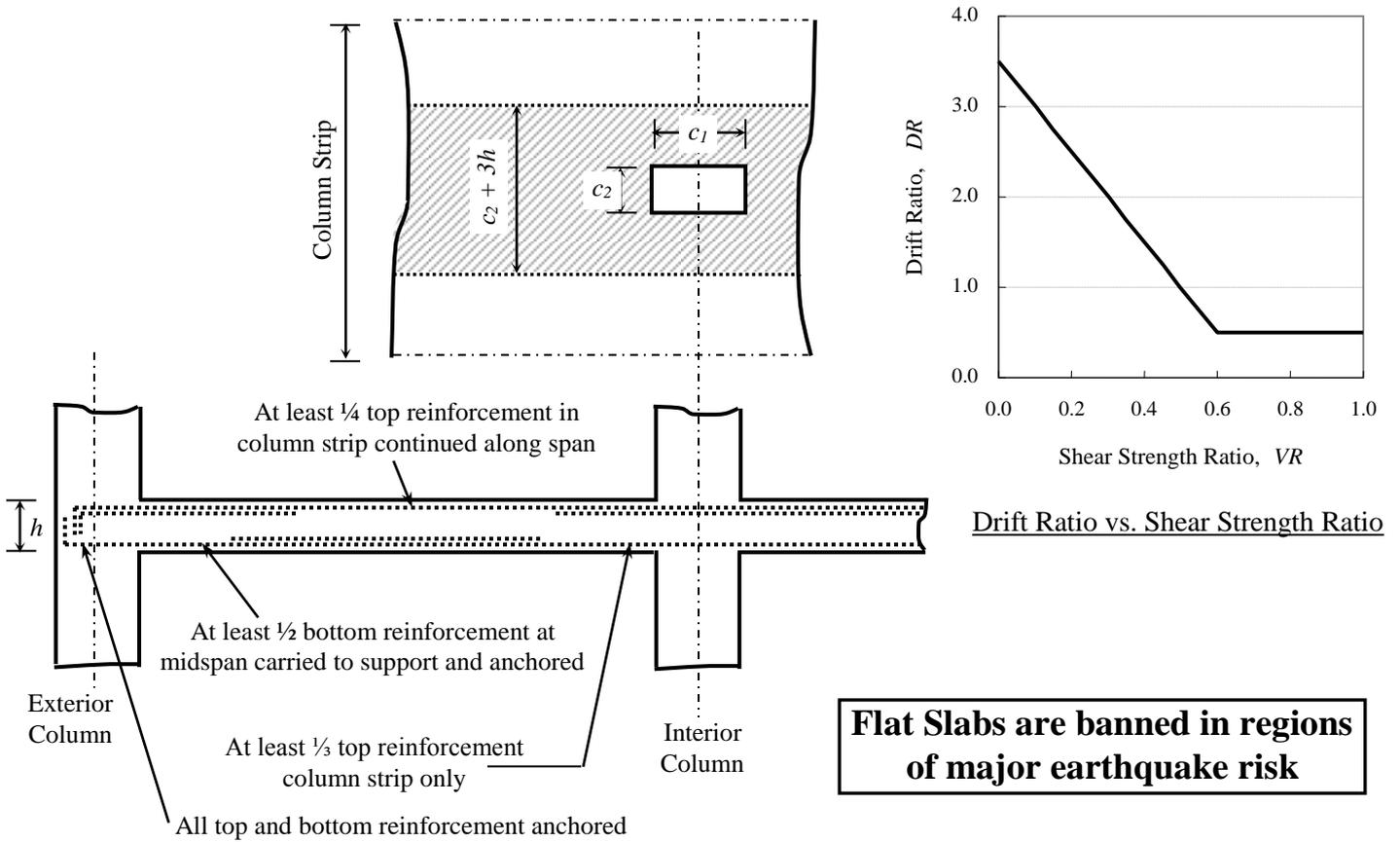
If 45° inclined stirrups are used, $A_v = 35.91 / (20 \sin 45^\circ) = 2.54$ in², requiring at least 12-#4 bars around the column

All these bars should be extended full development length beyond the critical sections.

Additional Seismic Risks and Design Provisions of Flat Slabs



Transfer of moment from slab to column (a) Forces resulting from vertical load and unbalanced moment; (b) Critical section for an interior column; (c) Shear stress distribution for an interior column



Reinforcement requirements in regions of moderate seismic risk

Questions and Problems (2)

1.
 - (i) What is flat slab? Mention the differences between flat slab and flat plate.
 - (ii) Mention the advantages and disadvantages of flat slabs.
 - (iii) Write short notes on the role of (a) Drop Panel, (b) Column Capital, (c) Longitudinal and Transverse Beam in the structural design of flat slabs.
 - (iv) Explain why flat slabs are divided into Column Strips and Middle Strips for design purpose.
 - (v) Distinguish between the structural analyses
 - (a) of flat slab and beam-supported slab, (b) by Direct Design Method and Equivalent Frame Method.
 - (vi) Mention the conditions necessary for using the Direct Design Method of flat slab analysis.
 - (vii) Define the factors α , β , and explain their effect on the structural analysis of flat slabs.
 - (viii) What is punching shear and pancake failure of flat slabs?

Explain why the punching shear strength of concrete is considered larger than its flexural shear strength.
 - (ix) Briefly outline the design provisions for two types of shear reinforcement in flat slabs.
2. For the flat slab shown in Example 2.1, use USD to
 - (i) calculate the bending moments in the short direction,
 - (ii) complete the flexural design of the entire slab [Given: $f_c' = 3$ ksi, $f_y = 50$ ksi].
3. Use WSD/USD to design shear reinforcements for the corner and edge columns of the flat slab of Example 2.1, using (i) Bent bars, (ii) Vertical stirrups.

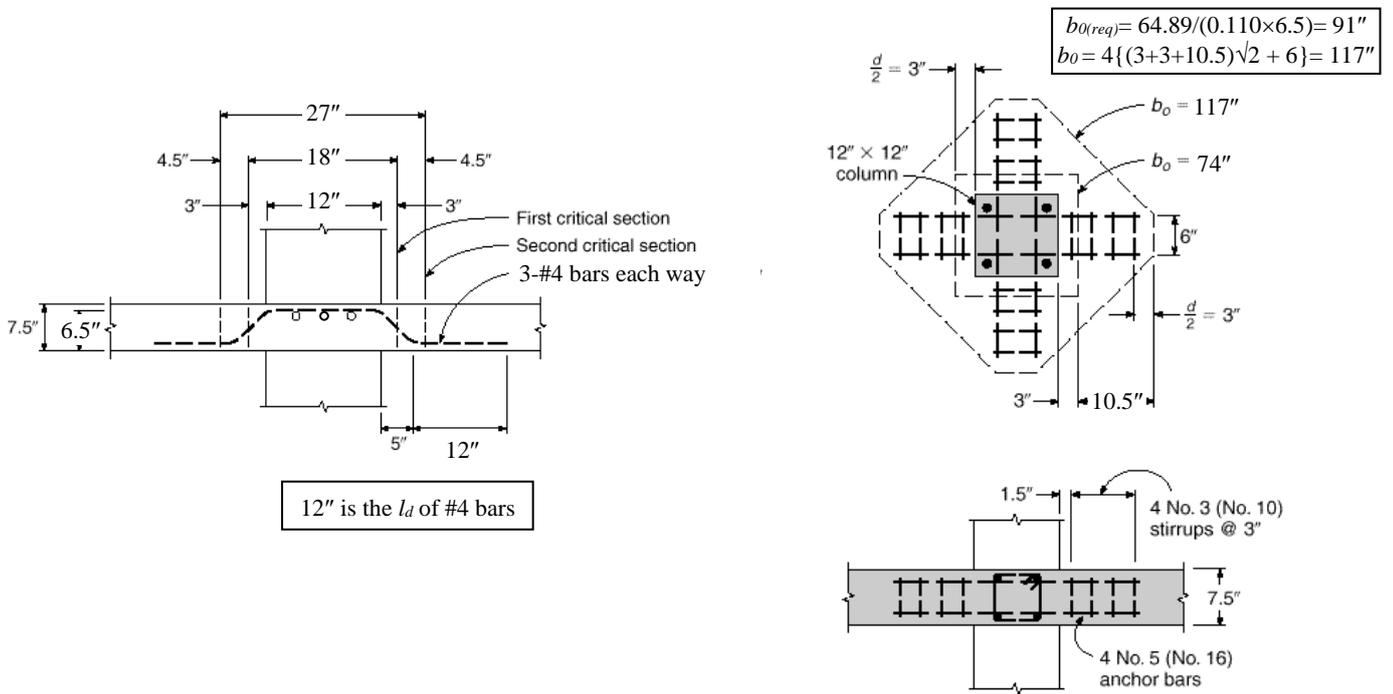


Fig. 2.13: Details of (a) Bent Bars, (b) Integral Beams with Vertical Stirrups



Fig. 2.14: (a) Shear Band, and (b) Placement around column

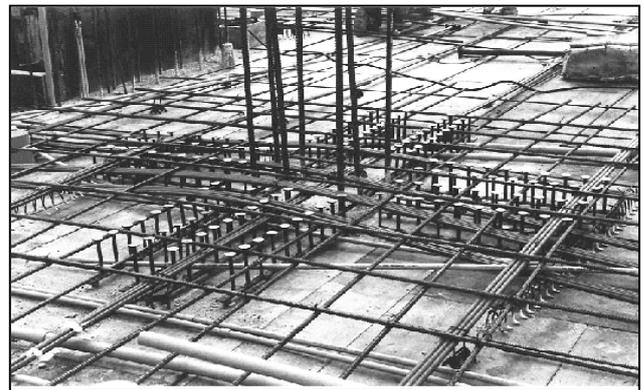
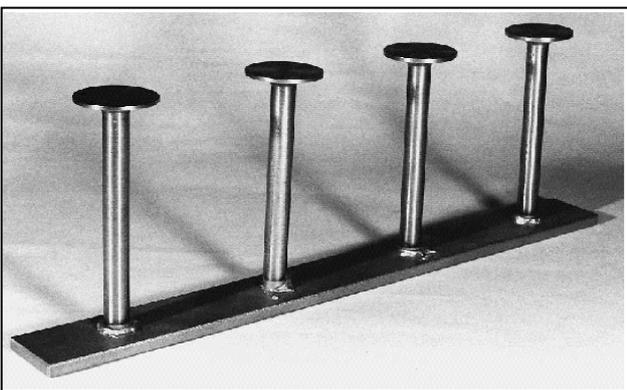


Fig. 2.15: (a) Shear Studs, and (b) Application in Flat Slab

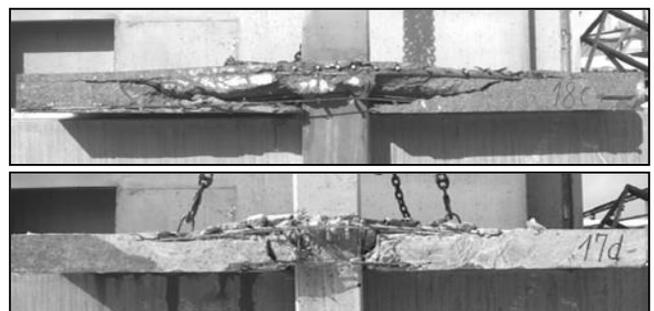
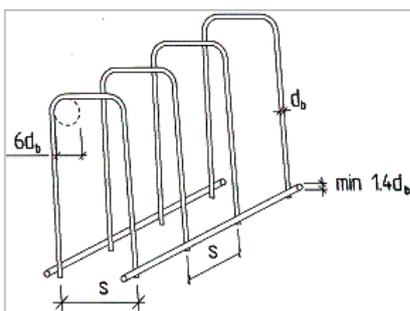


Fig. 2.16: (a) Ductile Shear Reinforcement, (b) Punching Shear with Shear Stud and Ductile Shear Reinforcement

Design of Short Columns

Columns are usually defined as structural members that carry axial forces, more commonly compression; although many columns carry tension and their axial forces are often accompanied by bending moments. So RC columns are designed principally to resist compressive forces and are called *short columns* if their strength is governed entirely by the strength of the materials and cross-section, rather than geometric nonlinearity caused by high slenderness ratio.

The main reinforcements in RC columns are longitudinal; i.e., parallel to the direction of the axial force, and consists of bars arranged in a square, rectangular or circular pattern (Fig. 3.1). It is more economical to make the concrete carry most of the column loads, and minimum column dimensions of 10-in (12-in for seismic design) are specified for this purpose, but some reinforcements are provided.

ACI Code recommends the ratio of longitudinal steel area (A_s) to gross cross-sectional area (A_g) to be in the range of 1~8% (although ratios of 2~4% are more commonly used). The lower limit is necessary to ensure minimum resistance to bending and reduce effects of creep and shrinkage. Ratios higher than 0.08 are uneconomical and would cause congestion of reinforcements.

ACI further specifies that a minimum of four longitudinal bars (size at least #5) should be used for rectangular columns and six for circular columns.

As shown in Fig. 3.1, two types of RC column are most commonly used; i.e., those with lateral ties and spiral reinforcements.

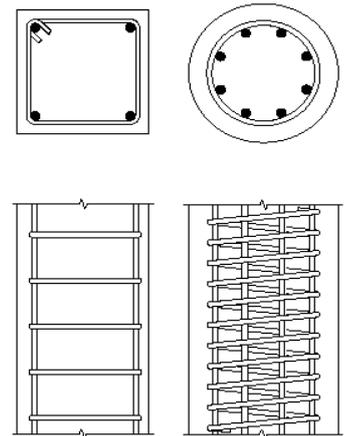


Fig. 3.1: Columns with Lateral Ties and Spiral Reinforcements

Design for Axial Force

Elastic Range

Both concrete and steel behave nearly elastically at low stresses; i.e., which are proportional to strain. Therefore, using 'transformed' RC section, the axial force (P_e) resisted elastically by the section is derived to be

$$P_e = f_c A_c + f_s A_s = f_c (A_c + n A_s) = f_c [A_g + (n - 1) A_s] \quad \dots\dots\dots(3.1)$$

where $n = E_s/E_c$ is the modular ratio, A_s = Area of reinforcing bars, A_c = Net area of concrete = Gross area minus A_s .

Load Carrying Capacity

One quantity of particular interest to the structural designer is the ultimate strength, the maximum load that the column will carry. Tests (at different loading conditions and rates) have shown that concrete and steel can be assumed to carry maximum stresses of $0.85f_c'$ and f_y under all circumstances. So the nominal load that the member can carry is

$$P_n = 0.85f_c' A_c + f_y A_s = A_g [0.85f_c' + \rho_s (f_y - 0.85f_c')] \quad \dots\dots\dots(3.2)$$

The ultimate load capacity is obtained by multiplying the nominal capacity by the resistance factor ϕ , which is taken as 0.70 for tied columns and 0.75 for spirally reinforced columns (due to the greater toughness of the latter).

The greater structural importance, explosive nature of failure, its greater dependence on concrete strength, relatively casual construction process and anomalous usage within the structure results in the reduced value of ϕ for columns.

A further limitation to column strength is imposed to allow for accidental eccentricities of loading not considered in the analysis, using a factor $\alpha = 0.80$ for tied columns and 0.85 for spirally reinforced columns. Therefore, the ultimate load carrying capacity of columns is

$$P_u = \alpha \phi A_g [0.85f_c' + \rho_s (f_y - 0.85f_c')] \quad \dots\dots\dots(3.3)$$

$$\text{The corresponding allowable load according to WSD, } P_{all} = \phi' (0.25f_c' A_g + f_{sall} A_s) = \phi' A_g (0.25f_c' + \rho_s f_{sall}) \quad \dots\dots\dots(3.4)$$

where $\phi' = 0.85$ for tied columns and 1.00 for spirally reinforced columns.

While Eqs. (3.3) and (3.4) are both used for design, Eq. (3.1) is used more for stress analysis of a given section.

Example 3.1

Use (i) WSD, (ii) USD to design tied square column sections for applied compressive force of 100 kips and 500 kips, assuming steel ratio $\rho_s = 0.02 = 2\%$ [Given: $f_c' = 3$ ksi, $f_y = 50$ ksi, $f_{sall} = 20$ ksi].

(i) For $P = 100$ kips, Eq. (3.4) $\Rightarrow 100 = 0.85 A_g (0.25f_c' + \rho_s f_{sall}) = A_g (0.25 \times 3 + 0.02 \times 20) \Rightarrow A_g = 102.3 \text{ in}^2$
 \therefore Choose (10.5" \times 10.5") section, $A_s = (100/0.85 - 0.75 \times 110.25)/20 = 1.75 \text{ in}^2$, i.e., use 4 #6 bars

For $P = 500$ kips $\Rightarrow A_g = 511.5 \text{ in}^2 \Rightarrow$ Take (23" \times 23") section

And $A_s = (500/0.85 - 0.75 \times 529)/20 = 9.57 \text{ in}^2$, i.e., use 12 #8 bars

(ii) Taking overload factor 1.4 for $P = 100$ kips, Eq. (3.3) \Rightarrow

$$140 = 0.8 \times 0.7 \times A_g [0.85f_c' + \rho_s (f_y - 0.85f_c')] = 0.56 A_g [(2.55 + 0.02 \times (47.45))] \Rightarrow A_g = 71.4 \text{ in}^2$$

\therefore Take smallest possible column section; i.e., (10" \times 10"), $A_s = (250 - 2.55 \times 100)/47.45 = -ve$, i.e., use 4 #5 bars

For $P = 500$ kips $\Rightarrow A_g = 357.2 \text{ in}^2 \Rightarrow$ Take (19" \times 19") section, $A_s = 6.94 \text{ in}^2$, i.e., use 4 #9, 4 #8 bars

Lateral Ties and Spirals

As mentioned, longitudinal column bars are always confined transversely either by lateral ties or spirals. These lateral reinforcements hold the longitudinal bars in position, and also prevent the highly stressed slender bars from buckling.

Ties

To achieve adequate tying yet to hold the ties to a minimum (so as not to interfere with concrete placing), ACI recommends the following rules for *Lateral Ties*.

- * Longitudinal bars up to #10 should be enclosed by at least #3 ties, and larger bars enclosed by at least #4 ties
- * Longitudinal spacing of the ties should not exceed 16 times longitudinal bar diameter ($16d_l$), 48 times tie bar diameter ($48d_t$), nor the least column dimension.
- * Every corner and alternate longitudinal bar should be supported laterally by ties with included angle not more than 135° , and no bar should be more than 6-in clear from such a bar.

Fig. 3.2 shows the tie arrangements for rectangular and square columns of various sizes, while Fig. 3.3 shows seismic performance of columns without adequate ties.

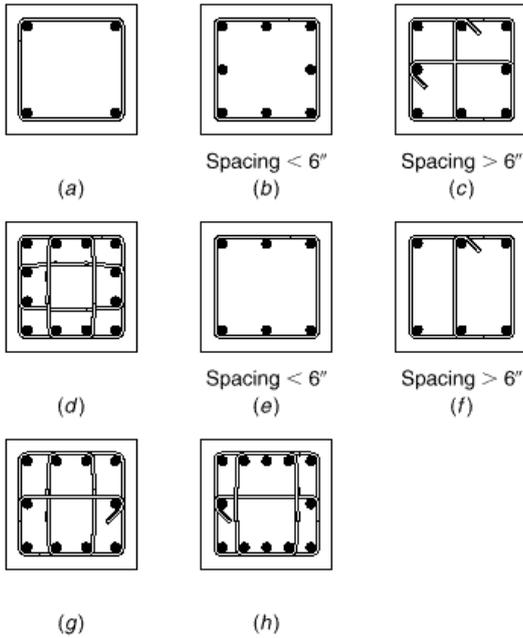


Fig. 3.2: Tie arrangements for square and rectangular columns (Nilson)



Fig. 3.3: Columns without adequate ties in earthquake

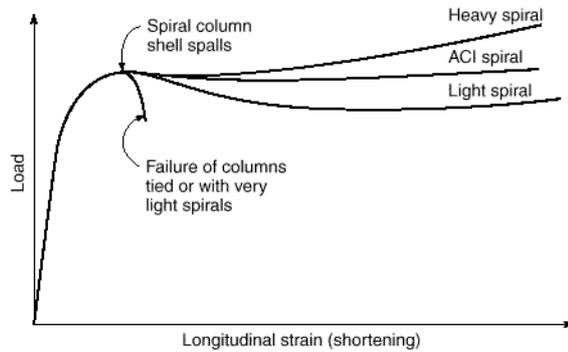


Fig. 3.4: Behavior of tied and spirally reinforced columns (Nilson)

Spirals

ACI also recommends that spirals should be continuous bars not less than #3, clear spacing between turns of the spiral should be between 1" and 3". It further stipulates that the strength lost due to concrete spalling $[= 0.85f_c(A_g - A_{core})]$ should be compensated by the strength gained by providing the spiral [which is $= 2\rho_s f_y A_{core}$, based on experimental results], from which the required minimum ratio of spiral reinforcement is given as

$$\rho_s = 0.45(A_g/A_{core} - 1)(f_c/f_y) \quad \dots\dots\dots(3.5)$$

$$\text{and their spacing is given by } S = 4A_{sp}/(\rho_s d_{core}) \quad \dots\dots\dots(3.6)$$

where A_{sp} = Cross-sectional area of spiral wire, and d_{core} = Outside diameter of spiral.

Fig. 3.3 shows the structural behavior of a tied column compared to a spirally reinforced column, the concrete spalling load of both columns being equal. However, it follows that the former fails in a sudden and brittle manner and the latter in a more gradual manner, which explains the relatively greater value of ϕ for spirally reinforced columns ($= 0.75$ compared to 0.70 for tied columns).

Example 3.2

Calculate the required spacing of spiral reinforcements of a 20"-dia column with $d_{core} = 17"$ [$f_c' = 3$ ksi, $f_y = 50$ ksi].

$$A_g = \pi(20/2)^2 = 314 \text{ in}^2, \quad A_{core} = \pi(17/2)^2 = 227 \text{ in}^2$$

$$\therefore \text{Eq. (3.5)} \Rightarrow \rho_s = 0.45(A_g/A_{core} - 1)(f_c/f_y) = 0.45 \times (314/227 - 1)(3/50) = 0.0104, \text{ and}$$

$$\text{Eq. (3.6)} \Rightarrow S = 4A_{sp}/(\rho_s d_{core}) = 4 \times 0.11/(0.0104 \times 17) = 2.50" \text{ for \#3 spirals.}$$

Columns Subjected to Compression and Bending

Although columns chiefly carry compressive forces, they are almost always accompanied by bending moments due to rigidity/continuity of the beam-column joints, imperfect construction of columns, eccentricity of loads, as well as due to presence of transverse loads like wind, earthquake etc.

In an elastic member under compression (P) and bending $M (= Pe$, where e is eccentricity), maximum fiber stress is

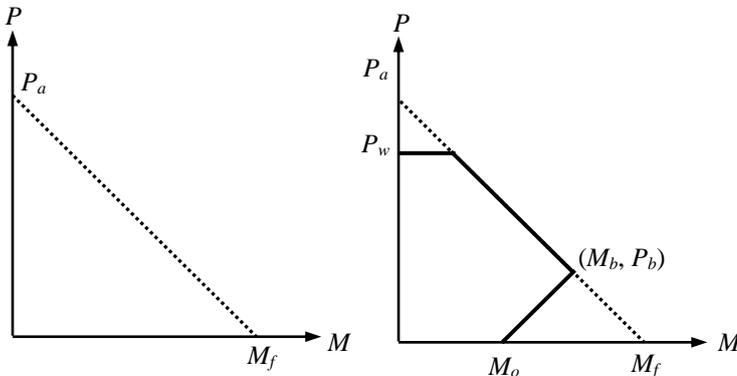
$$P/A + M/S = f_{max} \Rightarrow P/(f_{max}A) + M/(f_{max}S) = 1 \Rightarrow P/(P_a) + M/(M_f) = 1 \quad \dots\dots\dots(3.7)$$

The values of P_a and M_b are given in WSD as

$$P_a = 0.34f_c'(1 + \rho_g m) A_g \quad \text{and} \quad M_f = 0.45f_c' S_{ut} \quad \dots\dots\dots(3.8)$$

where A_g is the gross area of the column section and S_{ut} the uncracked transformed section modulus [= I_{ut}/c , assuming steel areas be multiplied by $2n$ rather than n , as partial recognition of inelastic action].

Interaction Diagram



A plot of Eq. (3.7) is shown in Fig. 3.5(a), which represents a straight line of negative slope. This is called the Interaction Diagram for column design, as each point on it represents a combination of design loads resulting in allowable stress on the column section.

However, the section can neither take an axial load greater than its allowable load P_w as concentrically loaded column [Eq. (3.4)] nor a bending moment greater than its allowable capacity M_o as a pure beam. In the latter situation, the 'failure' of the section is governed by steel tension rather than concrete compression.

Fig. 3.5: Interaction Diagram showing (a) compression 'failure', (b) compression and tension 'failures'

This moment is given by ACI as

$$M_o = 0.40 A_s f_y (d - d'), \text{ for symmetrical tied columns} \quad \dots\dots\dots(3.9a)$$

$$M_o = 0.12 A_{sTotal} f_y (D_s), \text{ for spiral columns} \quad \dots\dots\dots(3.9b)$$

In between, there is a load and moment where these two situations occur simultaneously; i.e., the allowable compressive stress in concrete and allowable tensile stress in steel coincide at the particular combination of axial force and moment (M_b, P_b). These two values are related by balanced eccentricity, specified by ACI as

$$e_b = (0.17 + 0.67 \rho_g m) d, \text{ for symmetrical tied columns} \quad \dots\dots\dots(3.10a)$$

$$e_b = (0.14t + 0.43 \rho_g m D_s), \text{ for spiral columns} \quad \dots\dots\dots(3.10b)$$

The interaction diagram in USD is not linear because the stresses are superimposed as in WSD. However, the nature of the graph is still quite similar, with a maximum ultimate force P_u [given by Eq. (3.3)], a moment M_u [obtained from analysis of beam section by USD] and a balanced condition (M_b, P_b), represented by the balanced compression depth of the section c_b , given by

$$c_b = [87/(87 + f_y)] d \quad \dots\dots\dots(3.11)$$

upon which the strains, stresses and forces over the section (in concrete and steel) are obtained using strain compatibility of plane sections (assumed to remain plane after bending).

For design purpose, each force and moment is multiplied by an appropriate resistance factor ϕ and α , as mentioned.

Columns under Biaxial Bending

There are several approximations suggested to analyze/design columns under biaxial bending which is often a more practical situation than uniaxial bending. An extension of Eq. (3.7) is used for situations where compression governs;

$$P/(P_a) + M_x/(M_{fx}) + M_y/(M_{fy}) \leq 1 \quad \dots\dots\dots(3.12)$$

while an equation suggested by Bresler can be used to calculate the allowable/ultimate load capacity under biaxial bending, from concentric (P_0) and uniaxial load capacities (P_x, P_y); i.e.,

$$1/P_{xy} = 1/P_x + 1/P_y - 1/P_0 \quad \dots\dots\dots(3.13)$$

Example 3.3

For the tied column section shown below [with $f_c' = 3$ ksi, $f_y = 60$ ksi], use the WSD to

(i) Draw the interaction diagram about x -axis

Width $b = 25''$, Depth $h = 20'' \Rightarrow$ Gross Area $A_g = 25 \times 20 = 500 \text{ in}^2$

Also, given $A_s = 14 \times 1.0 = 14.0 \text{ in}^2$

\therefore Steel ratio $\rho_g = A_s/A_g = 14.0/500 = 0.028$, $m = f_y / 0.85 f_c' = 60/2.55 = 23.53$

$\therefore m\rho_g = 0.659$

The important parameters include

$P_a = 0.34 f_c' (1 + m\rho_g) A_g = 1.02 \times 1.659 \times 500 = 846 \text{ k}$

$P_w = \phi' (0.25 f_c' + \rho_g f_{sall}) A_g = 0.85 \times (0.75 + 0.028 \times 24) \times 500 = 604.4 \text{ k}$

$I_{utx} = bh^3/12 + (2n - 1) \sum A_s(y - y_c)^2$
 $= 25 \times 20^3/12 + (2 \times 9 - 1) \times 2 \times (5 \times 1.0 \times 7.5^2 + 2 \times 1.0 \times 2.5^2)$
 $= 26654 \text{ in}^4$

$S_{utx} = I_{utx}/c = 26654/10.0 = 2665.4 \text{ in}^3$

$\therefore M_{fx} = 0.45 f_c' S_{utx} = 1.35 \times 2665.4 = 3598 \text{ k}''$

Also, using steel stresses proportional to distance from neutral axis,

$M_{0x} = \sum A_s f_s (d - d') = 5.0 \times 24 \times 15 + 2.0 \times (24 \times 2.5/7.5) \times 5 = 1880 \text{ k}''$

\therefore Equation of the compression-line of the diagram is $P/P_a + M_0/M_{fx} = 1$

Also, balanced eccentricity $e_b = (0.17 + 0.67 m\rho_g) d$
 $= (0.17 + 0.67 \times 0.659) \times 17.5 = 10.70''$

$\therefore P_b/846 + P_b \times 10.70/3598 = 1$

$\Rightarrow P_b = 240.6 \text{ kip}$, $M_b = P_b e_b = 240.64 \times 10.70 = 2575 \text{ k}''$

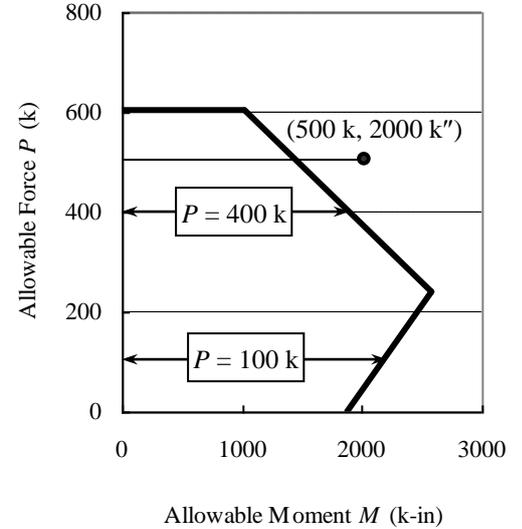
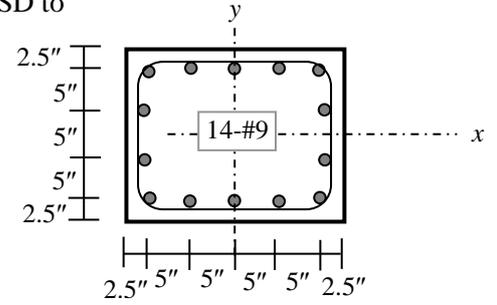


Fig. 3.6: Interaction Diagram (WSD)

(ii) Calculate the allowable moment on the section if it is subjected to axial force P equal to (a) 400 k, (b) 100 k.

(a) When $P = 400 \text{ k} > P_b$; i.e., compression governs

$\Rightarrow P/846 + M/3598 = 1 \Rightarrow 400/846 + M/3598 = 1 \Rightarrow M = 1897 \text{ k}''$

(b) When $P = 100 \text{ k} < P_b$, i.e., tension governs

$\Rightarrow P/P_b = (M - M_0)/(M_b - M_0) \Rightarrow 100/240.6 = (M - 1880)/(2575 - 1880) \Rightarrow M = 2169 \text{ k}''$

(iii) Verify if the section is allowed to take $P = 500 \text{ k}$, at an eccentricity $e = 4''$

Here the point representing $P = 500 \text{ k}$, and $M = 500 \times 4 = 2000 \text{ k}''$, is put outside the interaction diagram.

\therefore The section is not allowed to take the load.

(iv) Verify if the section is allowed to take $P = 250 \text{ k}$, along with bending moments $M_x = 1000 \text{ k}''$, and $M_y = 1500 \text{ k}''$.

Using Eq. (3.12) as the design condition; i.e., $P/(P_a) + M_x/(M_{fx}) + M_y/(M_{fy}) \leq 1$

Here $P_a = 846 \text{ k}$, $M_{fx} = 3598 \text{ k}''$ and

$S_{uty} = I_{uty}/c = \{20 \times 25^3/12 + (2 \times 9 - 1) \times 2 \times (4.0 \times 10^2 + 2.0 \times 5^2)\}/12.5 = 3307 \text{ in}^3$, $M_{fy} = 4465 \text{ k}''$

$\therefore 250/846 + 1000/3598 + 1500/4465 = 0.30 + 0.28 + 0.34 = 0.91 < 1.0$, i.e., within allowable limits

This can be verified using Bresler's equation also

For $M_x = 1000 \text{ k}''$, $P_x = (1 - 1000/3598) \times 846 = 611 \text{ k}$

and for $M_y = 1500 \text{ k}''$, $P_y = (1 - 1500/4465) \times 846 = 562 \text{ k}$

$\therefore 1/P_{xy} = 1/611 + 1/562 - 1/846$

$\Rightarrow P_{xy} = 447 \text{ k} > 250 \text{ k}$; i.e., OK

Example 3.4

(i) Use USD to draw the interaction diagram (about x-axis) for the tied column section shown in Example 3.3 [Given: $f_c' = 3$ ksi, $f_y = 60$ ksi].

The important parameters include

$$P_{0u} = \phi \alpha (0.85 f_c' A_c + A_s f_y) = 0.70 \times 0.80 \times \{2.55 \times (500 - 14.0) + 14.0 \times 60\} = 0.56 \times 2079.3 = 1164.4 \text{ k}$$

$$M_{0u} \text{ is calculated approximating section as singly reinforced (as } \rho_s < \rho_b \text{)}$$

For two layers of steel, $d \cong 20 - 4 = 16''$
 $\therefore a = A_s f_y / (0.85 f_c' b) = 7.0 \times 60 / (2.55 \times 25) = 6.59''$
 $\therefore M_{x0u} = \phi A_s f_y (d - a/2) = 0.90 \times 7.0 \times 60 (16 - 6.59/2) = 4803 \text{ k''}$

For the balanced condition, $c = [87 / (87 + f_y)] d = [87 / 147] 16 = 9.47''$
 \therefore Stresses in steel layers are
 $f_{s1} = 87(c - 2.5) / c = 64.0$, i.e., $= f_y = 60$ ksi; $f_{s2} = 87(c - 7.5) / c = 18.1$ ksi;
 $f_{s3} = 87(c - 12.5) / c = -27.8$ ksi (i.e., tension);
 $f_{s4} = 87(c - 17.5) / c = -73.8$ ksi, i.e., $= -60$ ksi (tension)

Concrete compressive force $= \alpha f_c' b c = 0.72 \times 3 \times 25 \times 9.47 = 511.3 \text{ k}$

\therefore Total force on section $P_b = \sum P$
 $= 60 \times 5.0 + 18.1 \times 2.0 - 27.8 \times 2.0 - 60 \times 5.0 + 511.3 = 491.8 \text{ k}$

and Moment on section $M_b = \sum M$
 $= 300 \times 7.5 + 36.2 \times 2.5 - 55.6 \times (-2.5) - 300 \times (-7.5)$
 $+ 511.3 (10 - 0.425 \times 9.47) = 7785 \text{ k''}$

\therefore The corresponding design force $P_u = 0.7 \times 491.8 = 344.3 \text{ k}$
and design moment $M_u = 0.7 \times 7785 = 5450 \text{ k''}$

\therefore The non-dimensional values are calculated as

$$K_n = P_u / f_c' A_g = P_u / (0.70 \times 3 \times 500) = P_u / (1050), \text{ and } R_n = M_u / f_c' A_g h = M_u / (30000) = M_u / (\phi 30000)$$

$$\Rightarrow K_{n0} = 1164.4 / 1050 = 1.10, R_{n0} = 4803 / (0.90 \times 30000) = 0.18$$

while the balanced condition is given by $K_{nb} = 491.8 / 1500 = 0.33, R_{nb} = 7785 / 30000 = 0.26$

(ii) Show corresponding results in the interaction diagrams given in Nilson.

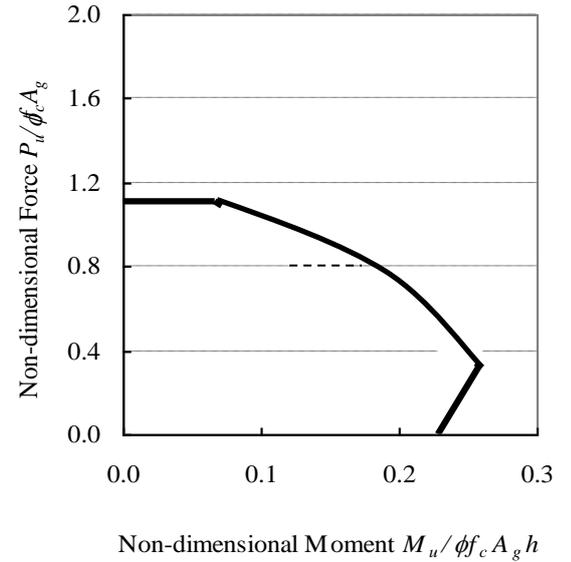


Fig. 3.7: Non-dimensional Interaction Diagram (USD)

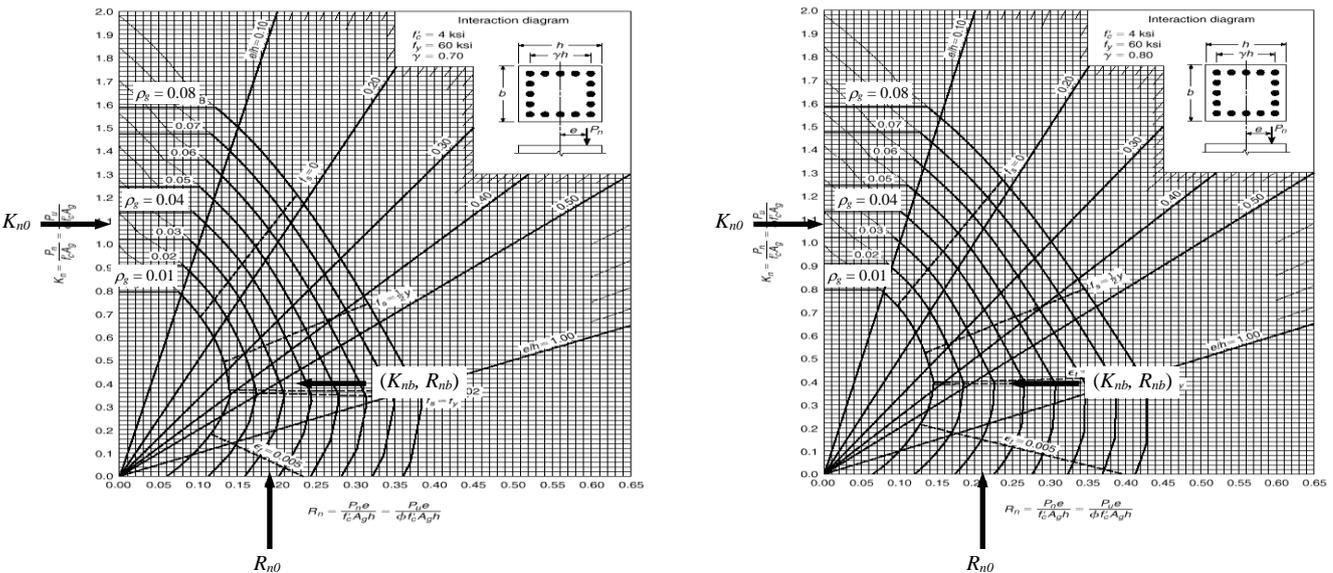


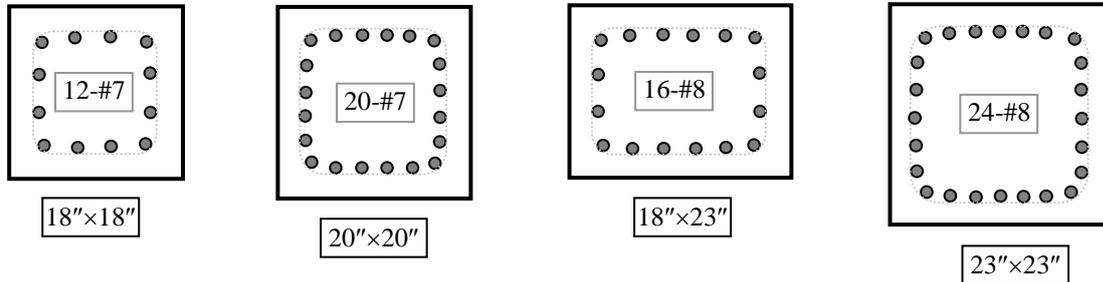
Fig. 3.8: Non-dimensional Interaction Diagrams for (a) $\gamma = 0.70$, (b) $\gamma = 0.80$

The steel ratio is $\rho_g = 14.0 / 500 = 0.028$ here

However this is equivalent to $0.028 \times (60/3) \div (60/4) = 0.037$ in the interaction diagram of Fig. 3.8 since material properties $f_c' = 3$ ksi, $f_y = 60$ ksi are chosen instead of the assumed $f_c' = 4$ ksi, $f_y = 60$ ksi for Fig. 3.8. Moreover, since $\gamma = 0.75$ here, the results should be interpolated from given diagrams for $\gamma = 0.70$ and $\gamma = 0.80$.

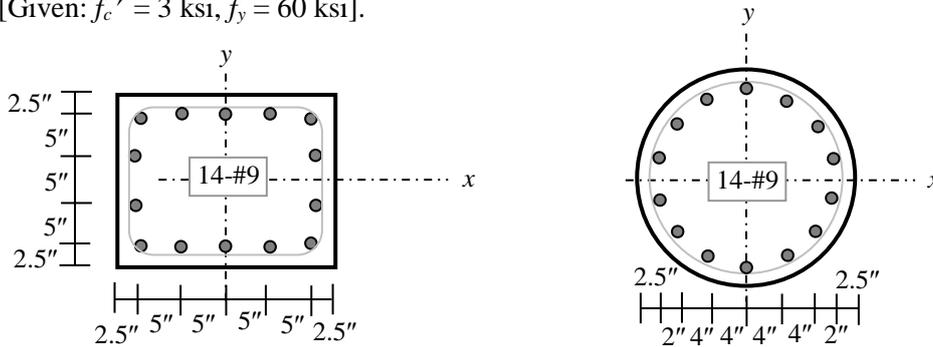
Questions and Problems (3)

1. (i) Mention and justify the maximum and minimum steel ratios specified by ACI for RC columns.
 (ii) Explain why transverse reinforcements are used in RC columns.
 Also compare between the effect of ties and spirals on the strength and ductility of columns.
 (iii) Mention the ACI recommendations for the size, spacing and arrangement of lateral ties and spirals.
 (iv) Explain why the factors ϕ and α are used for column design.
 Also explain why a smaller value of ϕ is used for columns compared to beams.
 (v) Explain why the application of compressive load may increase or decrease the moment capacity of columns.
2. (i) Draw suitable tie arrangements for the following column sections, as per ACI Code.

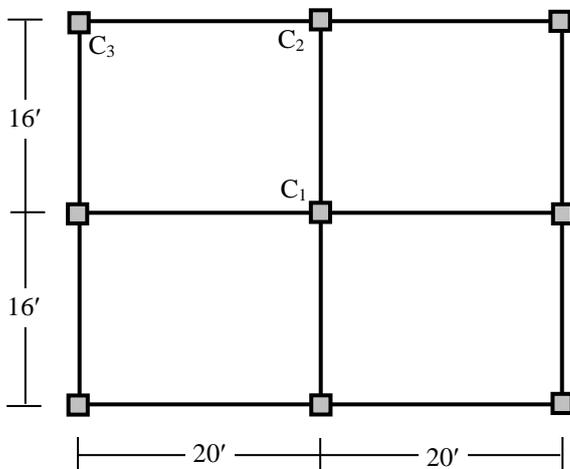


- (ii) Calculate the required spacing of spiral reinforcements for circular column sections of (a) 18"-dia, (b) 30"-dia, with $d_{core} = d_{col} - 3"$ [Given: $f_c' = 4$ ksi, $f_y = 60$ ksi].

3. Use WSD/USD to calculate the balanced axial force (P_b) and bending moment (M_{by}) for the column sections shown below [Given: $f_c' = 3$ ksi, $f_y = 60$ ksi].



4. Figure below shows the plan of a 6-storied RC structure, with 5" thick slabs and 12" x 18" beams and 5" thick partition walls along all column lines. Floor loads also include working FF = 30 psf, RW = 50 psf, LL = 40 psf.



- (i) Design the central column C_1 , if it is subjected to axial force only [i.e., no moments].
 - (ii) Use the section of (i) to check if it is adequate for
 - (a) C_2 [subjected to axial force and working $M_x = 20$ k']
 - (b) C_3 [subjected to axial force and working $M_x = M_y = 20$ k']
- [Given: $f_c' = 3$ ksi, $f_y = 60$ ksi].

Following are some of the theoretical questions discussed so far, with guidelines for their answers.

- The examination questions can be different or mixed or parts (based on the same topics/concepts).
 - Don't copy this language, just follow the points and read books to prepare your own answers.
1. What is two-way slab? Mention the differences between one-way and two-way slabs.
 - Slab where load is transferred in two directions.
 - Differences include supports (both directions), span ratios (0.5~1.0), and reinforcements (flexural, corner)
 2. What is Coefficient Method of two-way slab design? Mention the parameters governing slab moment coefficients.
 - Moments are obtained using coefficients (and the formula $M = C_w l^2$)
 - Parameters include support conditions (nine cases), span ratios ($l_a/l_b = 0.5\sim 1.0$) and type of moment (D/L/-)
 3. Specify the moment coefficient for a simply supported square slab and explain how it is derived.
 - Explain how to get 0.0625 (half of 1/8), 0.048 (plate action) and 0.036 (inelastic redistribution).
 4. Explain why the slab coefficients (C_a) decrease while the slab coefficients (C_b) increase with span ratio.
 - $w_b/w_a \propto (\text{Span ratio})^4 \Rightarrow$ Increasing span ratio results in larger share of w_b ; i.e., more equitable distribution of applied slab loads
 5. Explain why the slab coefficients for DL and LL are different for M^+ but equal for M^- .
 - Draw influence lines for M^+ , M^- for a two-span continuous beam and place the loads for M_{max}^+ and M_{max}^- [LL for M^+ are placed differently than DL, but similarly for M^-]
 6. What are corner reinforcements in two-way slabs? Mention the ACI provisions for corner reinforcements.
 - Reinforcements to resist torsional stresses caused by unbalanced moments at the discontinuous ends (corners).
 - Similar (size, spacing) to the reinforcements for maximum moments in both directions.
 7. What is flat slab? Mention the differences between flat slab and flat plate.
 - Slabs supported directly on columns.
 - Differences include drop panel and column capital (draw figures).
 8. Mention the advantages and disadvantages of flat slabs.
 - Advantages include aesthetic, brightness, fire resistance, less height, easier formwork, construction (no beam).
 - Disadvantages include punching shear, consequent pancake failure, seismic vulnerability.
 9. Mention role of Drop Panel, Column Capital, Longitudinal and Transverse Beam in structural design of flat slabs.
 - Drop Panels help against punching shear by increasing effective slab thickness.
 - Column Capitals help against punching shear by increasing punching perimeter.
 - Longitudinal Beams reduce middle strip moment by increasing column strip moment but eventually shares it almost fully; Transverse Beams reduce exterior negative moment.
 10. Explain why flat slabs are divided into Column Strips and Middle Strips for design purpose.
 - Moments are distributed unevenly across the width of slab (show figure), and are much larger near columns.
 - Would have been unsafe and uneconomical to assume it uniform over the entire transverse direction.
 11. Compare between the structural analyses (a) of beam-supported slab vs. flat slab, (b) by Direct Design Method vs. Equivalent Frame Method.
 - - Moments calculated by coefficients (based on supports and span ratio) vs. two-step distribution.
 - - Moments in longer direction smaller vs. greater.
 - Moment distribution in longitudinal direction is direct vs. using structural analysis.
 12. Mention the conditions necessary for using the Direct Design Method of flat slab analysis.
 - Number of spans (at least 3), rectangular panels (span ratio ≥ 0.5), adjacent spans (within 1/3), column offset (less than 10%), LL/DL (≤ 2), beam stiffness ratio (≤ 5).
 13. Define the factors α , β_t and explain their effect on the structural analysis of flat slabs.
 - Mention Equations for α , β_t .
 - α (relative stiffness of longitudinal beam) increases column strip moments (reducing middle strip moments).
 - β_t (relative stiffness of transverse beam) decreases exterior negative column strip moments.
 14. What is punching shear and pancake failure of flat slabs?

Explain why the punching shear strength of concrete is considered larger than its flexural shear strength.

 - - Shear failure around the periphery of columns (within $d/2$ of all sides) leading to punching through the slab.
 - - May cause one slab to collapse upon the one at bottom, which also fails to take the impact load and leads to similar failure of successive slabs one upon another.
 - Elements within the punching zone are stronger in shear due to the large compressive stresses they are under (mention equations).

15. Briefly outline the design provisions for two types of shear reinforcement in flat slabs.
- Draw figures for best bar reinforcement and inclined stirrups.
 - Mention equations for spacing, maximum allowable V , V_c .
16. Mention and justify the maximum and minimum steel ratios specified by ACI for RC columns.
- Minimum ratio (1%) to protect against possible moments
 - Maximum ratio (8%) to prevent congestion of longitudinal bars.
17. Explain why transverse reinforcements are used in RC columns.
Also compare between the effect of ties and spirals on the strength and ductility of columns.
- Transverse reinforcements keep longitudinal bars in position and prevent their buckling.
 - Spirals are designed to compensate for the loss of strength due to concrete spalling, so strength is quite similar to tied columns (mention ϕ factors), but ductility is much greater (show figure) due to closely spaced spirals.
18. Mention the ACI recommendations for the size, spacing and arrangement of lateral ties and spirals.
- Mention tie size (#3 or #4), spacing (minimum of $16d_b$, $48d_t$ or least column dimension) and arrangement (provisions for supporting longitudinal bars).
 - Spirals (at least #3) spacing 1~3 in.
 - Show figures (longitudinal view).
19. Explain why the factors ϕ and α are used for column design.
Also explain why a smaller value of ϕ is used for columns compared to beams.
- ϕ is the resistance factor to account for possible reduction of member strength (due to material, section, etc), while α is incorporated for possible moments.
 - Greater structural importance, explosive nature of failure, greater dependence on f'_c , casual construction process (concrete segregation, minimal supervision), anomalous usage (e.g., electric wire) result in reduced ϕ for columns.
20. Explain why the application of compressive load may increase or decrease the moment capacity of columns.
- Draw interaction diagram and explain its two zones of failure (i.e, concrete compression and steel tension)
 - Additional compressive load means more compressive stress, so it causes more distress in concrete and decreases moment capacity when concrete compression governs, while it reduces tension in steel and increases moment capacity when steel tension governs.

Design of Shallow Foundations

The substructure, or foundation, is the part of the structure that is placed below the ground surface and transmits the load to the underlying soil. Since soil is much weaker and flexible compared to the structural materials (concrete and soil) that endure the loads above the ground, the main function of the foundation is to distribute the load over a large area to keep soil pressure and deformation (settlement) within 'allowable' limits (without harming the superstructure).

This can be achieved by using large bearing area of shallow footings or transferring the loads to firmer strata using deep foundations, at the same time through friction with surrounding soil.

Soil Pressure

The load on a wall or column is often transmitted vertically to the footing, which in turn is supported by the upward pressure of the underlying soil. If the load is symmetrical with respect to the bearing area, the bearing pressure is assumed uniformly distributed, which is also only approximately true.

Allowable bearing pressures are established from principles of soil mechanics, on the basis of load tests both in the field and within the laboratories. The allowable bearing pressures q_a under service loads are usually based on a factor of safety of 2.5~3.0 against exceeding the bearing capacity of the particular soil and to keep support settlements within tolerable limits.

For concentrically loaded footings, the required area is determined from

$$A_{req} = (DL + LL)/q_a \dots\dots\dots(4.1)$$

Wall Footings

Simple principles of beam action apply to wall footings, which are designed for the shear force and bending moments arising out of the soil pressure on the footing. Fig. 4.1 shows a wall footing (of footing width b and wall thickness t_w) with forces acting on it. For design purpose the maximum bending moment in footings at the face of the walls is given by

$$M_{max} = q_a \{(B-t_w)/2\}^2/2 = q_a(B-t_w)^2/8 \dots\dots\dots(4.2)$$

while governing shear force (computed at a distance d from wall face) is

$$V_{max} = q_a \{(B-t_w)/2-d\} \dots\dots\dots(4.3)$$

Flexural reinforcements are put at the bottom of wall footings in the direction of the B , while temperature reinforcements are placed transversely above them.

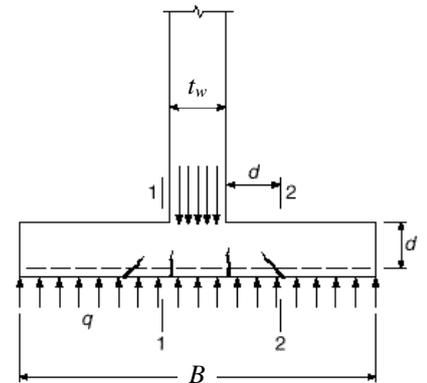
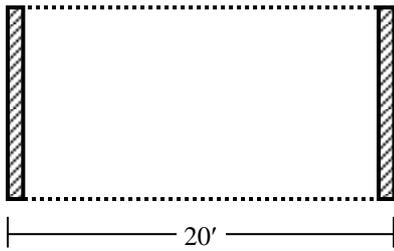


Fig. 4.1: Wall footing

Example 4.1

Use the WSD/USD to design a footing for the 10"-thick brick wall supporting RC slabs shown below (in a 5-storied building with 10'-high stories), if they carry loads including FF = 30 psf, RW = 50 psf and LL = 40 psf [Given: Depth of footing = 4 ft, Allowable soil bearing pressure = 3 ksf, $f_c' = 3$ ksi, $f_y = 50$ ksi].



Clear span $L = 20 - 20/12 = 18.33'$
 Required slab thickness $t_{req} = L/20 (0.4 + f_y/100) = 18.33 \times 12/20 \times 0.9 = 9.9''$
 \therefore Assume slab thickness $t_s = 10''$
 \therefore Total distributed load on slab = $10/12 \times 150 + 30 + 50 + 40 = 245$ psf
 Wall height = $10 - 10/12 = 9.17'$
 \Rightarrow Self-weight of wall = $10/12 \times 9.17 \times 120 = 917$ lb/ft
 \therefore Load per length of wall = $245 \times 20/2 + 917 = 3367$ lb/ft = 3.367 k/ft
 \therefore For 5-stories, the total load is $w = 5 \times 3.367 = 16.83$ k/ft

At a depth of 4' (filled by approximately 3' soil and 1' concrete),
 The allowable soil bearing pressure $q_{all} = 3 - 3 \times 0.11 - 1 \times 0.15 = 2.52$ ksf

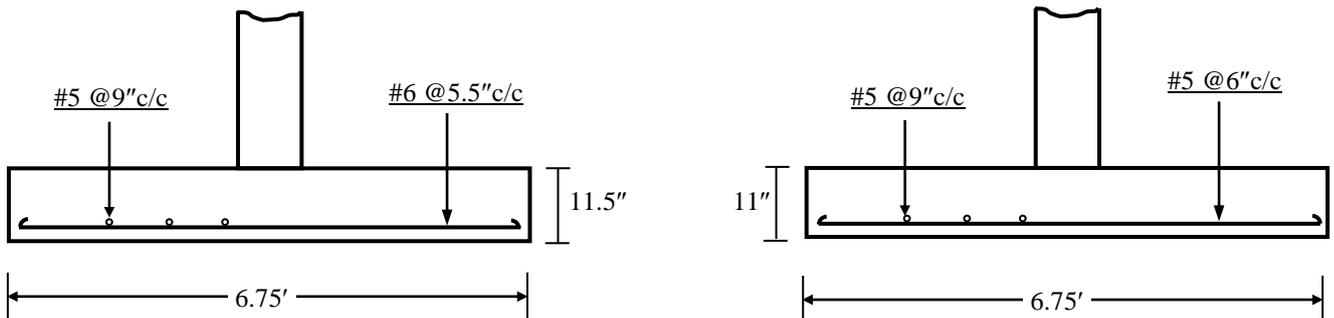
\therefore Required width of the footing $B_{req} = w/q_{all} = 16.83/2.52 = 6.68' \Rightarrow$ Footing width $B = 6.75'$

For WSD, $f_{call} = 0.45 \times 3 = 1.35$ ksi, $f_{sall} = 0.4 \times 50 = 20$ ksi, $n = 9$, $r = 14.81$
 $\Rightarrow k = n/(n + r) = 0.378$, $j = 1 - k/3 = 0.874$, $R_w = 0.5f_{call}kj = 0.5 \times 1.35 \times 0.378 \times 0.874 = 0.223$ ksi
 Also $v_c = 1.1\sqrt{f_c'} = 1.1\sqrt{(3/1000)} = 0.060$ ksi

For USD, $\rho_b = 87/(87 + f_y) (\alpha f_c'/f_y) = 87/137 \times 0.72 \times 3/50 = 0.0274$, $\rho_{max} = 0.75\rho_b = 0.0206$
 $\Rightarrow R_u = \phi \rho_{max} f_y (1 - 0.59\rho_{max} f_y/f_c') = 0.90 \times 0.206 \times 50 (1 - 0.59 \times 0.0206 \times 50/3) = 0.739$ ksi
 Also $v_{cu} = 2\phi\sqrt{f_c'} = 2 \times 0.85\sqrt{(3/1000)} = 0.093$ ksi

Table 4.1: Design Table for Wall Footing (using WSD and USD)

Parameter	Equation	WSD	USD
w_{Slab} (psf)	$SW + FF + RW + LL$	$125 + 30 + 50 + 40 = 245$	$1.4 (205) + 1.7 (40) = 355$
w_{Wall} (lb/')	$t_{Wall} \times h_{Wall} \times \gamma_{Wall}$	$10/12 \times 9.17 \times 120 = 917$	$1.4 (917) = 1283$
w_f (k/')	$N (w_{Slab}L/2 + w_{Wall})$	$5 \times (2450 + 917)/1000 = 16.83$	$5 \times (3550 + 1283)/1000 = 24.17$
q_{net} (ksf)	w_{Wall}/B	$16.83/6.75 = 2.49$	$24.17/6.75 = 3.58$
V_{max} (k/')	$q_{net} \{(B - t_{Wall})/2 - d\}$	$2.49 \{(6.75 - 10/12)/2 - d/12\}$	$3.58 \{(6.75 - 10/12)/2 - d/12\}$
v_{max} (ksi)	V_{max}/bd	$(7.38 - 0.208d)/12d$	$(10.59 - 0.298d)/12d$
d_{reqs} (")	$v_{max} = v_c$	$7.38/(0.208 + 0.060 \times 12) = 7.93$	$10.59/(0.298 + 0.093 \times 12) = 7.48$
M_{max} (k/')	$q_{net} \{(B - t_{Wall})/2\}^2/2$	$2.49 \{2.96\}^2/2 = 10.91$	$3.58 \{2.96\}^2/2 = 15.67$
d_{reqm} (")	$\sqrt{(M_{max}/R_b)}$	$\sqrt{(10.91/0.223)} = 7.00$	$\sqrt{(15.67/0.739)} = 4.61$
d (")	$\geq d_{req}$	8.0	7.5
t (")	$d + 3.5$	$8.0 + 3.5 = 11.5$	$7.5 + 3.5 = 11.0$
A_s (in ² /ft)	$M_{max}/(f_{sall}jd)$, OR $0.85f_c'/f_y [1 - \sqrt{(1 - 2M_n/(0.85f_c'bd^2))}]bd$	$10.91 \times 12 / (20 \times 0.874 \times 8) = 0.94$	$0.051 [1 - \sqrt{(1 - 2 \times 15.67 / (0.9 \times 2.55 \times 7.5^2))}] \times 12 \times 7.5 = 0.60$
A_{st} (in ² /ft)	$0.003bt$	$0.003 \times 12 \times 11.5 = 0.41$	$0.003 \times 12 \times 11.0 = 0.40$
L_d (")	$0.020 f_y/\sqrt{f_c'} d_b$ [Anchored]	$0.020 \times [50/\sqrt{(3/1000)}] (6/8) = 13.7$	13.7



Designed Wall footing in WSD and USD

Individual Column Footings

Square Footings

In plan, individual column footings are usually square or rectangular in shape. The design of column footings is somewhat similar to design of wall footings in that bending moment and shear force are checked respectively at the face of column and at a distance d from it. However, since footings support the column loads directly, they are also to be checked against punching shear (Fig. 4.2), using the principles and equations [2.3(a)~(c)] presented for flat slabs. Since the critical sections for moment, shear as well as punching shear are all close to the column, the footing thickness may be varied accordingly (shown in Fig. 4.3), including pedestal and stepped footing.

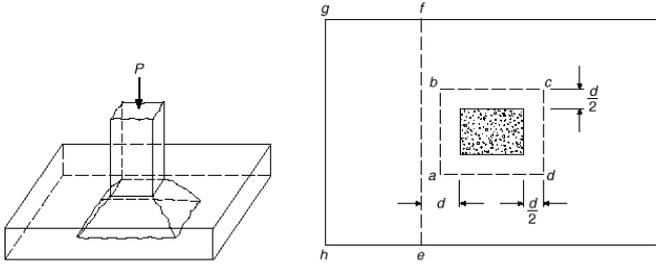


Fig. 4.2: Punching shear in column footing

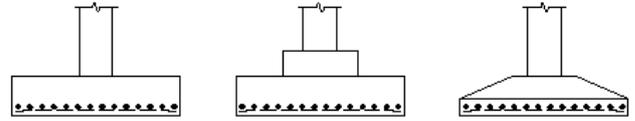


Fig. 4.3: Types of individual-column footing

Flexural main reinforcements are put in both directions at the bottom of column footings; however, they are to be checked against the minimum temperature/shrinkage reinforcements, as well as the minimum reinforcements $[A_{smin} = (200/f_y)bd]$ used to prevent early tensile collapse of beams.

The footings are also to be checked against bearing pressure from column loads (illustrated subsequently) and need to be wide enough to accommodate the development lengths of longitudinal bars.

Rectangular Footings

Space limitations (e.g., property lines or other footings) may not permit the use of square footings in many cases, requiring the designer to opt for rectangular footings. Design of rectangular footings follows the same procedure as the design of square footings, with particular additional detailing in bar placement.

As in square footings, reinforcements in the long direction of rectangular footings are uniformly distributed in the long direction. For bars in the short direction, however, one has to consider that the support provided to the footing by the column is concentrated near the middle, resulting in larger curvatures under column that decreases with the distance from the column.

Therefore, larger steel area per width is needed in the central portion (B of length L) than near the ends of the footing. ACI Code suggests a portion [given by Eq. (4.4)] of reinforcement in the short direction to be uniformly distributed over bandwidth equal to the short side B

$$A_{s(B)}/A_{s(L)} = 2/(\beta + 1) \quad \dots\dots\dots(4.4)$$

where $\beta = L/B$ and rest of the reinforcement in the short direction to be uniformly distributed outside the center bandwidth B .

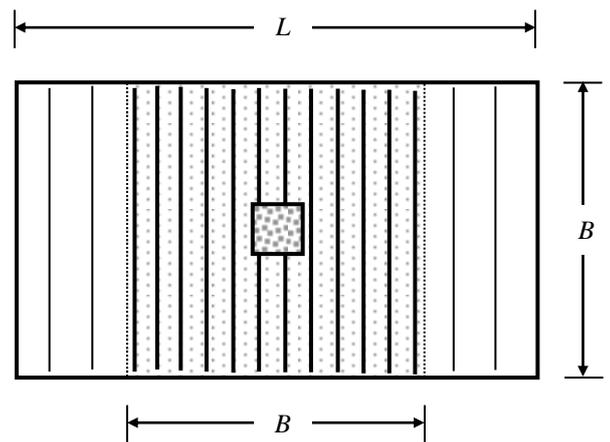
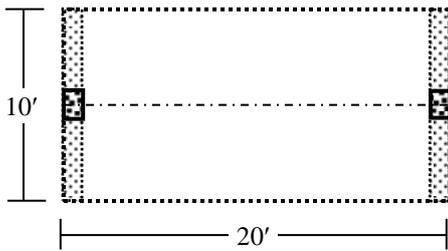


Fig. 4.4: Central bandwidth for reinforcement in rectangular footing

Example 4.2

Use the WSD/USD to design a footing for the 12"×18" RC columns supporting RC slabs shown below (in a 5-storied building with 10'-high stories), if they carry loads including FF = 30 psf, RW = 50 psf and LL = 40 psf

[Given: Depth of footing = 4 ft, Allowable soil bearing pressure = 3 ksf, $f_c' = 3$ ksi, $f_y = 50$ ksi].



Clear span $L = 20 - 2 = 18'$

Required slab thickness $t_{req} = L/20 (0.4 + f_y/100) = 18 \times 12/20 \times 0.9 = 9.7''$

\therefore Assume slab thickness $t_s = 10'' \Rightarrow$ Total distributed load on slab = 245 psf

Beam height below slab = 8'' \Rightarrow Height of 5'' wall = 10 - (10+8)/12 = 8.5'

\Rightarrow Weight of beam = (8 × 12)/12² × 150 × 10 = 1000 lb

Weight of column and wall = (12 × 18)/12² × 8.5 × 150 + 5/12 × 8.5 × 120 × 8.5 = 5525 lb

\therefore Load on column = 245 × 10 × 20/2 + 1000 + 5525 = 31025 lb = 31.03 k

Factored load on the column = 355 × 100 + 1.4 (6525) = 44635 lb = 44.64 k

\therefore For 5-stories, total load is $P = 5 \times 31.03 = 155.1$ k (WSD), and $P_u = 5 \times 44.64 = 223.2$ k (USD)

Allowable soil bearing pressure $q_{all} = 2.52$ ksf (from Example 4.1)

\therefore Required footing area $A_{req} = P/q_{all} = 155.1/2.52 = 61.6$ ft²

Using square footing \Rightarrow Footing size 8' × 8'

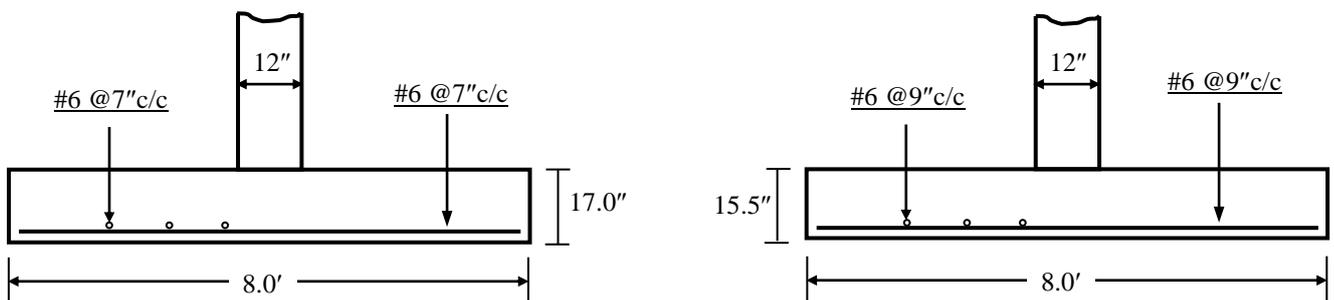
Concrete and steel properties (for both USD and WSD) are used from Example 4.1.

For punching shear, choosing between, 4.0, $(2 + 4/\beta_c) = 2 + 4/(18/12) = 4.67$, $(2 + \alpha_s d/b_o) = 2 + 40d/[2(30+2d)]$

$v_{pu} = 4\phi\sqrt{f_c'} = 4 \times 0.85\sqrt{(3/1000)} = 0.186$ ksi and $v_{pw} = 2\sqrt{f_c'} = 2\sqrt{(3/1000)} = 0.110$ ksi

Table 4.2: Design Table for Square Footing (using WSD and USD)

Parameter	Equation	WSD	USD
P_f (k)	Calculations above	155.1	223.2
q_{net} (ksf)	P_f/A	155.1/64.0 = 2.42	223.2/64.0 = 3.49
V_p (k)	$P_f - q_{net} (c_1+d) (c_2+d)$	155.1 - 2.42 (12+d)(18+d)/12 ²	223.2 - 3.49 (12+d)(18+d)/12 ²
A_p (in ²)	$2(c_1+d + c_2+d) d$	$2(12+d + 18+d)d = 60d + 4d^2$	$2(12+d + 18+d)d = 60d + 4d^2$
Equation	$v_p A_p = V_p$	$0.110 (60d + 4d^2) = 155.1 - 0.0168(12+d)(18+d)$	$0.186 (60d + 4d^2) = 223.2 - 0.0242(12+d)(18+d)$
d_{reqp} (")	Solving $V_p/A_p = v_p$	12.06	10.79
V_{max} (k')	$q_{net} \{(B - c_1)/2 - d\}$	$2.42 \{(8 - 12/12)/2 - d/12\}$	$3.49 \{(8 - 12/12)/2 - d/12\}$
v_{max} (ksi)	V_{max}/bd	$(8.48 - 0.202d)/12d$	$(12.20 - 0.291d)/12d$
d_{reqs} (")	$v_{max} = v_c$	$8.48/(0.202 + 0.060 \times 12) = 9.32$	$12.20/(0.291 + 0.093 \times 12) = 8.85$
M_{max} (k')	$q_{net} \{(B - c_1)/2\}^2/2$	$2.42 \{3.50\}^2/2 = 14.85$	$3.49 \{3.50\}^2/2 = 21.36$
d_{reqm} (")	$\sqrt{(M_{max}/Rb)}$	$\sqrt{(14.85/0.223)} = 8.16$	$\sqrt{(21.36/0.739)} = 5.38$
d (")	$\geq d_{req}$	12.5, 13.5	11.0, 12.0
t (")	$d + 3.5$	13.5 + 3.5 = 17.0	12.0 + 3.5 = 15.5
A_s (in ² /ft)	$M_{max}/(f_{sall} jd)$, OR $0.85f_c' / f_y [1 - \sqrt{(1 - 2M_u / (0.85f_c' bd^2))}] bd$	$14.85 \times 12 / (20 \times 0.874 \times 13.5) = 0.75$	$0.051 [1 - \sqrt{(1 - 2 \times 21.36 / (0.9 \times 2.55 \times 12^2))}] \times 12 \times 12 = 0.49$
A_{st}, A_{smin} (in ² /ft)	$0.003bt$, $(200/f_y)bd$	$0.003 \times 12 \times 17 = 0.61$ $(0.20/50) \times 12 \times 13.5 = 0.65$	$0.003 \times 12 \times 15.5 = 0.56$ $(0.20/50) \times 12 \times 12 = 0.58$
L_d (")	$0.050 f_y / \sqrt{f_c'} d_b$ [Un-Anchored]	$0.050 \times [50 / \sqrt{(3/1000)}] (6/8) = 34.3$	34.3
$\sigma_{bearing}$ (ksi)	$P/A_c = 2 \times 0.375f_c'$, OR $P_u/A_c = 2\phi 0.85f_c'$	$155.1 / (12 \times 18) = 0.72$ $< 2 \times 0.375 \times 3 = 2.25$ (OK)	$223.2 / (12 \times 18) = 1.03$ $< 2 \times 0.70 \times 0.85 \times 3 = 4.34$ (OK)

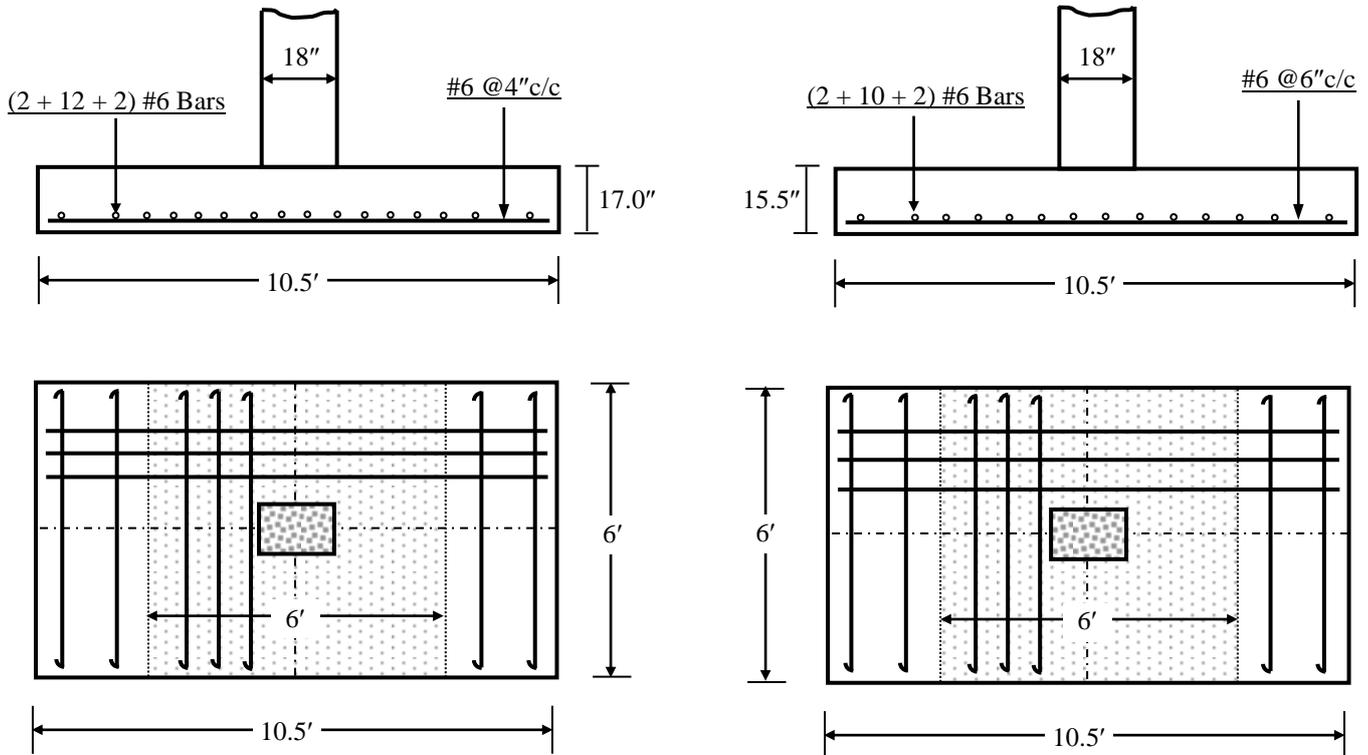


Designed Square footing in WSD and USD

Using rectangular footing \Rightarrow Footing size $6' \times 10.5'$

Table 4.3: Design Table for Rectangular Footing (using WSD and USD)

Parameter	Equation	WSD	USD
P_f (k)	Calculations before	155.1	223.2
q_{net} (ksf)	P_f/A	$155.1/63.0 = 2.46$	$223.2/63.0 = 3.54$
V_p (k)	$P_f - q_{net} (c_1+d) (c_2+d)$	$155.1 - 2.46 (12+d)(18+d)/12^2$	$223.2 - 3.54 (12+d)(18+d)/12^2$
A_p (in ²)	$2(c_1+d + c_2+d) d$	$2(12+d + 18 + d)d = 60d + 4d^2$	$2(12+d + 18 + d)d = 60d + 4d^2$
Equation	$V_p/A_p = v_p$	$0.110 (60d + 4d^2) = 155.1 - 0.0171(12+d)(18+d)$	$0.186 (60d + 4d^2) = 223.2 - 0.0246(12+d)(18+d)$
d_{reqp} (")	$V_p/A_p = v_p$	12.05	10.78
V_{max} (k')	$q_{net} \{(B - c_1)/2 - d\}$	$2.46 \{(10.5 - 18/12)/2 - d/12\}$	$3.54 \{(10.5 - 18/12)/2 - d/12\}$
v_{max} (ksi)	V_{max}/bd	$(11.08 - 0.205d)/12d$	$(15.94 - 0.295d)/12d$
d_{reqs} (")	$v_{max} = v_c$	$11.08/(0.205 + 0.060 \times 12) = 11.95$	$15.94/(0.295 + 0.093 \times 12) = 9.90$
M_{max} (k')	$q_{net} \{(B - c_1)/2\}^2/2$	$2.46\{4.50\}^2/2 = 24.93$ $2.46\{2.50\}^2/2 = 7.69$	$3.54\{4.50\}^2/2 = 35.87$ $3.54\{2.50\}^2/2 = 11.07$
d_{reqm} (")	$\sqrt{(M_{max}/Rb)}$	$\sqrt{(25.44/0.223)} = 10.57$	$\sqrt{(35.87/0.739)} = 6.97$
d (")	$\geq d_{req}$	12.5, 13.5	11.0, 12.0
t (")	$d + 3.5$	$12.5 + 3.5 = 16.0$	$11.0 + 3.5 = 14.5$
A_s (in ² /ft)	$M_{max}/(f_{sal}jd)$, OR $0.85f_c'f_y[1 - \sqrt{(1 - 2M_n)/(0.85f_c'bd^2)}]bd$	$24.93 \times 12 / (20 \times 0.874 \times 13.5) = 1.29$, AND 0.43	$0.051[1 - \sqrt{(1 - 2 \times 35.87)/(0.9 \times 2.55 \times 12^2)}] \times 12 \times 12 = 0.87$, AND 0.28
A_{st}, A_{smin} (in ² /ft)	$0.003bt, (200/f_y)bd$	$0.003 \times 12 \times 17 = 0.61$ $(0.20/50) \times 12 \times 13.5 = 0.65$	$0.003 \times 12 \times 15.5 = 0.56$ $(0.20/50) \times 12 \times 12 = 0.58$
$A_{s(band)}$ (in ²)	$\{2/(1+\beta)\}A_{s(short)}$	$A_{s(short)} = 0.65 \times 10.5 = 6.80$ $A_{s(b)} = 2 \times 6.80 / (1 + 10.5/6) = 4.95$	$A_{s(short)} = 0.58 \times 10.5 = 6.05$ $A_{s(b)} = 2 \times 6.05 / (1 + 10.5/6) = 4.40$
L_d (")	$0.050 f_y/\sqrt{f_c'} d_b$ [Un-Anchored]	$0.050 \times [50/\sqrt{(3/1000)}] (6/8) = 34.3$	34.3
$\sigma_{bearing}$ (ksi)	$P/A_c = 2 \times 0.375f_c'$, OR $P_u/A_c = 2\phi 0.85f_c'$	$158.3/(12 \times 18) = 0.73$ $< 2 \times 0.375 \times 3 = 2.25$ (OK)	$229.1/(12 \times 18) = 1.06$ $< 2 \times 0.70 \times 0.85 \times 3 = 4.34$ (OK)



Designed Rectangular footing (Side and Top view) in WSD and USD

Combined Column Footings

Use of individual footings under exterior columns becomes difficult if property rights prevent them from projecting beyond the exterior walls or if interior footings are so large and so close that they overlap. Footings supporting single columns may need to be combined in such cases. Spread footings that support more than one column or wall are called combined footings. Fig. 4.5 shows possible arrangements of various types of combined (and individual) footings.

It is desirable to design combined footing so that its centroid coincides with the resultant of column loads, thereby producing uniform bearing pressure over the entire area preventing soil overstress and tilting tendency of the footing. Fig. 4.6 shows various convenient shapes of a two-column footing, depending on the relative column loads. Depending on column loads, soil condition and available area, combined footings can take the forms of *strap footing* (Fig. 4.7), *grid foundation* (Fig. 4.8), or *mat foundation* (Fig. 4.9).

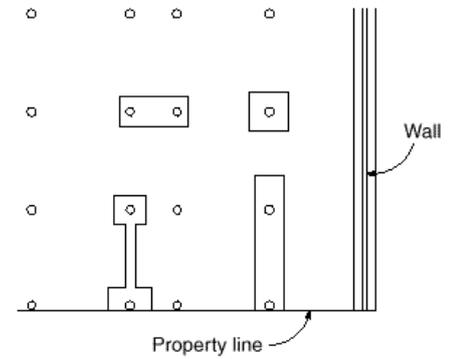


Fig. 4.5: Types of individual and combined shallow footing (Nilson)

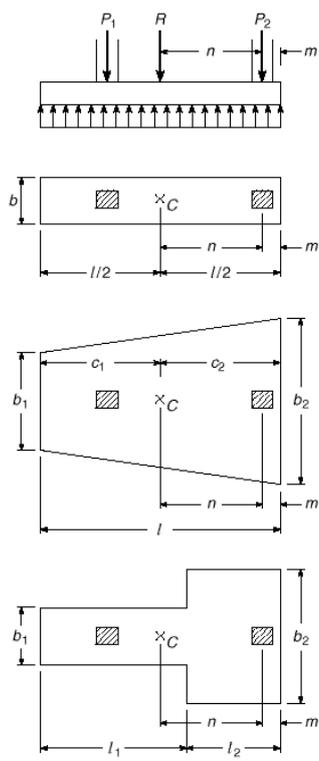


Fig. 4.6: Two-column footings



Fig. 4.7: Strap footing (Nilson)

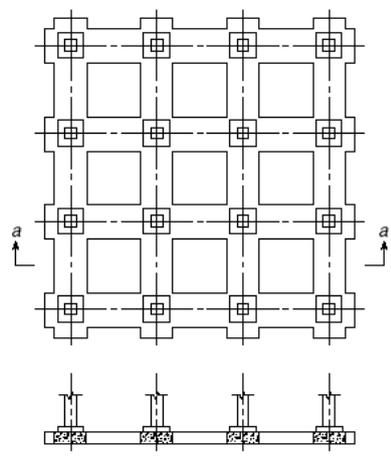


Fig. 4.8: Grid foundation (Nilson)

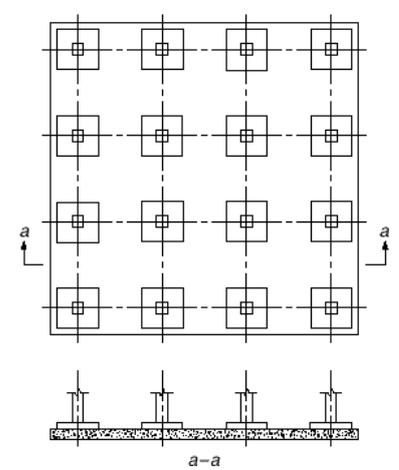
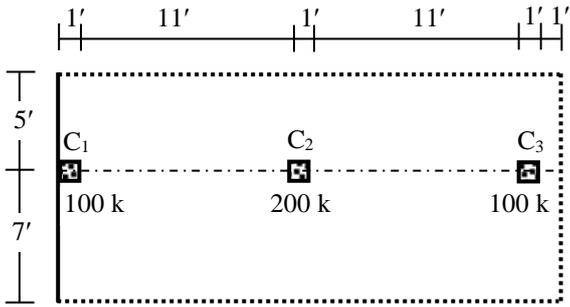


Fig. 4.9: Mat foundation (Nilson)

Example 4.3

Figure below shows the working loads (half DL, half LL), arrangement of 12"×12" columns C₁, C₂, C₃ and boundaries of other footings (firm lines for property lines and dotted lines for adjacent footing boundaries).

Design the footing(s) for the columns [Given: Allowable soil bearing pressure = 2.0 ksf, $f_c' = 3$ ksi, $f_y = 50$ ksi].



Possible individual footing areas under columns C₁, C₂ and C₃ are $(100/2.0 =) 50 \text{ ft}^2$, $(200/2.0 =) 100 \text{ ft}^2$, 50 ft^2 respectively.

Individual footing is not advisable particularly under C₁ and C₃.
 \therefore A combined footing is suggested under the three columns.

Center of the footing should be at
 $\bar{x} = (100 \times 0.5 + 200 \times 12.5 + 100 \times 24.5)/400 = 12.5'$
 \therefore Footing length = $2 \times 12.5 = 25'$
 Required footing area = $400/2.0 = 200 \text{ ft}^2 \Rightarrow$ Width = $8'$
 \Rightarrow Load per unit length = $2.0 \text{ ksf} \times 8' = 16 \text{ k/ft}$

Assuming half DL and half LL, the column loads for USD are $(1.4 \times 50 + 1.7 \times 50 =)$ 155 k, 310 k and 155k and the net soil pressure = $620/200 = 3.1 \text{ ksf}$; i.e., 24.8 k/ft.

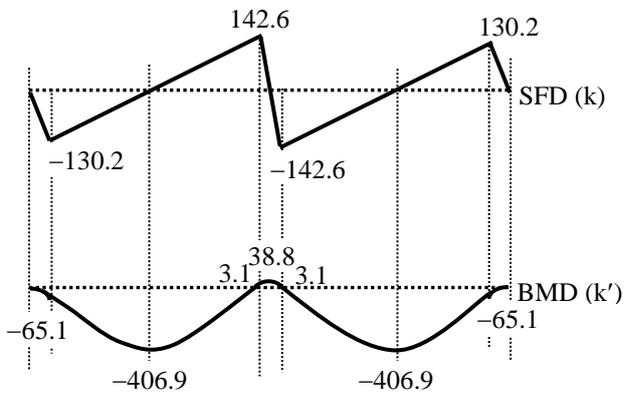
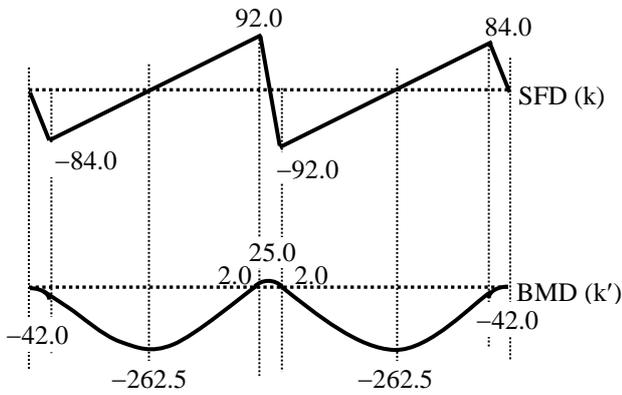
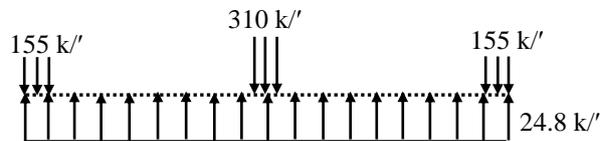
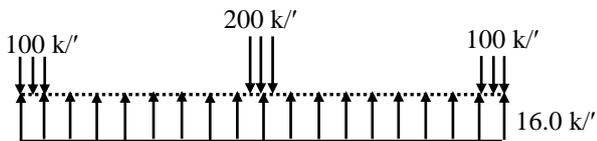
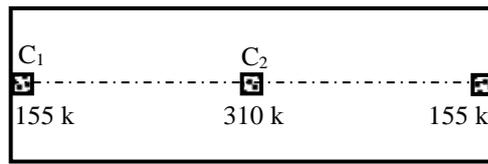
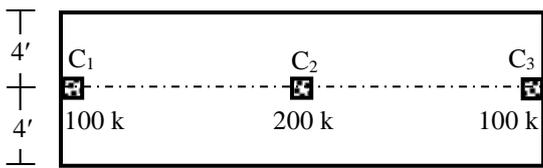
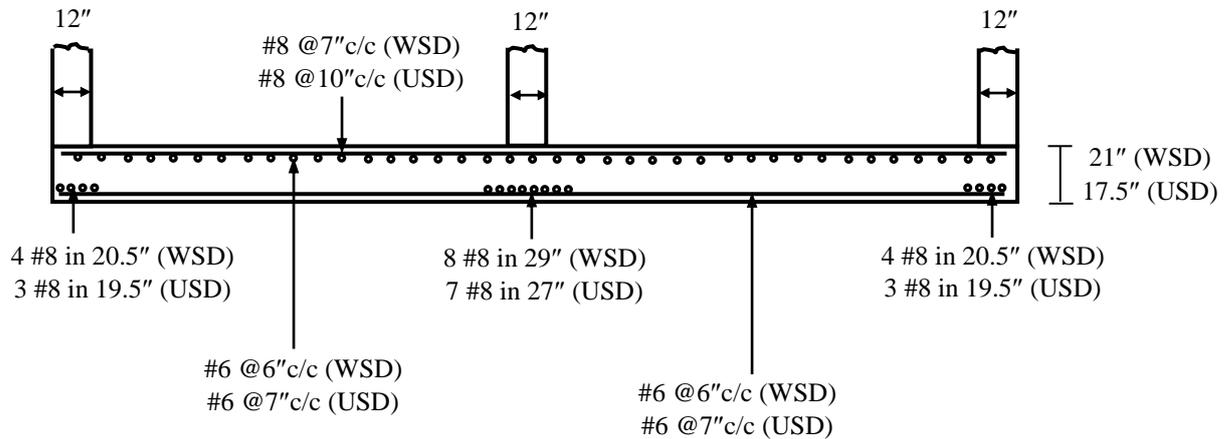


Table 4.4: Design Table for Combined Footing (using WSD and USD)

Parameter	Equation	WSD	USD
V_p (k)	$P_c - q_{net} (c_1+d) (c_2+d)$	200-2.0 (12+d)(12+d)/12 ² OR 100-2.0 (12+d)(12+d/2)/12 ²	310-3.1 (12+d)(12+d)/12 ² OR 155-3.1 (12+d)(12+d/2)/12 ²
A_p (in ²)	$\frac{2(c_1+d + c_2+d)d}{2}$ OR $\{2(c_1+d) + c_2+d\}d$	$\frac{2(12+d + 12+d)d}{2} = 48d + 4d^2$ OR $\{2(12+d/2) + (12+d)\}d = 36d+2d^2$	$\frac{2(12+d + 12+d)d}{2} = 48d + 4d^2$ OR $\{2(12+d/2) + (12+d)\}d = 36d+2d^2$
Equation	$V_p/A_p = v_p$	0.110 (48d + 4d ²) = 200-0.0139(12+d)(12+d) OR 0.110 (36d + 2d ²) = 100-0.0139(12+d)(12+d/2)	0.186 (48d + 4d ²) = 310-0.0215(12+d)(12+d) OR 0.186 (36d + 2d ²) = 155-0.0215(12+d)(12+d/2)
d_{reqp} (")	$V_p/A_p = v_p$	15.60, OR 13.48	14.78, OR 12.71
V_f (k)	$V_{max} - w_{net} d$	92.0-16.0 d/12	142.6-24.8 d/12
v_f (ksi)	V_f/bd	(92.0-1.33d)/(8×12)d	(142.6-2.07d)/(8×12)d
d_{reqs} (")	$v_f = v_c$	92.0/(1.33+0.060×96) = 12.93	142.6/(2.07+0.093×96) = 12.96
M_{max} (k')	From BMD	262.5	406.9
d_{reqm} (")	$\sqrt{M_{max}/Rb}$	$\sqrt{\{262.5/(0.223×8)\}} = 12.13$	$\sqrt{\{406.9/(0.739×8)\}} = 8.30$
w_{ibm} (k')	$P_{c2}/B, P_{c1}/B$	200/8 = 25.0, AND 12.5	310/8 = 38.8, AND 19.4
M_{ibm} (k')	$w_{ibm}L^2/2$	25.0×(4-6.0/12) ² /2 = 153.1, 76.6	38.8×(4-6.0/12) ² /2 = 237.3, 118.6
d_{reqt} (")	$\sqrt{M_{ibm}/Rb}$	$\sqrt{\{153.1×12/(0.223×28)\}} = 17.15$	$\sqrt{\{237.3×12/(0.739×27)\}} = 12.00$
d (")	$\geq d_{req}$	17.5	15.0
t (")	$d + 3.5, \text{ OR } d + 2.5$	17.5 + 3.5 = 21.0	15.0 + 2.5 = 17.5
A_s (in ² /ft)	$\frac{M_{max}/(f_{sall}jd), \text{ OR } 0.85f_c/f_y[1-\sqrt{1-2M_n/(0.85f_c bd^2)}]}{bd}$	$\frac{262.5×12/(20×0.874×18.5)}{8×12×15} = 9.74; \text{ i.e., } 9.74/8 = 1.22 \text{ in}^2/\text{ft}$	$\frac{0.051[1-\sqrt{1-2×406.9/(0.9×2.55×8×15^2)}]}{8×12×15} = 7.63; \text{ i.e., } 0.95 \text{ in}^2/\text{ft}$
A_{st}, A_{smin} (in ² /ft)	$0.003bt, (200/f_y)bd$	$0.003×12×21.0 = 0.76$ $(0.20/50)×12×18.5 = 0.89$	$0.003×12×17.5 = 0.63$ $(0.20/50)×12×15 = 0.72$
$A_{s(tbm)}$ (in ²)	$\frac{M_{max}/(f_{sall}jd), \text{ OR } 0.85f_c/f_y[1-\sqrt{1-2M_n/(0.85f_c bd^2)}]}{bd}$	$\frac{153.1×12/(20×0.874×17.5)}{8×12×14} = 6.01, \text{ AND } 3.00$	$\frac{0.051[1-\sqrt{1-2×237.3×12/(0.9×2.55×28×14^2)}]}{28×14} = 5.19, \text{ AND } 2.40$
$\sigma_{bearing}$ (ksi)	$P/A_c = 2×0.375f_c', \text{ OR } P_u/A_c = 2\phi 0.85f_c'$	$200/(12×12) = 1.39$ $< 2×0.375×3 = 2.25 \text{ (OK)}$	$310/(12×12) = 2.15$ $< 2×0.70×0.85×3 = 3.57 \text{ (OK)}$



Designed Combined footing in WSD and USD

Questions and Problems (4)

1. (i) Mention different types of RC foundations and also mention when they are used.
 (ii) Distinguish between allowable soil pressure and net soil pressure in the design of footings by USD.
 (iii) Specify the minimum reinforcements required in the design of wall footings and column footings.
 (iv) Explain why punching shear is considered in the design of column footings but not for wall footings.
 (v) Define the bandwidth in placing reinforcements for rectangular footings and explain why it is used.
 (vi) What is Transverse Beam in combined footings? Narrate how it is designed.

[Given: $f_c' = 4$ ksi, $f_y = 60$ ksi and allowable bearing capacity of soil = 4 ksf for all questions]

2. Fig. 1 shows the floor plan of a 6-storied RC structure, consisting of 5"-thick slab S (supported on 12"×16" beams), 7"-thick flat-slab FS and 5" thick partition walls along all column lines.

Floor loads also include working $FF = 30$ psf, $RW = 40$ psf and $LL = 50$ psf.

Use the USD to determine the

- (i) Size of circular column C_1 (for axial force only)
- (ii) Size and thickness of a square footing supporting column C_1 .

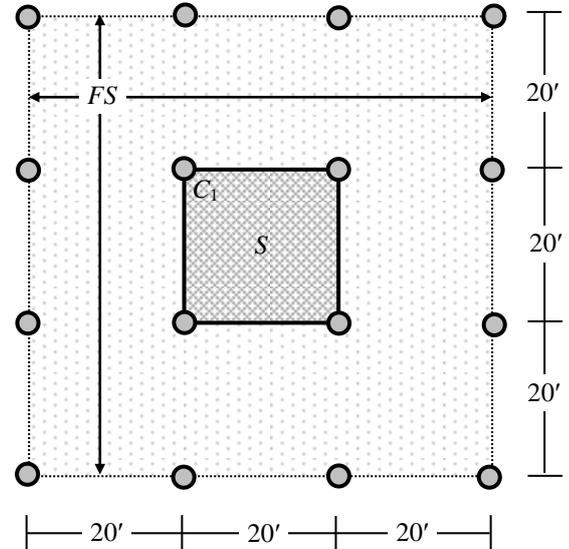


Fig. 1

3. Figures below show two options of transferring a vertical load of $P = 120$ kip (equal DL and LL), to the soil underneath; i.e., through a (6'×10') wall footing [Fig. 2(a)] and a (6'×10') column footing [Fig. 2(b)].

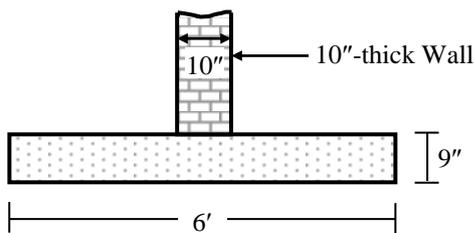


Fig. 2(a)

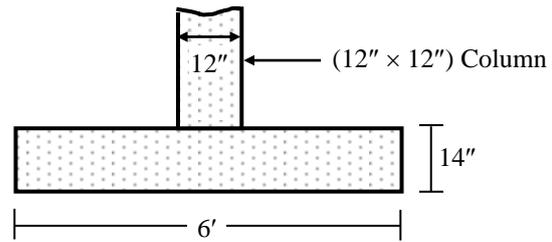


Fig. 2(b)

Use the USD to calculate and show the reinforcements required (in both directions) for the

- (i) 9"-thick wall footing, (ii) 14"-thick column footing.

4. Fig. 3 shows the axial force and bending moment on (12"×12") column C_1 and (18"×18") column C_2 . Calculate the

- (i) Maximum and minimum soil pressure under footing F_1 (supporting C_1) and footing F_2 (supporting C_2)
- (ii) Required length, width and thickness (considering punching shear around C_2 only for WSD) of a combined footing supporting C_1 and C_2 .

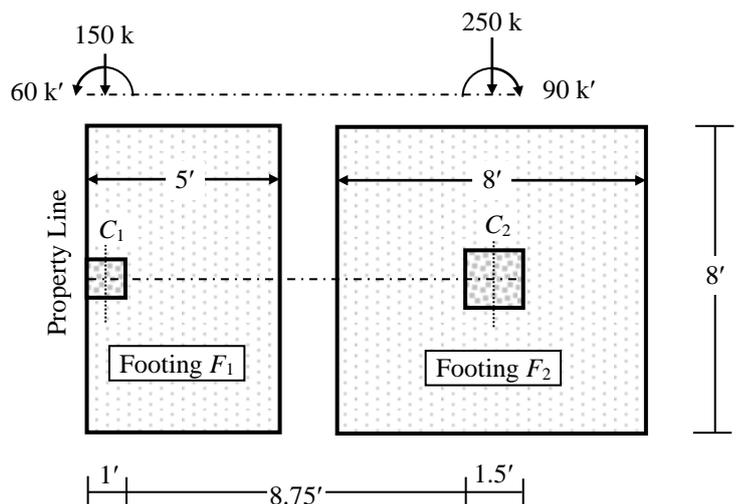


Fig. 3

2. Self-wt of $S = 5/12 \times 150 = 62.5$ psf, and of $FS = 7/12 \times 150 = 87.5$ psf

\therefore Factored load on $S = 1.4(62.5 + 30 + 40) + 1.7(50) = 270.5$ psf

and on $FS = 1.4(87.5 + 30 + 40) + 1.7(50) = 305.5$ psf

Factored load from beam and wall = $1.4 \{ (12 \times 11) / 144 \times 150 + (5/12 \times (10 - 16/12) \times 120) \} = 1.4 \times 570.8 = 799.2$ lb/ft

Factored load from FS wall = $1.4 \{ (5/12 \times (10 - 7/12) \times 120) \} = 1.4 \times 470.8 = 659.2$ lb/ft

\therefore Column Load per floor = $[\{ 270.5 \times 10 \times 10 + 305.5 \times (20 \times 20 - 100) \} + 799.2 \times 20 + 659.2 \times 20]$ lb = 147.87 k

\therefore Total Load on column $C_1 = 6 \times 147.87 = 887$ k

(i) Assuming 3% reinforcement, $P_u = \alpha \phi \{ 0.85 f_c' (1 - \rho_g) + \rho_g f_y \} A_g$

$\Rightarrow 887 = 0.8 \times 0.7 \{ 3.40 \times 0.97 + 0.03 \times 60 \} A_g \Rightarrow A_g = 310.8$ in²; i.e., Assume 20"-dia column

(ii) (a) Working floor loads are $(62.5 + 30 + 40 + 50) = 182.5$ psf and $(87.5 + 30 + 40 + 50) = 207.5$ psf

Working load from beam and wall = 570.8 lb/ft, Working load from FS wall = 470.8 lb/ft

\therefore Column Load per floor = $182.5 \times 10 \times 10 + 207.5 \times (20 \times 20 - 100) + 570.8 \times 20 + 470.8 \times 20$ lb = 101.33 k

\therefore Total Load on footing = $6 \times 101.33 = 608$ k

Assuming 4'-depth of footing (optional), allowable bearing capacity = $4 - 3 \times 0.11 - 1 \times 0.15 = 3.52$ ksf

\therefore Required footing size = $608 / 3.52 = 172.73$ ft²; i.e. (13.5' \times 13.5')

\Rightarrow Net soil pressure = $887 / (13.5 \times 13.5) = 4.87$ ksf

(b) Punching shear force = $887 - 4.87 \times \pi/4(20+d)^2/12^2 = 887 - 0.0266(20+d)^2$, and Area = $\pi(20+d)d$

$\Rightarrow 887 - 0.0266(20+d)^2 = \pi(20+d)d \times 4 \times 0.85 \sqrt{(3/1000)} \Rightarrow d_{req} = 28.84"$

\therefore Beam shear force = $4.87 \times \{ (13.5 - 20/12) / 2 - 28.9 / 12 \} = 26.40$ k, and Area = $12 \times 28.84 = 346.1$ in²

\Rightarrow Beam shear stress = $26.40 / 346.1 = 0.076$ ksi; which is $< 2 \times 0.85 \sqrt{(3/1000)} = 0.093$ ksi \Rightarrow OK

Maximum bending moment $M_{u(max)} = 4.87 \times \{ (13.5 - 20/12) / 2 \}^2 / 2 = 85.21$ k-ft/ft

$\rho_{max} = 0.75 \times 87 / (87 + f_y)$, $\alpha f_c' / f_y = 0.75 \times 87 / (87 + 60) \times 0.72 \times 4 / 60 = 0.0213$

$R_u = \phi \rho_{max} f_y (1 - 0.59 \rho_{max} f_y / f_c') = 0.90 \times 0.0213 \times 60 (1 - 0.59 \times 0.0213 \times 60 / 4) = 0.934$ ksi

$\Rightarrow d_{req} = \sqrt{(M_u / R_u)} = \sqrt{(85.21 / 0.934)} = 9.55"$

\therefore Choose $d = 29.0"$; i.e., $t = 29.0 + 3.5 = 32.5"$

3. Working load $P = 120$ k (i.e., $60 + 60$); \therefore Ultimate load $P_u = 1.4 \times 60 + 1.7 \times 60 = 186$ k

\Rightarrow Net soil pressure = $186 / (6 \times 10) = 3.10$ ksf

(i) For the wall footing, Maximum bending moment $M_{u(max)} = 3.10 \times \{ (6 - 10/12) / 2 \}^2 / 2 = 10.34$ k-ft/ft

Also $d = 9 - 2.5 = 6.5"$, $\therefore A_s = 0.85 f_c' / f_y [1 - \sqrt{(1 - 2M_u / (0.85 f_c' b d^2))}] b d$

$= (3.4 / 60) [1 - \sqrt{(1 - 2 \times 10.34 / (3.4 \times 6.5^2))}] \times 12 \times 6.5 = 0.33$ in²/ft

while $A_{s(temp)} = 0.036 t = 0.036 \times 9 = 0.32$ in²/ft

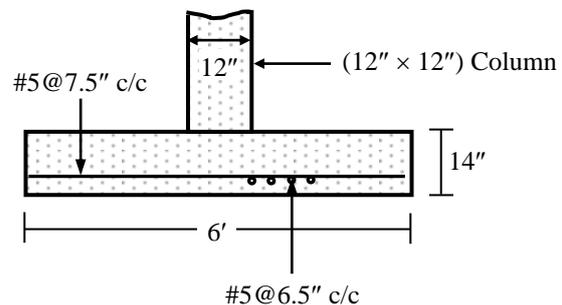
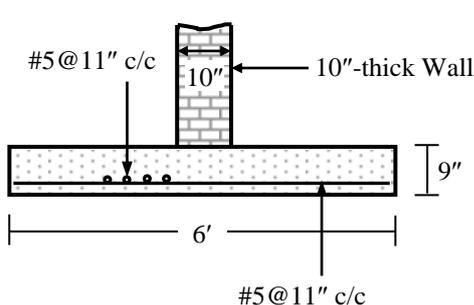
(ii) For the column footing, Maximum bending moments are

$M_{u1(max)} = 3.10 \times \{ (6 - 12/12) / 2 \}^2 / 2 = 9.69$ k-ft/ft, with $d = 14 - 3.5 = 10.5"$

$\therefore A_{s1} = (3.4 / 60) [1 - \sqrt{(1 - 2 \times 9.69 / (3.4 \times 10.5^2))}] \times 12 \times 10.5 = 0.19$ in²/ft, while $A_{s(temp)} = 0.036 \times 14 = 0.50$ in²/ft

and $M_{u2(max)} = 3.10 \times \{ (10 - 12/12) / 2 \}^2 / 2 = 31.69$ k-ft/ft, with $d = 14 - 2.5 = 11.5"$

$\therefore A_{s2} = (3.4 / 60) [1 - \sqrt{(1 - 2 \times 31.39 / (3.4 \times 11.5^2))}] \times 12 \times 11.5 = 0.57$ in²/ft



4. (i) For Footing F_1 , $P_1 = 150$ k, $M_1 = 60 + 150 \times (5/2 - 1/2) = 360$ k-ft
 \therefore Maximum and minimum pressure = $150/(5 \times 8) \pm 360 \times 6/(5^2 \times 8) = 3.75 \pm 10.8 = 14.55$ and -7.05 ksf;
 i.e., Maximum > Allowable soil pressure, and Negative Minimum indicates overturn

For Footing F_2 , $P_2 = 250$ k, $M_2 = 90$ k-ft

\therefore Maximum and minimum pressure = $250/(8 \times 8) \pm 90 \times 6/(8^2 \times 8) = 3.91 \pm 1.05 = 4.96$ and 2.85 ksf;
 i.e., Maximum > Allowable soil pressure

- (ii) For combined footing, $\bar{x} = \{150 \times 0.5 + 250(1 + 8.75 + 0.75) - 60 + 90\}/(150 + 250) = 6.83'$



\therefore Length of footing (L) = $6.83 + 6.83 = 13.65'$

\therefore Required footing area = $(150 + 250)/3.52 = 113.6$ ft² \Rightarrow Required Width = $113.6/13.65 = 8.33'$, let $B = 8.5'$

\therefore Pressures = $400/(13.65 \times 8.5) \pm \{150 \times 6.33 + 60 - 250 \times (10.0 - 6.33) - 90\} \times 6/(13.65^2 \times 8.5) = 3.45$ ksf

Considering punching shear under C_2

$\Rightarrow 250 - (3.45/12)^2(18+d)^2 = 4(18+d)d \times 2 \times \sqrt{(3/1000)} \Rightarrow d_{req} = 15.3''$; i.e., let $d = 15.5''$

\therefore Thickness $t = 15.5 + 3.5 = 19''$

Design of Pile Foundations

If the bearing capacity of upper soil layers is insufficient for a spread foundation, but firmer strata are available at greater depth, piles are used to transfer the loads to these deeper strata. [Fig. 5.1(i)]. Piles can be classified based on

- * Construction process (*Pre-cast Piles* are constructed in factory and carried to the site [Fig. 5.1(ii)], while *Cast-in-situ Piles* are constructed on site), or
- * Nature of resistance (*Friction Piles* resist loads predominantly by ‘skin’ friction between pile and soil, while *End-bearing Piles* primarily resist by bearing force at the end of pile).

Circular, rectangular, hexagonal are some of the more common cross-sectional shapes of piles.

Analysis of Piles in a Group

Piles are generally arranged in groups or clusters under each column (Fig. 5.2). The group is capped by a spread footing or cap that distributes column load to all piles in the group. Pile caps differ from spread footings in two distinctive manners; i.e., reactions on pile caps act as concentrated loads at piles (instead of as distributed pressures), and pile group can redistribute overturning moment among piles in the group.

If a group of N piles is subjected to axial force P and biaxial bending moments M_x and M_y , force in a pile at distances x and y from the centroid of the group is

$$F(x, y) = P/N + M_x y / (\sum y_i^2) + M_y x / (\sum x_i^2) \quad \dots\dots\dots(5.1)$$

where $\sum x_i^2$ and $\sum y_i^2$ are sums of (pile-distance)² in x - and y -directions.

Design of Piles

Bending moments may govern the design of piles when they are subjected to lateral loads or when pre-cast piles are carried to sites [Fig. 5.1(i)]. In such cases, the structural design of piles would be quite similar to the design of beams.

However, piles are subjected primarily to axial forces under vertical loading [Fig. 5.1(ii)]. In such cases, their structural design is quite similar to the design of columns. Eq. (5.1) is used to determine the cross-sectional area (a_p) and reinforcement (a_s) required, assuming f_{call} and f_{sall} to be the allowable stress (or ultimate or yield strength in USD) in concrete and steel, and ϕ the strength reduction (or safety) factor.

$$F = \phi (a_p f_{call} + a_s f_{sall}) \quad \dots\dots\dots(5.2)$$

A relatively small amount of longitudinal reinforcement (often 1% or less of the cross-sectional area) is used in piles. The longitudinal bars are confined by ties (or spirals), amounting to a small percentage (about 0.2%) of pile volume

The pile-length should be determined considering the shear strength of the surrounding soil acting over the pile perimeter. Therefore the required pile-length for axial force F is

$$L_p = F / \{ \alpha_2 \tau_s (\pi d_p) \} \quad \dots\dots\dots(5.3)$$

where d_p = Pile diameter, τ_s = Shear strength of surrounding soil, α_2 = Strength reduction factor for soil disturbance

Design of Pile-Caps

Piles are generally arranged in tight patterns, minimizing the cost of caps, but they cannot be placed closer than the conditions of driving and of undisturbed carrying capacity permit. A spacing of about 3 times the top diameter of the pile (but no less than 2.5') is customary. Fig. 5.3 shows typical spacing between piles in a group.

Like spread footings, pile caps should also be designed for bending moment (at column faces), punching shear (at $d/2$ from faces of piles and columns) and flexural shear (at d from column faces).

Flexural shear at any section through pile cap should include the following

- (a) Entire reaction from any pile whose center is $d_p/2$ or more outside it
- (b) No reaction from any pile whose center is $d_p/2$ or more inside it
- (c) Linearly interpolated reaction from any pile with center between (a) and (b).

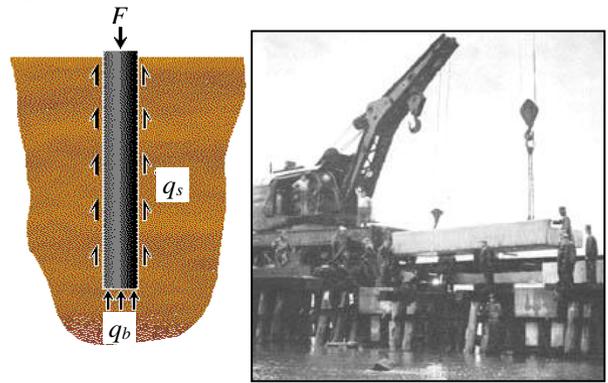


Fig. 5.1: (i) Axially loaded pile, (ii) Carrying pre-cast pile

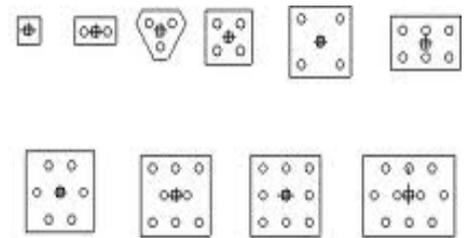


Fig. 5.2: Piles arranged in groups of 1~10

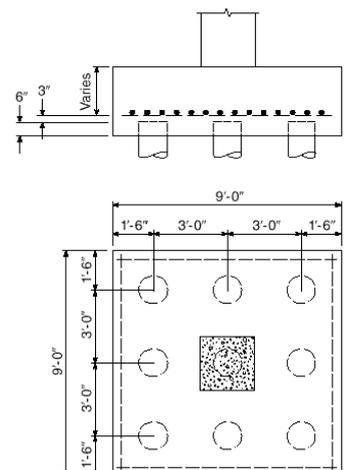


Fig. 5.3: Clear covers in Pile-Cap

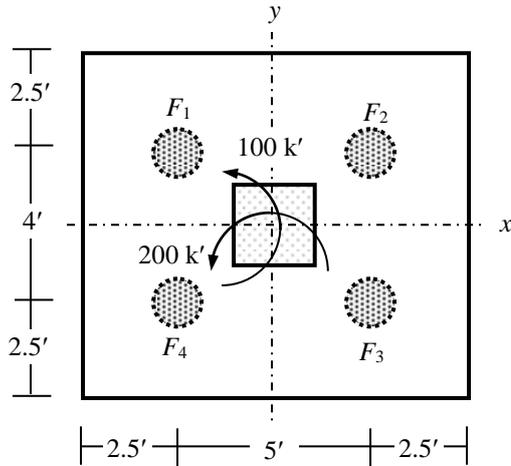
Example 5.1

A 25" × 25" column carrying working loads $DL = 300$ k, and $LL = 200$ k is underlain by soil with allowable bearing capacity = 2 ksf. The column also carries biaxial moments (due to LL) of $M_x = 100$ k-ft and $M_y = 200$ k-ft.

The footing area beneath the column must not exceed (10' × 10') because of proximity to adjacent columns. Use the WSD/USD to analyze and design the footing [Given: $f_c' = 3$ ksi, $f_y = 50$ ksi].

Total working load = 300 + 200 = 500 k, with moments $M_x = 100$ k-ft and $M_y = 200$ k-ft

The ultimate force $P_u = 1.4 \times 300 + 1.7 \times 200 = 760$ k, with moments $M_{xu} = 1.7 \times 100 = 170$ k-ft and $M_y = 340$ k-ft
 \Rightarrow Required area of footing $A_{req} = 500/2.0 = 250$ ft² even without moments, which is too large and does not satisfy the adjacent space requirements. Therefore, deep foundation (i.e., pile) is the best option here.



Assuming four pile arrangement as shown, with (10' × 9') pile cap
 $N = 4$, $\sum x^2 = 4 \times 2.5^2 = 25$ ft², and $\sum y^2 = 4 \times 2^2 = 16$ ft²

$$\therefore \text{Maximum pile force } F_1 = 500/4 + 100 \times 2/16 + 200 \times 2.5/25 = 125 + 12.5 + 20 = 157.5 \text{ k}$$

$$\text{Minimum pile force } F_3 = 125 - 12.5 - 20 = 92.5 \text{ k (checking if tension)}$$

$$\text{while the other pile forces are } F_2 = 125 + 12.5 - 20 = 117.5 \text{ k, and } F_4 = 125 - 12.5 + 20 = 132.5 \text{ k}$$

For USD, the corresponding forces are

$$F_1 = 760/4 + 170 \times 2/16 + 340 \times 2.5/25 = 190 + 21.1 + 34 = 245.1 \text{ k}$$

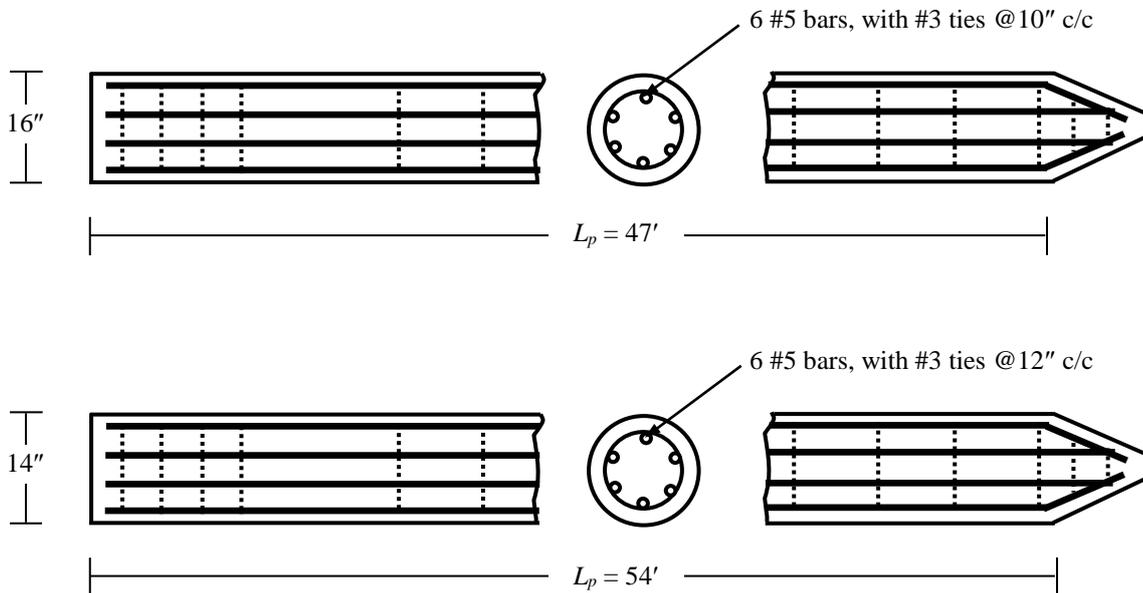
$$F_3 = 190 - 21.1 - 34 = 134.9 \text{ k, } F_2 = 177.1 \text{ k, } F_4 = 202.9 \text{ k}$$

Table 5.1: Design Table for Pile (using WSD and USD)

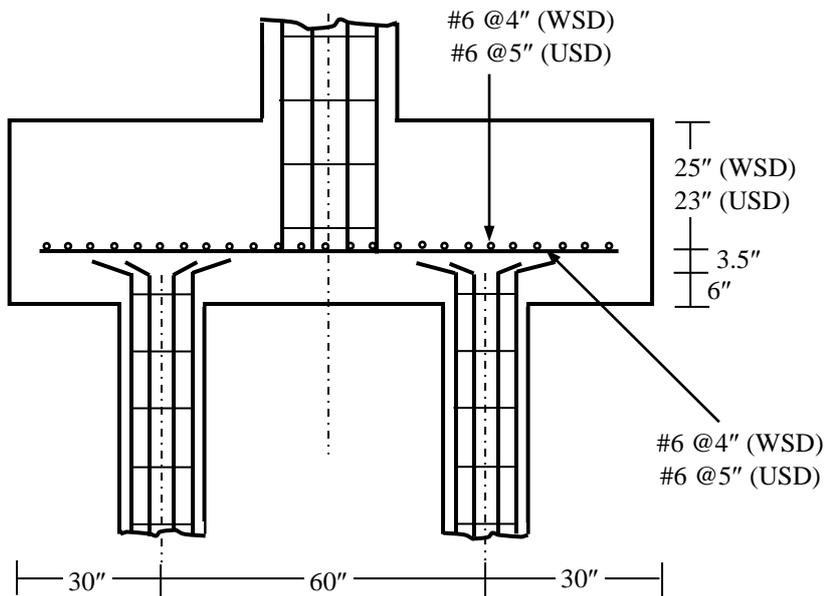
Parameter	Equation	WSD	USD
d_p (")	$F = \phi(a_p f_{calt} + a_s f_{sall})$	$157.5 = 0.85(0.25 \times 3 + 0.01 \times 20) \pi d_p^2 / 4$ $\Rightarrow d_p = 15.8$; i.e., 16"	$245.1 = 0.80 \times 0.70(0.85 \times 3 + 0.01 \times 50) \pi d_p^2 / 4$ $\Rightarrow d_p = 13.5$; i.e., 14"
a_s (in ²)	$a_s = (P/\phi - a_p f_{calt})/f_{sall}$	$(157.5/0.85 - 201 \times 0.75)/20 = 1.72$	$(245.1/0.56 - 154 \times 2.55)/50 = 0.90$
S_t (")	0.2% of Pile volume; i.e., $S_t = (100/0.2)(a_t/a_p) \pi d_c$	$(100/0.2)(0.375/16)^2 \pi (12) \square = 10.4$; i.e., #3@10" c/c	$(100/0.2)(0.375/14)^2 \pi \square (10) = 11.3$; i.e., #3@11" c/c
L_p (')	$L_p = P/\{\alpha_2(\pi d_p) \tau_u\}$	$157.5/\{0.8 \pi (16/12)(2.0/2)\} = 47.0$	$157.5/\{0.8 \pi (14/12)(2.0/2)\} = 53.7$

Table 5.2: Design Table for Pile Cap (using WSD and USD)

Parameter	Equation	WSD	USD
V_p (k)	P_f	500	760
A_p (in ²)	$4(c+d)d$	$4(25+d)d = 100d + 4d^2$	$100d + 4d^2$
Equation	$V_p/A_p = v_p$	$500/(100d + 4d^2) = 0.110$	$760/(100d + 4d^2) = 0.186$
d_{reqp} (")	$V_p/A_p = v_p$	23.52	21.80
d_{shear} (")	$\{(x + d_p/2) - (c/2 + d)\}$	$(30 + 16/2) - (12.5 + d) = 25.5 - d$	$(30 + 14/2) - (12.5 + d) = 24.5 - d$
V_{max} (k)	$(F_1 + F_4)(d_{shear}/d_p)$	$(157.5 + 132.5)(25.5 - d)/16$	$(245.1 + 202.9)(24.5 - d)/14$
v_{max} (ksi)	V_{max}/bd	$18.13(25.5 - d)/\{(9 \times 12)d\}$	$32(24.5 - d)/\{(9 \times 12)d\}$
d_{reqs} (")	$v_{max} = v_c$	$462.2/(18.13 + 0.060 \times 108) = 18.76$	$784.0/(32.0 + 0.093 \times 108) = 18.64$
M_{max} (k')	$F_1 d_1 + F_4 d_4$ $F_1 d_1 + F_2 d_2$	$(290.0)(2.5 - 12.5/12) = 422.9$ $(275.0)(2.0 - 12.5/12) = 263.5$	$(448.0)(2.5 - 12.5/12) = 653.3$ $(422.2)(2.0 - 12.5/12) = 404.6$
d_{reqm} (")	$\sqrt{(M_{max}/Rb)}$	$\sqrt{[422.9/(0.223 \times 9)]} = 14.52$	$\sqrt{[653.3/(0.739 \times 9)]} = 9.92$
d (")	$\geq d_{req}$	24.0, 25.0	22.0, 23.0
t (")	$d + 3.5 + 6$	$25.0 + 3.5 + 6 = 34.5$	$23.0 + 3.5 + 6 = 32.5$
A_s (in ² /ft)	$M_{max}/(f_{sall} j d)$, OR $0.85 f_c' / f_y [1 - \sqrt{1 - 2M_n/(0.85 f_c' b d^2)}] b d$	$422.9 \times 12 / (20 \times 0.874 \times 25) = 11.6$ i.e., 11.6/9 = 1.29 in ² /ft AND 7.5/10 = 0.75 in ² /ft	$0.051 [1 - \sqrt{1 - 2 \times 653.3 / (0.9 \times 2.55 \times 9 \times 23^2)}] \times 108 \times 23 = 7.0$; i.e., 7.0/9 = 0.78 in ² /ft AND 5.0/10 = 0.50 in ² /ft
A_{st}, A_{smin} (in ² /ft)	$0.003 b t$, $(200/f_y) b d$	$0.003 \times 12 \times 28.5 = 1.03$ $(0.20/50) \times 12 \times 24.0 = 1.15$	$0.003 \times 12 \times 26.5 = 0.95$ $(0.20/50) \times 12 \times 22.0 = 1.06$
L_d (")	$0.050 f_y \sqrt{f_c'} d_b$ [Un-Anchored]	$0.050 \times [50/\sqrt{(3/1000)}] (6/8) = 34.3$	34.3
$\sigma_{bearing}$ (ksi)	$P/A_c = 2 \times 0.375 f_c'$, OR $P_u/A_c = 2 \phi 0.85 f_c'$	$500/(25 \times 25) = 0.80$ $< 2 \times 0.375 \times 3 = 2.25$ (OK)	$760/(25 \times 25) = 1.22$ $< 2 \times 0.70 \times 0.85 \times 3 = 4.34$ (OK)



Designed Pile in WSD and USD



Designed Pile Cap (Side view) in WSD and USD

Questions and Problems (5)

1. (i) Narrate the utility of using pile foundations, explaining the mechanisms of pile resistance to applied loads.
 (ii) Classify pile foundations based on their construction process as well as resistance mechanism.
 (iii) Why is it not advisable to use single piles under columns?
 Show pile arrangements in different pile groups and mention the recommendations for pile spacing.
 (iv) Derive the expression of pile force in a pile group subjected to axial force and biaxial bending.
 (v) Outline the procedure for the structural design of piles and pile caps.

2. For the bridge abutment shown in Fig. 1, V and H are the total vertical and horizontal forces per unit width.

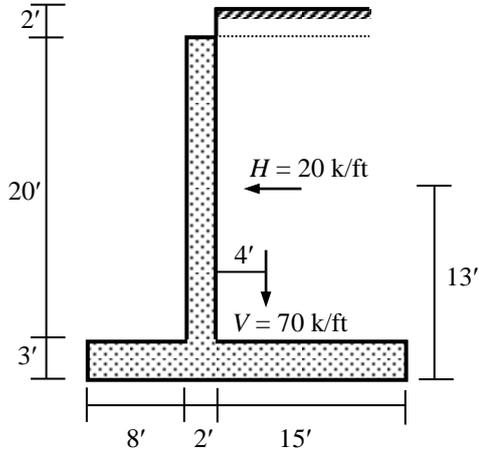


Fig. 1

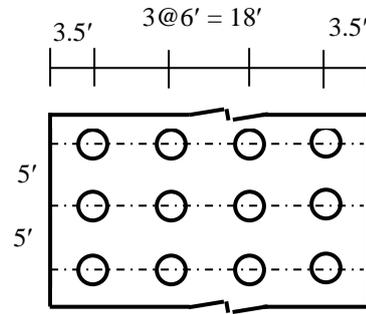


Fig. 2

- (i) Draw the pressure diagram on the soil below the base of the abutment and comment on the prospect of foundation uplift and necessity of using piles [Given: Allowable bearing capacity of the soil = 3 ksf].
- (ii) Design the pile group arranged as in Fig. 2 for the foundation of the abutment shown in Fig. 1.
- (iii) Design the pile cap for the pile loads calculated in (ii).

Design of Retaining Walls

Retaining walls are structures designed to restrain masses of soil or other loose material to unnatural slopes, where conditions make it impossible to let them assume their natural slopes. Such conditions occur when the width of excavation, cut or embankment is restricted; e.g., the width of railway or highway is fixed, as is the substructure underneath bridge, area within the basement walls of buildings and the must also retain the surrounding soil.

Types of Retaining Walls

Free-standing retaining walls are of various types, the most common of which are shown in Fig. 6.1. The gravity type wall [Fig. 6.1(a)] retains earth entirely by its own weight and generally contains no reinforcement. The reinforced cantilever wall [Fig. 6.1(b)] consists of vertical arm that retains earth and is held in position by a footing or base slab (with the filling earth on top of 'heel' contributes to the stability of the wall, with a 'toe' in front). Counterfort type walls [Fig. 6.1(c)] are used to reduce the bending moments on the cantilevers (similar to the action of two-way slabs). The suitability of the three types of walls depends on the price of material and construction, but gravity types are typically suitable for low walls (up to 10 ft), cantilever types for heights of 10~20 ft, while counterforts are suitable for greater heights.

Fig. 6.2 shows some other types of retaining walls used quite commonly, including anchored wall, braced excavation, bored pile and sheet pile, the last three used more for basement walls.

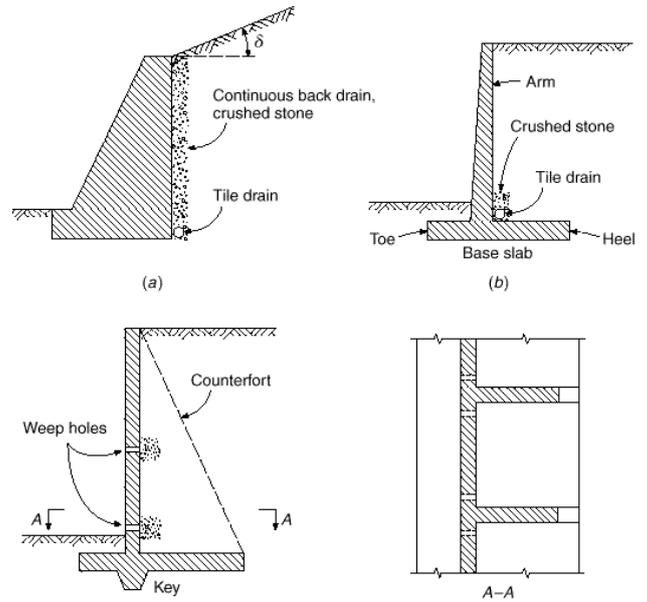


Fig. 6.1: (a) Gravity type, (b) Cantilever type, (c) Counterfort retaining wall



Fig. 6.2: (a) Anchored wall, (b) Braced excavation, (c) Bored pile, (d) Sheet pile

Lateral Earth Pressure

The most important consideration in proper design and installation of retaining walls is to recognize and counteract the tendency of the retained material to move downslope due to gravity. This creates lateral earth pressure behind the wall which depends on the angle of internal friction (ϕ) and the cohesive strength (c) of the retained material, as well as the direction and magnitude of movement the retaining structure undergoes.

Lateral earth pressures are zero at the top of the wall and increase proportionally (in homogenous ground) to a maximum value at the lowest depth. Earth pressures will push the wall forward or overturn it if not properly addressed.

To describe the pressure a soil will exert, an earth pressure coefficient C , is used, which is a function of the soil properties and has a horizontal component C_h with a value between 0 (completely solid) and 1 (completely liquid). Horizontal earth pressure is assumed to be directly proportional to the vertical pressure at any given point in the soil profile, and can also depend on the stress history of the soil. Lateral earth pressure coefficients are classified into three categories: at-rest, active, and passive. The pressure coefficient C_0 is the coefficient of earth pressure at rest, which depends both on the nature of the backfill and the method of depositing and compacting it. Empirical relations have been created to predict at rest pressure. Two of the more commonly used formulae are

$$C_{0(NC)} = 1 - \sin \phi, \text{ for normally consolidate soil and } C_0 = C_{0(NC)} OCR^{\sin(\phi)} \dots\dots\dots(6.1)$$

and the corresponding lateral earth pressure at rest

$$p_0 = C_0 \gamma_s h \dots\dots\dots(6.2)$$

If the wall moves away from the fill, a sliding plane ab forms in the soil mass, and the wedge abc sliding along the plane exerts pressure against the wall. The corresponding pressure is called the *active earth pressure*. This pressure is less than the pressure at rest, and for horizontal backfills, the pressure coefficient is given by

$$C_a = (1 - \sin \phi)/(1 + \sin \phi) \dots\dots\dots(6.3)$$

If, on other hand, the wall is pushed against the fill, a sliding plane ad is formed and the wedge acd is pushed upward by the wall along that plane. The pressure exerted by this larger wedge is called the *passive earth pressure*, and the pressure coefficient (for horizontal backfills) is

$$C_p = (1 + \sin \phi)/(1 - \sin \phi) \dots\dots\dots(6.4)$$

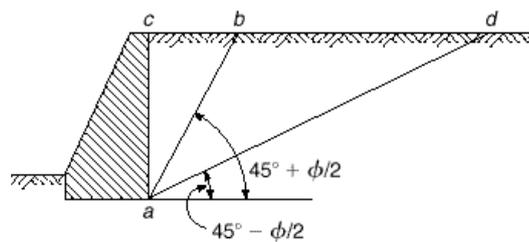


Fig. 6.3: Basis of active and passive earth pressure

Also, any groundwater behind the wall that is not dissipated by a drainage system causes hydrostatic pressure on the wall. Unless the wall is designed to retain water, it is important to have proper drainage behind the wall in order to limit the pressure to the wall's design value. Drainage materials will reduce or eliminate the hydrostatic pressure and improve the stability of the material behind the wall. Fig. 6.1 shows three types of drainage systems for retaining walls; i.e., crushed stone (soil with large pores), tile drain and weep hole (to drain out water) [also shown in Fig. 6.4].



Fig. 6.4: (a) Granular soil and Tube, (b) Weep holes in retaining wall

Weep holes consisting of 6~8 in diameter pipe embedded in the wall are usually spaced 5~10 ft horizontally. Additional rows of such holes should be provided in walls of substantial height. To facilitate drainage and prevent clogging, 1 ft³ or more of crushed stone is placed at the end of each weeper. Care must be taken that the outflow from weep holes is carried off safely so as not to seep into and soften the soil under the wall. To prevent this, *longitudinal drains* embedded in crushed stone or gravel can be provided along the rear face of the wall at one or more levels. A layer of crushed stone or gravel covering the entire rear face of the wall provides a *continuous backdrain*.

If, for some reason (e.g., no provision for drainage, or clogged drains, or heavy rainfall), water is not drained out from the earth-fill, hydrostatic pressure ($\gamma_w h$) should be added to the effective earth pressure [corresponding to effective soil unit weight of ($\gamma_s - \gamma_w$) instead of γ_s].

Design Criteria of Retaining Walls

Retaining walls should be designed against the lateral loads (Fig. 6.5) due to horizontal component of the earth pressure as well as surcharge pressure due to other loads (e.g., weights of backfill, vehicles). Building codes require walls to be designed to ensure stability against sliding, overturning, and excessive foundation pressure; and that they be designed for a safety factor of 1.5 against lateral sliding and overturning.

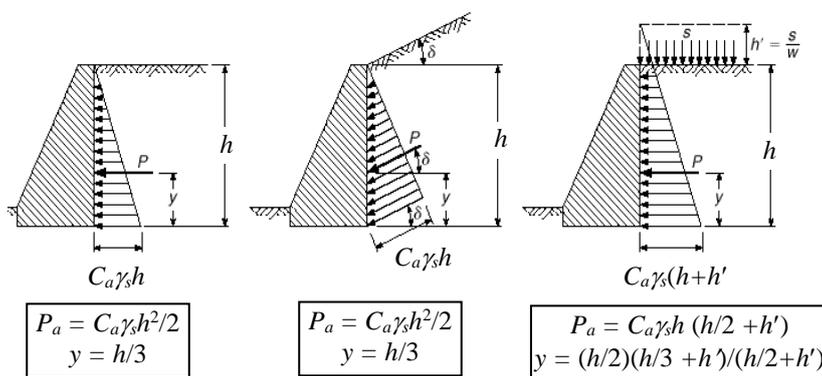


Fig. 6.5: Earth pressure for horizontal surface, sloping surface and horizontal surface with surcharge

In considering the criterion of sliding, the sliding resistance of retaining walls is derived from the base friction between the wall base and the foundation soils. To increase the sliding resistance of retaining walls, other than providing a large self-weight or a large retained soil mass, *shear keys* are installed at the wall base. The main purpose of installation of shear keys is to increase the extra passive resistance developed by the height of shear keys.

Friction between the wall base and the foundation soils is normally about a fraction of the angle of internal resistance within foundation soil. When a shear key is installed at the base of the retaining wall, the failure surface is changed from the wall base/soil horizontal plane to a plane within foundation soil (Fig. 6.6). Therefore, the friction angle mobilized in this case is ϕ and the sliding resistance can be enhanced.

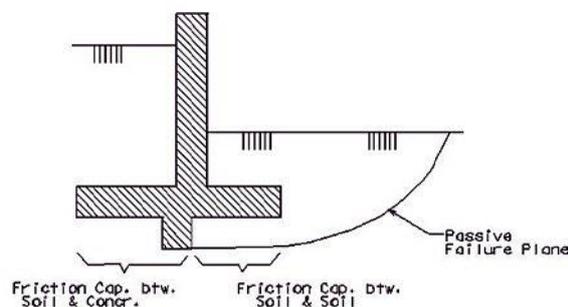


Fig. 6.6: Mechanism of shear key

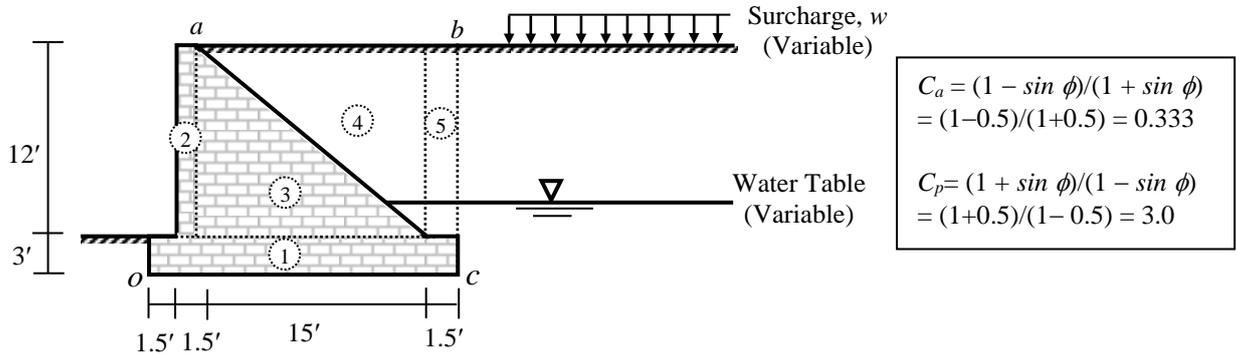
Example 6.1

Calculate the factor of safety against sliding and overturning for the retaining wall shown below, if

- (i) Water table is at c , and surcharge $w = 400$ psf is at a , or b [whichever is more critical]
- (ii) Surcharge $w = 0$, and water table is at surface level b [There is no drainage facility]

Also calculate the maximum/minimum soil pressure underneath the footing.

[Given: Unit weight of brick = 120 lb/ft^3 , Unit weight of soil = 110 lb/ft^3 ,
Angle of friction (ϕ) for the soil = 30° , Friction factor (f) between soil and base of wall = 0.4].



Self-weight of brick-wall = $(19.5 \times 3 + 1.5 \times 12 + 15/2 \times 12) \times 0.120 = 7.02 + 2.16 + 10.80 = 19.98 \text{ k/ft}$

Weight of soil = $(15/2 \times 12 + 1.5 \times 12) \times 0.110 = 9.90 + 1.98 = 11.88 \text{ k/ft}$

\therefore Total vertical load $W = 19.98 + 11.88 = 31.86 \text{ k/ft} \Rightarrow$ Frictional resistance $F = fW = 0.4 \times 31.86 = 12.74 \text{ k/ft}$

Passive pressure = $3.0 \times 0.11 \times 3 = 0.99 \text{ ksf} \Rightarrow$ Passive force = $0.99 \times 3/2 = 1.485 \text{ k/ft}$

\therefore Total Resisting force $R = 12.74 + 1.485 = 14.23 \text{ k/ft}$

Resisting Moment (about o) M_R

$$= 7.02 \times 19.5/2 + 2.16 \times (1.5 + 1.5/2) + 10.80 \times (3 + 15/3) \\ + 9.90 \times (3 + 2 \times 15/3) + 1.98 \times (18 + 1.5/2) + 1.485 \times 3/3 \\ = 327.02 \text{ k-ft/ft}$$

(i) For surcharge $w = 400$ psf at b (most critical)

Uniform $p_o = 0.333 \times 400 = 133.33 \text{ psf} = 0.133 \text{ ksf}$

Linearly varying Active pressure from 0 to $(0.333 \times 0.11 \times 15) = 0.55 \text{ ksf}$

\therefore Horizontal force $H = 0.133 \times 15 + 0.55/2 \times 15 = 2.0 + 4.125 = 6.13 \text{ k/ft}$

Overturning moment (about o) $M_O = 2.0 \times 15/2 + 4.125 \times 15/3 = 35.63 \text{ k-ft/ft}$

$\therefore FS_{(Sliding)} = F/H = 14.23/6.13 = 2.32$

$FS_{(OTurn)} = M_R/M_O = 327.02/35.63 = 9.18$

Total vertical force = 31.86 k/ft , and moment about $O = 327.02 - 35.63 = 291.39 \text{ k-ft/ft}$

\therefore Moment about midpoint = $31.86 \times 9.75 - 291.39 = 19.24 \text{ k-ft/ft}$

\Rightarrow Soil Pressure underneath = $31.86/19.5 \pm 6 \times 19.24/19.5^2 = 1.63 \pm 0.30 = 1.93 \text{ ksf}$ and 1.33 ksf

(ii) For $w = 0$ and water-table at b

Linearly varying Active pressure from zero to $\{0.333 \times (110 - 62.5) + 62.5\} \times 15 = 1175 \text{ psf} = 1.175 \text{ ksf}$

\therefore Horizontal force $H = 1.175/2 \times 15 = 8.81 \text{ k/ft}$

Overturning moment (about o) $M_O = 8.81 \times 15/3 = 44.06 \text{ k-ft/ft}$

$\therefore FS_{(Sliding)} = F/H = 14.23/8.81 = 1.61$

$FS_{(OTurn)} = M_R/M_O = 327.02/44.06 = 7.42$

Total vertical force = 31.86 k/ft , and moment about $o = 327.02 - 44.06 = 282.96 \text{ k-ft/ft}$

\therefore Moment about midpoint = $31.86 \times 9.75 - 282.96 = 27.68 \text{ k-ft/ft}$

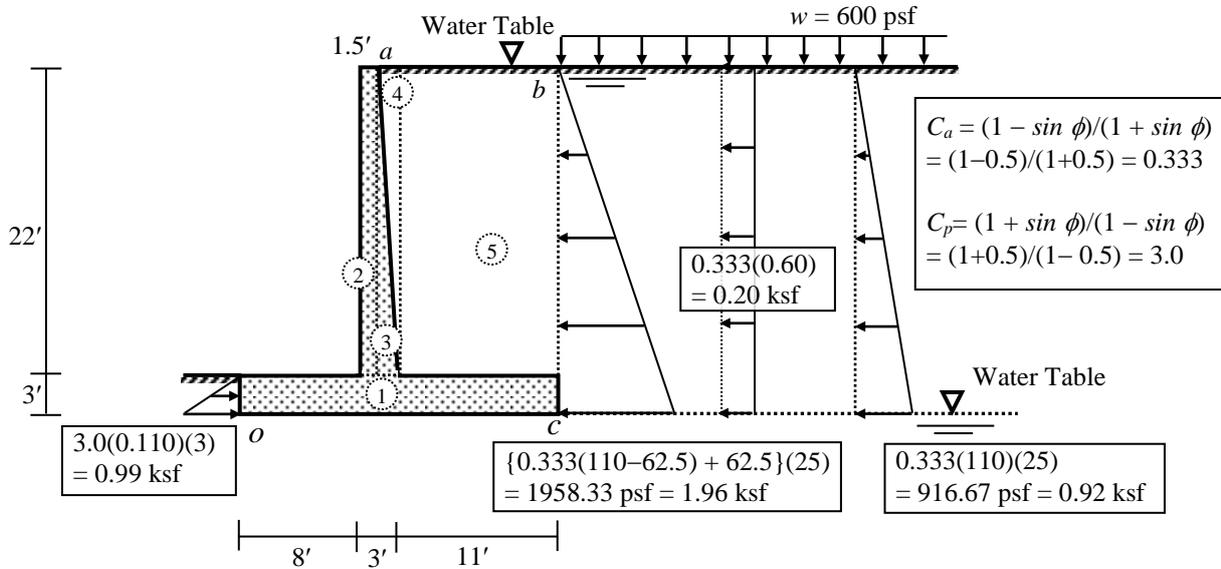
\Rightarrow Soil Pressure underneath = $31.86/19.5 \pm 6 \times 27.68/19.5^2 = 1.63 \pm 0.44 = 2.07 \text{ ksf}$ and 1.19 ksf

Example 6.2

If the water table is at surface level *b* for the RC retaining wall shown below

- (i) Calculate the factor of safety against sliding and overturning, and suggest possible options to improve the *FS*.
- (ii) Calculate the maximum/minimum soil pressure underneath the footing for the option chosen in (i).
- (iii) Use the USD to design the retaining wall, showing the reinforcements with neat sketches.

[Given: Unit weight of concrete = 150 lb/ft³, Unit weight of soil = 110 lb/ft³, Angle of friction (ϕ) for the soil = 30°, Friction factor (*f*) between soil and wall-base = 0.5].



- (i) Total horizontal force, $H = 1.96/2 \times 25 + 0.20 \times 25 = 24.5 + 5.0 + 29.5 \text{ k/ft}$ and Overturning Moment, $M_o = 24.5 \times 25/3 + 5.0 \times 25/2 = 266.67 \text{ k-ft/ft}$

	W (k/ft)	x (ft)	Wx (k-ft/ft)
1	$22 \times 3 \times 0.15 = 9.90$	11.0	108.90
2	$1.5 \times 22 \times 0.15 = 4.95$	$8 + 1.5/2 = 8.75$	43.31
3	$1.5/2 \times 22 \times 0.15 = 2.48$	$8 + 1.5 + 1.5/3 = 10.0$	24.75
4	$1.5/2 \times 22 \times 0.11 = 1.82$	$11 - 1.5/3 = 10.5$	18.15
5	$11 \times 22 \times 0.11 = 26.62$	$11 + 11/2 = 16.5$	439.23
Total	45.76		635.25

\therefore Total resisting force $F = 45.76 \times 0.5 + 0.99/2 \times 3 = 22.88 + 1.49 = 24.37 \text{ k/ft}$
 $\Rightarrow FS_{(Sliding)} = F/H = 24.37/29.5 = 0.83$, Not OK

and Resisting moment $M_R = 635.25 + 1.485 \times 3/3 = 636.74 \text{ k-ft/ft} \Rightarrow FS_{(OTurn)} = M_R/M_o = 636.74/266.67 = 2.38$

Options to improve sliding resistance may include

(a) Additional 4-ft Foundation

\Rightarrow Additional weight = $4 \times 3 \times 0.15 + 4 \times 22 \times 0.11 = 11.48 \text{ k/ft} \Rightarrow$ Additional friction = $11.48 \times 0.5 = 5.74 \text{ k/ft}$
 $\therefore FS_{(Sliding)} = F/H = (24.37 + 5.74)/29.5 = 1.02$, Not OK

(b) Additional 1.5-ft Shear Key

\Rightarrow Total passive resistance = $3.0 \times 0.11 \times 4.5^2/2 = 3.34 \text{ k/ft}$, instead of 1.49 k/ft
 $\therefore FS_{(Sliding)} = F/H = (22.88 + 3.34)/29.5 = 0.89$, Not OK

Moreover, sliding between soil layers increase the friction factor to $\tan \phi = 0.577$, instead of 0.50 (i.e., by 15.4%, which, however, will not be applicable to the heel), but it is still not OK

(c) Drainage of water

\Rightarrow Total horizontal force, $H = 0.92/2 \times 25 + 5.0 = 11.46 + 5.0 = 16.46 \text{ k/ft}$
 and Overturning Moment, $M_o = 11.46 \times 25/3 + 5.0 \times 25/2 = 157.99 \text{ k-ft/ft}$
 $\Rightarrow FS_{(Sliding)} = F/H = 24.37/16.46 = 1.48$, Almost OK

- (ii) Using Option(c), total vertical force = 45.76 k/ft, and Moment about *o* = $636.74 - 157.99 = 478.75 \text{ k-ft/ft}$
 \therefore Moment about midpoint = $45.76 \times 11.0 - 478.75 = 24.61 \text{ k-ft/ft}$
 \Rightarrow Soil Pressure underneath = $45.76/22 \pm 6 \times 24.61/22^2 = 2.08 \pm 0.31 = 2.39 \text{ ksf}$ and 1.77 ksf

(iii) USD of Cantilever Type Retaining Wall

(a) Stem

Assuming all horizontal loads are from LL

Wall pressures consist of $0.20 \times 1.7 = 0.34$ ksf (uniform) and $0 \sim 0.807 \times 1.7 = 1.37$ ksf (triangular)

Maximum horizontal force = $0.34 \times 22 + 1.37 \times 22/2 = 7.48 + 15.08 = 22.56$ k/ft

Assuming $d = 32.5'' = 2.71'$ \Rightarrow Maximum Shear force = $0.34 \times 19.29 + 1.20 \times 19.29/2 = 6.56 + 11.57 = 18.13$ k/ft

$\therefore v_c = 2\phi\sqrt{f'_c} = 2 \times 0.85\sqrt{3/1000} = 0.093$ ksi $\Rightarrow d_{req} = 18.13/(12 \times 0.093) = 16.23''$

Maximum Bending Moment = $7.48 \times 22/2 + 15.08 \times 22/3 = 192.90$ k-ft/ft

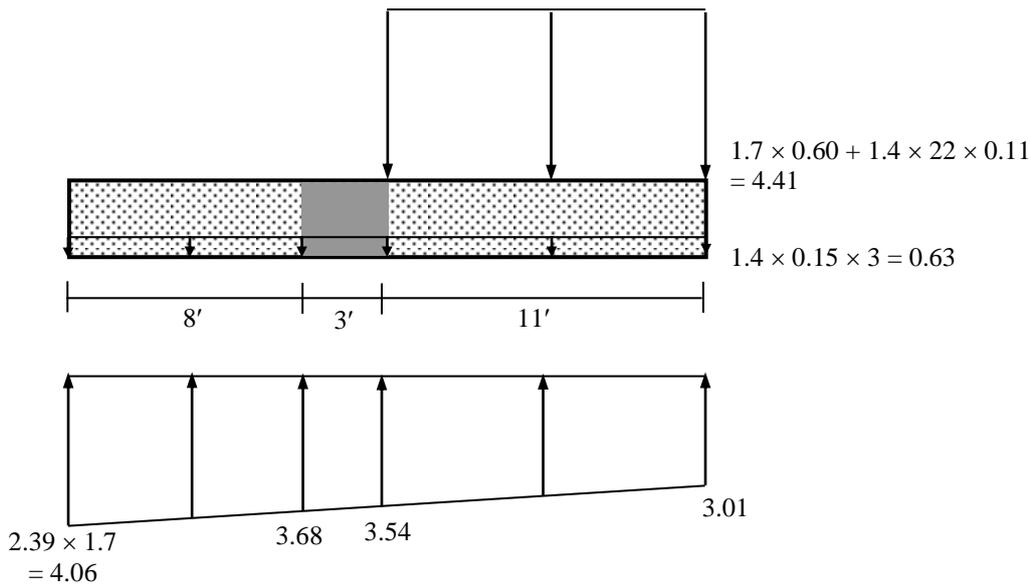
$\therefore R_u = 0.739$ ksi $\Rightarrow d_{req} = \sqrt{(192.90/0.739)} = 16.16''$

$\therefore A_{s(req)} = 0.85f'_c/f_y [1 - \sqrt{(1 - 2M_u/(0.85f'_c bd^2))}]bd$
 $= (2.55/50) [1 - \sqrt{(1 - 2 \times 192.90/(0.9 \times 2.55 \times 12 \times 32.5^2))}] \times 12 \times 32.5 = 1.65$ in²/ft

$A_{s(temp)} = 0.036t = 0.036 \times 36 = 1.30$ in²/ft, $A_{s(min)} = (0.2/f_y)bd = (0.2/50) \times 12 \times 32.5 = 1.56$ in²/ft

\therefore #8@5.5" c/c on the soil-side can be used as main reinforcement, and #8@6" c/c in the other direction.

(b) Toe and Heel

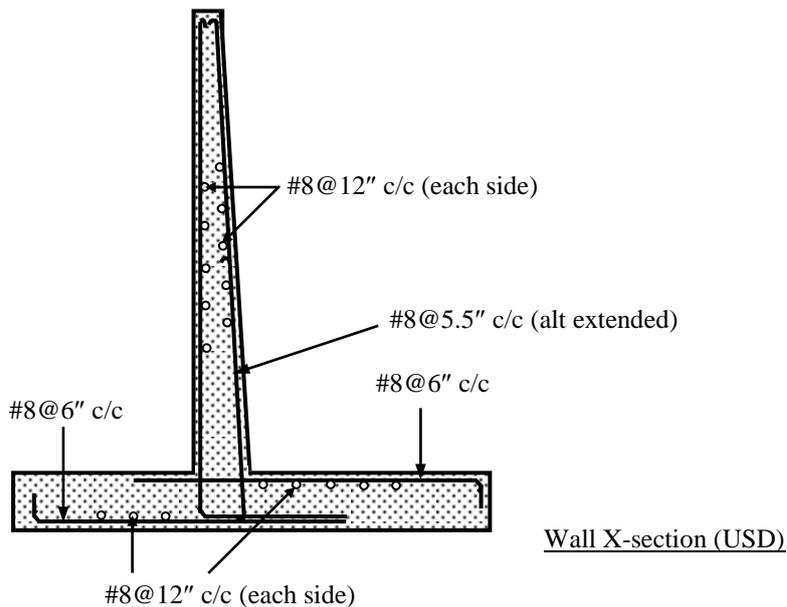


M_{max} for toe = $(3.68 - 0.63) \times 8^2/2 + 0.38 \times 8^2/3 = 105.77$ k-ft/ft

M_{max} for heel = $(4.41 + 0.63 - 3.01) \times 11^2/2 - 0.53 \times 11^2/6 = 112.12$ k-ft/ft

Since the thickness is identical with the stem and M_{max} in both cases is considerably smaller, $A_{s(req)}$ will be smaller than $A_{s(min)} = 1.56$ in²/ft, which governs here. In fact, a smaller thickness can also be considered.

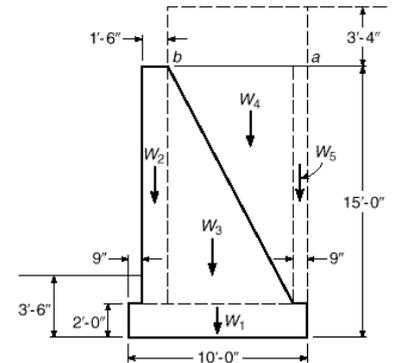
\therefore #8@6" c/c can be used as main reinforcement as well as in the other direction.



Questions and Problems (6)

1. (i) What is retaining wall? Mention some practical examples of where they are used.
 (ii) Name different types of retaining walls and explain their relative advantages.
 (iii) Distinguish between earth pressure at rest, active earth pressure and passive earth pressure.
 (iv) Explain why it is important to prevent the accumulation of water behind retaining walls.
 Also mention some possible measures to be taken in this regard.
 (v) Explain the importance of shear key and weep holes for the stability of retaining walls.

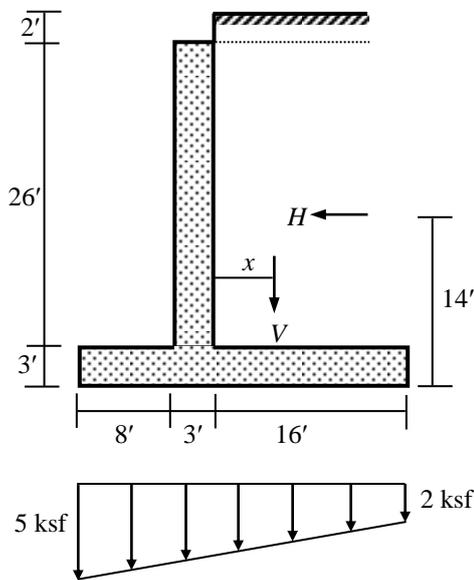
2. Figure on right shows the cross-section of a gravity-type retaining wall, which is subjected to a surcharge of 400 psf in addition to the lateral earth pressure.



- (i) Check the stability of the wall against sliding and overturning,
 - (ii) Draw the pressure diagram on the soil underneath the footing and comment on the prospect of overstress and foundation uplift
- [Given: Unit weight of soil = 120 lb/ft³, angle of internal friction $\phi = 30^\circ$, Friction factor between soil and wall = 0.50].

3. V and H are the total vertical and horizontal forces per unit width of the retaining wall shown below. If the friction factor between soil and wall = 0.50 and the distance $x = 5'$, calculate the values of

- (i) V and H
- (ii) Factor of safety of the wall against sliding and overturning.



4. Use the WSD/USD to design the stem, toe and heel of the retaining wall shown in Question 3
 [Given: $f_c' = 3$ ksi, $f_y = 50$ ksi].

5. Factor of safety against sliding is 1.50 and against overturning is 3.50 for the retaining wall shown in Question 3. The friction factor between soil and wall = 0.50 and allowable bearing capacity of the soil is = 3 ksf.
 - (i) Calculate the values of vertical load V and distance x .
 - (ii) Draw the pressure diagram on the soil below the base of the wall and comment on the prospect of foundation uplift and necessity of using deep foundation.

Prestressed Concrete

As dead load is a substantial part of the total load on RC structures, any saving in depth of members can represent a substantial saving in material cost, in terms of total height of structures, load on foundations, heating, electrical, plumbing, wall and partition surfaces. While this can be achieved by using high-strength materials, these are limited by considerations of cracking and deflection. These limitations of RC are largely overcome by prestressed concrete.

Prestressed concrete is a concrete that is compressed axially to overcome concrete's natural weakness in tension. Prestressing tendons (generally of high tensile steel cable or rods) are used to provide a clamping load, which produces a compressive stress that balances or reduces the tensile stress that the concrete member would otherwise experience due to a bending load.

Therefore, cracking under service loads can be minimized or avoided altogether. Deflections may be limited to an acceptable value, or even eliminated by proper load combination. This may permit the use of efficient and economical high-strength steels in the form of strands, wires or bars in conjunction with high-strength concrete.

Prestressed concrete is used for beams, floors or bridges with a longer span than is practical with ordinary RC.

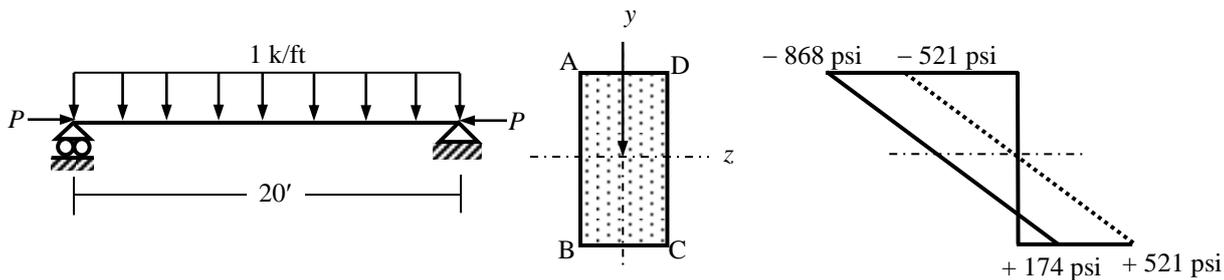


Fig. 7.1: Prestressed Concrete (a) Beam, (b) Floor, (c) Bridge

Example 7.1

Calculate the maximum stresses at the midspan section of a 20' long simply supported rectangular beam of (12" × 24") section under a uniformly distributed load of 1 k/ft if it is subjected to an additional compressive force of $P = 100$ kips.

Also calculate the pre-stressing force P_0 necessary to avoid tension in the section altogether.



With $b = 12''$, $h = 24''$

\Rightarrow Cross-sectional Area $A = 12'' \times 24'' = 288 \text{ in}^2$, Moment of Inertia about z -axis $I_z = 12 \times 24^3 / 12 = 13824 \text{ in}^4$

Maximum midspan bending moment $M_z = 1 \times 20^2 / 8 = 50 \text{ k}' = 600 \text{ k}''$

$\therefore \sigma_{A,D} = -100/288 - 600 \times 12 / 13824 = (-0.347 - 0.521) \text{ ksi} = -868 \text{ psi}$, $\sigma_{B,C} = (-0.347 + 0.521) \text{ ksi} = +174 \text{ psi}$

If the necessary pre-stressing force to avoid tension in the section is P_0 , then

$\sigma_{I(max)} = \sigma_{B,C} = -P_0/288 + 0.521 = 0 \Rightarrow P_0 = 150 \text{ kips}$

Materials

Pre-stressing Steel

As mentioned, prestressing is done by using high-strength steels in the form of strands, wires or bars (Fig. 7.3), whose typical stress-strain diagrams are shown in Fig. 7.4.



Fig. 7.3: Prestressing strands and bar

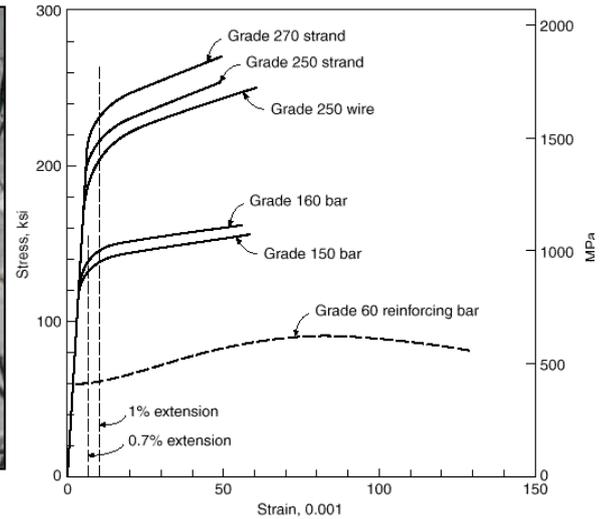


Fig. 7.4: Stress vs. Strain of Reinforcing and Prestressing Tendons

Concrete

Concrete of substantially higher compressive strength (often above 5 ksi) is used for pre-stressed structures, which may be more practical because a large proportion of prestressed construction is precast, with the concrete mixed, placed and cured under carefully controlled conditions. High-strength concrete also has

- * Greater modulus of elasticity, reducing initial elastic strain due to prestress force and also reducing creep strain
- * Greater bearing strength, which can be necessary to withstand the high bearing stresses when prestressing force is transferred from prestressing tendons
- * Greater bond strength, resulting in better transfer of stress between prestressing steel and concrete.

The maximum permissible stresses in prestressing steel and concrete at various phases of loading conditions are listed in Table 7.1 and 7.2 respectively.

Table 7.1: Maximum permissible stresses in prestressing steel

Due to tendon jacking force but not greater than $0.80f_{pu}$ and the maximum recommended by manufacturer	$0.94f_{py}$
Immediately after prestress transfer, but not greater than $0.74f_{pu}$	$0.82f_{py}$
Post-tensioning tendons at anchorage devices and couplers, immediately after tendon anchorage	$0.70f_{pu}$

Table 7.2: Permissible stresses in concrete in prestressed flexural members

Extreme fiber stress in compression immediately after transfer	$0.60f_{ci}$
Extreme fiber stress in tension immediately after transfer	$3\sqrt{f_{ci}}$
Extreme fiber stress in tension immediately after transfer at the end of a simply supported member	$6\sqrt{f_{ci}}$
Extreme fiber stress in compression due to prestress plus sustained load	$0.45f_c'$
Extreme fiber stress in compression due to prestress plus total load	$0.60f_c'$
Extreme fiber stress in tension in precompressed tensile zone under service load	$7.5\sqrt{f_c'}$

[where f_c' = Ultimate strength of concrete, f_{ci} = Concrete strength at the time of prestressing]

Methods of prestressing: Pre-tension and Post-tension

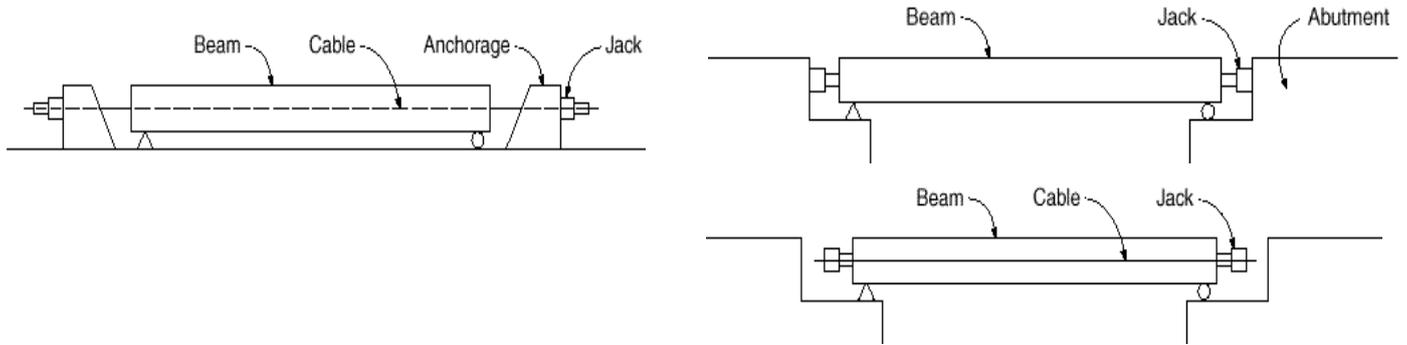


Fig. 7.2: Method of prestressing by (a) Pre-tension, (b) Post-tension

Pre-tension	Post-tension
Prestressing strands are tensioned prior to casting of concrete	Using jacks against abutments
Concrete is placed around tension strands	Tie jack bases with wires/cables passed through hollow conduit
Jacketing pressure released after concrete has gained sufficient strength	One end of the tendon is anchored, applying force at other end
	After wires reach desired stress, tendon is wedged against concrete and jack is removed

Loss of Prestress

- * Elastic shortening + Creep + Shrinkage of concrete
- * Creep of steel
- * Friction between concrete and steel

Prestressed Concrete Sections

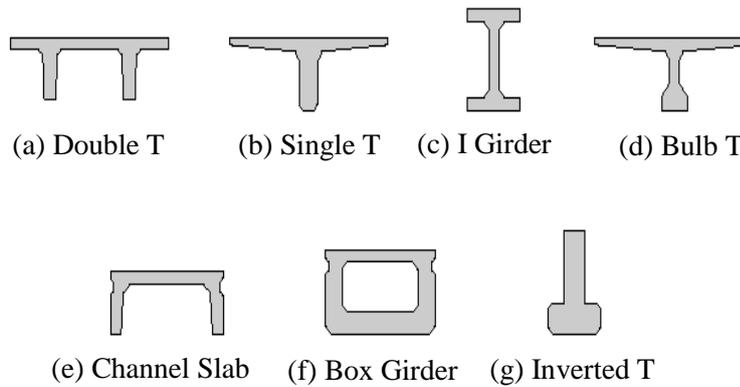


Fig. 7.5: Typical PC Sections

Example 7.2

Pretensioned I beam with constant eccentricity. A simply supported symmetrical I beam shown in cross section in Fig. 19.12a will be used on a 40 ft simple span. It has the following section properties:

- Moment of inertia: $I_c = 12,000 \text{ in}^4$
- Concrete area: $A_c = 176 \text{ in}^2$
- Radius of gyration: $r^2 = 68.2 \text{ in}^2$
- Section modulus: $S = 1000 \text{ in}^3$
- Self-weight: $w_o = 0.183 \text{ kips/ft}$

and is to carry a superimposed dead plus live load (considered "sustained," not short-term) of 0.750 kips/ft in addition to its own weight. The beam will be pretensioned with multiple seven-wire strands with the centroid at a constant eccentricity of 7.91 in. The prestress force P_i immediately after transfer will be 158 kips; after time-dependent losses, the force will reduce to $P_e = 134$ kips. The specified strength of the concrete $f'_c = 5000$ psi, and at the time of prestressing the strength will be $f'_{ci} = 3750$ psi. Calculate the concrete flexural stresses at the midspan section of the beam at the time of transfer, and after all losses with full service load in place. Compare with ACI allowable stresses for a Class U member.

SOLUTION. Stresses in the concrete resulting from the initial prestress force of 158 kips may be found by Eqs. (19.1a) and (19.1b):

$$f_1 = -\frac{158,000}{176} \left(1 - \frac{7.91 \times 12}{68.2} \right) = +352 \text{ psi}$$

$$f_2 = -\frac{158,000}{176} \left(1 + \frac{7.91 \times 12}{68.2} \right) = -2147 \text{ psi}$$

The self-weight of the beam causes the immediate superposition of a moment of

$$M_o = 0.183 \times \frac{40^2}{8} = 36.6 \text{ ft-kips}$$

and corresponding stresses of $36,600 \times 12/1000 = 439$ psi, so that the net stresses at the top and bottom of the concrete section due to initial prestress and self-weight, from Eqs. (19.2a) and (19.2b), are

$$f_1 = +352 - 439 = -87 \text{ psi}$$

$$f_2 = -2147 + 439 = -1708 \text{ psi}$$

After losses, the prestress force is reduced to 134 kips, and the concrete stresses due to that force plus self-weight are

$$f_1 = +352 \times \frac{134}{158} - 439 = 140 \text{ psi}$$

$$f_2 = -2147 \times \frac{134}{158} + 439 = -1382 \text{ psi}$$

and stresses at the end of the beam are

$$f_1 = +352 \cdot \frac{134}{158} = 299$$

$$f_2 = -2147 \cdot \frac{134}{158} = 1821$$

The superimposed load of 0.750 kips/ft produces a midspan moment of $M_d + M_l = 0.750 \times 40^2/8 = 150$ ft-kips, and the corresponding stresses of $150,000 \times 12/1000 = 1800$ psi in compression and tension at the top and bottom of the beam, respectively. Thus, the service load stresses at the top and bottom faces are

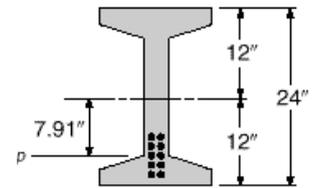
$$f_1 = -140 - 1800 = -1940 \text{ psi}$$

$$f_2 = -1382 + 1800 = +418 \text{ psi}$$

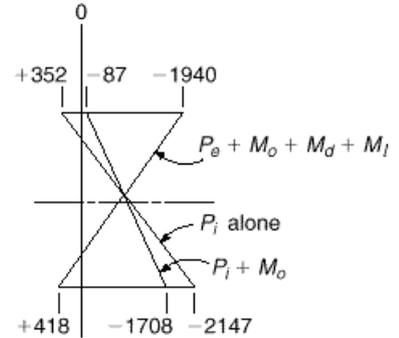
Concrete stresses at midspan are shown in Fig. 19.12b and at the beam end in Fig. 19.12c. According to the ACI Code (see Table 19.2), the stresses permitted in the concrete are

- Tension at transfer: $f_{ti} = 3\sqrt{3750} = +184$ psi
- Compression at transfer: $f_{ci} = 0.60 \times 3750 = -2250$ psi
- Tension at service load: $f_{ts} = 7.5\sqrt{5000} = +530$ psi
- Compression at service load: $f_{cs} = 0.45 \times 5000 = -2250$ psi

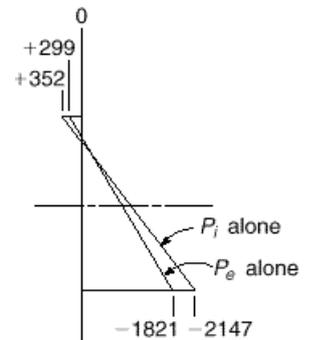
At the initial stage, with prestress plus self-weight in place, the actual compressive stress of 1708 psi is well below the limit of 2250 psi, and no tension acts at the top, although 184 psi is allowed. While more prestress force or more eccentricity might be suggested to more fully utilize the section, to attempt to do so in this beam, with constant eccentricity, would violate limits at the support, where self-weight moment is zero. It is apparent that at the supports, the initial prestress force acting alone produces tension of 352 psi at the top of the beam (Fig. 19.12c), barely below the permitted value of $6\sqrt{3750} = 367$, so very little improvement can be made. Finally, at full service load, the tension of 418 psi is under the allowed 530 psi, and compression of 1940 psi is well below the permitted 2250 psi.



Cross-section



Stresses at midspan

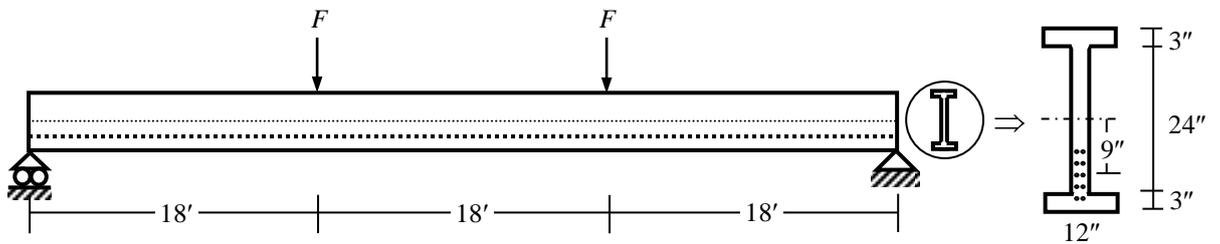


Stresses at ends

Questions and Problems (7)

1.
 - (i) What is pre-stressed concrete? Mention some practical examples of where they are used.
 - (ii) Mention the advantages and disadvantages of pre-stressed concrete compared to reinforced concrete.
 - (iii) Narrate the distinctive features of the materials (i.e., concrete and steel) used in pre-stressed concrete.
 - (iv) Briefly describe the methods of pre-stressing concrete.
 - (v) What is loss of pre-stress? Mention the possible reasons for pre-stress loss.

2. Figure below shows a concrete beam subjected to pre-stressing force $P = 200$ kips immediately after pre-stressing that reduces to 170 kips due to losses. Compute the extreme fiber stresses at midspan of the beam for the following design conditions.
 - (i) At initial condition with full prestress only,
 - (ii) At initial condition with full prestress and beam self-weight,
 - (iii) At working condition with effective prestress and beam self-weight.
 - (iv) Check the stresses calculated with the allowable tensile and compressive stresses in concrete.
 [Given: $f_c' = 5$ ksi, $f_{ci} = 3.5$ ksi].



3. For the beam shown in Question 2, calculate the working loads F to keep the tensile stress in concrete equal to
 - (i) zero, (ii) modulus of rupture of concrete.

Following are some of the theoretical questions discussed after midterm exam, with guidelines for their answers.

- The examination questions can be different or mixed or parts (based on the same topics/concepts).
 - Don't copy this language, just follow the points and read books to prepare your own answers.
1. Mention different types of RC foundations and also mention when they are used.
 - Wall footings support load-bearing walls
 - Individual column footings: Usually they are square (preferred) or rectangular shaped (when property or other foundation lines do not allow square)
 - Combined column footings: Under multiple columns, when property or other foundation lines do not allow individual footings
 - Pile foundations: To transfer load (through friction and end bearing) to surrounding and underlying hard soil, instead of the soft soil at the top
 2. Distinguish between allowable soil pressure and net soil pressure in the design of footings by USD.
 - Allowable soil pressure is the pressure that can be allowed on soil to prevent bearing failure or large settlement and is obtained by dividing the ultimate pressure by a factor of safety (usually 2.5 or 3.0)
 - Net soil pressure is the pressure on underlying soil (due to design loads, but without self-weight of foundation) under design condition
 3. Specify the minimum reinforcements required in the design of wall footings and column footings.
 - Shrinkage reinforcement for wall footings (mention equation), because there is no bending moment along length of wall
 - Minimum reinforcement for beams (mention equation) also recommended for column footings, because of bi-directional moment, the possible catastrophic consequence of their failure and also because footing is not as well supported as slabs
 4. Explain why punching shear is considered in the design of column footings but not for wall footings.
 - Punching area around column footing is not as large, making it vulnerable to punching, similar to flat slab
 - Possible punching area around wall footing is too large, because of the large length of the walls, similar to beam-supported slab
 - Provide figures
 5. Define the band-width in placing reinforcements for rectangular footings and explain why it is used.
 - Bending Moments near columns are significantly larger compared to those away from columns (show BMD)
 - Also show the equation for band-width and corresponding reinforcements (with Figure)
 6. What is Transverse Beam in combined footings? Narrate how it is designed.
 - This is a hypothetical 'equivalent beam' under column in the transverse direction in order to consider the bending in transverse (in addition to longitudinal) direction
 - Assumed width of Transverse Beam is width of column plus $d/2$ in both directions
 - Load on Transverse Beam assumed to be uniformly distributed over footing width, and obtained by dividing the corresponding column load by width of footing
 - Upward soil pressure means positive moment and bottom reinforcements
 - Provide figures
 7. Narrate the utility of using pile foundations, explaining the mechanism of pile resistance to applied loads.
 - Pile foundations transfer load to surrounding through friction and to underlying hard soil by end bearing, instead of the soft soil at the top
 - Piles are subjected primarily to axial forces under vertical loading. In such cases, their structural design is quite similar to the design of columns
 - Bending moments may govern the design of piles when they are subjected to lateral loads or when pre-cast piles are carried to sites, when their structural design would be quite similar to the design of beams
 - Provide Figures
 8. Classify pile foundations based on their construction process as well as resistance mechanism.
 - Construction process (*Pre-cast Piles* are constructed in factory and carried to the site, while *Cast-in-situ Piles* are constructed on site)
 - Resistance mechanism (*Friction Piles* resist loads predominantly by 'skin' friction between pile and soil, while *End-bearing Piles* primarily resist by bearing force at the end of pile).
 - Provide Figures
 9. Why is it not advisable to use single piles under columns?
Show pile arrangements in different pile groups and mention the recommendations for pile spacing.
 - Piles may be subjected to uniaxial/biaxial bending, which a single pile cannot resist (unless it is designed like beams for moments)
 - Show pile groups with 2~5 piles with recommended spacings (Fig. 5.2 in class-note)

10. Derive the expression of pile force in a pile group subjected to axial force and biaxial bending.
 - This is Eq. (5.1) in class-note.
 - Derive from $\sigma(x,y) = P/A + M_x y/I_x + M_y x/I_y$, using $\sigma(x,y) = F(x,y)/a_p$, $I_x = a_p \sum y_i^2$, $I_y = a_p \sum x_i^2$
11. Outline the procedure for the structural design of piles and pile caps.
 - Pile: Mention equations (similar to column design) for pile area, longitudinal reinforcement and lateral ties
 - Pile caps: Mention concentrated pile-forces, and design for bending moment, flexural and punching shear
Also mention the share of pile forces (zero/full/partial) to be considered for checking against flexural shear
12. What is retaining wall? Mention some practical examples of where they are used.
 - Retaining walls are structures designed to restrain masses of soil or other loose material to unnatural slopes, where conditions make it impossible to let them assume their natural slopes.
 - Such conditions occur when the width of excavation, cut or embankment is restricted; e.g., the width of railway or highway is fixed, as is the foundation under bridges and the basement walls of buildings.
13. Name different types of retaining walls and explain their relative advantages.
 - Gravity type (for $h \leq 10'$): Resists loads by its own weight (and no reinforcements).
 - Cantilever type (for $h = 10\sim 20'$): Much thinner walls than gravity type (but needs reinforcements).
 - Counterfort type (for $h \geq 20'$): Redistributes the lateral earth pressure in two directions, thereby reducing cantilever moments and shears
 - Provide figures
14. Distinguish between earth pressure at rest, active earth pressure and passive earth pressure.
 - The pressure coefficient C_0 is the coefficient of earth pressure at rest, which depends both on the nature of the backfill and the method of depositing and compacting it.
 - If the wall moves away from the fill, the corresponding pressure is called the *active earth pressure*
 - If the wall is pushed against the fill, the pressure exerted is called the *passive earth pressure*
 - Mention equations
15. Explain why it is important to prevent the accumulation of water behind retaining walls.
Also mention some possible measures to be taken in this regard.
 - Hydrostatic pressure significantly increases the lateral pressure on wall (mention equation)
 - Possible measures include crushed stone, longitudinal drain and weep holes
 - Proved figures
16. Explain the importance of shear key for the stability of retaining walls.
 - *Shear keys* are installed at the wall base to increase the sliding resistance of retaining walls.
 - The main purpose is to increase the extra passive resistance developed by the height of shear keys.
 - Therefore, the friction angle mobilized in this case is ϕ and the sliding resistance can be enhanced.
 - Proved figures
17. What is pre-stressed concrete? Mention some practical examples of where they are used.
 - A concrete compressed axially to overcome its natural weakness in tension. Prestressing tendons (generally of high tensile steel cable or rods) are used to provide a clamping load, which produces a compressive stress that balances the tensile stress that the concrete member would otherwise experience due to a bending load.
 - Used to produce beams, floors or bridges with a longer span than is practical with ordinary RC.
18. Mention the advantages and disadvantages of pre-stressed concrete compared to reinforced concrete.
 - No/less tension means no crack \Rightarrow Stiffer and smaller sections, less deflection, no corrosion of steel.
 - Pre-stressing is not a readily available/applicable technology and is less used in smaller constructions.
Pre-stressing forces may cause crushing/damage to concrete
19. Narrate the distinctive features of the materials (i.e., concrete and steel) used in pre-stressed concrete.
 - Steel should be high strength to minimize the effect of pre-stressing loss (show stress vs. strain along with ordinary reinforcing bars)
 - Concrete should also be high strength for greater E_c (less strain), greater strength in compression (particularly the pre-stressing force) and tension (particularly to avoid crack) and greater bond with steel.
20. Briefly describe the methods of pre-stressing concrete.
 - Pre-tensioning applied on steel before casting of concrete and transferred to concrete after gains strength
 - Post-tensioning applied on steel and transferred to concrete after casting and strength-gain of concrete (which is cast with a tube around wire)
21. What is loss of pre-stress? Mention the possible reasons for pre-stress loss.
 - Elastic shortening of concrete (due to immediate strain from pre-stressing force)
 - Creep + Shrinkage of concrete (due to long-term deformations)
 - Creep of steel (also due to long-term deformations)
 - Frictional loss between in concrete and steel

1. (i) Clear span of two-way slab $S = 20 - 12/12 = 19'$

(a) From deflection considerations, required thickness of the slab $S = 4 \times 19 \times 12/180 = 5.07''$; i.e., 5''

(b) Balanced steel ratio $= 87/(87 + f_y) \alpha f_c' / f_y = 87/(87 + 60) 0.72 \times 4/60 = 0.0284$

\therefore Steel ratio $\rho_s = 0.25 \rho_b = 0.0071$

$R_u = \phi \rho_s f_y (1 - 0.59 \rho_s f_y / f_c') = 0.90 \times 0.0071 \times 60 (1 - 0.59 \times 0.0071 \times 60/4) = 0.359$ ksi

\therefore Minimum $d = 5 - 1.5 = 3.5'' \Rightarrow M_u = R_u b d^2 = 0.359 \times 1 \times 3.5^2 = 4.40$ k''

(c) Maximum slab moment coefficient $= 0.045$

$\Rightarrow 0.045 \times w_u \times 19^2 = 4.40 \Rightarrow w_u = 0.271$ ksf $= 271$ psf

Self-wt of $S = 5/12 \times 150 = 62.5$ psf

\therefore Factored load on $S = 1.4 (62.5 + 30 + 40) + 1.7(LL) = 185.5 + 1.7LL = 271 \Rightarrow LL = 50.3$ psf

(ii) Given: $C_{a(D)+} = C_{b(D)+} = 0.018$, $C_{a(L)+} = C_{b(L)+} = 0.027$, $C_{a-} = C_{b-} = 0.045$

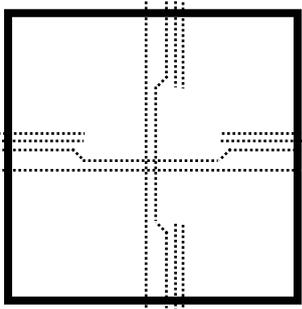
$\therefore M_{a+} = M_{b+} = (0.018 \times 185.5 + 0.027 \times 85.5) \times 19^2 = 2.04$ k'', and $M_{a-} = M_{b-} = 4.40$ k''

\therefore Using $f_c = 0.85 f_c'$, $A_s = (f_c / f_y) [1 - \sqrt{1 - 2M_u / (\phi f_c b d^2)}] b d$

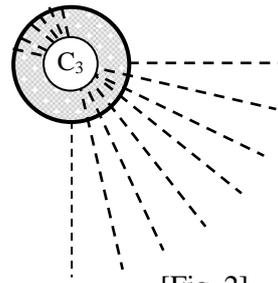
$A_{sb+} = (3.40/60) [1 - \sqrt{1 - 2 \times 2.04 / (0.90 \times 3.40 \times 3.5^2)}] (12 \times 3.5) = 0.133$ in²/ft

and $A_{sb-} = \rho_s b d = 0.071 \times 12 \times 3.5 = 0.298$ in²/ft, while $A_{s(temp)} = 0.036 \times 5 = 0.180$ in²/ft

\therefore Use #3@7'' c/c, alt ckd + 2-#3 extra top [Fig. 1]



[Fig. 1]



[Fig. 2]

2. (i) Clear span of flat slab $FS = 20 - 20/12 = 18.33'$

(a) From deflection considerations,

Required thickness of flat plate $FS = 18.33 \times 12/33 \times (0.8 + 60/200) = 7.33''$; i.e., 7.5''

(b) Using solution of Question 1, Steel ratio $\rho_s = 0.0071$, and $R_u = 0.359$ ksi

\therefore Minimum $d = 7.5 - 1.5 = 6.0'' \Rightarrow M_u = R_u b d^2 = 0.359 \times 1 \times 6^2 = 12.39$ k''

(c) Given: Distribution Factors applied to Moment M_0 for exterior slab are (0.00, +0.63, -0.75)

\therefore Total Moment $M_0 = w_u L_2 L^2/8 = w_u \times 20 \times 18.33^2/8 = 840.3 w_u \Rightarrow M_{Int(-)} = 0.75 M_0 = 630.2 w_u$

Total Column strip moment $= 75\%$ of $M_{Int(-)} = 472.7 w_u$

Total width of Column strip $= 20/4 \times 2 = 10'$ \Rightarrow Column strip moment/width $= 472.7 w_u/10 = 47.27 w_u$

$\Rightarrow 47.27 \times w_u = 12.39 \times 1000 \Rightarrow w_u = 273.7$ psf

Self-wt of $FS = 7.5/12 \times 150 = 93.75$ psf

\therefore Factored load on $FS = 1.4 (93.75 + 30 + 40) + 1.7(LL) = 229.3 + 1.7LL = 273.7 \Rightarrow LL = 26.2$ psf

(ii) Factored load from FS wall $= 1.4 \{ (5/12 \times (10 - 7.5/12) \times 120) \} = 1.4 \times 468.8 = 656.3$ lb/ft

\therefore Column C_3 load per floor $= [273.7 \times 10 \times 10 + 656.3 \times (10 + 10)]$ lb $= 40.50$ k

\therefore Ultimate Punching force $V_u = 40.50 - 273.7/1000 \times \pi/4 \{ (10 + 6/2)/12 \}^2 = 40.25$ k

\Rightarrow Nominal force $V_n = 40.25/0.75 = 53.66$ k

\therefore Maximum punching force for concrete $V_c = 4 \sqrt{f_c'} b_o d = 4 \times \sqrt{4/1000} \times (\pi \times 13)/2 \times 6 = 31.00$ k

If shear reinforcements are provided

Maximum allowable punching shear force $= 2 \sqrt{f_c'} b_o d = 15.50$ k

\therefore Shear reinforcements are to be provided for the additional force $(V_n - V_c) = 53.66 - 15.50 = 38.17$ k

If 45° inclined bars are used, $A_v = (V_n - V_c)/(f_y \sin \alpha) = 38.17/(60 \sin 45^\circ) = 0.90$ in²

Requiring at least 8-#3 bars around the column [Fig. 2]

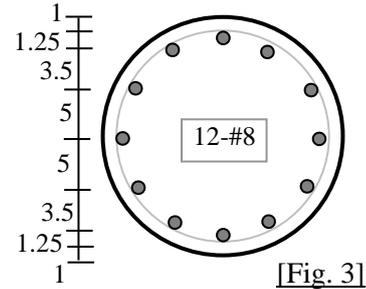
[Note: Spacing of 4-legged #3 vertical stirrups $S = A_v f_y d / (V - V_c) = 0.44 \times 60 \times 6.0 / 38.17 = 4.15''$

$\Rightarrow d/2 = 3''$ around column]

All these bars should be extended full development length beyond the critical sections.

3. (i) Self-wt of $S = 5/12 \times 150 = 62.5$ psf, and of $FS = 7/12 \times 150 = 87.5$ psf
 \therefore Total load on $S = 62.5 + 30 + 40 + 50 = 182.5$ psf, and on $FS = 87.5 + 30 + 40 + 50 = 207.5$ psf
 Working load from beam and wall $= (12 \times 11)/144 \times 150 + (5/12 \times (10 - 16/12)) \times 120 = 570.8$ lb/ft
 Working load from FS wall $= (5/12 \times (10 - 7/12)) \times 120 = 470.8$ lb/ft
 \therefore Column C_1 Load/floor $= [182.5 \times 10 \times 10 + 207.5 \times (20 \times 20 - 100)] + 570.8 \times 20 + 470.8 \times 20$ lb $= 101.33$ k
 \therefore Total Load on column $C_1 = 6 \times 101.33 = 608$ k

Assuming 3% reinforcement, $P_w = \phi'(0.25f_c' + \rho_g f_s) A_g$
 $\Rightarrow 608 = 1.0 \times (1.00 + 0.03 \times 24) A_g$
 $\Rightarrow A_g = 353.5$ in²; i.e., Assume 21.5"-dia column
 with $A_s = (608 - 1.0 \times \pi/4 \times 21.5^2)/24 = 10.21$ in²;
 i.e., Use 12#8 bars [Fig. 3]



Assuming column core diameter $= 18.5''$

Steel ratio of spiral reinforcement $\rho_s = 0.45(A_g/A_{core} - 1)(f_c'/f_y)$
 $= 0.45 [(21.5/18.5)^2 - 1] 4/60 = 0.0105$

and Spacing of #4 spirals, $S = 4A_{sp}/(\rho_s d_{core}) = 4 \times 0.20/(0.0105 \times 18.5) = 4.11$; i.e., Use #4 spirals @3" c/c

- (ii) Column C_3 Load/floor $= [207.5 \times 10 \times 10 + 470.8 \times 20]$ lb $= 30.17$ k
 \therefore Total Load P on column $C_1 = 6 \times 30.17 = 181$ k

For biaxial bending, $P/(P_a) + M_x/(M_{fx}) + M_y/(M_{fy}) \leq 1$

Steel ratio $\rho_g = A_s/A_g = 12 \times 0.79/(\pi/4 \times 21.5^2) = 9.48/363.05 = 0.026$, $m = f_y/0.85f_c' = 60/3.40 = 17.65$
 $\therefore m\rho_g = 0.461$

$P_a = 0.34f_c'(1 + m\rho_g)A_g = 1.36 \times 1.461 \times 363.05 = 721$ k

$M_{fx} = M_{fy} = 0.45f_c' \{ \pi D^4/64 + (2n-1) \sum A_s (y-y)^2 \} / y_{max}$
 $= 1.80 \{ \pi \times 21.5^4/64 + (2 \times 8 - 1) \times 1.57 (1 \times 0^2 + 2 \times 5^2 + 2 \times 8.5^2 + 1 \times 9.75^2) \} / 10.75 = 2898$ k"

If $M_x = M_y$, $P/(P_a) + M_x/(M_{fx}) + M_y/(M_{fy}) = 181/721 + M_x/2898 (1 + 1) = 1$

$\Rightarrow M_x = M_y = 1085$ k" $= 90.45$ k'

7. (ii) Using the results from Example 6.2 [also obtained from (i) of this question] for the same wall,
 Total vertical force $= 45.76$ k/ft
 and Moment about midpoint $= 24.61$ k-ft/ft

Since pile-rows are @4' c/c, each row is subjected to

Vertical force $V = 5.76 \times 4 = 183.04$ k, and Moment $M = 24.61 \times 4 = 98.44$ k'

\Rightarrow Maximum pile force $P_{max} = 183.04/3 + 98.44 \times 8/\{8^2 + 0^2 + (-8)^2\} = 61.01 + 6.15 = 67.17$ k
 and Minimum pile force $P_{min} = 61.01 - 6.15 = 54.86$ k (checked for no uplift)

Assuming 1% reinforcement, $P_{max} = \phi'(0.25f_c' + \rho_g f_s) a_p$

$\Rightarrow 67.17 = 0.85 (1.0 + 0.01 \times 24) a_p$

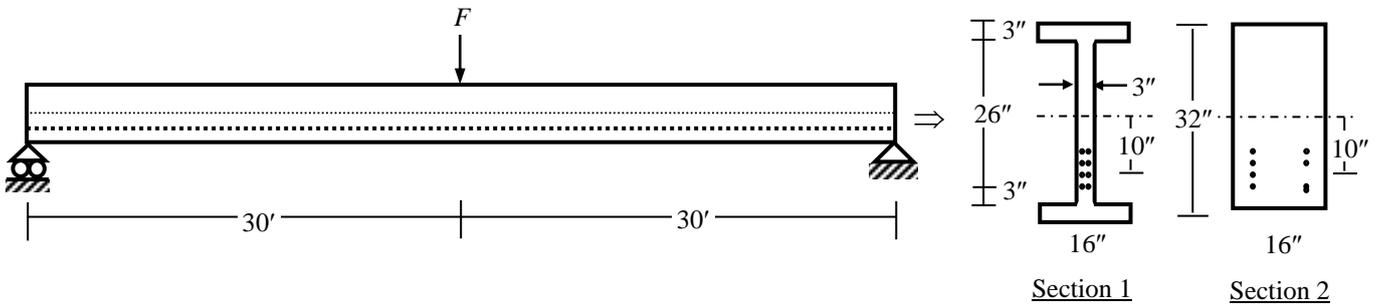
$\Rightarrow a_p = 63.72$ in²; i.e., Required $d_p = 9.01''$ is too small

[So, pile rows can be placed at larger intervals; e.g., @6~8' c/c, instead of @4' c/c]

\therefore Take $d_p = 12''$ [Otherwise the required clear covers make the reinforcements too congested]

Required pile-length $L_p = P/\{ \alpha_2 \tau_s (\pi d_p) \} = 67.17/(0.80 \times 1.0 \times \pi \times 12/12) = 21.38'$; i.e., Use 22-ft long piles.

10.



- (i) At working condition, $P = 250$ k (i.e., effective prestress force) and bending moment $M_{max} = wL^2/8$, where w = Beam self-weight

For Section 1

Area $A_1 = 16 \times 32 - 13 \times 26 = 174 \text{ in}^2 \Rightarrow w_1 = 174/144 \times 0.150 = 0.181 \text{ k/ft}$

$\Rightarrow M_{max} = 0.181 \times 60^2/8 = 81.56 \text{ k'}$

Moment of Inertia $I_1 = (16 \times 32^3 - 13 \times 26^3)/12 = 26764 \text{ in}^4 \Rightarrow S_1 = I_1/c = 26764/16 = 1673 \text{ in}^3$

$\Rightarrow f_{top} = -250/174 + 250 \times 10/1673 - 81.56 \times 12/1673 = -1.437 + 1.495 - 0.585 = -0.527 \text{ ksi}$

and $f_{bot} = -1.437 - 1.495 + 0.585 = -2.346 \text{ ksi}$

For Section 2

Area $A_2 = 16 \times 32 = 512 \text{ in}^2 \Rightarrow w_1 = 512/144 \times 0.150 = 0.533 \text{ k/ft}$

$\Rightarrow M_{max} = 0.533 \times 60^2/8 = 240.0 \text{ k'}$

Moment of Inertia $I_2 = 16 \times 32^3/12 = 43691 \text{ in}^4 \Rightarrow S_2 = I_2/c = 43691/16 = 2731 \text{ in}^3$

$\Rightarrow f_{top} = -250/512 + 250 \times 10/2731 - 240 \times 12/2731 = -0.488 + 0.916 - 1.055 = -0.627 \text{ ksi}$

and $f_{bot} = -0.488 - 0.916 + 1.055 = -0.349 \text{ ksi}$

- (ii) Allowable tensile stress in concrete $= 7.5\sqrt{f'_c} = 7.5\sqrt{(6/1000)} = 0.581 \text{ ksi}$
 and Allowable compressive stress in concrete $= 0.45f'_c = 0.45 \times 6 = 2.70 \text{ ksi}$; i.e., -2.70 ksi
 \Rightarrow All f_{top} and f_{bot} stresses are within the allowable limits.
- (iii) Cracking \Rightarrow Maximum tensile stress in concrete $= 0.581 \text{ ksi}$

For the loading case shown, there will be compression at top and tension at bottom fibers of the beam sections. So the check for cracking will be for bottom fibers only.

\therefore For Section 1, $f_{bot} = -2.346 + f_{LL} = 0.581 \Rightarrow f_{LL} = 2.927 \text{ ksi}$

$\Rightarrow M_{LL} = 2.927 \times 1673/12 = 408.03 \text{ k'}$ $\Rightarrow F \times 60/4 = 408.03 \Rightarrow F = 27.20 \text{ k}$

and for Section 2, $f_{bot} = -0.349 + f_{LL} = 0.581 \Rightarrow f_{LL} = 0.930 \text{ ksi}$

$\Rightarrow M_{LL} = 0.930 \times 2731/12 = 211.64 \text{ k'}$ $\Rightarrow F \times 60/4 = 211.64 \Rightarrow F = 14.11 \text{ k}$