# Solutions to Assignment 5

CIVE 612 Open-Channel Flow
Spring 2013
Prof. T. K. Gates

# CE 612 OPEN-CHANNEL FLOW

ASSIGNMENT 5

Due 2 April 2013

Work all the following problems. You are responsible to know how to work all of these problems but they do not have to be turned in for grading.

(1) The flow depth on the upstream side of a hydraulic jump in a rectangular channel is 1.5 ft with a corresponding Froude number of 5. Find the depth on the downstream side of the jump and the energy loss per unit weight of fluid flowing through the jump. Also, plot the water surface profile through the jump assuming the channel bed is approximately horizontal.

(2) A hydraulic jump forms just downstream of a vertical sluice gate controlling flow from a reservoir into an open channel. The flow depth and velocity in the channel on the downstream side of the jump are 5.2 ft and 4.3 ft/s, respectively. If the energy head loss under the gate is assumed negligible, what would be the depth in the reservoir just upstream of the gate?

(3) A hydraulic jump occurs in a trapezoidal canal with a bottom width of 3.0 m and side slopes of 1.5:1. The supercritical depth is 0.25 m for a steady flow rate of 14.4 $m^3/s$. Find the sequent depth and the energy head loss in the jump.

(4) A hydraulic jump occurs in a 3-ft diameter storm sewer with a steady flow of 5 $ft^3/s$. If the depth on the downstream side of the jump is 0.81 ft, find the depth on the upstream side.

(5) A steady flow of 10,500 $ft^3/s$ flows over a spillway on a dam with a 40-ft wide crest. The flow exits off the spillway chute into a rectangular stilling basin having the same width. The reservoir water surface elevation just behind the spillway is 205 ft above a reference datum, and the elevation of the tailwater in the river just downstream of the stilling basin is 105 ft. Assuming a 10% energy head loss along the spillway chute, design the elevation of the stilling basin invert and the length of the stilling basin to insure that the hydraulic jump occurs within the stilling basin.

(6) A hydraulic jump occurs in a triangular canal with side slopes of 2:1. The Froude number on the upstream side of the jump is 2.6 and the height (difference between upstream and downstream depths) of the jump is 1 m. Find the power lost in the jump.

For a hydraulic jump in a rectangular channel,

$$h_2 = (h_1/2)\left[(1+8Fr_1^2)^{1/2} - 1\right]$$
$$= (1.5/2)\left[(1+8(5)^2)^{1/2} - 1\right] = \underline{\underline{9.88 \text{ ft}}}$$

The energy head loss across the jump is

$$h_{Lj} = H_1 - H_2$$
$$= (h_1 + \bar{u}_1^2/2g) - (h_2 + \bar{u}_2^2/2g) \quad \text{for } z_1 \simeq z_2,\ \alpha_1 = \alpha_2 \simeq 1$$

Now

$$Fr_1 = \bar{u}_1/(g y_1)^{1/2}$$
$$\therefore \bar{u}_1 = F_1 (g y_1)^{1/2} = 5\left[(32.2)(1.5)\right]^{1/2} = 34.75 \text{ ft/s}$$
$$\bar{u}_2 = \bar{u}_1 A_1/A_2 = \bar{u}_1 (b h_1)/b h_2 = \bar{u}_1 h_1/h_2 = 34.75(1.5)/9.88$$
$$= 5.28 \text{ ft/s}$$

$$\therefore h_{Lj} = \left[1.5 + \frac{(34.75)^2}{2(32.2)}\right] - \left[9.88 + \frac{(5.28)^2}{2(32.2)}\right] = \underline{\underline{9.94 \text{ ft}}}$$

Now, the total jump length is computed as

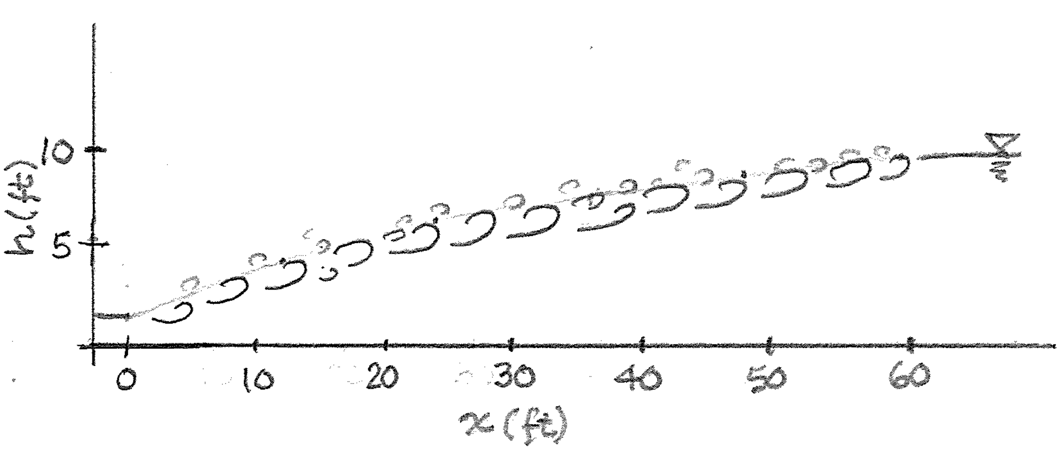
$$L_j \simeq h_1\left[220 \tanh\left(\frac{Fr_1 - 1}{22}\right)\right] = 1.5\left[220 \tanh\left(\frac{5-1}{22}\right)\right]$$
$$\simeq 59.3 \text{ ft}$$

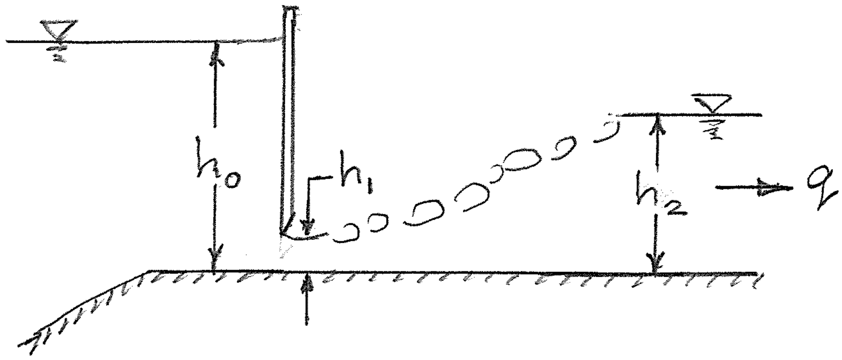
The flow depth along the jump is computed from

$$h_i = h_1 + (h_2 - h_1)\tanh\left\{1.5x/\left[h_1\left(-1.2 + 160 \tanh \frac{Fr_1}{20}\right)\right]\right\}$$
$$= 1.5 + (8.38)\tanh(0.0263x) \quad \text{where } x = \text{dist. from beginning of jump}$$

The depth at ~12 ft intervals is summarized in the table and profile plot below:

| $x_i$ (ft) | $h_i$ (ft) |
|---|---|
| 0 | 1.5 |
| 12 | 4.1 |
| 24 | 6.2 |
| 36 | 7.7 |
| 48 | 8.6 |
| 59.3 | 9.9 |






Assume a rectangular cross section and a horizontal sill beneath the gate leading from the reservoir into the channel. Also, assume that the hydraulic jump occurs just downstream of the gate.

The steady unit discharge can be computed as

$$q = \bar{u}_2 h_2 = 4.3\,\text{ft/s}\,(5.2\,\text{ft}) = 22.36\,(\text{ft}^3/\text{s})/\text{ft}$$

The supercritical depth, $h_1$, on the u.s. side of the jump can be calculated from the classical jump equation:

$$h_1 = (h_2/2)\left[(1+8Fr_2^2)^{1/2} - 1\right]$$

$$Fr_2 = \bar{u}_2/(gh_2)^{1/2} = 4.3/[(32.2)(5.2)]^{1/2} = 0.33$$

$$= (5.2/2)\left[(1+8(0.33)^2)^{1/2} - 1\right] = 0.97\text{ ft}$$

The associated supercritical velocity is

$$\bar{u}_1 = q/h_1 = 22.36/0.97 = 23.09\text{ ft/s}$$

Applying conservation of energy for negligible energy local loss under the gate gives

$$H_0 = H_1$$

$$h_0 + \alpha_0\bar{u}_0^2/2g = h_1 + \alpha_1\bar{u}_1^2/2g \quad \text{since } z_0 = z_1$$

For negligible velocity head in the reservoir and $\alpha_1 \simeq 1$,

$$h_0 = h_1 + \bar{u}_1^2/2g = 0.97 + \frac{(23.09)^2}{2(32.2)} = \underline{9.25\text{ ft}}$$

Note: Typically, the head loss under the gate would not be negligible.

Assuming negligible friction and gravity forces and applying conservation of linear momentum for steady flow across the jump ($\beta_1 = \beta_2 \simeq 1$),

$$\bar{h}_1 A_1 + Q^2/gA_1 = \bar{h}_2 A_2 + Q^2/gA_2$$

which for a trapezoidal channel with bottom width, $b$, and side slopes $z_s$ gives

$$\frac{bh_1^2}{2} + \frac{z_s h_1^3}{3} + \frac{Q^2}{gh_1(b+z_s h_1)} = \frac{bh_2^2}{2} + \frac{z_s h_2^3}{3} + \frac{Q^2}{gh_2(b+z_s h_2)}$$

since $\bar{h} = (h/3)\,\frac{2b+T_W}{b+T_W}$, $T_W = b + 2z_s h$

$A = h(b + z_s h)$

So that

$$\frac{3(0.25)^2}{2} + \frac{1.5(0.25)^3}{3} + \frac{(14.4)^2}{(9.81)(0.25)[3+1.5(0.25)]}$$

$$= \frac{3h_2^2}{2} + \frac{1.5h_2^3}{3} + \frac{(14.4)^2}{9.81\,h_2(3+1.5h_2)}$$

Solving gives $h_2 = \underline{\underline{2.87\ m}}$

For no appreciable change in the bed elevation across the jump,

$$h_{L_J} = (h_1 + \bar{u}_1^2/2g) - (h_2 + \bar{u}_2^2/2g) \quad \text{for } \alpha_1 = \alpha_2 \simeq 1$$

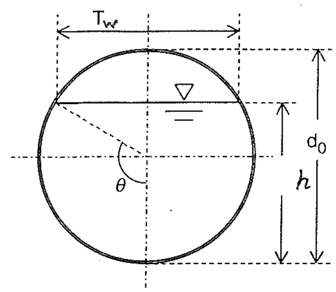
$$\bar{u}_1 = Q/A_1 = 14.4/0.25[3+1.5(0.25)] = 17.07\ m/s$$

$$\bar{u}_2 = Q/A_2 = 14.4/2.87[3+1.5(2.87)] = 0.69\ m/s$$

$$= \left[0.25 + \frac{(17.07)^2}{2(9.81)}\right] - \left[2.87 + \frac{(0.69)^2}{2(9.81)}\right] = \underline{\underline{12.38m}}$$

Assuming negligible friction and gravity forces and applying conservation of linear momentum for steady flow across the jump ($\beta_1 = \beta_2 \simeq 1$),

$$\bar{h}_1 A_1 + Q^2/gA_1 = \bar{h}_2 A_2 + Q^2/gA_2$$




Now for free surface flow in a circular channel, it can be shown that

$$\bar{h}A = \frac{d_0^3}{24}(3\sin\theta - \sin^3\theta - 3\theta\cos\theta)$$

with $A = \frac{1}{8}(2\theta - \sin\theta)d_0^2$

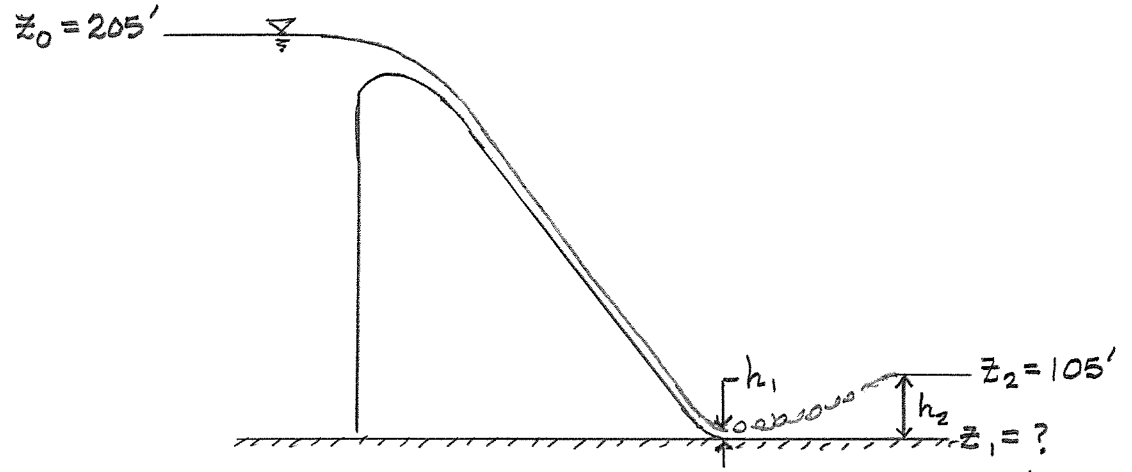
$$\theta = \pi - \cos^{-1}\{(h - d_0/2)/(d_0/2)\}$$

Thus,

$$\frac{d_0^3}{24}(3\sin\theta_1 - \sin^3\theta_1 - 3\theta_1\cos\theta_1) + \frac{Q^2}{(g/8)(2\theta_1 - \sin\theta_1)d_0^2} =$$
$$\frac{d_0^3}{24}(3\sin\theta_2 - \sin^3\theta_2 - 3\theta_2\cos\theta_2) + \frac{Q^2}{(g/8)(2\theta_2 - \sin\theta_2)d_0^2}$$

Now, $d_0 = 3\text{ ft}$; $\theta_2 = \pi - \cos^{-1}\{(0.81 - 3/2)/(3/2)\} = 1.09\text{ rad}$
with $h_2 = 0.81\text{ m}$; $Q = 5\text{ ft}^3/\text{s}$

Solving for $h_1$ gives $h_1 = \underline{\underline{0.6\text{ ft.}}}$




Assuming negligible velocity head in the reservoir just upstream of the spillway and applying conserv. of energy for steady flow from just upstream of the spillway to the stilling basin at the base of the spillway chute gives

$$205 = z_1 + h_1 + q^2/2gh_1^2 + h_{L_{0\to1}} \quad \text{for } \alpha_1 \simeq 1$$

Defining the energy head at pt. 0 relative to the stilling basin floor so that $h_{L_{0\to1}} = 0.10(205 - z_1)$ gives

$$205 = z_1 + h_1 + \frac{(10{,}500/40)^2}{2(32.2)h_1^2} + 0.10(205 - z_1)$$

$$0.9(z_1) = 184.5 - h_1 - 1070/h_1^2$$

$$z_1 = 205 - 1.11h_1 - 1189/h_1^2 \qquad (1)$$

Now, the classical jump equation gives

$$h_2 = h_1/2\left[(1+8Fr_1^2)^{1/2} - 1\right]$$
$$= h_1/2\left[\left(1+8\frac{q^2}{gh_1^3}\right)^{1/2} - 1\right] = \frac{h_1}{2}\left[\left(1+8\frac{(10{,}500/40)^2}{32.2h_1^3}\right)^{1/2} - 1\right]$$
$$= h_1/2\left[(1+17{,}120/h_1^3)^{1/2} - 1\right] \qquad (2)$$

Also, the tailwater level in the river must match the water surface elev. on the d.s. side of the jump in the stilling basin:

$$z_1 + h_2 = 105 \qquad (3)$$

Solving Eqs (1), (2), and (3) simultaneously gives

$h_1 = 2.03$ ft
$h_2 = 44.94$ ft
$z_1 = \underline{\underline{60.06 \text{ ft}}}$

Now, assuming a Type I stilling basin (no chute blocks, ~~baffle~~ piers, end sill),

$$Fr_1 = q/\sqrt{g h_1^3} = \frac{(10,500/40)}{[32.2(2.03)^3]^{1/2}} = 16$$

$$\therefore L \simeq h_1\left[220 \tanh\left(\frac{Fr_1 - 1}{22}\right)\right]$$

$$\simeq 2.03\left[220 \tanh\left(\frac{16-1}{22}\right)\right] = \underline{\underline{264.7 \text{ ft}}}$$

Assuming negligible friction and gravity forces and applying conserv. of linear momentum for steady flow across the jump ($\beta_1 = \beta_2 \simeq 1$) gives

$$\bar{h}_1 A_1 + Q^2/gA_1 = \bar{h}_2 A_2 + Q^2/gA_2$$

Now for a triangular cross section with side slopes $z_s$,

$$\bar{h}A = z_s h^3/3$$

$$A = z_s h^2$$

Also,

$$Fr^2 = \frac{Q^2 T_W}{g A^3}$$

or $Q^2 = Fr^2 g A^3/T_W$ with $T_W = 2z_s h$ for a triangular section

Thus, $Q^2 = 2g\,Fr^2 h^5$ for a triangular section.

So that,

$$\frac{z_s h_1^3}{3} + \frac{2Fr_1^2 h_1^3}{z_s} = \frac{z_s h_2^3}{3} + \frac{2Fr_1^2 h_1^5}{z_s h_2^2}$$

$$\left(\frac{2}{3}\right)h_1^3 + \frac{2(2.6)^2 h_1^3}{2} = \left(\frac{2}{3}\right)h_2^3 + \frac{2(2,6)^2 h_1^5}{2 h_2^2}$$

$$7.43\,h_1^3 = 0.67\,h_2^3 + 6.76 h_1^5/h_2^3 \qquad (1)$$

Also,

$$h_2 = h_1 + 1 \qquad (2)$$

Solving Eqs (1) and (2) simultaneously gives

$h_1 = 0.87$ m

$h_2 = 1.87$ m

Assuming negligible change in bed elev. across the jump, the head loss is ($\alpha_1 = \alpha_2 \simeq 1$):

$$h_{L_J} = (h_1 + \bar{u}_1^2/2g) - (h_2 + \bar{u}_2^2/2g)$$

$$\bar{u}_1 = \frac{Q}{A_1} = \frac{[2g\,Fr_1^2 h_1^5]^{1/2}}{2h_1^2} = \frac{8.13}{1.51} = 5.37 \text{ m/s}$$

$$\bar{u}_2 = Q/A_2 = 8.13/2(1.87)^2 = 1.16 \text{ m/s}$$

Thus

$$h_{L_J} = \left[0.87 + \frac{(5.37)^2}{2(9.81)}\right] - \left[1.87 + \frac{(1.16)^2}{2(9.81)}\right] = 0.40\ m$$

The power lost is

$$\mathbb{P} = \gamma Q h_{L_J} = (9792\ N/m^3)(8.13\ m^3/s)(0.40\ m) = \underline{\underline{31{,}844\ \frac{N \cdot m}{s}}}$$

assuming Temp. of water $\simeq$ 20 °C