

TWO-WAY SLABS

Two-way Slabs:

When the ratio (L/S) is less than 2.0, it is called two-way slab, shown in Figure 1. Bending will take place in the two directions in a dish-like form. Accordingly, main reinforcement is required in the two directions.

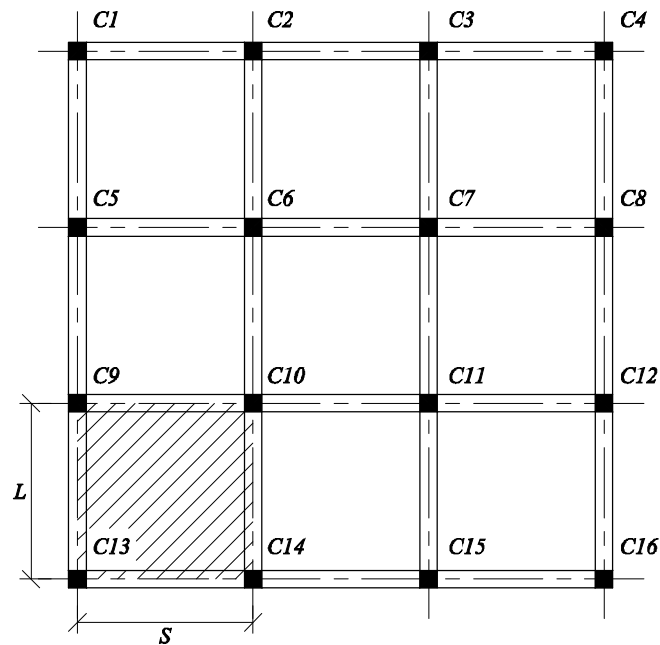


Figure 1: Two way slabs

Types

A- Flat Plates:

A flat plate floor is a two-way slab with no supporting beams, only columns. The slab has smooth under surfaces without column capitals or drop panels, shown in Figure 2.a. This type of two-way slabs is suited for light loads associated with apartment construction and can be economically used for spans up to 6.0 meters. Negative moment reinforcement is placed near the top of the plate close to the columns while the positive moment reinforcement is placed near the bottom surface in the middle of the plate in both directions.

Use of flat plates is limited by their punching shear capacities where large thickness is required in case of heavy loads or large spans, thus rendering the system uneconomical.

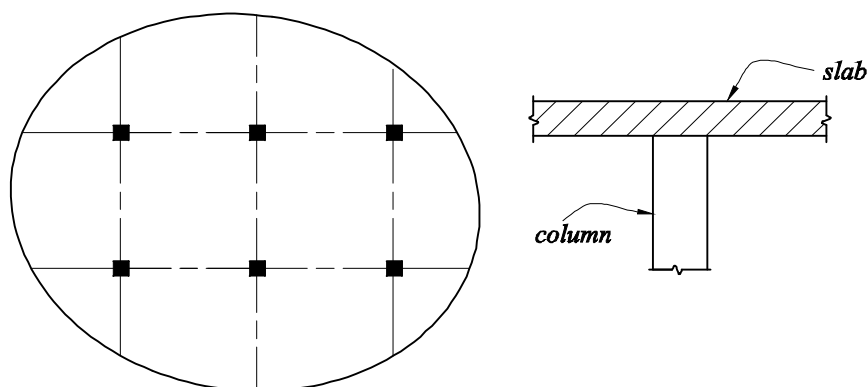


Figure 2.a: Flat plate

B- Flat Slabs:

A two-way slab with column capitals or drop panels, or both, shown in Figure 2.b. This system is appropriate for heavier loads encountered in office or industrial buildings. The system is used for spans up to 8.0 meters, due to the increase in slab thickness around the columns.

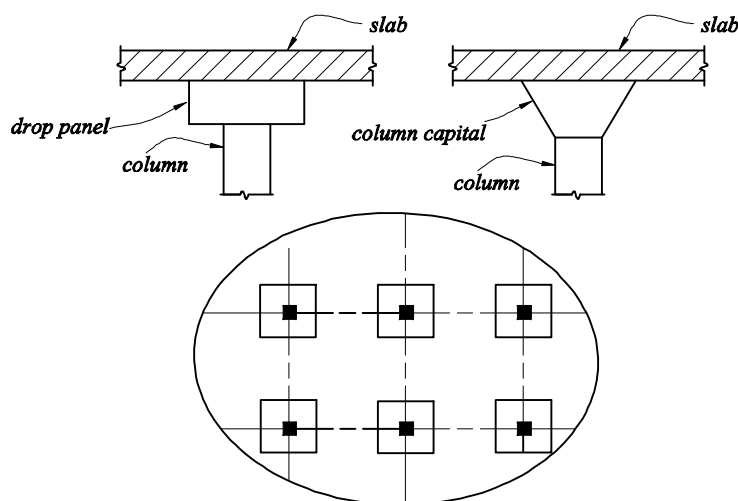


Figure 2.b: Flat Slab

C- Two-way Edge-supported Solid Slabs:

In this system, shown in Figure 2.c, beams provide moment interaction with the columns especially when moment resisting frames are used to resist lateral loads. The slab overall thickness is greater because beams project downward, thus the system becomes inflexible

in terms of mechanical layout. The system can be used economically for spans up to 7.0 meters.

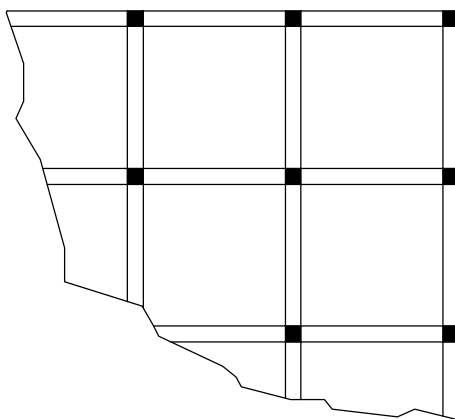


Figure 2.c: Two-way edge supported solid slab

D- Two-way Ribbed Slabs:

1. Waffle Slabs:

The waffle slab is capable of providing the largest spans of the conventional concrete floor systems and can be economically used for spans up to 12.0 meters. This is attributed to the considerable reduction in dead load as compared to other floor systems. Waffle slab construction consists of orthogonal sets of ribs with solid parts at the columns, as shown in Figure 2.d. The ribs are formed with fiberglass or metal dome forms. The ribs are usually 0.60 to 0.90 meter on center. Shear is transferred to the columns by using beams or shear heads.

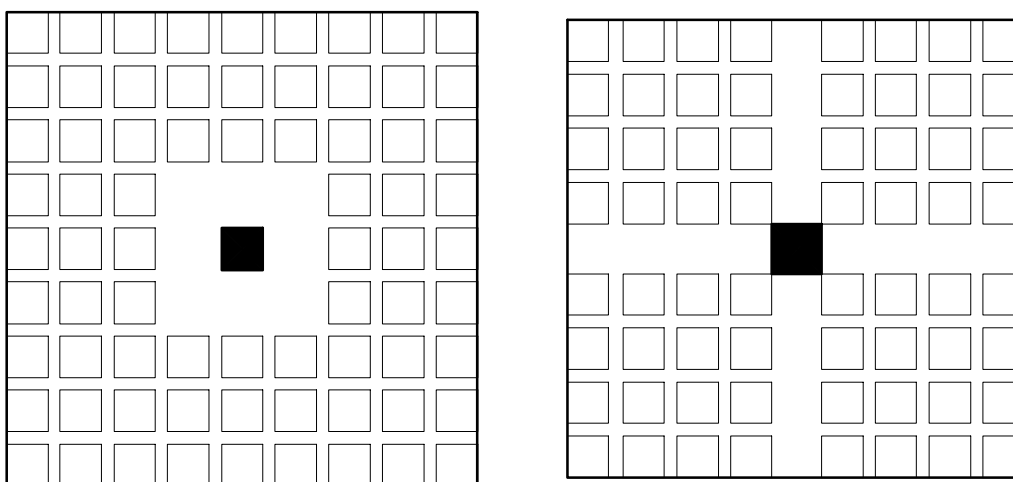


Figure 2.d: Waffle slabs

2. Two-way Edge-supported Ribbed Slabs:

This system, shown in Figure 2.e, can be economically used for spans up to 7.0 meters. It is similar to the waffle slab but the voids between ribs are filled with hollow blocks. Hidden or drop beams can be used with this system depending on their spans.

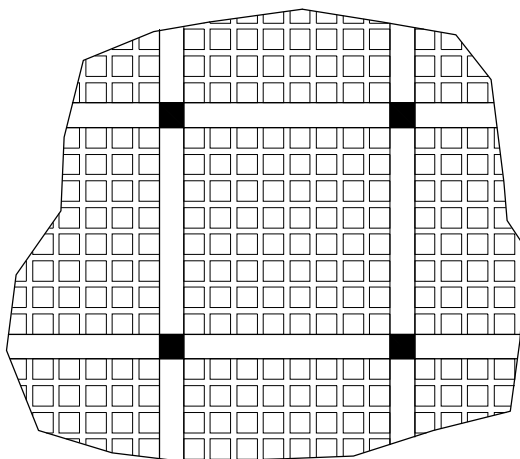


Figure 2.e: Two-way edge supported ribbed slabs

Design Methods

In the *ACI Code 318-08*, two empirical methods are given for analysis and design of two-way slabs, regardless of their types. The first of these methods is the *Direct Design Method* which has to meet certain limiting conditions. This method will be elaborately dealt with later on in this chapter. The second method is called the Equivalent Frame Method, which has no restrictions on its use. This method is rather complex and will not be covered in this book. Detailed coverage of the method can be found in the *ACI Code 318-08*.

For design of edge supported slabs, simplified design methods will be used. These design methods are basically similar in nature to the method provided in the *ACI Code 318-1963* and older versions. These methods are based on load distribution in two principal directions parallel to the short and long sides of edge supported panels, depending on long side to short side ratio and support conditions at slab edges.

Simplified Design Methods

The load carried to each support and the bending moments in two-way slabs are not easy to calculate if all the conditions are to be satisfied. Therefore, empirical formulae and

approximate theories have been formulated which give bending moments in slabs supported on all four edges. Bending moments can be obtained by one of the following methods:

- § Grashoff method: it is used for load distribution when the slab is simply supported on all four edges and the corners are not held down.
- § Marcus method: it is used for load distribution when the slab is simply supported on all four edges and the corners are held down. Slabs built into thin walls or monolithic with the beams come into this category.
- § *Egyptian Code* method: it is used for various support conditions associated with continuous slabs.

Grashoff Method

Consider a slab panel of length L and width S supported on all four edges and subjected to uniformly distributed load w , as shown in Figure 3.a. This method is based on distribution of the load w to two loads, one in the short direction w_s , and the other in the long direction w_l , where

$$w = w_s + w_l \quad (1)$$

Consider two strips of unit width in the directions of L and S at the center of the slab. The deflection of each strip at the center will be the same.

The central deflection of the strip in L direction, shown in Figure 3.b, is given as

$$\Delta_l = \frac{5 w_l L^4}{384 EI} \quad (2)$$

The central deflection of the strip in S direction, shown in Figure 3.c, is given as

$$\Delta_s = \frac{5 w_s S^4}{384 EI} \quad (3)$$

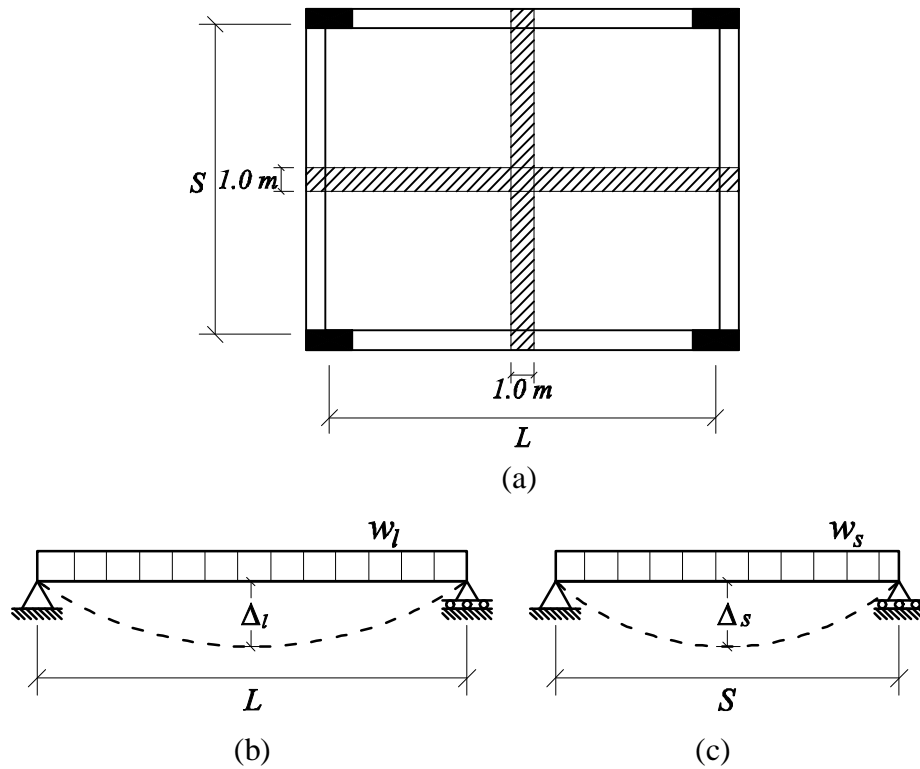


Figure 3: (a) Strips in the short and long directions; (b) deflection in the long direction; (c) deflection in the short direction

Equating Eqs. (2) and (3) gives

$$\frac{5 w_l L^4}{384 EI} = \frac{5 w_s S^4}{384 EI} \quad (4)$$

$$\text{or } \frac{w_s}{w_l} = \frac{L^4}{S^4} \quad (5)$$

Eq. (5) can be written as

$$\text{or } \frac{w_s}{w_s + w_l} = \frac{L^4}{L^4 + S^4} \quad (6)$$

Since $w = w_s + w_l$,

$$w_s = w \left(\frac{L^4}{L^4 + S^4} \right) \quad (7)$$

Dividing the right hand side of Eq. (7) by S^4 and letting $r = L/S$,

$$w_s = w \left(\frac{r^4}{1+r^4} \right) = a w \quad (8)$$

The load transferred in the long direction w_l is evaluated by as

$$w_l = w \left(\frac{1}{1+r^4} \right) = b w \quad (9)$$

The load coefficients a and b are given in Table 1.

Table 1: Grashoff load coefficients

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
a	0.500	0.594	0.675	0.741	0.793	0.835	0.868	0.893	0.913	0.929	0.941
b	0.500	0.406	0.325	0.259	0.207	0.165	0.132	0.107	0.087	0.071	0.059

The maximum bending moment occurs along shorter span and depth of the slab is designed for this bending moment. The reinforcement of such slabs is provided in two directions placed one above the other. The reinforcement in the shorter direction is placed closest to the concrete surface so as to provide greater effective depth to resist larger moment associated with this shorter direction.

Marcus Method

In Grashoff method, effect of restraint against lifting of the corners and effect of torsion in the slab were neglected. Marcus has given an approximate method for determining bending moments in slabs, simply supported on four edges with corners prevented from being lifted and considering torsion in the slab. The bending moments at midspan of such a slab are the midspan moments as calculated by Grashoff multiplied by a modification factor which is less than unity as the positive bending moments in the slab are reduced due to corner restraint and torsion. This modification factor is given by $1 - \left(\frac{5}{6} \right) \left(\frac{r^2}{1+r^4} \right)$.

Load distribution coefficients according to Marcus are given in Table 2.

Table 2: Marcus load coefficients

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
a	0.292	0.355	0.411	0.470	0.526	0.577	0.623	0.663	0.699	0.730	0.757
b	0.292	0.240	0.198	0.165	0.137	0.114	0.095	0.079	0.067	0.056	0.047

Egyptian Code Method

Grashoff and Marcus methods have basically dealt with analysis of simply supported slab panels. The *Egyptian Code* method has extended these methods for analysis of continuous slabs where individual panels have different supporting conditions. To account for the various supporting conditions, the rectangularity ratio $r = L/S$ is modified by multiplying the nominator and the dominator of this expression by the ratio of distance between inflection points to the length between supports for longer and shorter spans, respectively, or

$$r' = \frac{L m_l}{S m_s} = r \times \frac{m_1}{m_2}.$$

For simply supported spans, $m = 1.0$, for spans continuous from one side, $m = 0.87$, and for spans continuous from both sides, $m = 0.76$.

The load distribution factors for edge supported two way slabs according to the *Egyptian Code of Practice* are given in Table 3.

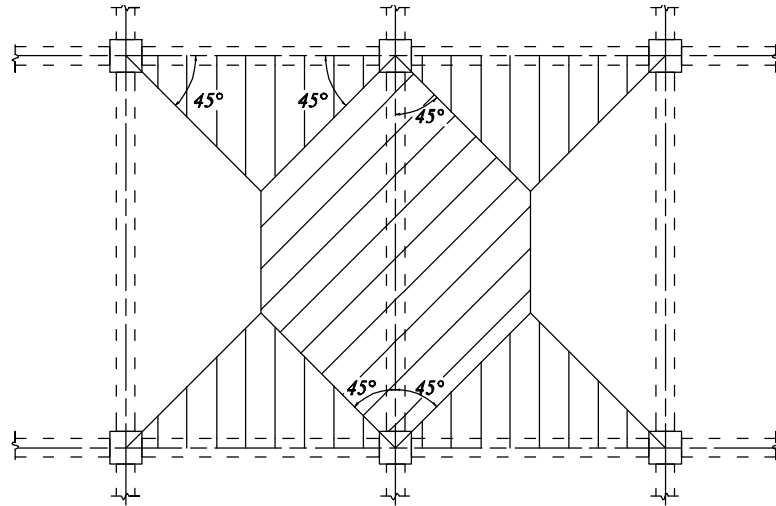
Table 3: Egyptian Code load coefficients

r'	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
a	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
b	0.35	0.29	0.25	0.21	0.18	0.16	0.14	0.12	0.11	0.09	0.08

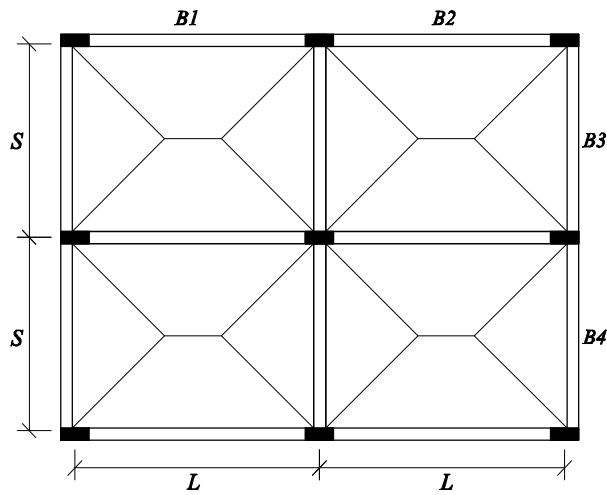
Beams for Two-way Slabs Designed by Approximate Methods

The load supported by each beam is considered to be the entire load on tributary areas bounded by 45-degree lines drawn from the corners of the Figure 4.a and Figure 4.b. Triangular or trapezoidal distributions are possible. If available analysis software can not cope with such load distributions, as shown in Figure 9.4.c it is possible to replace these load distributions by equivalent uniform loads that give same maximum shear and moment as the original loads do.

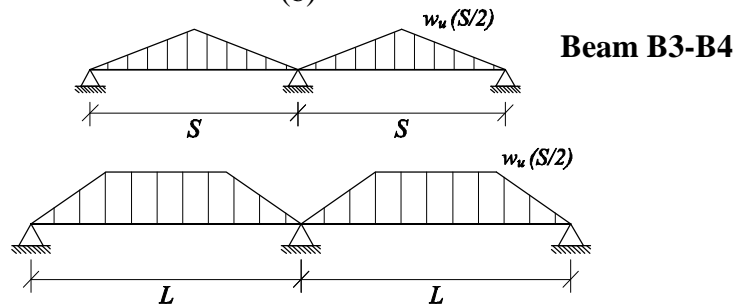
For triangular load distribution, the equivalent shear force coefficient C_s is equal to 0.50, and the equivalent bending moment coefficient C_b is equal to 2/3. For trapezoidal load distribution, C_s and C_b are given in Table 4.



(a)



(b)



(c)

Figure 9.4: (a) Tributary areas for slab loads on beams; (b) tributary areas for loads on continuous beams; (c) loads on beams *B3-B4* and *B1-B2*

Table 4: Shear and moment equivalent load coefficients for trapezoidal load distribution

r	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
C_s	0.500	0.545	0.583	0.615	0.643	0.667	0.688	0.706	0.722	0.737	0.750
C_b	0.667	0.725	0.769	0.803	0.830	0.852	0.870	0.885	0.897	0.908	0.917

In Table 4, $C_s = 1 - \frac{1}{2r}$, and $C_b = 1 - \frac{1}{3r^2}$.

The total load on a given beam consists of its own weight, weight of partitions, if any, and the loads resulting from the slab, as explained earlier.