Module 7

Influence Lines

Lesson 37

Moving Load and Its Effects on Structural Members

Instructional Objectives:

The objectives of this lesson are as follows:

- Understand the moving load effect in simpler term
- Study various definitions of influence line
- Introduce to simple procedures for construction of influence lines

37.1 Introduction

In earlier lessons, you were introduced to statically determinate and statically indeterminate structural analysis under non-moving load (dead load or fixed loads). In this lecture, you will be introduced to determination of maximum internal actions at cross-sections of members of statically determinate structured under the effects of moving loads (live loads).

Common sense tells us that when a load moves over a structure, the deflected shape of the structural will vary. In the process, we can arrive at simple conclusion that due to moving load position on the structure, reactions value at the support also will vary.

From the designer's point of view, it is essential to have safe structure, which doesn't exceed the limits of deformations and also the limits of load carrying capacity of the structure.

37.2 Definitions of influence line

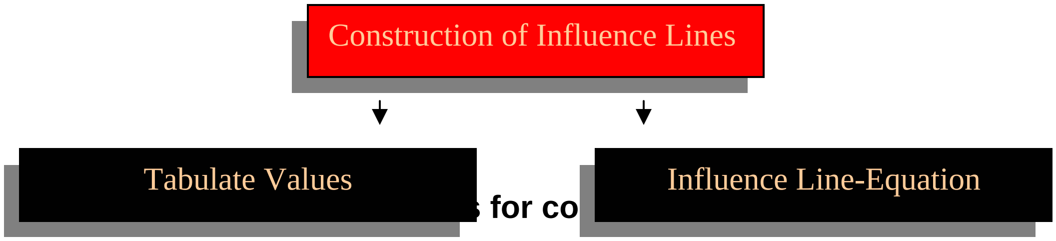
In the literature, researchers have defined influence line in many ways. Some of the definitions of influence line are given below.

- An influence line is a diagram whose ordinates, which are plotted as a function of distance along the span, give the value of an internal force, a reaction, or a displacement at a particular point in a structure as a unit load move across the structure.
- An influence line is a curve the ordinate to which at any point equals the value of some particular function due to unit load acting at that point.
- An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a unit concentrated force moves over the member.

37.3 Construction of Influence Lines

In this section, we will discuss about the construction of influence lines. Using any one of the two approaches (Figure 37.1), one can construct the influence line at a specific point P in a member for any parameter (Reaction, Shear or

Moment). In the present approaches it is assumed that the moving load is having dimensionless magnitude of unity. Classification of the approaches for construction of influence lines is given in Figure 37.1.

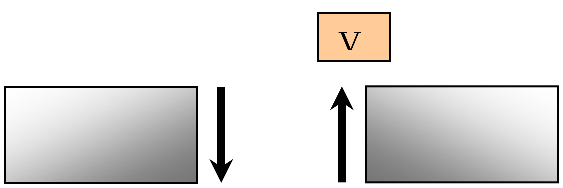
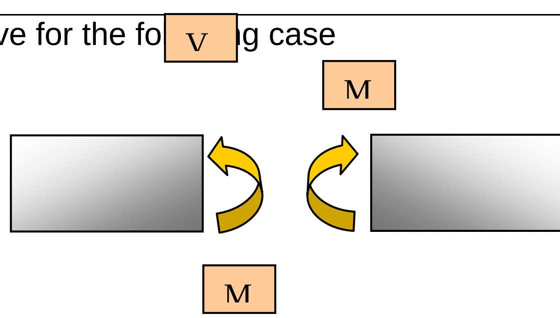


37.3.1 Tabulate Values

Apply a unit load at different locations along the member, say at ***x***. And these locations, apply statics to compute the value of parameter (reaction, shear, or moment) at the specified point. The best way to use this approach is to prepare a table, listing unit load at x versus the corresponding value of the parameter calculated at the specific point (i.e. Reaction R, Shear V or moment M) and plot the tabulated values so that influence line segments can be constructed.

37.3.2 Sign Conventions

Sign convention followed for shear and moment is given below.

Parameter	Sign for influence line
Reaction R	Positive at the point when it acts upward on the beam.
Shear V	Positive for the following case 
Moment M	Positive for the following case 

37.3.3 Influence Line Equations

Influence line can be constructed by deriving a general mathematical equation to compute parameters (e.g. reaction, shear or moment) at a specific point under the effect of moving load at a variable position ***x***.

The above discussed both approaches are demonstrated with the help of simple numerical examples in the following paragraphs.

37.4 Numerical Examples

Example 1:

Construct the influence line for the reaction at support B for the beam of span 10 m. The beam structure is shown in Figure 37.2.

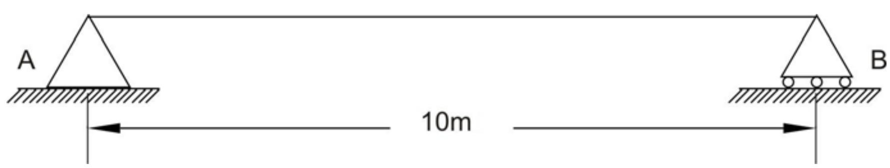


Figure 37.2: The beam structure

Solution:

As discussed earlier, there are two ways this problem can be solved. Both the approaches will be demonstrated here.

Tabulate values:

As shown in the figure, a unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A. Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows (Figure 37.3).

$$\Sigma M_A = 0 : \; R_B \times 10 - 1 \times 2.5 = 0 \Rightarrow R_B = 0.25$$

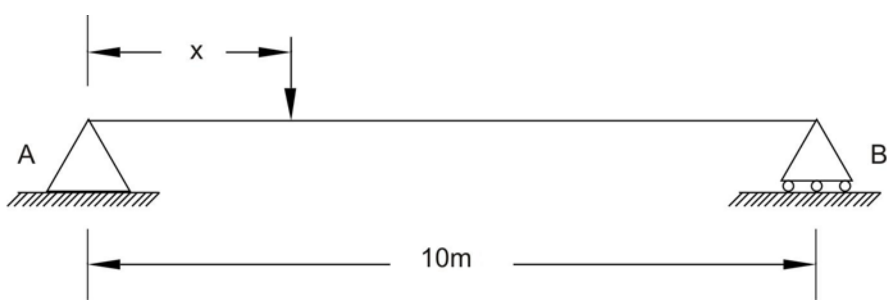


Figure 37.3: The beam structure with unit load

Similarly, the load can be placed at 5.0, 7.5 and 10 m. away from support A and reaction R_B can be computed and tabulated as given below.

x	R_B
0	0.0
2.5	0.25
5.0	0.5
7.5	0.75
10	1

Graphical representation of influence line for R_B is shown in Figure 37.4.

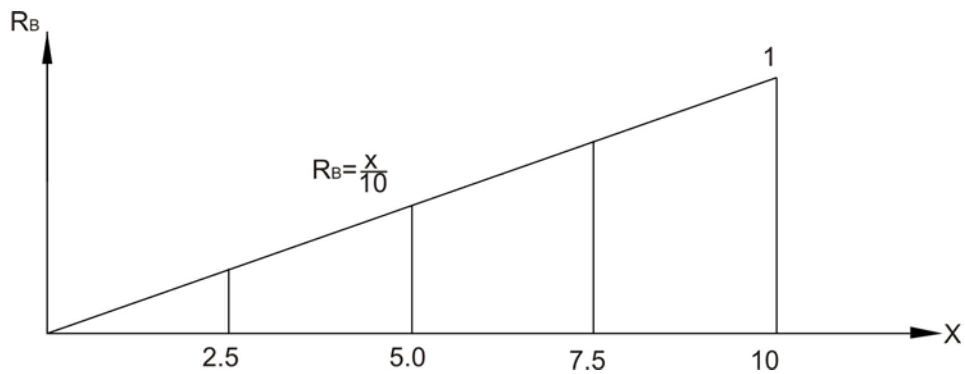


Figure 37.4: Influence line for reaction R_B.

Influence Line Equation:

When the unit load is placed at any location between two supports from support A at distance x then the equation for reaction R_B can be written as

$$\Sigma M_A = 0 : \; R_B \times 10 - x = 0 \Rightarrow R_B = x/10$$

The influence line using this equation is shown in Figure 37.4.

Example 2:

Construct the influence line for support reaction at B for the given beam as shown in Fig 37.5.

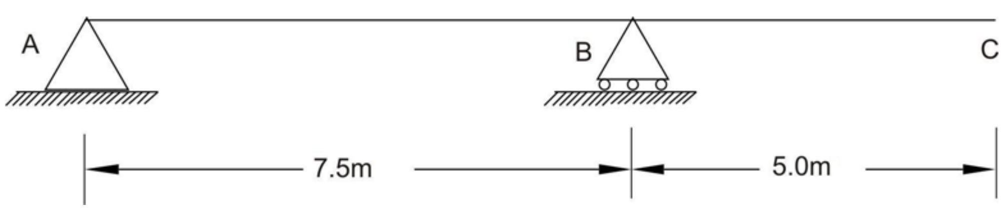


Figure 37.5: The overhang beam structure

Solution:

As explained earlier in example 1, here we will use tabulated values and influence line equation approach.

Tabulate Values:

As shown in the figure, a unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A. Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows.

$$\Sigma M_A = 0 : R_B \times 7.5 - 1 \times 2.5 = 0 \Rightarrow R_B = 0.33$$

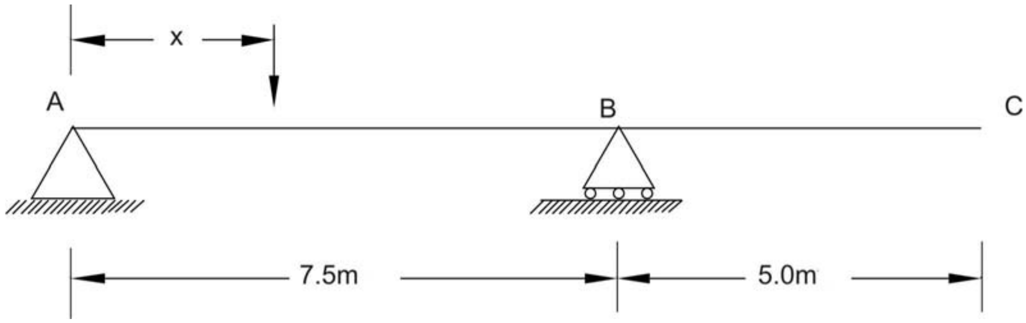


Figure 37.6: The beam structure with unit load

Similarly one can place a unit load at distances 5.0 m and 7.5 m from support A and compute reaction at B. When the load is placed at 10.0 m from support A, then reaction at B can be computed using following equation.

$$\Sigma M_A = 0 : R_B \times 7.5 - 1 \times 10.0 = 0 \Rightarrow R_B = 1.33$$

Similarly a unit load can be placed at 12.5 and the reaction at B can be computed. The values of reaction at B are tabulated as follows.

x	R_B
0	0.0
2.5	0.33
5.0	0.67
7.5	1.00
10	1.33
12.5	1.67

Graphical representation of influence line for R_B is shown in Figure 37.7.

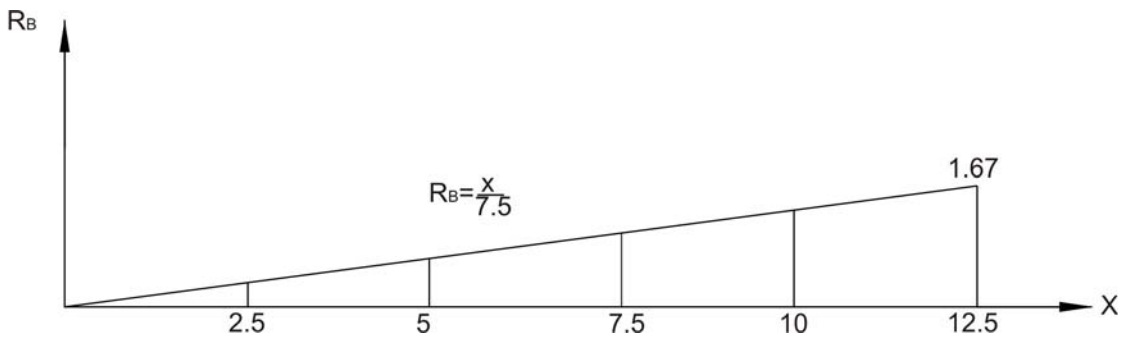


Figure 37.7: Influence for reaction R_B.

Influence line Equation:

Applying the moment equation at A (Figure 37.6),

$\Sigma M_A = 0 : \; R_B \times 7.5 - 1 \times x = 0 \;\Rightarrow\; R_B = x/7.5$

The influence line using this equation is shown in Figure 37.7.

Example 3:

Construct the influence line for shearing point C of the beam (Figure 37.8)

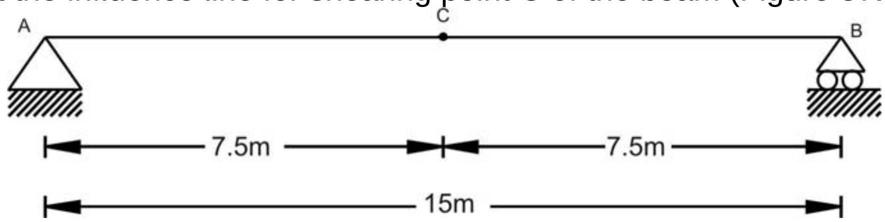


Figure 37.8: Beam Structure

Solution:

Tabulated Values:

As discussed earlier, place a unit load at different location at distance x from support A and find the reactions at A and finally computer shear force taking section at C. The shear force at C should be carefully computed when unit load is placed before point C (Figure 37.9) and after point C (Figure 37.10). The resultant values of shear force at C are tabulated as follows.

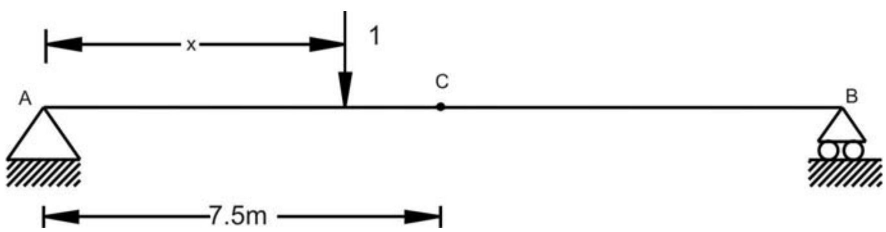


Figure 37.9: The beam structure – a unit load before section

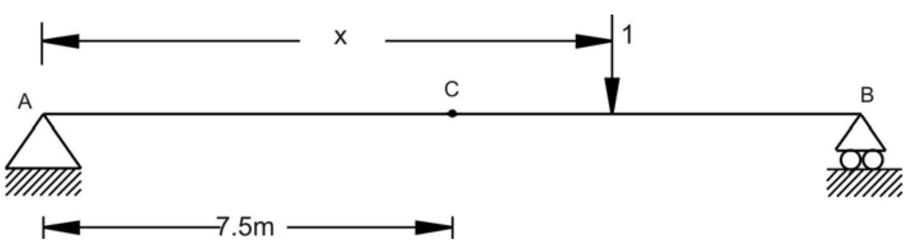


Figure 37.10: The beam structure - a unit load before section

X	V_c
0	0.0
2.5	-0.16
5.0	-0.33
7.5(-)	-0.5
7.5(+)	0.5
10	0.33
12.5	0.16
15.0	0

Graphical representation of influence line for V_c is shown in Figure 37.11.

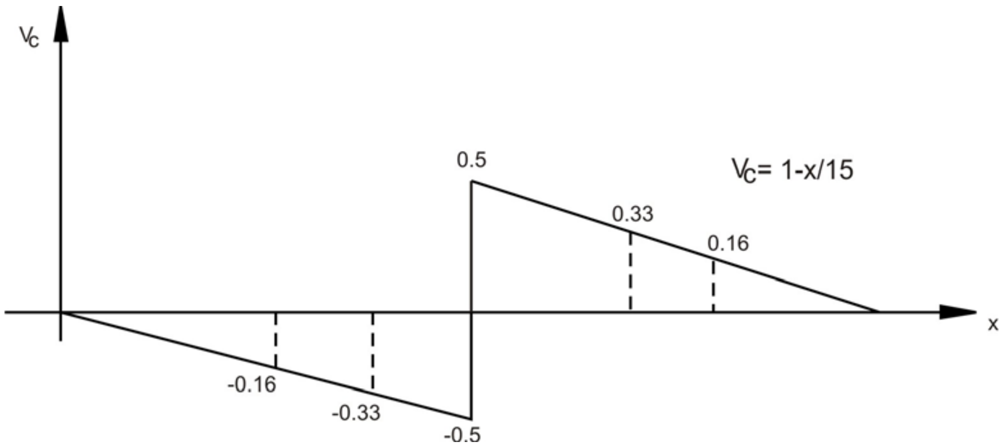


Figure 37.11: Influence line for shear point C

Influence line equation:

In this case, we need to determine two equations as the unit load position before point C (Figure 37.12) and after point C (Figure 37.13) will show different shear force sign due to discontinuity. The equations are plotted in Figure 37.11.

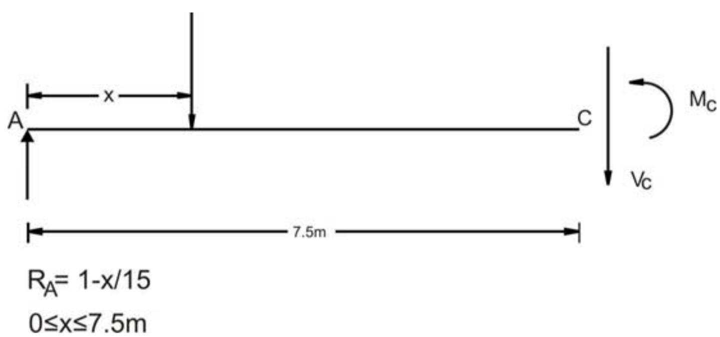


Figure 37.12: Free body diagram – a unit load before section

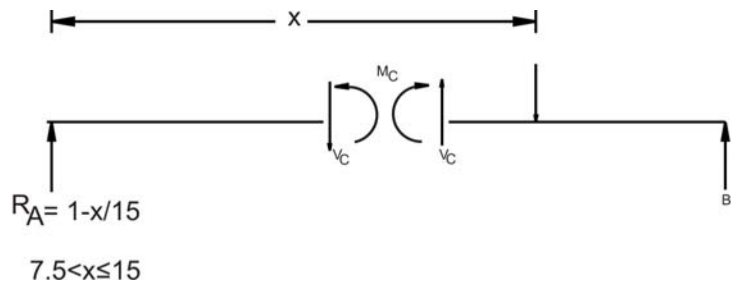


Figure 37.13: Free body diagram – a unit load after section

Influence Line for Moment:
Like shear force, we can also construct influence line for moment.

Example 4:
Construct the influence line for the moment at point C of the beam shown in Figure 37.14

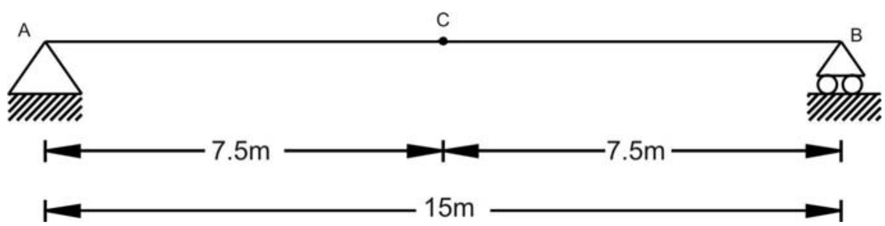


Figure 37.14: Beam structure

Solution:
Tabulated values:
Place a unit load at different location between two supports and find the support reactions. Once the support reactions are computed, take a section at C and compute the moment. For example, we place the unit load at x=2.5 m from support A (Figure 37.15), then the support reaction at A will be 0.833 and support reaction B will be 0.167. Taking section at C and computation of moment at C can be given by

$\Sigma M_c = 0 : \; -M_c + R_B \times 7.5 - = 0 \Rightarrow -M_c + 0.167 \times 7.5 - = 0 \Rightarrow M_c = 1.25$

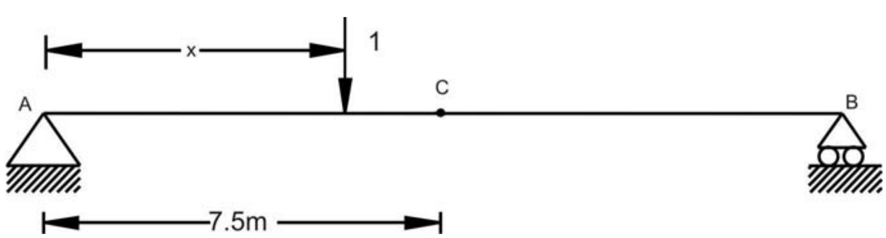


Figure 37.15: A unit load before section

Similarly, compute the moment M_c for difference unit load position in the span. The values of Mc are tabulated as follows.

X	M_c
0	0.0
2.5	1.25
5.0	2.5
7.5	3.75
10	2.5
12.5	1.25
15.0	0

Graphical representation of influence line for M_c is shown in Figure 37.16.

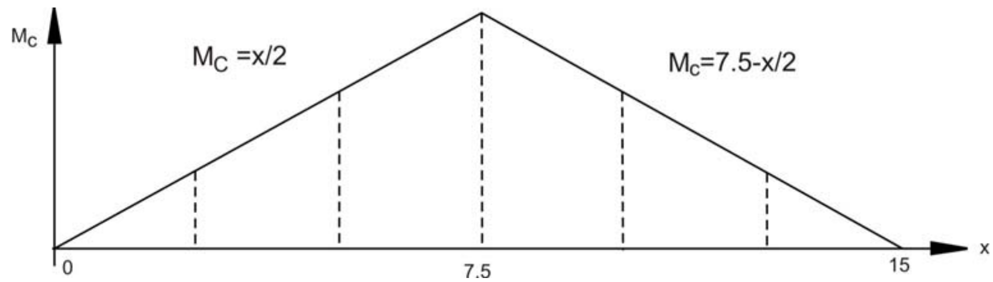


Figure 37.16: Influence line for moment at section C

Influence Line Equations:

There will be two influence line equations for the section before point C and after point C.

When the unit load is placed before point C then the moment equation for given Figure 37.17 can be given by

$\Sigma M_c = 0: \; M_c + 1(7.5 - x) - (1 - x/15)x7.5 = 0 \Rightarrow M_c = x/2$, where $0 \le x \le 7.5$

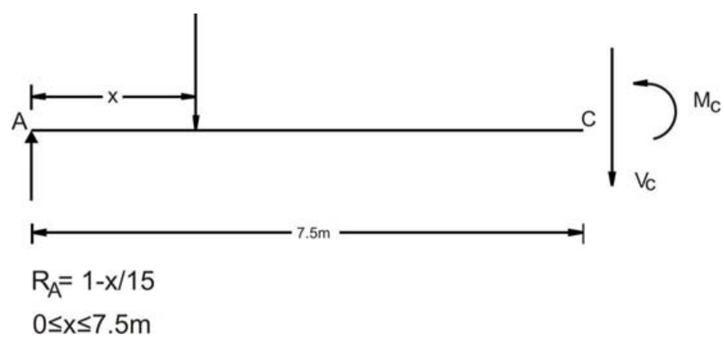


Figure 37.17: Free body diagram - a unit load before section

When the unit load is placed after point C then the moment equation for given Figure 37.18 can be given by

$\Sigma M_c = 0: \; M_c - (1 - x/15) \times 7.5 = 0 \Rightarrow M_c = 7.5 - x/2$, where $7.5 < x \le 15.0$

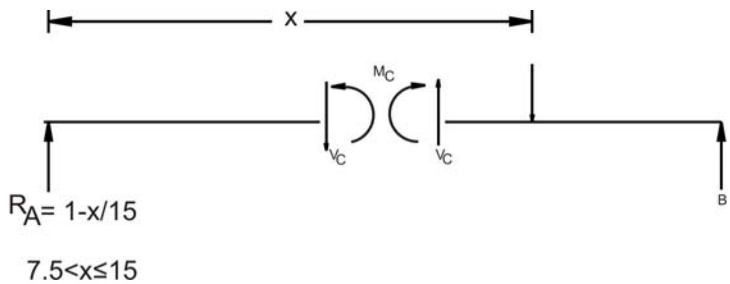


Figure 37.18: Free body diagram - a unit load before section

The equations are plotted in Figure 37.16.

Example 5:
Construct the influence line for the moment at point C of the beam shown in Figure 37.19.

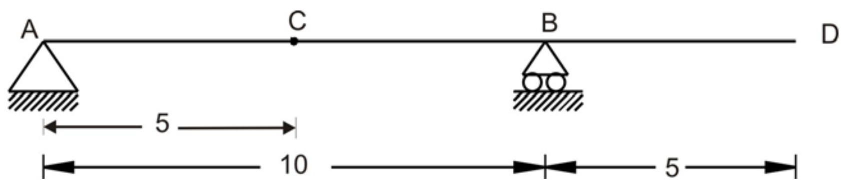


Figure 37.19: Overhang beam structure

Solution:
Tabulated values:
Place a unit load at different location between two supports and find the support reactions. Once the support reactions are computed, take a section at C and compute the moment. For example as shown in Figure 37.20, we place a unit load at 2.5 m from support A, then the support reaction at A will be 0.75 and support reaction B will be 0.25.

Figure 37.20: A unit load before section C

Taking section at C and computation of moment at C can be given by

$$\Sigma M_c = 0 : \ -M_c + R_B \times 5.0 - = 0 \Rightarrow -M_c + 0.25 \times 5.0 = 0 \Rightarrow M_c = 1.25$$

Similarly, compute the moment M_c for difference unit load position in the span. The values of Mc are tabulated as follows.

x	M_c
0	0
2.5	1.25
5.0	2.5
7.5	1.25
10	0
12.5	-1.25
15.0	-2.5

Graphical representation of influence line for M_c is shown in Figure 37.21.

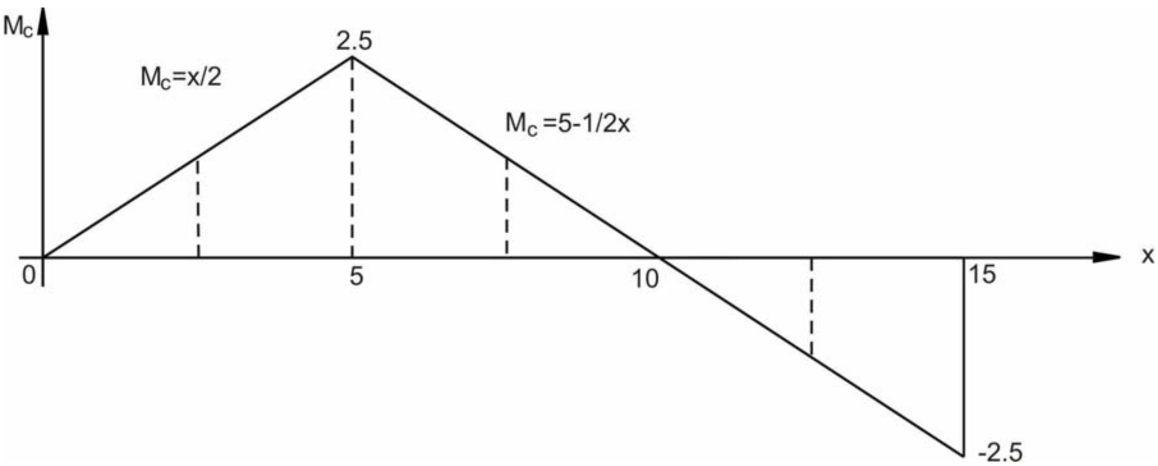


Figure 37.21: Influence line of moment at section C

Influence Line Equations:

There will be two influence line equations for the section before point C and after point C.

When a unit load is placed before point C then the moment equation for given Figure 37.22 can be given by

$\Sigma M_c = 0: \ M_c + 1(5.0 - x) - (1 - x/10)x5.0 = 0 \Rightarrow M_c = x/2$, where $0 \le x \le 5.0$

Figure 37.22: A unit load before section C

When a unit load is placed after point C then the moment equation for given Figure 37.23 can be given by

$\Sigma M_c = 0: \ M_c - (1 - x/10) \times 5.0 = 0 \Rightarrow M_c = 5 - x/2$, where $5 < x \le 15$

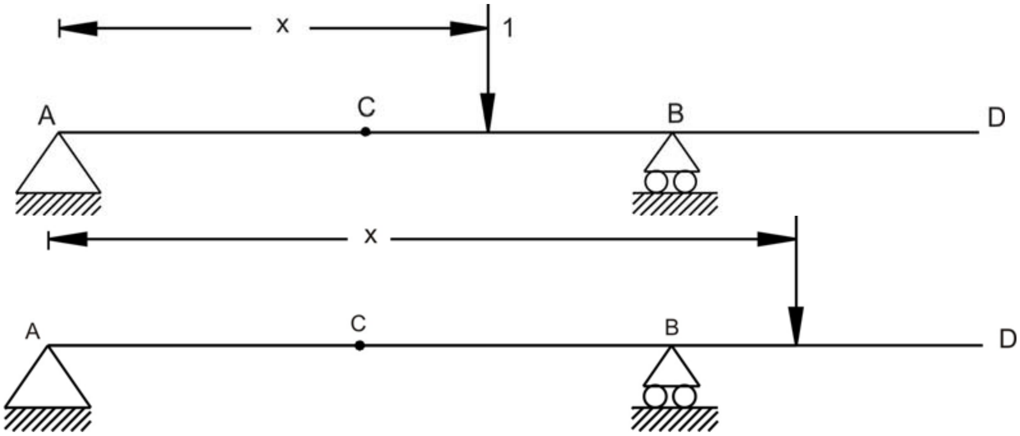


Figure 37.23: A unit load after section C

The equations are plotted in Figure 37.21.

37.5 Influence line for beam having point load and uniformly distributed load acting at the same time

Generally in beams/girders are main load carrying components in structural systems. Hence it is necessary to construct the influence line for the reaction, shear or moment at any specified point in beam to check for criticality. Let us assume that there are two kinds of load acting on the beam. They are concentrated load and uniformly distributed load (UDL).

37.5.1 Concentrated load

As shown in the Figure 37.24, let us say, point load P is moving on beam from A to B. Looking at the position, we need to find out what will be the influence line for reaction B for this load. Hence, to generalize our approach, like earlier examples, let us assume that unit load is moving from A to B and influence line for reaction A can be plotted as shown in Figure 37.25. Now we want to know, if load P is at the center of span then what will be the value of reaction A? From Figure 37.24, we can find that for the load position of P, influence line of unit load gives value of 0.5. Hence, reaction A will be 0.5xP. Similarly, for various load positions and load value, reactions A can be computed.

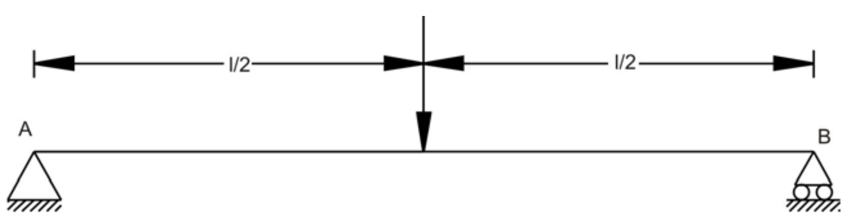


Figure 37.24: Beam structure

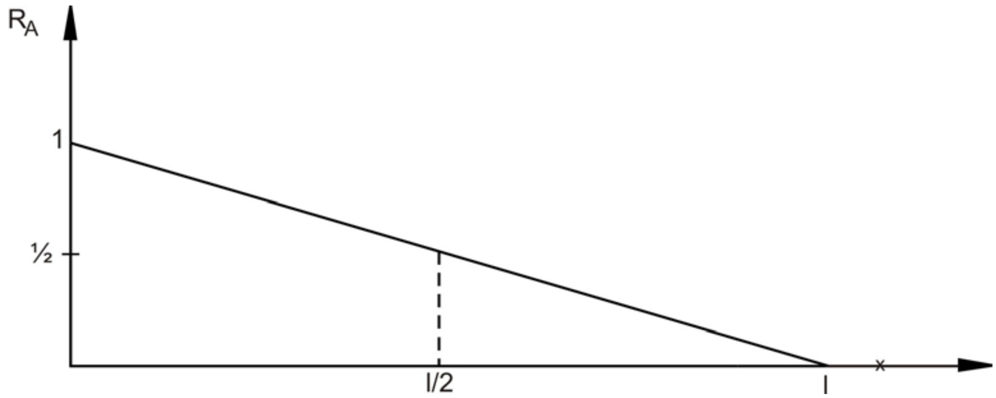


Figure 37.25: Influence line for support reaction at A

37.5.2 Uniformly Distributed Load

Beam is loaded with uniformly distributed load (UDL) and our objective is to find influence line for reaction A so that we can generalize the approach. For UDL of w on span, considering for segment of dx (Figure 37.26), the concentrated load dP can be given by w.dx acting at x. Let us assume that beam's influence line ordinate for some function (reaction, shear, moment) is y as shown in Figure 37.27. In that case, the value of function is given by (dP)(y) = (w.dx).y. For computation of the effect of all these concentrated loads, we have to integrate over the entire length of the beam. Hence, we can say that it will be ∫ w.y.dx = w ∫ y.dx. The term ∫ y.dx is equivalent to area under the influence line.

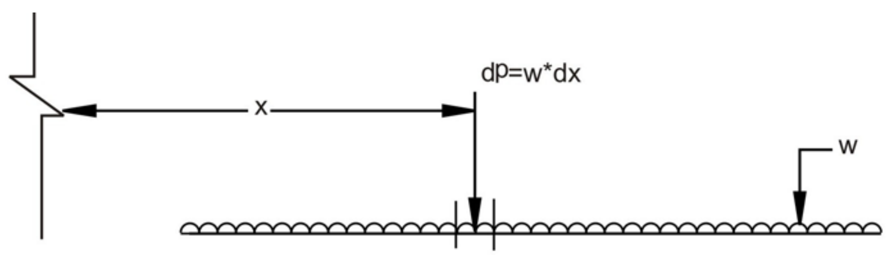


Figure 37.26: Uniformly distributed load on beam

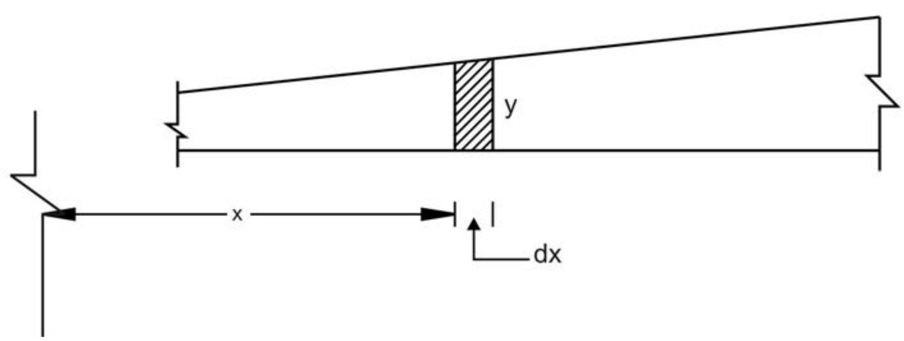


Figure 37.27: Segment of influence line diagram

For a given example of UDL on beam as shown in Figure 37.28, the influence line (Figure 37.29) for reaction A can be given by area covered by the influence line for unit load into UDL value. i.e. [0.5x (1)xl] w = 0.5 w.l.

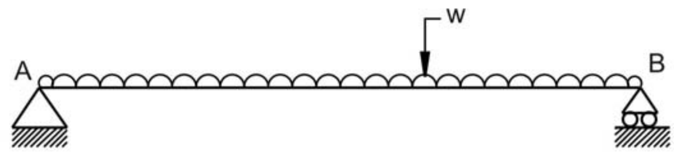


Figure 37.28: UDL on simply supported beam

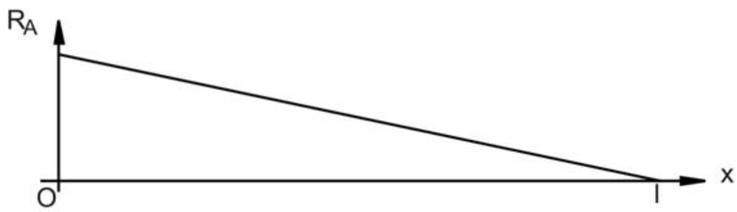


Figure 37.29: Influence line for support reaction at A.

37.6 Numerical Example

Find the maximum positive live shear at point C when the beam (Figure 37.30) is loaded with a concentrated moving load of 10 kN and UDL of 5 kN/m.

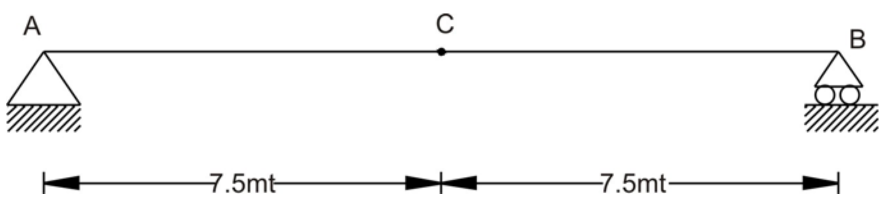


Figure 37.30: Simply supported beam

Solution:

As discussed earlier for unit load moving on beam from A to B, the influence line for the shear at C can be given by following Figure 37.31.

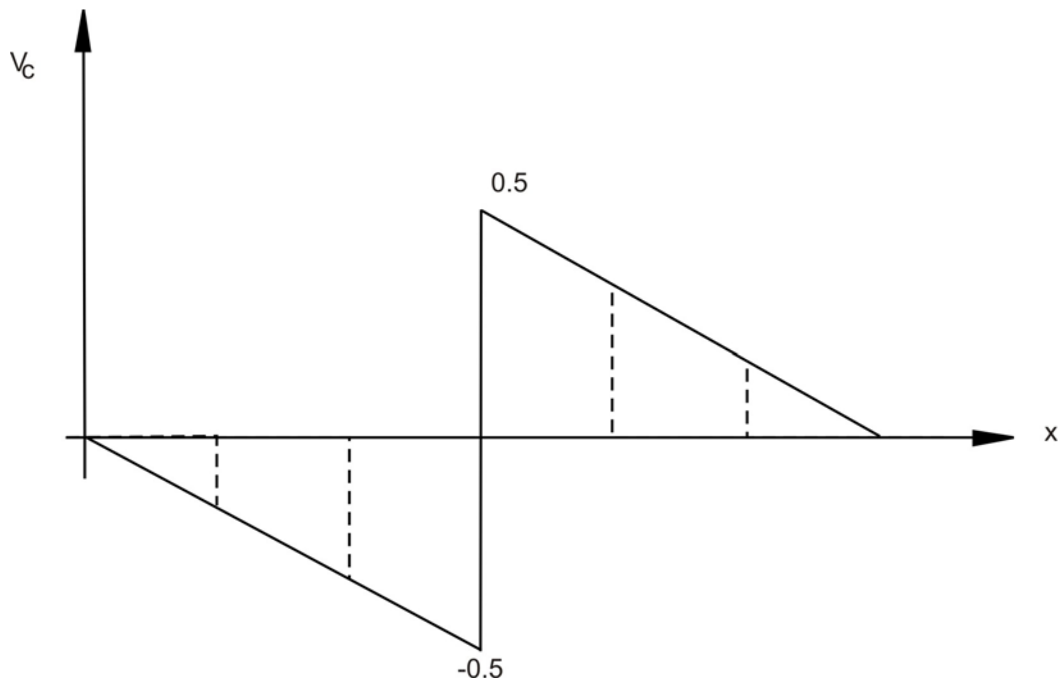


Figure 37.31: Influence line for shear at section C.

Concentrated load: As shown in Figure 37.31, the maximum live shear force at C will be when the concentrated load 10 kN is located just before C or just after C. Our aim is to find positive live shear and hence, we will put 10 kN just after C. In that case,

$$V_c = 0.5 \times 10 = 5 \text{ kN.}$$

UDL: As shown in Figure 37.31, the maximum positive live shear force at C will be when the UDL 5 kN/m is acting between x = 7.5 and x = 15.

$$V_c = [\, 0.5 \times (15 - 7.5)\,(0.5)] \times 5 = 9.375$$

Total maximum Shear at C:

$$(V_c)\text{ max} = 5 + 9.375 = 14.375.$$

Finally the loading positions for maximum shear at C will be as shown in Figure 37.32. For this beam one can easily compute shear at C using statics.

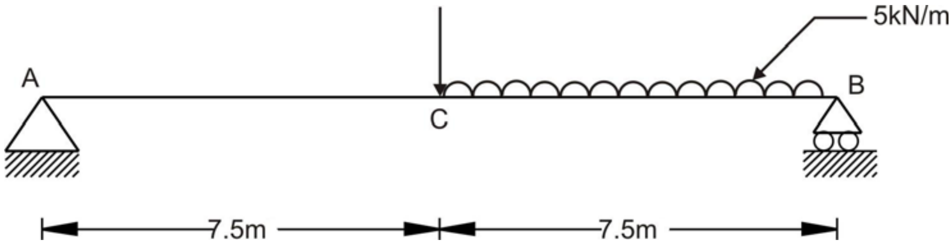


Figure 37.32: Simply supported beam

37.7 Closing Remarks

In this lesson we have studied the need for influence line and their importance. Further we studied the available various influence line definitions. Finally we studied the influence line construction using tabulated values and influence line equation. The understanding about the simple approach was studied with the help of many numerical examples.

Suggested Text Books for Further Reading

- Armenakas, A. E. (1988). *Classical Structural Analysis – A Modern Approach*, McGraw-Hill Book Company, NY, ISBN 0-07-100120-4

- Hibbeler, R. C. (2002). *Structural Analysis*, Pearson Education (Singapore) Pte. Ltd., Delhi, ISBN 81-7808-750-2

- Junarkar, S. B. and Shah, H. J. (1999). *Mechanics of Structures – Vol. II*, Charotar Publishing House, Anand.

- Leet, K. M. and Uang, C-M. (2003). *Fundamentals of Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-058208-4

- Negi, L. S. and Jangid, R.S. (2003). *Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-462304-4

- Norris, C. H., Wilbur, J. B. and Utku, S. (1991). *Elementary Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-058116-9

Module 7

Influence Lines

Lesson 38

Influence Lines for Beams

Instructional Objectives:

The objectives of this lesson are as follows:

- How to draw qualitative influence lines?
- Understand the behaviour of the beam under rolling loads
- Construction of influence line when the beam is loaded with uniformly distributed load having shorter or longer length than the span of the beam.

38.1 Müller Breslau Principle for Qualitative Influence Lines

In 1886, Heinrich Müller Breslau proposed a technique to draw influence lines quickly. The Müller Breslau Principle states that the ordinate value of an influence line for any function on any structure is proportional to the ordinates of the deflected shape that is obtained by removing the restraint corresponding to the function from the structure and introducing a force that causes a unit displacement in the positive direction.

Let us say, our objective is to obtain the influence line for the support reaction at A for the beam shown in Figure 38.1.

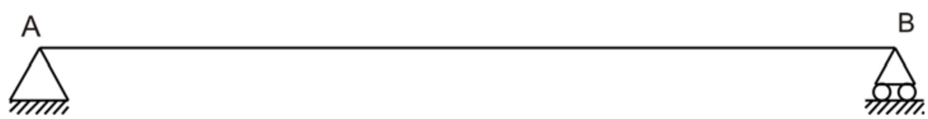


Figure 38.1: Simply supported beam

First of all remove the support corresponding to the reaction and apply a force (Figure 38.2) in the positive direction that will cause a unit displacement in the direction of R_A. The resulting deflected shape will be proportional to the true influence line (Figure 38.3) for the support reaction at A.

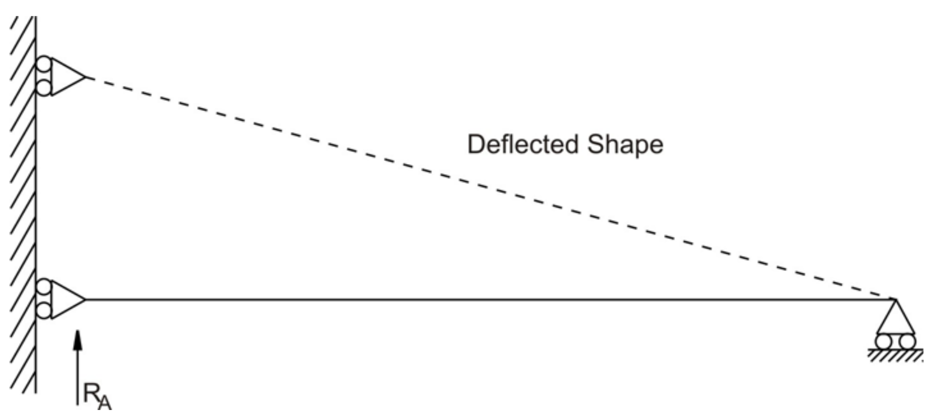


Figure 38.2: Deflected shape of beam

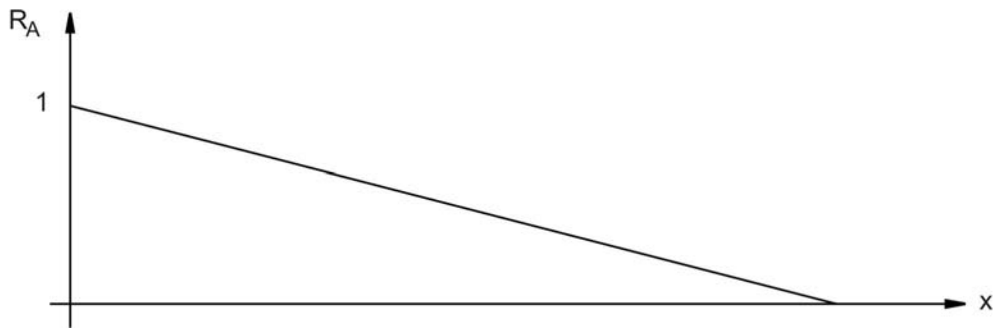


Figure 38.3: Influence line for support reaction A

The deflected shape due to a unit displacement at A is shown in Figure 38.2 and matches with the actual influence line shape as shown in Figure 38.3. Note that the deflected shape is linear, i.e., the beam rotates as a rigid body without any curvature. This is true only for statically determinate systems.

Similarly some other examples are given below.

Here we are interested to draw the qualitative influence line for shear at section C of overhang beam as shown in Figure 38.4.

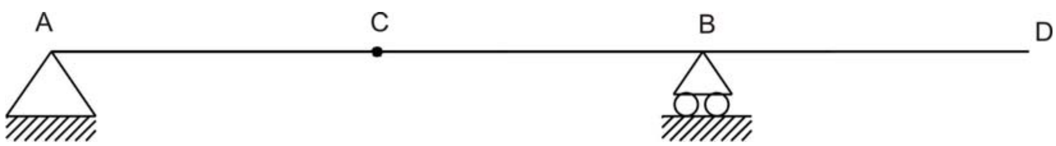


Figure 38.4: Overhang beam

As discussed earlier, introduce a roller at section C so that it gives freedom to the beam in vertical direction as shown in Figure 38.5.

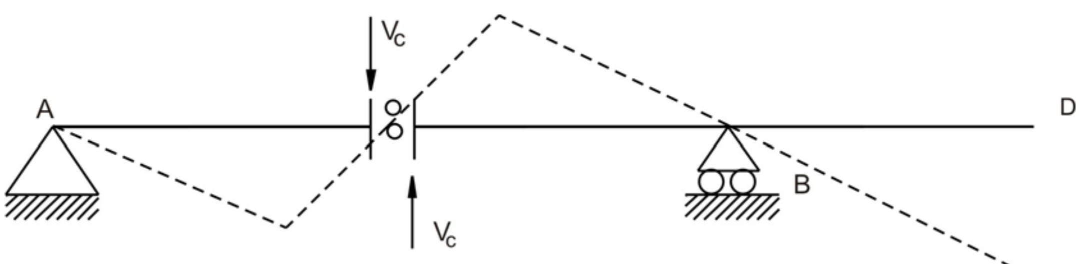


Figure 38.5: Deflected shape of beam

Now apply a force in the positive direction that will cause a unit displacement in the direction of V_C. The resultant deflected shape is shown in Figure 38.5. Again, note that the deflected shape is linear. Figure 38.6 shows the actual influence, which matches with the qualitative influence.

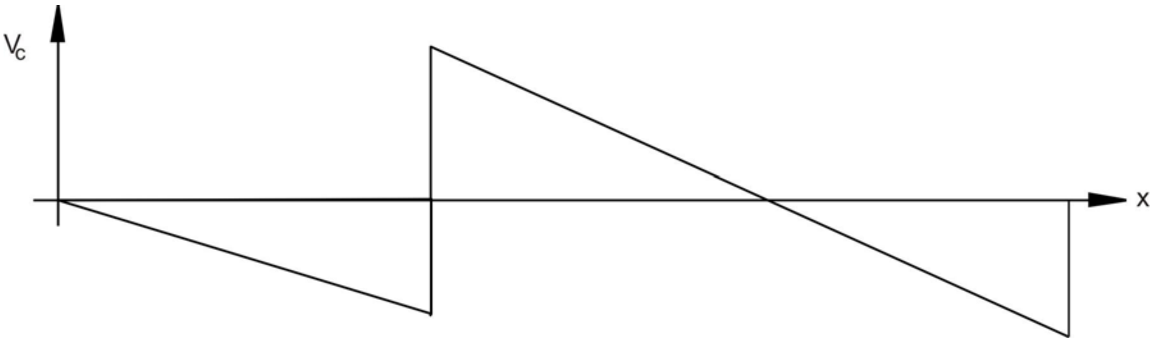


Figure 38.6: Influence line for shear at section C

In this second example, we are interested to draw a qualitative influence line for moment at C for the beam as shown in Figure 38.7.

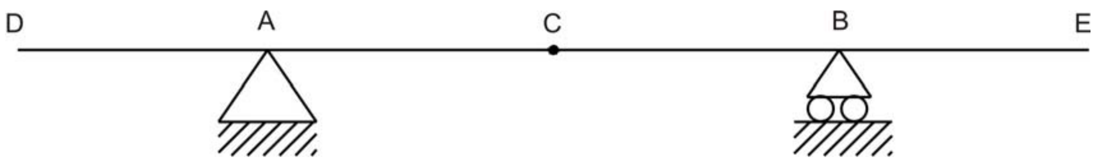


Figure 38.7: Beam structure

In this example, being our objective to construct influence line for moment, we will introduce hinge at C and that will only permit rotation at C. Now apply moment in the positive direction that will cause a unit rotation in the direction of M_c. The deflected shape due to a unit rotation at C is shown in Figure 38.8 and matches with the actual shape of the influence line as shown in Figure 38.9.

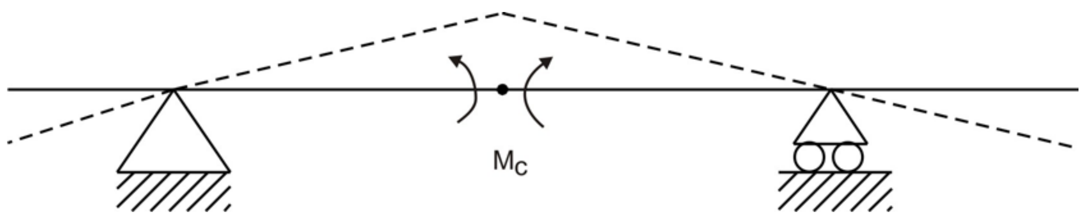


Figure 38.8: Deflected shape of beam

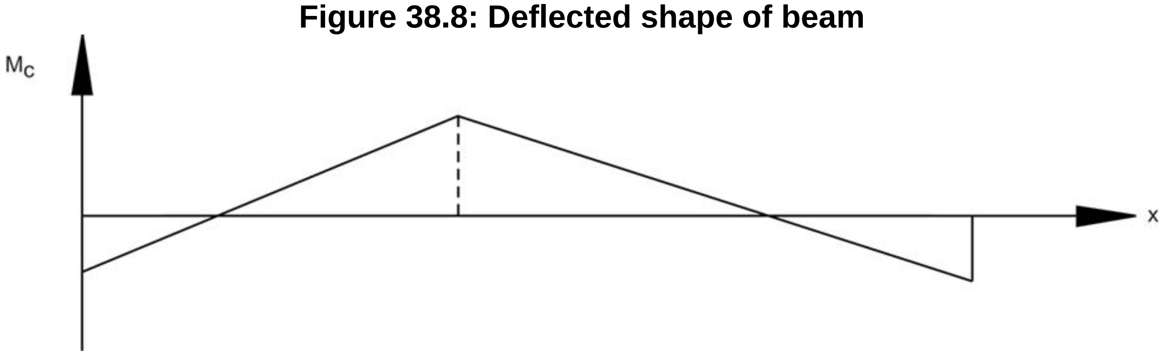


Figure 38.9: Influence line for moment at section C

38.2. Maximum shear in beam supporting UDLs

If UDL is rolling on the beam from one end to other end then there are two possibilities. Either Uniformly distributed load is longer than the span or uniformly distributed load is shorter than the span. Depending upon the length of the load and span, the maximum shear in beam supporting UDL will change. Following section will discuss about these two cases. It should be noted that for maximum values of shear, maximum areas should be loaded.

38.2.1 UDL longer than the span

Let us assume that the simply supported beam as shown in Figure 38.10 is loaded with UDL of w moving from left to right where the length of the load is longer than the span. The influence lines for reactions R_A, R_B and shear at section C located at x from support A will be as shown in Figure 38.11, 38.12 and 38.13 respectively. UDL of intensity w per unit for the shear at supports A and B will be given by

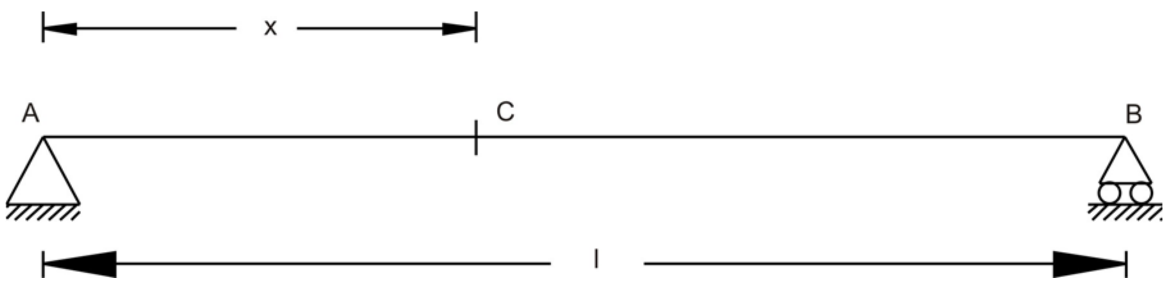


Figure 38.10: Beam Structure

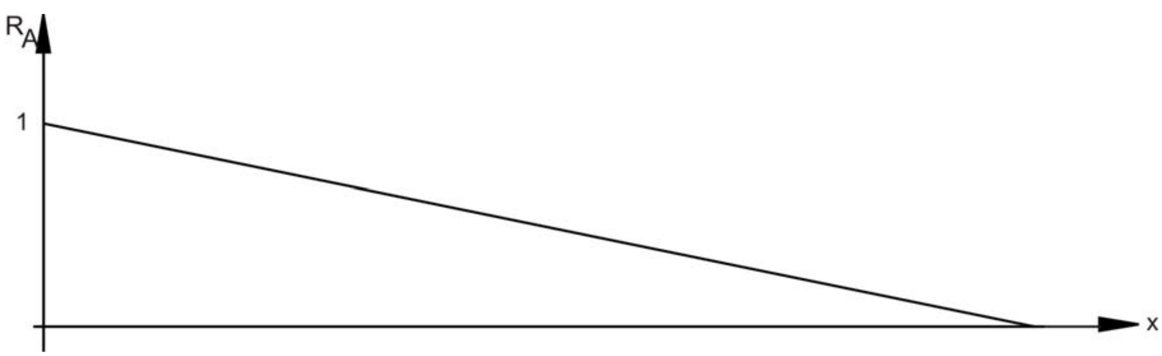


Figure 38.11: Influence line for support reaction at A

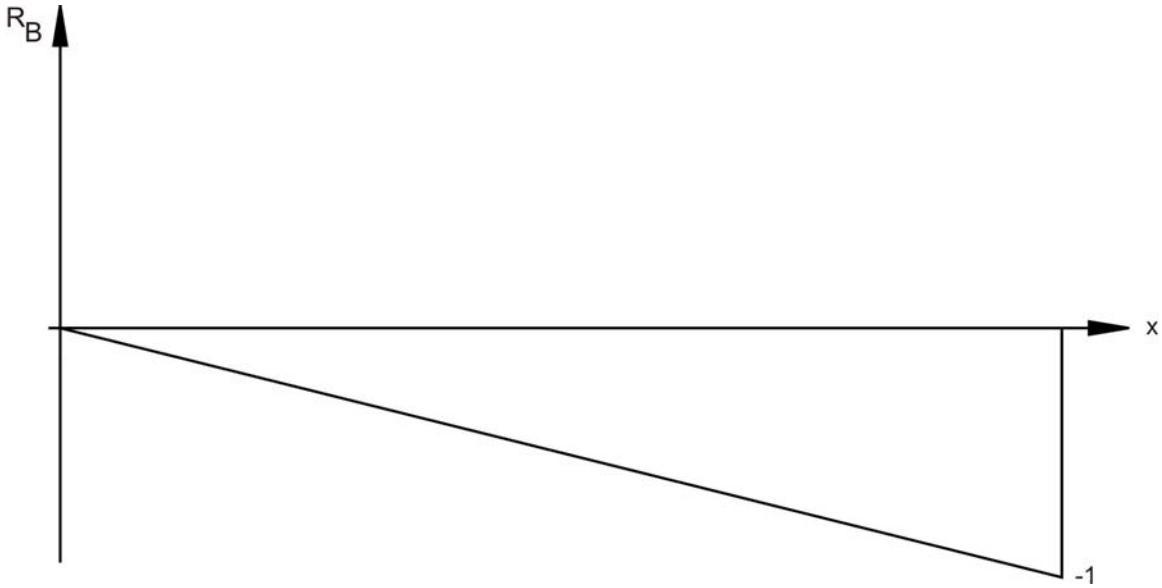


Figure 38.12: Influence line for support reaction at B

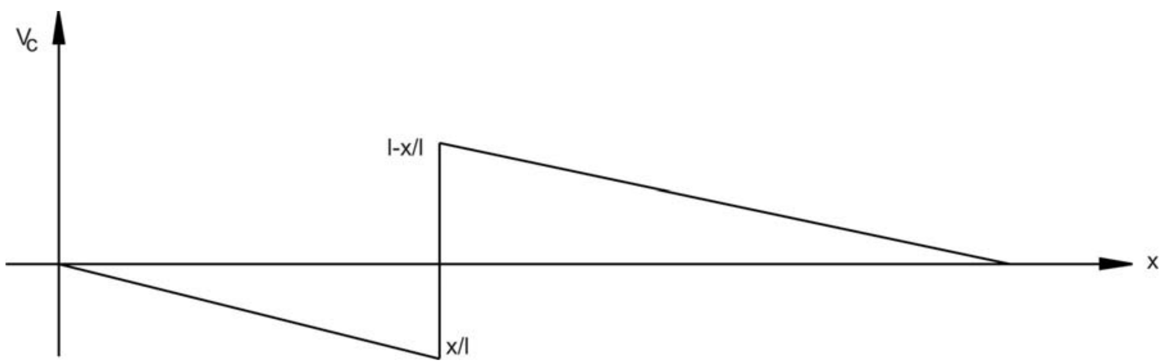


Figure 38.13: Influence line for shear at section C

$$R_A = w \times \frac{1}{2} \times l \times 1 = \frac{wl}{2}$$

$$R_B = -w \times \frac{1}{2} \times l \times 1 = \frac{-wl}{2}$$

Suppose we are interested to know shear at given section at C. As shown in Figure 38.13, maximum negative shear can be achieved when the head of the load is at the section C. And maximum positive shear can be obtained when the tail of the load is at the section C. As discussed earlier the shear force is computed by intensity of the load multiplied by the area of influence line diagram covered by load. Hence, maximum negative shear is given by

$$= -\frac{1}{2} \times x \times \frac{x}{l} \times w = -\frac{wx^2}{2l}$$

and maximum positive shear is given by

$$= \frac{1}{2} \times \left(\frac{l-x}{l} \right) \times (l-x) \times w = -\frac{w(l-x)^2}{2l}$$

38.2.2 UDL shorter than the span

When the length of UDL is shorter than the span, then as discussed earlier, maximum negative shear can be achieved when the head of the load is at the section. And maximum positive shear can be obtained when the tail of the load is at the section. As discussed earlier the shear force is computed by the load intensity multiplied by the area of influence line diagram covered by load. The example is demonstrated in previous lesson.

38.3 Maximum bending moment at sections in beams supporting UDLs.

Like the previous section discussion, the maximum moment at sections in beam supporting UDLs can either be due to UDL longer than the span or due to ULD shorter than the span. Following paragraph will explain about computation of moment in these two cases.

38.3.1 UDL longer than the span

Let us assume the UDL longer than the span is traveling from left end to right hand for the beam as shown in Figure 38.14. We are interested to know maximum moment at C located at x from the support A. As discussed earlier, the maximum bending moment is given by the load intensity multiplied by the area of influence line (Figure 38.15) covered. In the present case the load will cover the completed span and hence the moment at section C can be given by

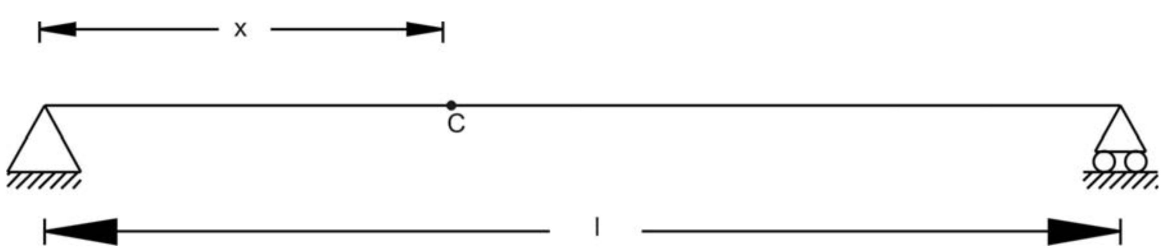


Figure 38.14: Beam structure

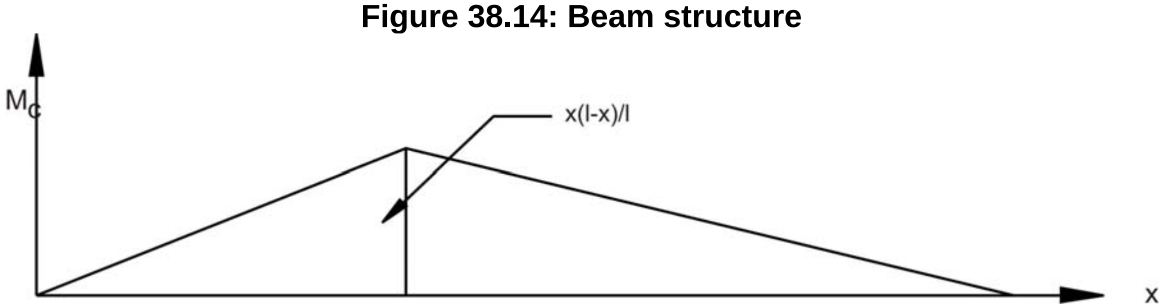


Figure 38.15: Influence line for moment at section C

$$w \times \frac{1}{2} \times l \times \frac{x(l-x)}{l} = -\frac{wx(l-x)}{2}$$

Suppose the section C is at mid span, then maximum moment is given by

$$\frac{w \times \frac{l}{2} \times \frac{l}{2}}{2} = \frac{wl^2}{8}$$

38.3.2 UDL shorter than the span

As shown in Figure 38.16, let us assume that the UDL length y is smaller than the span of the beam AB. We are interested to find maximum bending moment at section C located at x from support A. Let say that the mid point of UDL is located at D as shown in Figure 38.16 at distance of z from support A. Take moment with reference to A and it will be zero.

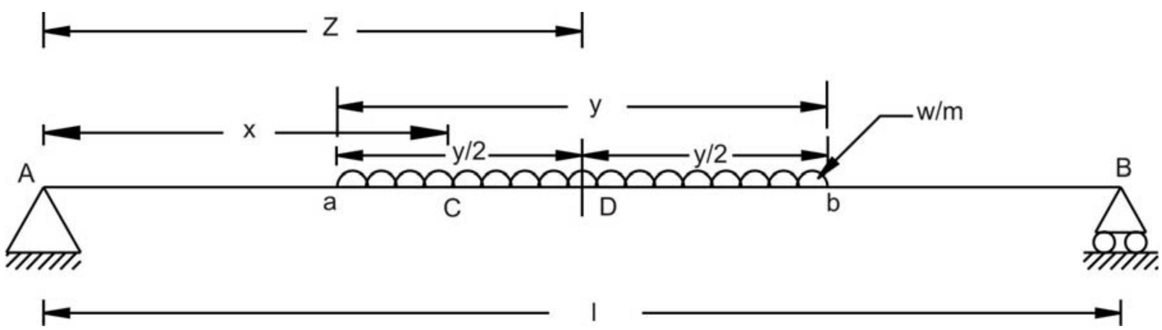


Figure 38.16: Beam loaded with UDL shorter in length than span

Hence, the reaction at B is given by

$$R_B = w \times y \times \frac{z}{l} = -\frac{wx(l-x)}{2}$$

And moment at C will be

$$M_C = R_B(l-x) - \frac{w}{2}(z + \frac{y}{2} - x)^2$$

Substituting value of reaction B in above equation, we can obtain

$$M_C = \frac{wyz}{l}(l-x) - \frac{w}{2}(z + \frac{y}{2} - x)^2$$

To compute maximum value of moment at C, we need to differentiate above given equation with reference to z and equal to zero.

$$\frac{dM_c}{dz} = \frac{wy}{l}(l - x) - w(z + \frac{y}{2} - x) = 0$$

Therefore,

$$\frac{y}{l}(l - x) = (z + \frac{y}{2} - x)$$

Using geometric expression, we can state that

$$\frac{ab}{AB} = \frac{Cb}{CB}$$

$$\therefore \frac{CB}{Cb} = \frac{AB}{ab} = \frac{AB - CB}{ab - Cb} = \frac{AC}{aC}$$

$$\therefore \frac{aC}{Cb} = \frac{AC}{CB}$$

The expression states that for the UDL shorter than span, the load should be placed in a way so that the section divides it in the same proportion as it divides the span. In that case, the moment calculated at the section will give maximum moment value.

38.4 Closing Remarks

In this lesson we studied how to draw qualitative influence line for shear and moment using Müller Breslau Principle. Further we studied how to draw the influence lines for shear and moment when the beam is loaded with UDL. Here, we studied the two cases where the UDL length is shorter or longer than span. In the next lesson we will study about two or more than two concentrated loads moving on the beam.

Suggested Text Books for Further Reading

- Armenakas, A. E. (1988). *Classical Structural Analysis – A Modern Approach*, McGraw-Hill Book Company, NY, ISBN 0-07-100120-4

- Hibbeler, R. C. (2002). *Structural Analysis*, Pearson Education (Singapore) Pte. Ltd., Delhi, ISBN 81-7808-750-2

- Junarkar, S. B. and Shah, H. J. (1999). *Mechanics of Structures – Vol. II*, Charotar Publishing House, Anand.

- Leet, K. M. and Uang, C-M. (2003). *Fundamentals of Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-058208-4

- Negi, L. S. and Jangid, R.S. (2003). *Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-462304-4

- Norris, C. H., Wilbur, J. B. and Utku, S. (1991). *Elementary Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-058116-9

Module 7

Influence Lines

Lesson 39

Influence Lines for Beams (Contd.)

Instructional Objectives:

The objectives of the present lesson are as follows.

- Construction of influence line for maximum shear at sections in a beam supporting two concentrated loads
- Construction of influence line for maximum moment at sections in a beam supporting two concentrated loads
- Construction of influence line for maximum end shear in a beam supporting a series of moving concentrated loads
- Construction of influence line for maximum shear at a section in a beam supporting a series of moving concentrated loads
- Construction of influence line for maximum moment at a section in a beam supporting a series of moving concentrated loads
- Construction of influence line for absolute maximum moment in s beam supporting a series of moving concentrated loads
- Understanding about the envelopes of maximum influence line values

39.1 Introduction

In the previous lessons, we have studied about construction of influence line for the either single concentrated load or uniformly distributed loads. In the present lesson, we will study in depth about the beams, which are loaded with a series of two or more then two concentrated loads.

39.2 Maximum shear at sections in a beam supporting two concentrated loads

Let us assume that instead of one single point load, there are two point loads P_1 and P_2 spaced at y moving from left to right on the beam as shown in Figure 39.1. We are interested to find maximum shear force in the beam at given section C. In the present case, we assume that $P_2<P_1$.

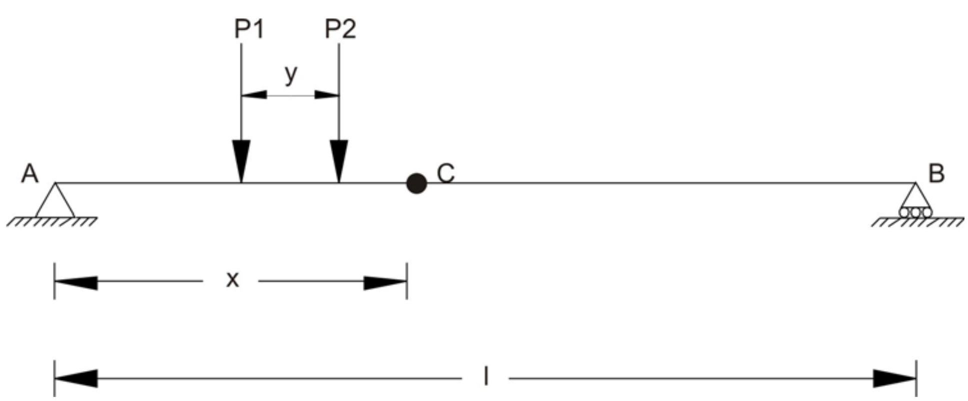


Figure 39.1: Beam loaded with two concentrated point loads

Now there are three possibilities due to load spacing. They are: $x<y$, $x=y$ and $x>y$.

Case 1: $x<y$

This case indicates that when load P_2 will be between A and C then load P_1 will not be on the beam. In that case, maximum negative shear at section C can be given by

$$V_C = -P_2 \frac{x}{l}$$

and maximum positive shear at section C will be

$$V_C = P_2 \frac{(l-x)}{l}$$

Case 2: $x=y$

In this case, load P_1 will be on support A and P_2 will be on section C. Maximum negative shear can be given by

$$V_C = -P_2 \frac{x}{l}$$

and maximum positive shear at section C will be

$$V_C = P_2 \frac{(l-x)}{l}$$

Case 3: $x>y$

With reference to Figure 39.2, maximum negative shear force can be obtained when load P_2 will be on section C. The maximum negative shear force is expressed as:

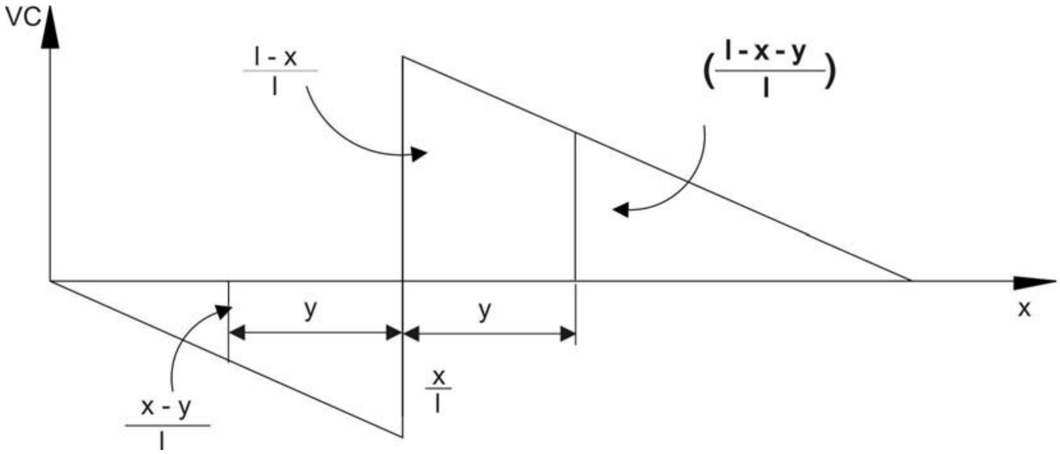


Figure 39.2: Influence line for shear at section C

$$V_C^{\,1} = -P_2\frac{x}{l} - P_1\left(\frac{x-y}{l}\right)$$

And with reference to Figure 39.2, maximum positive shear force can be obtained when load P_1 will be on section C. The maximum positive shear force is expressed as:

$$V_C^{\,2} = -P_1\frac{x}{l} + P_2\left(\frac{l-x-y}{l}\right)$$

From above discussed two values of shear force at section, select the maximum negative shear value.

39.3 Maximum moment at sections in a beam supporting two concentrated loads

Let us assume that instead of one single point load, there are two point loads P_1 and P_2 spaced at y moving left to right on the beam as shown in Figure 39.3. We are interested to find maximum moment in the beam at given section C.

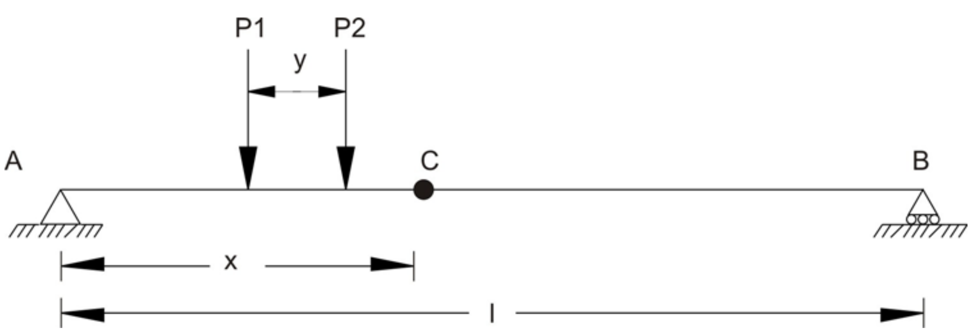


Figure 39.3: Beam loaded with two concentrated loads

With reference to Figure 39.4, moment can be obtained when load P_2 will be on section C. The moment for this case is expressed as:

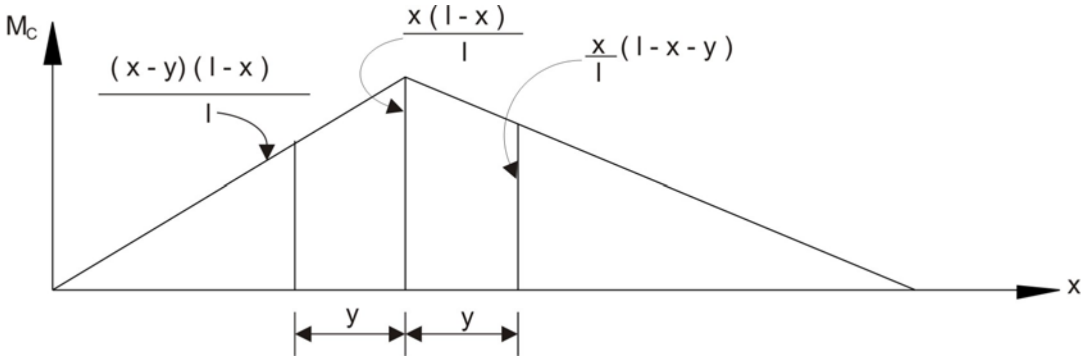


Figure 39.4: Influence line for moment at section C

$$M_C^{\,1} = P_1(x-y)\left(\frac{l-x}{l}\right) + P_2 x\left(\frac{l-x}{l}\right)$$

With reference to Figure 39.4, moment can be obtained when load P_1 will be on section C. The moment for this case is expressed as:

$$M_C^{\,2} = P_1 x\left(\frac{l-x}{l}\right) + P_2 x\left(\frac{l-x-y}{l}\right)$$

From above two cases, maximum value of moment should be considered for maximum moment at section C when two point loads are moving from left end to right end of the beam.

39.4 Maximum end shear in a beam supporting a series of moving concentrated loads

In real life situation, usually there are more than two point loads, which will be moving on bridges. Hence, in this case, our aim is to learn, how to find end shear in beam supporting a series of moving concentrated loads. Let us assume that as shown in Figure 39.5, four concentrated loads are moving from right end to left end on beam AB. The spacing of the concentrated load is given in Figure 39.5.

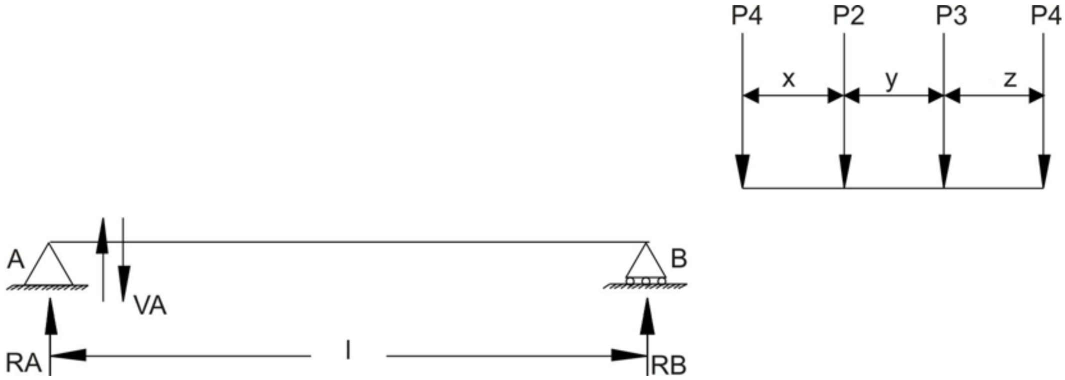


Figure 39.5: Beam loaded with a series of loads

As shown in figure, we are interested in end shear at A. We need to draw influence line for the support reaction A and a point away from the support at infinitesimal distance on the span for the shear V_A. The influence lines for these cases are shown in Figure 39.6 and 39.7.

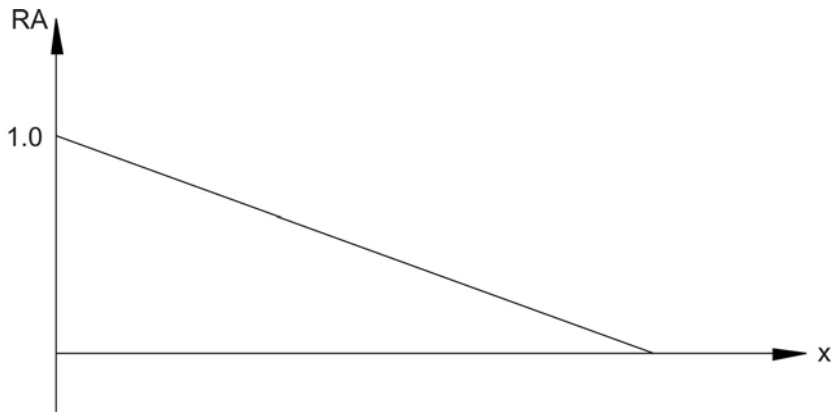


Figure 39.6: Influence line for reaction at support A

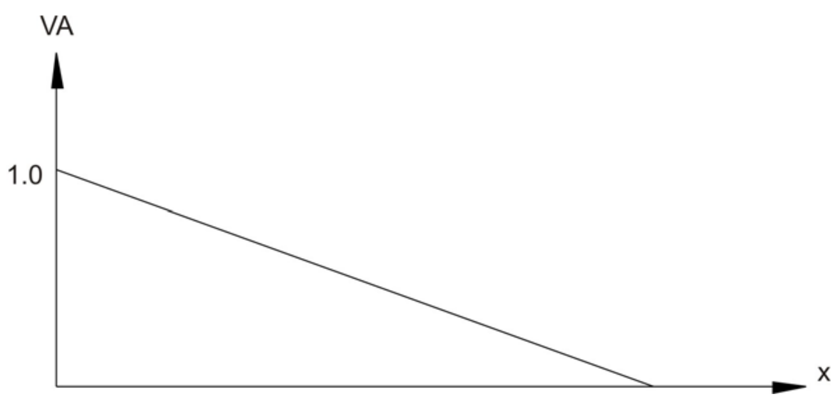


Figure 39.7: Influence line for shear near to support A.

When loads are moving from B to A then as they move closer to A, the shear value will increase. When load passes the support, there could be increase or decrease in shear value depending upon the next point load approaching support A. Using this simple logical approach, we will find out the change in shear value near support and monitor this change from positive value to negative value. Here for the present case let us assume that ΣP is summation of the loads remaining on the beam. When load P_1 crosses support A, then P_2 will approach A. In that case, change in shear will be expressed as

$$dV = \frac{\sum Px}{l} - P_1$$

When load P_2 crosses support A, then P_3 will approach A. In that case change in shear will be expressed as

$$dV = \frac{\sum Py}{l} - P_2$$

In case if *dV* is positive then shear at A has increased and if *dV* is negative, then shear at A has decreased. Therefore, first load, which crosses and induces negative changes in shear, should be placed on support A.

39.4.1 Numerical Example

Compute maximum end shear for the given beam loaded with moving loads as shown in Figure 39.8.

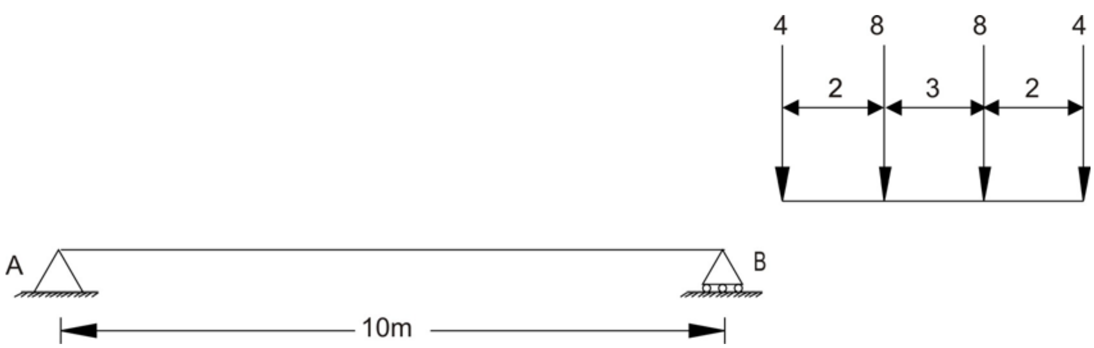


Figure 39.8: Beam loaded with a series of four concentrated loads

When first load of 4 kN crosses support A and second load 8 kN is approaching support A, then change in shear can be given by

$$dV = \frac{\sum (8+8+4)2}{10} - 4 = 0$$

When second load of 8 kN crosses support A and third load 8 kN is approaching support A, then change in shear can be given by

$$dV = \frac{\sum (8+4)3}{10} - 8 = -3.8$$

Hence, as discussed earlier, the second load 8 kN has to be placed on support A to find out maximum end shear (refer Figure 39.9).

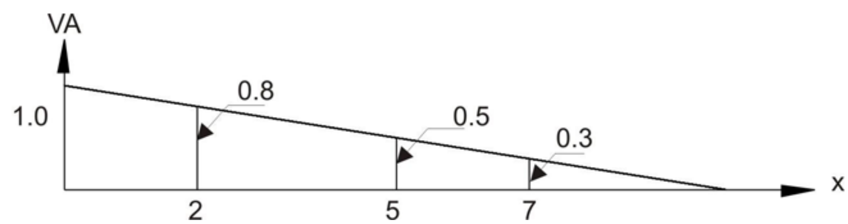


Figure 39.9: Influence line for shear at A.

$$V_A = 4\times1 + 8\times0.8 + 8\times0.5 + 4\times0.3 = 15.6kN$$

39.5 Maximum shear at a section in a beam supporting a series of moving concentrated loads

In this section we will discuss about the case, when a series of concentrated loads are moving on beam and we are interested to find maximum shear at a section. Let us assume that series of loads are moving from right end to left end as shown in Figure. 39.10.

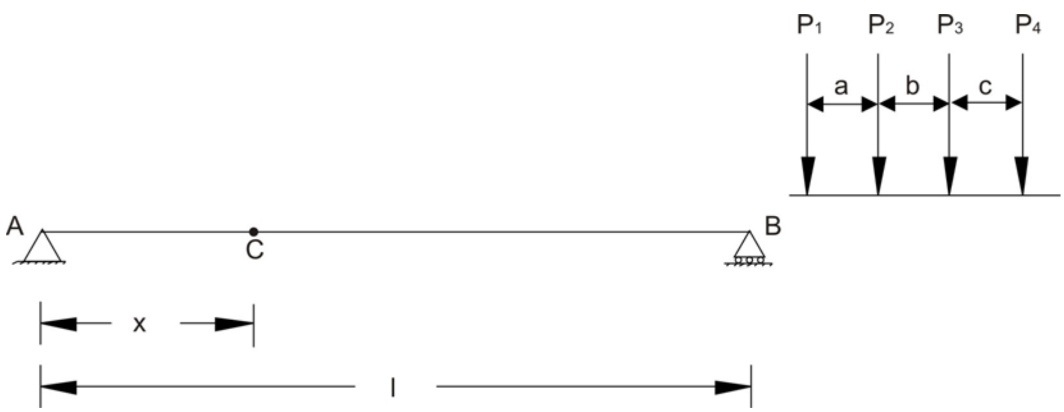


Figure 39.10: Beam loaded with a series of loads

The influence line for shear at the section is shown in Figure 39.11.

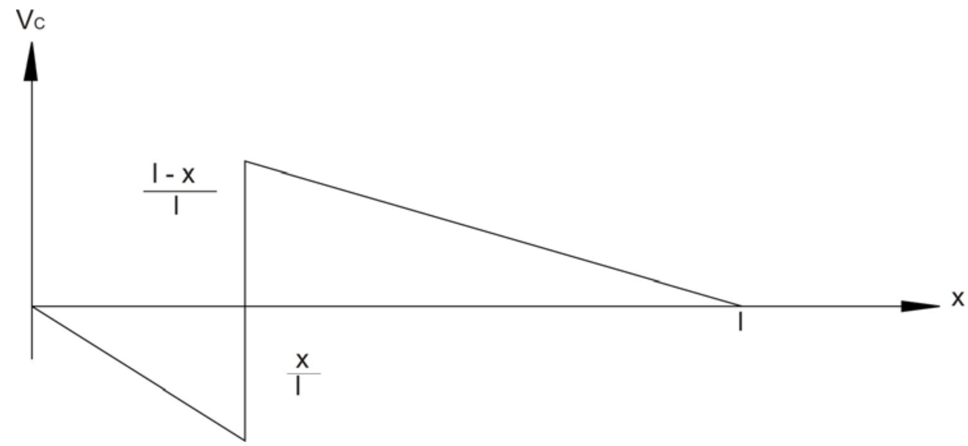


Figure 39.11: Influence line for shear at section C

Monitor the sign of change in shear at the section from positive to negative and apply the concept discussed in earlier section. Following numerical example explains the same.

39.5.1 Numerical Example

The beam is loaded with concentrated loads, which are moving from right to left as shown in Figure 39.12. Compute the maximum shear at the section C.

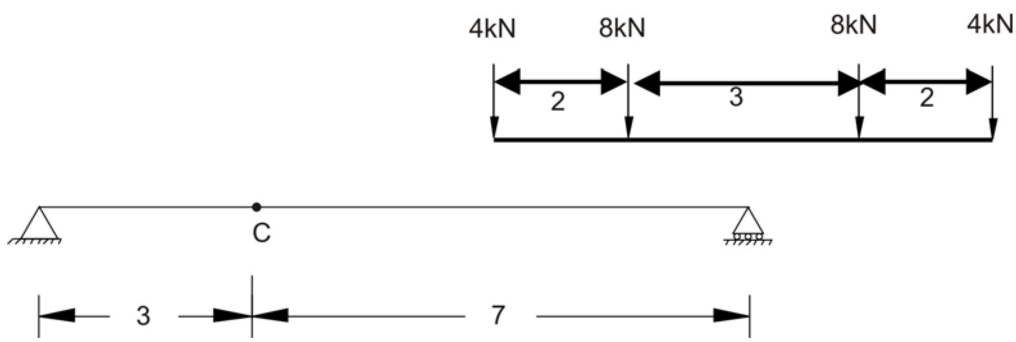


Figure 39.12: Beam loaded with a series of loads

The influence line at section C is shown in following Figure 39.13.

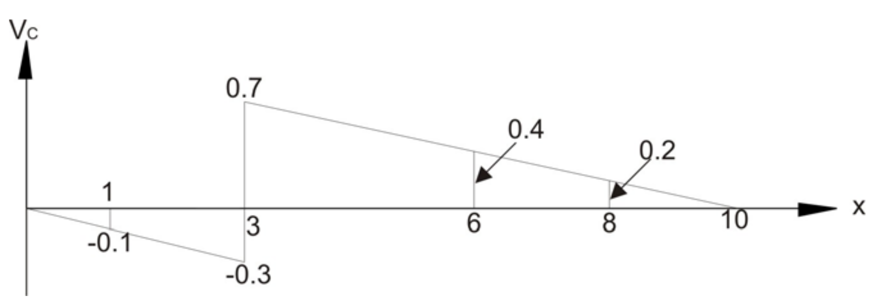


Figure 39.13: Influence line for shear at section C

When first load 4kN crosses section C and second load approaches section C then change in shear at a section can be given by

$$dV = \frac{20 \times 2}{10} - 4 = 0$$

When second load 8 kN crosses section C and third load approaches section C then change in shear at section can be given by

$$dV = \frac{12 \times 3}{10} - 8 = -4.4$$

Hence place the second concentrated load at the section and computed shear at a section is given by

$$V_C == 0.1 \times 4 + 0.7 \times 8 + 0.4 \times 8 + 0.2 \times 4 = 9.2 kN$$

39.6 Maximum Moment at a section in a beam supporting a series of moving concentrated loads

The approach that we discussed earlier can be applied in the present context also to determine the maximum positive moment for the beam supporting a series of moving concentrated loads. The change in moment for a load P_1 that moves from position x_1 to x_2 over a beam can be obtained by multiplying P_1 by the change in ordinate of the influence line i.e. $(y_2 - y_1)$. Let us assume the slope of the influence line (Figure 39.14) is S, then $(y_2 - y_1) = S(x_2 - x_1)$.

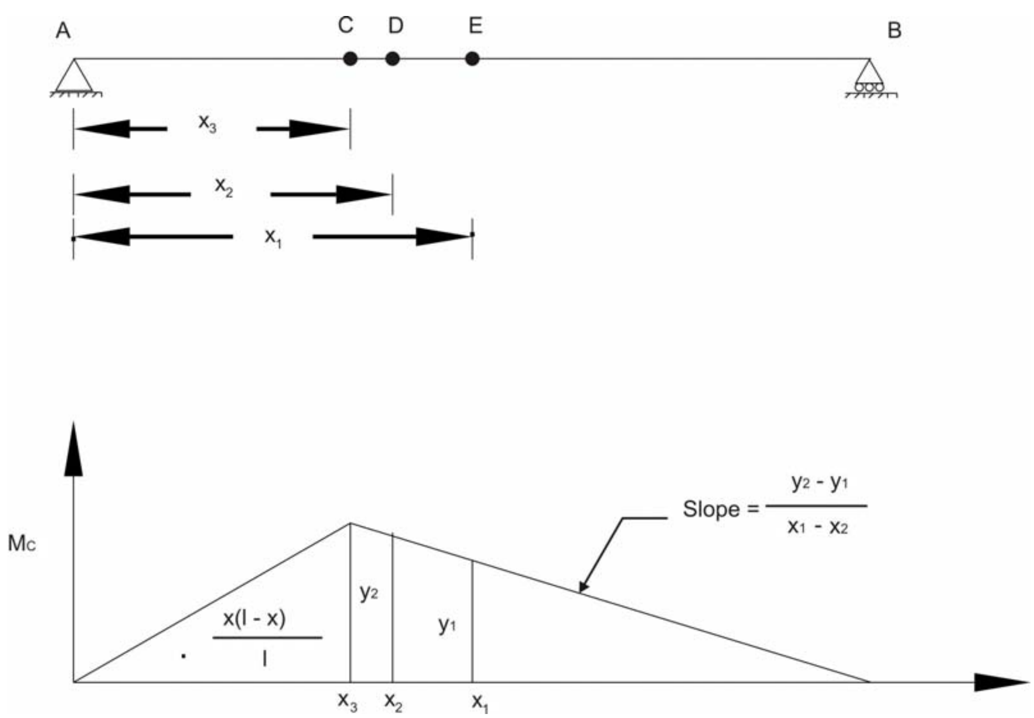


Figure 39.14: Beam and Influence line for moment at section C

Hence change in moment can be given by

$$dM = P_1 S(x_2 - x_1)$$

Let us consider the numerical example for better understanding of the developed concept.

39.6.1 Numerical Example

The beam is loaded with concentrated loads, which are moving from right to left as shown in Figure 39.15. Compute the maximum moment at the section C.

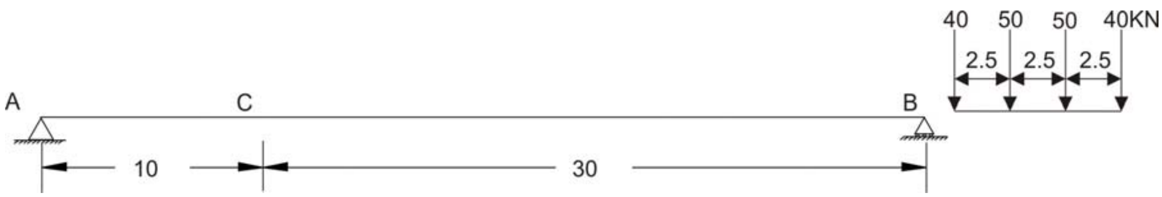


Figure 39.15: Beam loaded with a series of loads

The influence line for moment at C is shown in Figure 39.16.

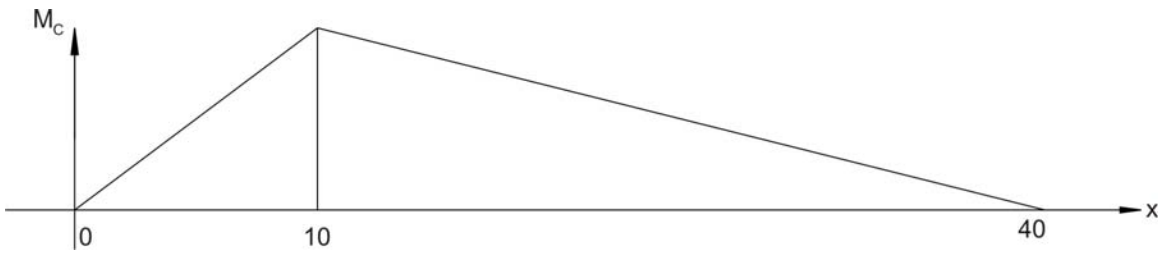


Figure 39.16: Beam loaded with a series of loads

If we place each of the four-concentrated loads at the peak of influence line, then we can get the largest influence from each force. All the four cases are shown in Figures 39.17-20.

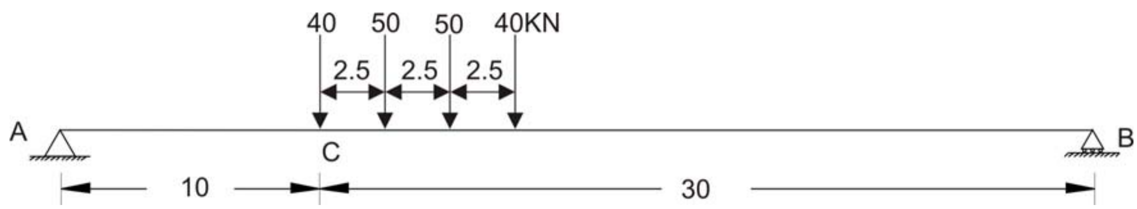


Figure 39.17: Beam loaded with a series of loads – First load at section C

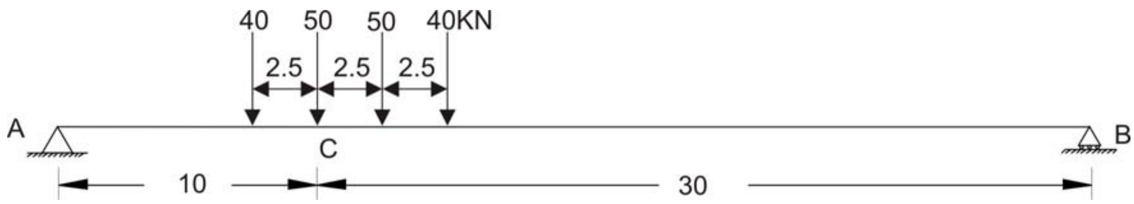


Figure 39.18: Beam loaded with a series of loads – Second load at section C

Figure 39.19: Beam loaded with a series of loads - – Third load at section C

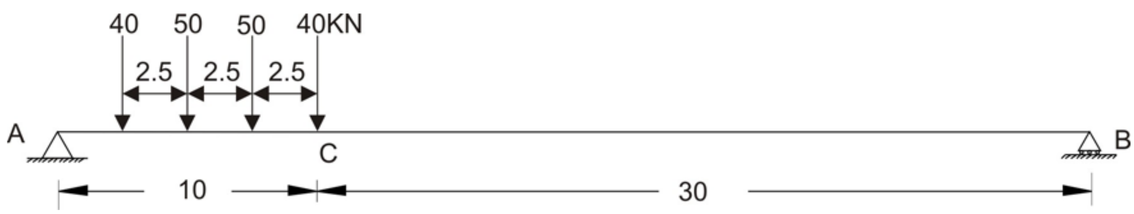


Figure 39.20: Beam loaded with a series of loads - – Third load at section C

As shown in Figure 39.17, when the first load crosses the section C, it is observed that the slope is downward (7.5/10). For other loads, the slope is upward (7.5/(40-10)). When the first load 40 kN crosses the section and second load 50 kN is approaching section (Figure 39.17) then change in moment is given by

$$dM = -40\left(\frac{7.5}{10}\right)2.5 + (50+50+40)\left(\frac{7.5}{(40-10)}\right)2.5 = 12.5 kN.m$$

When the second load 50 kN crosses the section and third load 50 kN is approaching section (Figure 39.18) then change in moment is given by

$$dM = -(40+50)\left(\frac{7.5}{10}\right)2.5 + (50+40)\left(\frac{7.5}{(40-10)}\right)2.5 = -112.5 kN.m$$

At this stage, we find negative change in moment; hence place second load at the section and maximum moment (refer Figure 39.21) will be given by

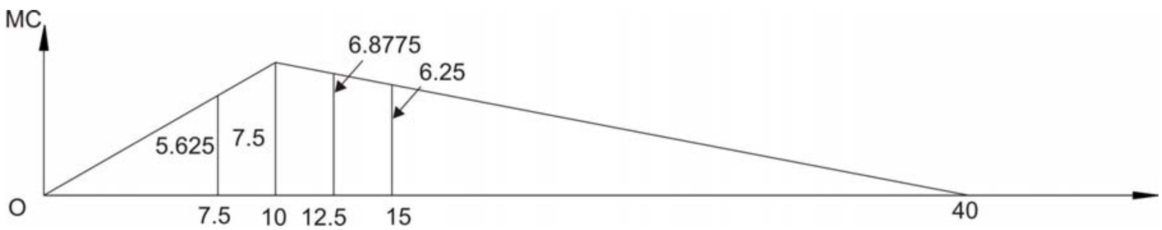


Figure 39.21: Influence line for moment at C

$$M_c = 40(5.625) + 50(7.5) + 50(6.8775) + 40(6.25) = 1193.87 kNm$$

39.7 Absolute maximum moment in s beam supporting a series of moving concentrated loads.

In earlier sections, we have learned to compute the maximum shear and moment for single load, UDL and series of concentrated loads at specified locations. However, from design point of view it is necessary to know the critical location of the point in the beam and the position of the loading on the beam to find maximum shear and moment induced by the loads. Following paragraph explains briefly for the cantilever beam or simply supported beam so that quickly maximum shear and moment can be obtained.

Maximum Shear: As shown in the Figure 39.22, for the cantilever beam, absolute maximum shear will occur at a point located very near to fixed end of the beam. After placing the load as close as to fixed support, find the shear at the section close to fixed end.

Figure 39.22: Absolute maximum shear case – cantilever beam

Similarly for the simply supported beam, as shown in Figure 39.23, the absolute maximum shear will occur when one of the loads is placed very close to support.

Figure 39.23: Absolute maximum shear – simply supported beam

Moment:

The absolute maximum bending moment in case of cantilever beam will occur where the maximum shear has occurred, but the loading position will be at the free end as shown in Figure 39.24.

Figure 39.24: Absolute maximum moment – cantilever beam

The absolute maximum bending moment in the case of simply supported beam, one cannot obtain by direct inspection. However, we can identify position analytically. In this regard, we need to prove an important proposition.

Proposition:

When a series of wheel loads crosses a beam, simply supported ends, the maximum bending moment under any given wheel occurs when its axis and the center of gravity of the load system on span are equidistant from the center of the span.

Let us assume that load P_1, $P_{2,}$ P_3 etc. are spaced shown in Figure 39.25 and traveling from left to right. Assume P_R to be resultant of the loads, which are on the beam, located in such way that it nearer to P_3 at a distance of d_1 as shown in Figure 39.25.

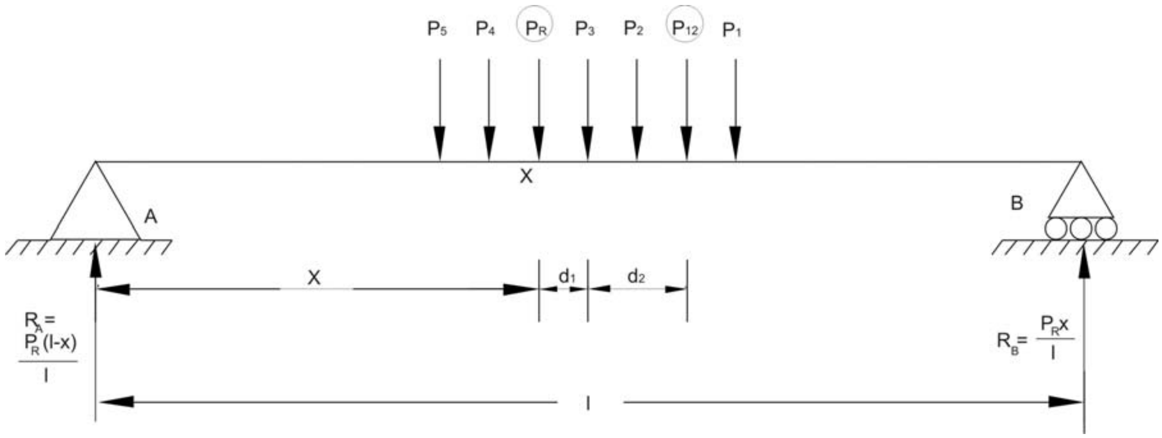


Figure 39.25: Absolute maximum moment case – simply supported beam

If P_{12} is resultant of P_1 and P_2, and distance from P_3 is d_2. Our objective is to find the maximum bending moment under load P_3. The bending moment under P_3 is expressed as

$$M = \frac{P_R x}{l}(l - x - d_1) - P_{12}(d_2)$$

Differentiate the above expression with respect to x for finding out maximum moment.

$$\frac{dM}{dx} = \frac{P_R}{l}(l - 2x - d_1) = 0 \Rightarrow l - 2x + d_1 = 0 \Rightarrow x = \frac{l}{2} - \frac{d_1}{2}$$

Above expression proves the proposition.

Let us have a look to some examples for better understanding of the above-derived proposition.

39.7.1 Numerical Examples

Example 1:

The beam is loaded with two loads 25 kN each spaced at 2.5 m is traveling on the beam having span of 10 m. Find the absolute maximum moment

Solution:

When the a load of 25kN and center of gravity of loads are equidistant from the center of span then absolute bending moment will occur. Hence, place the load on the beam as shown in Figure 39.26.

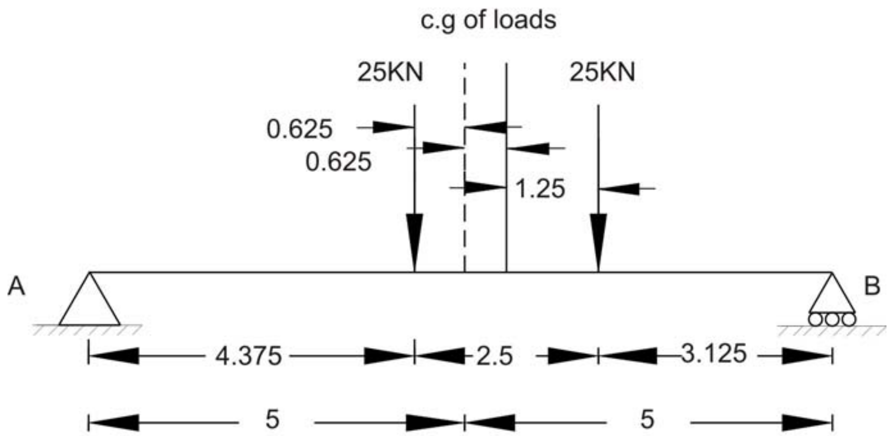


Figure 39.26: Simply supported beam (Example 1)

The influence line for M_x is shown in Figure 39.27

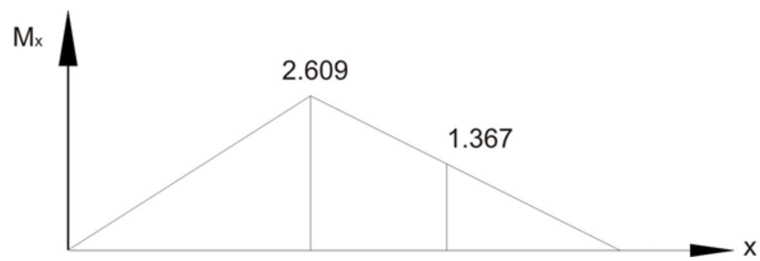


Figure 39.27: Influence line for moment at X (Example 1)

Computation of absolute maximum moment is given below.

$$M_x = 25(2.461) + 25(1.367) = 95.70 kN.m$$

Example 2:
Compute the absolute maximum bending moment for the beam having span of 30 m and loaded with a series of concentrated loads moving across the span as shown in Figure 39.28.

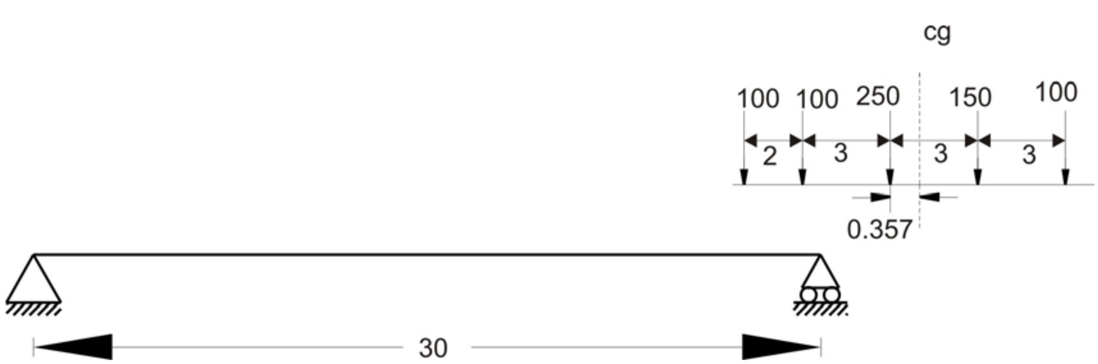


Figure 39.28: Simply supported beam (Example 2)

First of all compute the center of gravity of loads from first point load of 100 kN

$$= \frac{100(2) + 250(5) + 150(8) + 100(11)}{100 + 100 + 250 + 150 + 100} = \frac{3750}{700} = 5.357m$$

Now place the loads as shown in Figure 39.29.

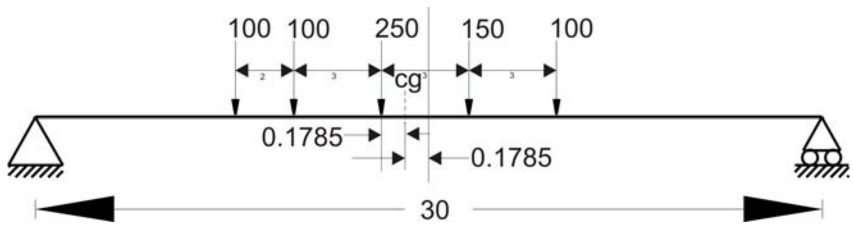


Figure 39.29: Simply supported beam with load positions (Example 2)

Also, draw the influence line as shown in Figure 39.30 for the section X.

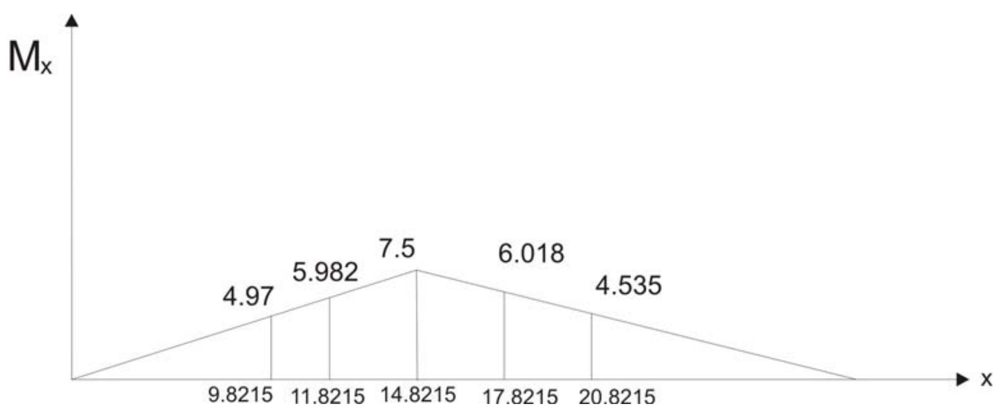


Figure 39.30: Influence Line for moment at section X (Example 2)

$$M_x = 100(4.97) + 100(5.982) + 250(7.5) + 150(6.018) + 100(4.535) = 4326.4 kN.m$$

39.8 Envelopes of maximum influence line values

For easy calculations steps of absolute maximum shear and moment rules for cantilever beam and simply supported beam were discussed in previous section. Nevertheless, it is difficult to formulate such rules for other situations. In such situations, the simple approach is that develop the influence lines for shear and moment at different points along the entire length of the beam. The values easily can be obtained using the concepts developed in earlier sections. After obtaining the values, plot the influence lines for each point under consideration in one plot and the outcome will be envelop of maximums. From this diagram, both the absolute maximum value of shear and moment and location can be obtained. However, the approach is simple but demands tedious calculations for each point. In that case, these calculations easily can be done using computers.

39.9 Closing Remarks

In this lesson, we have learned various aspects of constructing influence lines for the cases when the moving concentrated loads are two or more than two. Also, we developed simple concept of finding out absolute maximum shear and moment values in cases of cantilever beam and simply supported beam. Finally, we discussed about the need of envelopes of maximum influence line values for design purpose.

Suggested Text Books for Further Reading

- Armenakas, A. E. (1988). *Classical Structural Analysis – A Modern Approach*, McGraw-Hill Book Company, NY, ISBN 0-07-100120-4

- Hibbeler, R. C. (2002). *Structural Analysis*, Pearson Education (Singapore) Pte. Ltd., Delhi, ISBN 81-7808-750-2

- Junarkar, S. B. and Shah, H. J. (1999). *Mechanics of Structures – Vol. II*, Charotar Publishing House, Anand.

- Leet, K. M. and Uang, C-M. (2003). *Fundamentals of Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-058208-4

- Negi, L. S. and Jangid, R.S. (2003). *Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-462304-4

- Norris, C. H., Wilbur, J. B. and Utku, S. (1991). *Elementary Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-058116-9

Module 7

Influence Lines

Lesson

40

Influence Lines for Simple Trusses

Instructional Objectives:

The objectives of this lesson are as follows.

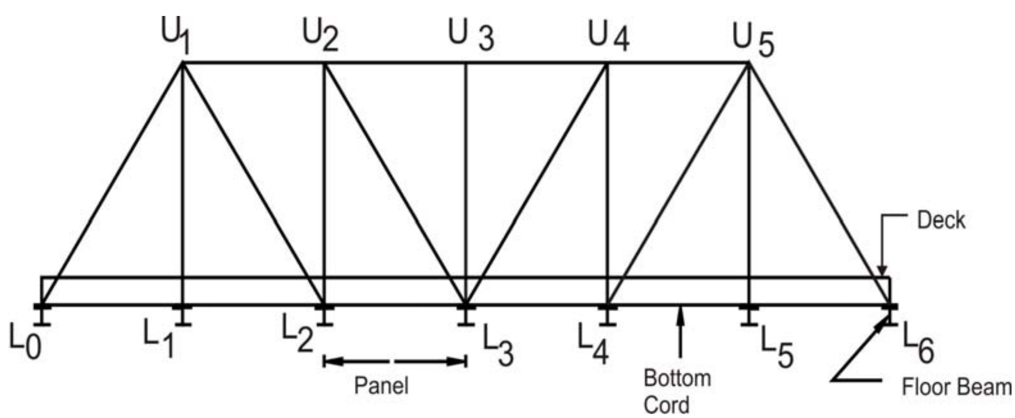
- Understand the bridge truss floor system and load transfer mechanism
- Draw the influence line for the truss reactions
- Draw the influence line for the truss member forces

40.1 Introduction

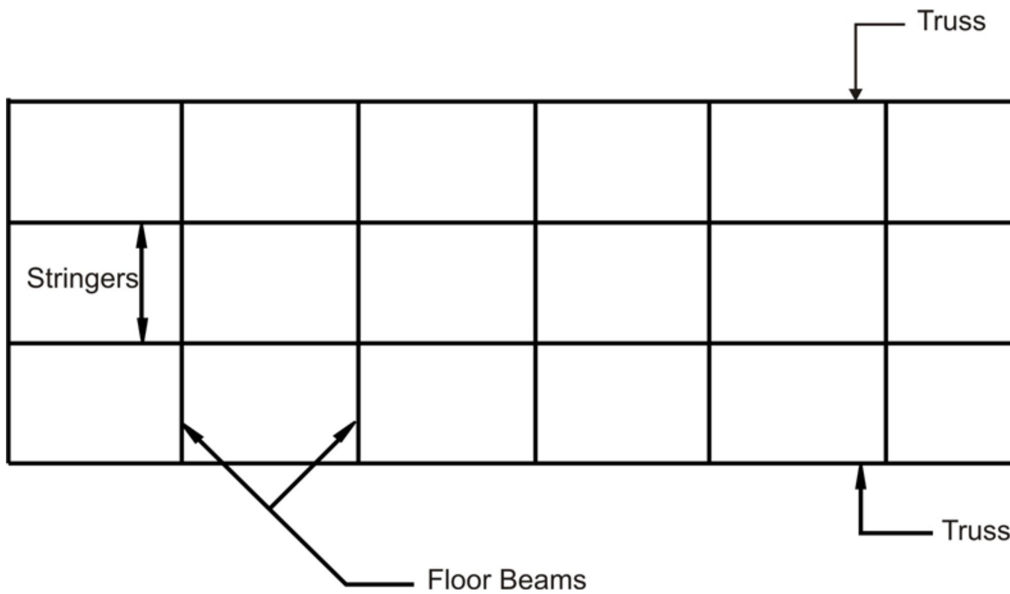
In previous lessons, we have studied the development of influence lines for beams loaded with single point load, UDL and a series of loads. In similar fashion, one can construct the influence lines for the trusses. The moving loads are never carried directly on the main girder but are transmitted across cross girders to the joints of bottom chord. Following section will explain load transmission to the trusses followed by the influence lines for the truss reactions and influence lines for truss member forces.

40.2 Bridge Truss Floor System

A typical bridge floor system is shown in Figure 40.1. As shown in Figure, the loading on bridge deck is transferred to stringers. These stringers in turn transfer the load to floor beams and then to the joints along the bottom chord of the truss.



Front view



Floor plan

Figure 40.1 Bridge floor system

It should be noted that for any load position; the truss is always loaded at the joint.

40.3 Influence lines for truss support reaction

Influence line for truss reactions are of similar to that a simply supported beam. Let us assume that there is truss with overhang on both ends as shown in Figure 40.2. In this case, the loads to truss joints are applied through floor beams as discussed earlier. These influence lines are useful to find out the support, which will be critical in terms of maximum loading.

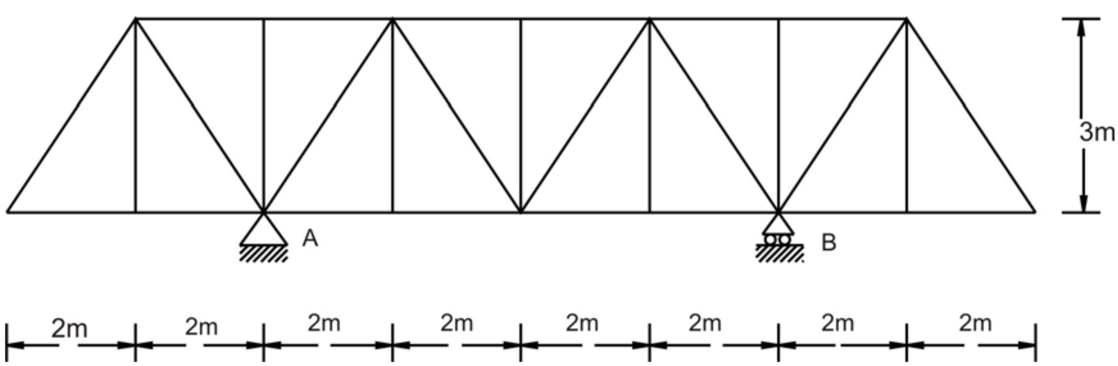
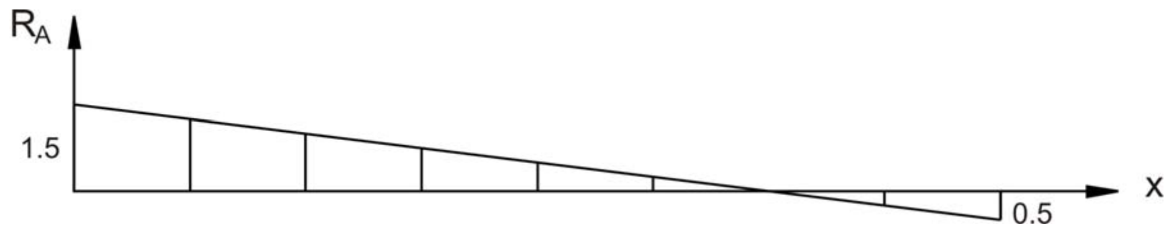
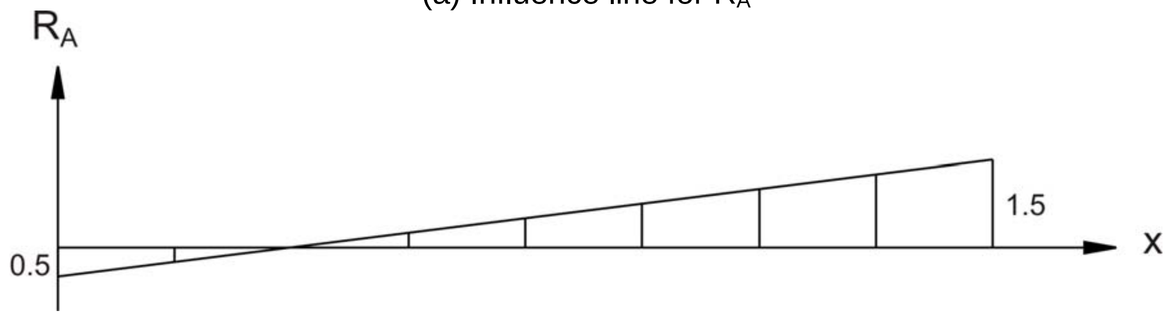


Figure 40.2 Bridge truss

The influence lines for truss reactions at A and B are shown in Figure 40.3.



(a) Influence line for R_A



(b) Influence line for R_B

Figure 40.3: Influence lines for support reactions

40.4 Influence lines for truss member forces

Influence lines for truss member force can be obtained very easily. Obtain the ordinate values of influence line for a member by loading each joint along the deck with a unit load and find member force. The member force can be found out using the method of joints or method of sections. The data is prepared in tabular form and plotted for a specific truss member force. The truss member carries axial loads. In the present discussion, tensile force nature is considered as positive and compressive force nature is considered as negative.

40.4.1 Numerical Examples

Example 1:

Construct the influence line for the force in member GB of the bridge truss shown in Figure 40.4.

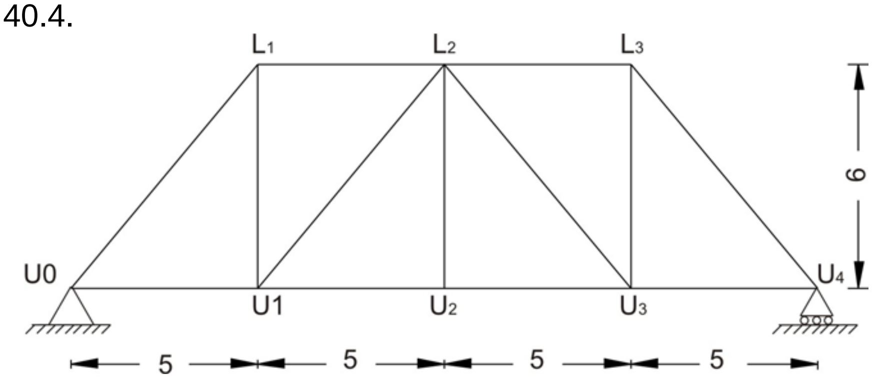


Figure 40.4: Bridge Truss (Example 1)

Solution:

Tabulated Values:

In this case, successive joints L_0, L_1, L_2, L_3, and L_4 are loaded with a unit load and the force F_{L2U3} in the member L_2U_3 are using the method of sections. Figure 40.5 shows a case where the joint load is applied at L_1 and force F_{L2U3} is calculated.

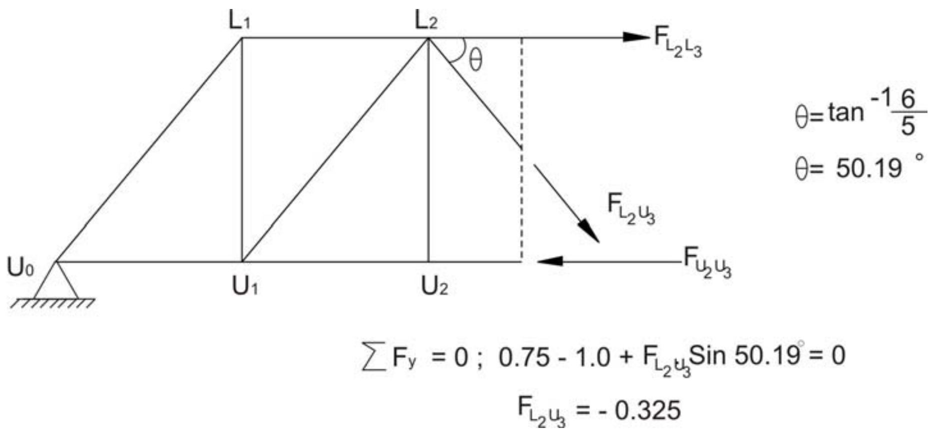


Figure 40.5: Member Force F_{L2U3} Calculation using method of sections.

The computed values are given below.

x	F_{L2U3}
0	0
5	-0.325
10	-0.650
15	0.325
20	0

Influence line: Let us plot the tabular data and connected points will give the influence line for member L_2U_3. The influence line is shown in Figure 40.6. The figure shows the behaviour of the member under moving load. Similarly other influence line diagrams can be generated for the other members to find the critical axial forces in the member.

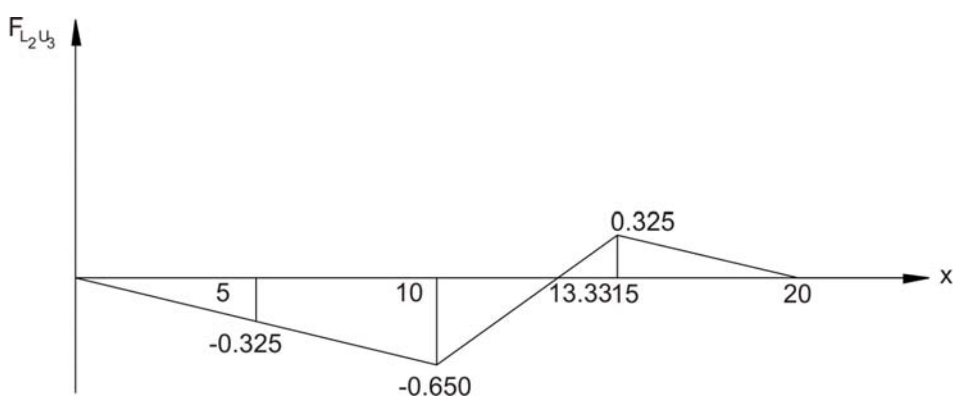


Figure 40.6: Influence line for member force F_{L2U3}

Example 2:

Tabulate the influence line values for all the members of the bridge truss shown in Figure 40.7.

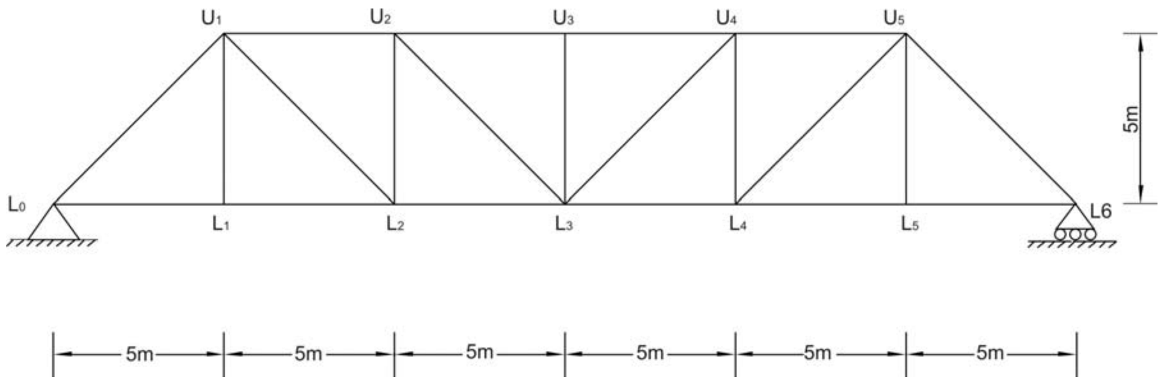


Figure 40.7: Bridge Truss (Example 2)

Solution:

Tabulate Values:

Here objective is to construct the influence line for all the members of the bridge truss, hence it is necessary to place a unit load at each lower joints and find the forces in the members. Typical cases where the unit load is applied at L_1, L_2 and L_3 are shown in Figures 40.8-10 and forces in the members are computed using method of joints and are tabulated below.

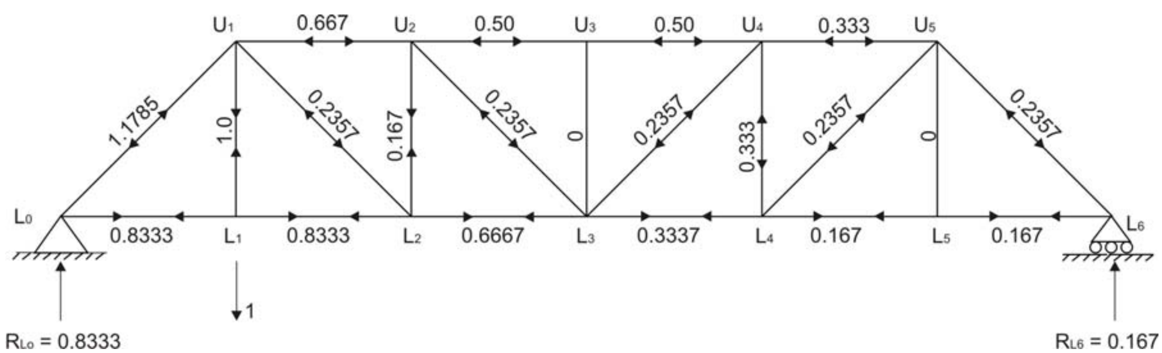


Figure 40.8: Member forces calculation when unit load is applied at L_1

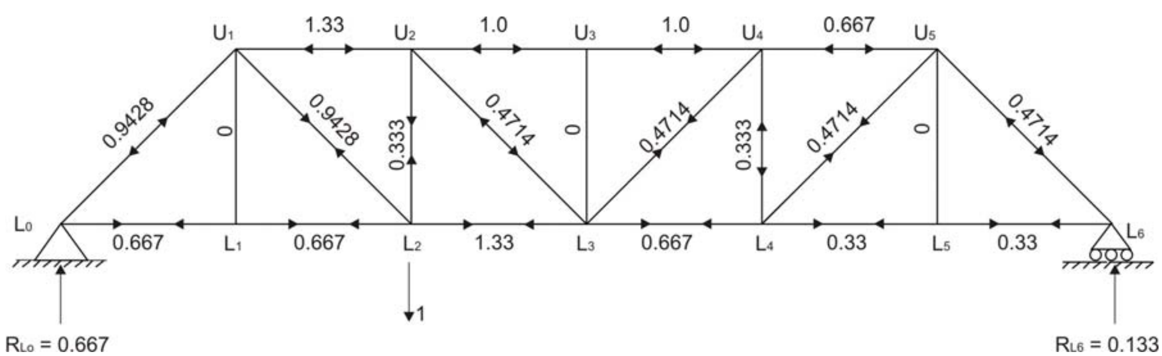


Figure 40.9: Member forces calculation when unit load is applied at L_2

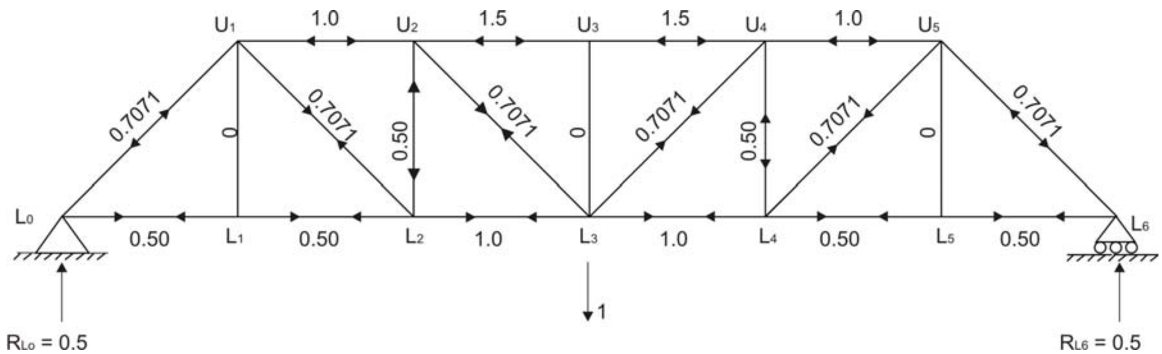


Figure 40.10: Member forces calculation when unit load is applied at L_3

Member	Member force due to unit load at:						
	L_0	L_1	L_2	L_3	L_4	L_5	L_6
L_0L_1	0	0.8333	0.6667	0.5	0.3333	0.1678	0
L_1L_2	0	0.8333	0.6667	0.5	0.3333	0.1678	0
L_2L_3	0	0.6667	1.3333	1.0	0.6667	0.3336	0
L_3L_4	0	0.3336	0.6667	1.0	1.3333	0.6667	0
L_4L_5	0	0.1678	0.3333	0.5	0.6667	0.8333	0
L_5L_6	0	0.1678	0.3333	0.5	0.6667	0.8333	0
U_1U_2	0	-0.6667	-1.333	-1.0	-0.6667	-0.333	0
U_2U_3	0	-0.50	-1.000	-1.5	-1.0	-0.50	0
U_3U_4	0	-0.50	-1.000	-1,5	-1.0	-0.50	0
U_4U_5	0	-0.333	-0.6667	-1.0	-1.333	-0.6667	0
L_0U_1	0	-1.1785	-0.9428	-0.7071	-0.4714	-0.2357	0
L_1U_1	0	1	0	0	0	0	0
L_2U_1	0	-0.2357	0.9428	0.7071	0.4714	0.2357	0
L_2U_2	0	0.167	0.3333	-0.50	-0.3333	-0.3333	0
L_3U_2	0	-0.2357	-0.4714	0.7071	0.4714	0.2357	0
L_3U_3	0	0	0	0	0	0	0
L_3U_4	0	0.2357	0.4714	0.7071	-0.4714	-0.2357	0
L_4U_4	0	-03333	-0.3333	-0.50	0.3333	0.167	0
L_4U_5	0	0.2357	0.4714	0.7071	0.9428	-0.2357	0
L_5U_5	0	0	0	0	0	1	0
L_6U_5	0	-0.2357	-0.4714	-0.7071	-0.9428	-1.1785	0

Influence lines:

Using the values obtained in the above given table, the influence line can be plotted very easily for truss members.

40.5 Closing Remarks

In this lesson we have studied how the loads are transferred in bridge truss floor system. Further, we found that there is similarity between the influence line of support reactions for simply supported beam and truss structures. Finally we studied the influence line for truss member forces. It was essential to know the method of sections and method of joints for the analysis of trusses while drawing influence lines.

Suggested Text Books for Further Reading

- Armenakas, A. E. (1988). *Classical Structural Analysis – A Modern Approach*, McGraw-Hill Book Company, NY, ISBN 0-07-100120-4

- Hibbeler, R. C. (2002). *Structural Analysis*, Pearson Education (Singapore) Pte. Ltd., Delhi, ISBN 81-7808-750-2

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- Negi, L. S. and Jangid, R.S. (2003). *Structural Analysis*, Tata McGraw-Hill Publishing Company Limited, New Delhi, ISBN 0-07-462304-4

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