2.2 Losses in Prestress (Part II)

This section covers the following topics

- Friction
- Anchorage Slip
- Force Variation Diagram

2.2.1 Friction

The friction generated at the interface of concrete and steel during the stretching of a curved tendon in a post-tensioned member, leads to a drop in the prestress along the member from the stretching end. The loss due to friction does not occur in pre-tensioned members because there is no concrete during the stretching of the tendons.

The friction is generated due to the curvature of the tendon and the vertical component of the prestressing force. The following figure shows a typical profile (laying pattern) of the tendon in a continuous beam.

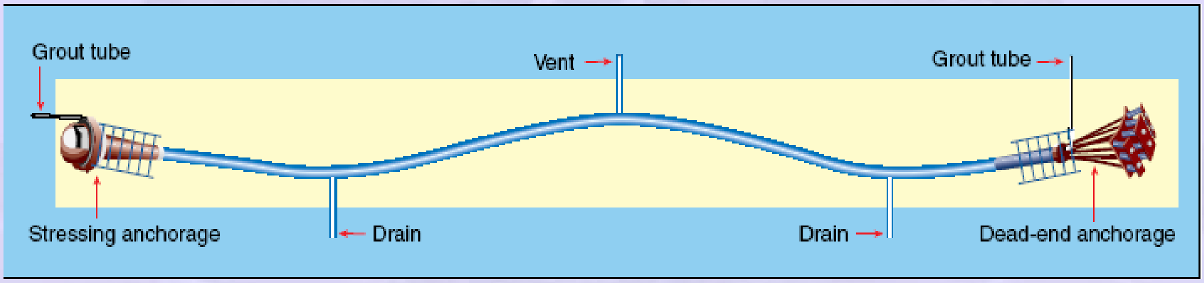


Figure 2-2.1 A typical continuous post-tensioned member

(Reference: VSL International Ltd.)

In addition to friction, the stretching has to overcome the **wobble** of the tendon. The wobble refers to the change in position of the tendon along the duct. The losses due to friction and wobble are grouped together under friction.

The formulation of the loss due to friction is similar to the problem of belt friction. The sketch below (Figure 2-2.2) shows the forces acting on the tendon of infinitesimal length *dx*.

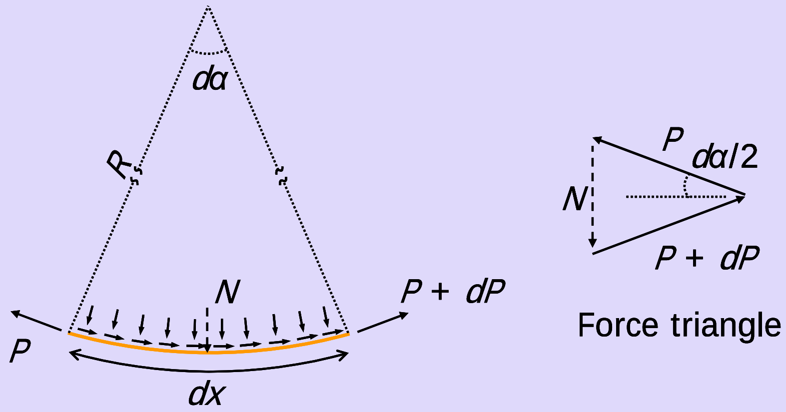


Figure 2-2.2 Force acting in a tendon of infinitesimal length

In the above sketch,

P = prestressing force at a distance *x* from the stretching end

R = radius of curvature

dα = subtended angle.

The derivation of the expression of *P* is based on a circular profile. Although a cable profile is parabolic based on the bending moment diagram, the error induced is insignificant.

The friction is proportional to the following variables.

- Coefficient of friction (*μ*) between concrete and steel.
- The resultant of the vertical reaction from the concrete on the tendon (*N*) generated due to curvature.

From the equilibrium of forces in the force triangle, *N* is given as follows.

$$N = 2P\sin\frac{d\alpha}{2}$$

$$\approx 2P\frac{d\alpha}{2} = Pd\alpha \qquad \textbf{(2-2.1)}$$

The friction over the length *dx* is equal to $\mu N = \mu P d\alpha$.

Thus the friction (*dP*) depends on the following variables.

- Coefficient of friction (*μ*)
- Curvature of the tendon (*dα*)
- The amount of prestressing force (*P*)

The wobble in the tendon is effected by the following variables.

- Rigidity of sheathing
- Diameter of sheathing
- Spacing of sheath supports
- Type of tendon
- Type of construction

The friction due to wobble is assumed to be proportional to the following.

- Length of the tendon
- Prestressing force

For a tendon of length *dx*, the friction due to wobble is expressed as *kPdx*, where *k* is the wobble coefficient or coefficient for wave effect.

Based on the equilibrium of forces in the tendon for the horizontal direction, the following equation can be written.

$$P = P + dP + (\mu P d\alpha + kPdx)$$

$$\text{or, } dP = -(\mu P d\alpha + kPdx) \qquad \textbf{(2-2.2)}$$

Thus, the total drop in prestress (*dP*) over length *dx* is equal to – (*μPdα* + *kPdx*). The above differential equation can be solved to express *P* in terms of *x*.

$$\int_{P_0}^{P_x} \frac{dP}{P} = -\left(\mu \int_0^{\alpha} d\alpha + k \int_0^{x} dx\right)$$

$$\text{or, } lnP\Big|_{P_0}^{P_x} = -(\mu\alpha + kx)$$

$$\text{or, } ln\frac{P_x}{P_0} = -(\mu\alpha + kx)$$

$$\text{or, } P_x = P_0 e^{-(\mu\alpha + kx)} \qquad \textbf{(2-2.3)}$$

Here,

P_0 = the prestress at the stretching end after any loss due to elastic shortening.

For small values of *μα* + *kx*, the above expression can be simplified by the Taylor series expansion.

$$P_x = P_0\,(1 - \mu\alpha - kx) \qquad \textbf{(2-2.4)}$$

Thus, for a tendon with single curvature, the variation of the prestressing force is linear with the distance from the stretching end. The following figure shows the variation of prestressing force after stretching. The left side is the stretching end and the right side is the anchored end.

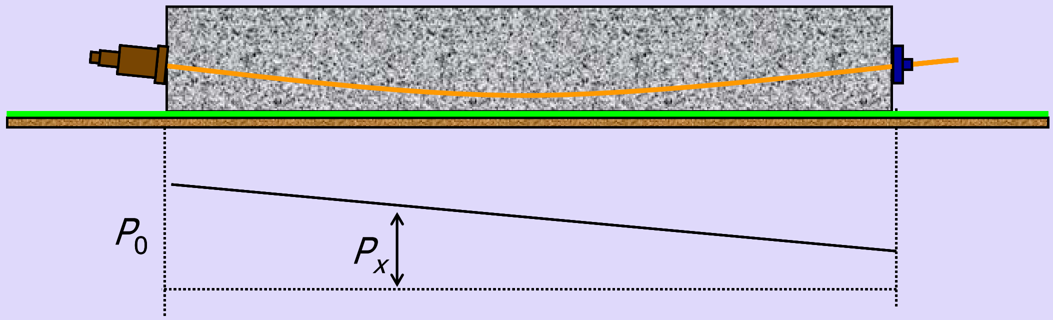


Figure 2-2.3 Variation of prestressing force after stretching

In the absence of test data, **IS:1343 - 1980** provides guidelines for the values of μ and k.

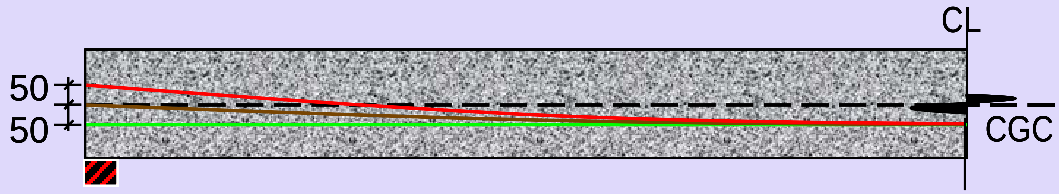
Table 2-2.1 Values of coefficient of friction

Type of interface	μ
For steel moving on smooth concrete	0.55.
For steel moving on steel fixed to duct	0.30.
For steel moving on lead	0.25.

The value of k varies from 0.0015 to 0.0050 per meter length of the tendon depending on the type of tendon. The following problem illustrates the calculation of the loss due to friction in a post-tensioned beam.

Example 2-2.1

A post-tensioned beam 100 mm × 300 mm ($b \times h$) spanning over 10 m is stressed by successive tensioning and anchoring of 3 cables A, B, and C respectively as shown in figure. Each cable has cross section area of 200 mm^2 and has initial stress of 1200 MPa. If the cables are tensioned from one end, estimate the percentage loss in each cable due to friction at the anchored end. Assume μ = 0.35, k = 0.0015 / m.



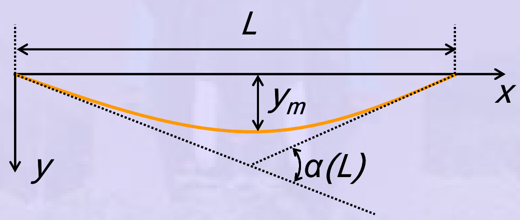
Solution

Prestress in each tendon at stretching end = 1200 × 200

= 240 kN.

To know the value of $\alpha(L)$, the equation for a parabolic profile is required.

$$\frac{dy}{dx} = \frac{4y_m}{L^2}(L - 2x)$$



Here,

y_m = displacement of the CGS at the centre of the beam from the ends

L = length of the beam

x = distance from the stretching end

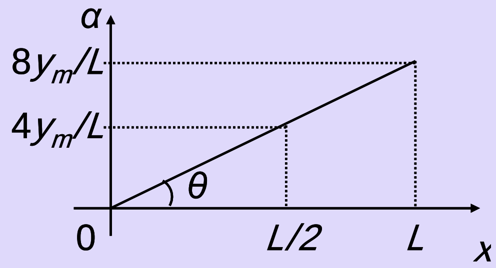
y = displacement of the CGS at distance x from the ends.

An expression of $\alpha(x)$ can be derived from the change in slope of the profile. The slope of the profile is given as follows.

$$\frac{dy}{dx} = \frac{4y_m}{L^2}(L - 2x)$$

At $x = 0$, the slope $dy/dx = 4y_m/L$. The change in slope $\alpha(x)$ is proportional to x.

The expression of $\alpha(x)$ can be written in terms of x as $\alpha(x) = \theta.x$, where, $\theta = 8y_m/L^2$. The variation is shown in the following sketch.



The total subtended angle over the length L is $8y_m/L$.

The prestressing force P_x at a distance x is given by

$$P_x = P_0 e^{-(\mu\alpha + kx)} = P_0 e^{-\eta x}$$

where, $\eta x = \mu\alpha + kx$

For cable A, $y_m = 0.1$ m.

For cable B, $y_m = 0.05$ m.

For cable C, $y_m = 0.0$ m.

For all the cables, $L = 10$ m.

Substituting the values of y_m and L

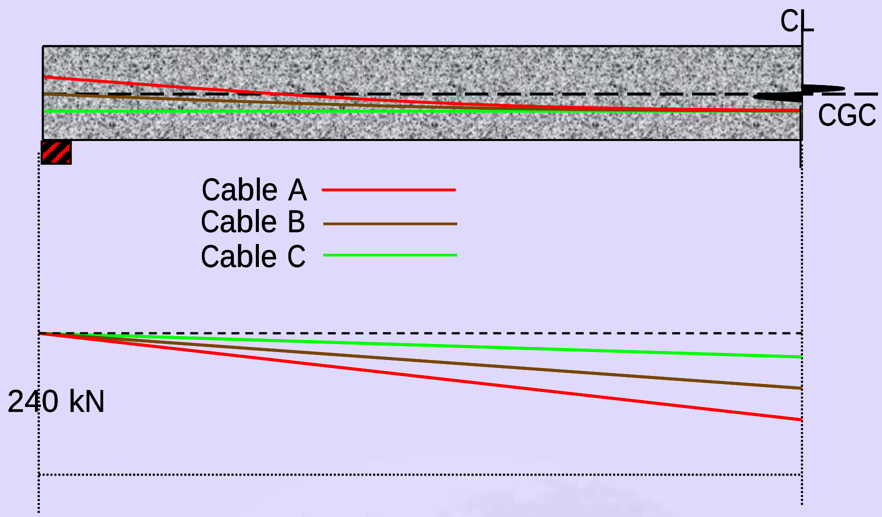
$$\eta x = \begin{cases} 0.0043x & \text{for cable A} \\ 0.0029x & \text{for cable B} \\ 0.0015x & \text{for cable C} \end{cases}$$

The maximum loss for all the cables is at $x = L = 10$, the anchored end.

$$e^{-\eta L} = \begin{cases} 0.958 & \text{for cable A} \\ 0.971 & \text{for cable B} \\ 0.985 & \text{for cable C} \end{cases}$$

Percentage loss due to friction = $(1 - e^{-\eta L}) \times 100\%$

$$= \begin{cases} 4.2\% & \text{for cable A} \\ 2.9\% & \text{for cable B} \\ 1.5\% & \text{for cable C} \end{cases}$$



Variation of prestressing forces

The loss due to friction can be considerable for long tendons in continuous beams with changes in curvature. The drop in the prestress is higher around the intermediate supports where the curvature is high. The remedy to reduce the loss is to apply the stretching force from both ends of the member in stages.

2-2.2 Anchorage Slip

In a post-tensioned member, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space. The anchorage block also moves before it settles on the concrete. There is loss of prestress due to the consequent reduction in the length of the tendon.

The total anchorage slip depends on the type of anchorage system. In absence of manufacturer's data, the following typical values for some systems can be used.

Table 2-2.2 Typical values of anchorage slip

Anchorage System	Anchorage Slip (Δs)
Freyssinet system	
12 - 5mm Φ strands	4 mm
12 - 8mm Φ strands	6 mm
Magnel system	8 mm
Dywidag system	1 mm

(Reference: Rajagopalan, N., *Prestressed Concrete*)

Due to the setting of the anchorage block, as the tendon shortens, there is a reverse friction. Hence, the effect of anchorage slip is present up to a certain length (Figure 2-2.4). Beyond this **setting length**, the effect is absent. This length is denoted as l_{set}.

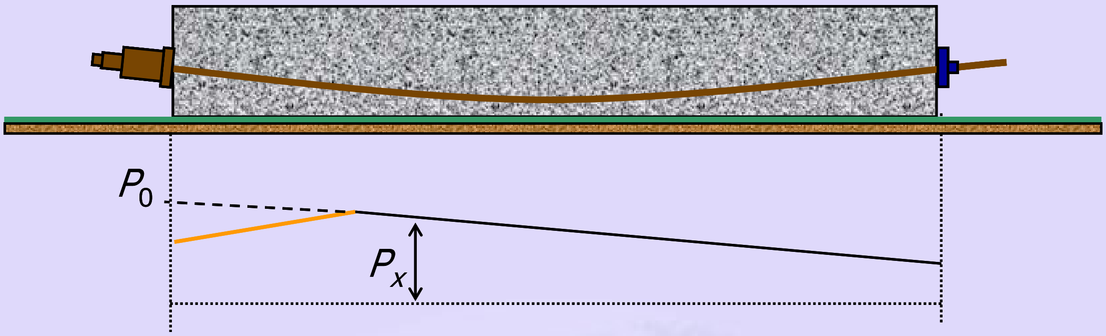


Figure 2-2.4 Variation of prestressing force after anchorage slip

2.2.3 Force Variation Diagram

The magnitude of the prestressing force varies along the length of a post-tensioned member due to friction losses and setting of the anchorage block. The diagram representing the variation of prestressing force is called the force variation diagram.

Considering the effect of friction, the magnitude of the prestressing force at a distance x from the stretching end is given as follows.

$$P_x = P_0 e^{-\eta x} \quad \textbf{(2-2.5)}$$

Here, $\eta x = \mu\alpha + kx$ denotes the total effect of friction and wobble. The plot of P_x gives the force variation diagram.

The initial part of the force variation diagram, up to length l_{set} is influenced by the setting of the anchorage block. Let the drop in the prestressing force at the stretching end be ΔP. The determination of ΔP and l_{set} are necessary to plot the force variation diagram including the effect of the setting of the anchorage block.

Considering the drop in the prestressing force and the effect of reverse friction, the magnitude of the prestressing force at a distance x from the stretching end is given as follows.

$$P_x' = (P_0 - \Delta P) e^{\eta' x} \quad \textbf{(2-2.6)}$$

Here, *η'* for reverse friction is analogous to η for friction and wobble.

At the end of the setting length ($x = l_{set}$), $P_x = P'_x$

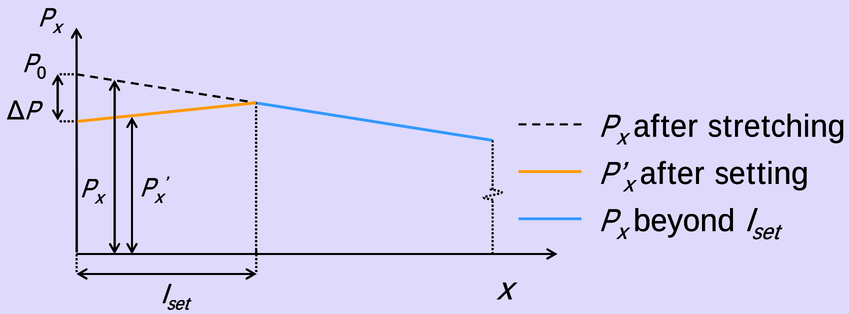


Figure 2-2.5 Force variation diagram near the stretching end

Substituting the expressions of P_x and P_x' for $x = l_{set}$

Since it is difficult to measure *η'* separately, *η'* is taken equal to *η*. The expression of ΔP simplifies to the following.

$$P_0 e^{-\eta l_{set}} = (P_0 - \Delta P)\, e^{\eta' l_{set}}$$

$$P_0 e^{-(\eta+\eta')l_{set}} = P_0 - \Delta P$$

$$P_0\left[1-(\eta+\eta')l_{set}\right] = P_0 - \Delta P$$

$$\Delta P = P_0(\eta+\eta')l_{set} = P_0 \eta l_{set}\left(1+\frac{\eta'}{\eta}\right) \quad \textbf{(2-2.7)}$$

$$\Delta P = 2P_0\eta l_{set} \quad \textbf{(2-2.8)}$$

The following equation relates l_{set} with the anchorage slip Δs.

$$\Delta_s = \frac{1}{2}\frac{\Delta P}{A_p E_p} l_{set}$$

$$\Delta_s = \frac{1}{2}\frac{l_{set}}{A_p E_p} P_0 \eta l_{set}\left(1+\frac{\eta'}{\eta}\right) \quad \textbf{(2-2.9)}$$

Transposing the terms,

$$l_{set}^2 = \Delta_s \frac{2A_p E_p}{P_0 \eta\left(1+\dfrac{\eta'}{\eta}\right)}$$

$$= \frac{\Delta_s A_p E_p}{P_0 \eta} \qquad \text{for } \eta' = \eta$$

Therefore,

$$l_{set} = \sqrt{\frac{\Delta_s A_p E_p}{P_0 \eta}} \qquad \textbf{(2-2.10)}$$

The term $P_0\eta$ represents the loss of prestress per unit length due to friction.

The force variation diagram is used when stretching is done from both the ends. The tendons are overstressed to counter the drop due to anchorage slip. The stretching from both the ends can be done simultaneously or in stages. The final force variation is more uniform than the first stretching.

The following sketch explains the change in the force variation diagram due to stretching from both the ends in stages.

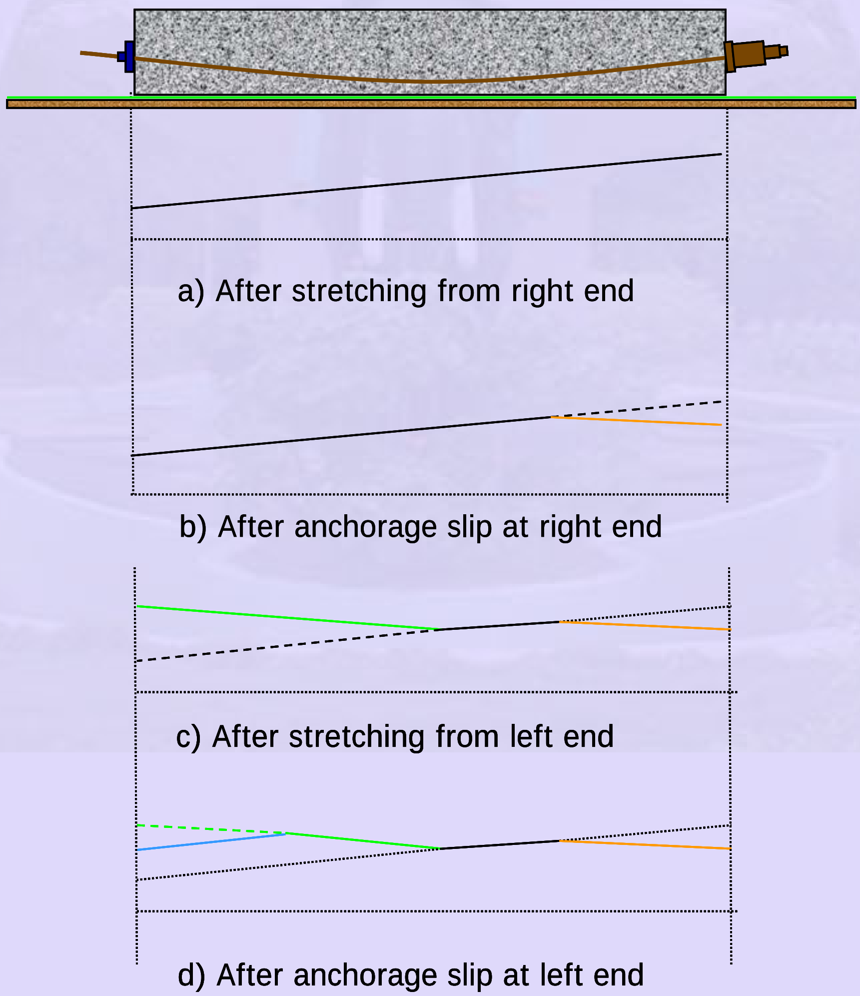


Figure 2-2.6 Force variation diagrams for stretching in stages

The force variation diagrams for the various stages are explained.

a) The initial tension at the right end is high to compensate for the anchorage slip. It corresponds to about 0.8 f_{pk} initial prestress. The force variation diagram (FVD) is linear.

b) After the anchorage slip, the FVD drops near the right end till the length l_{set}.

c) The initial tension at the left end also corresponds to about 0.8 f_{pk} initial prestress. The FVD is linear up to the centre line of the beam.

d) After the anchorage slip, the FVD drops near the left end till the length l_{set}. It is observed that after two stages, the variation of the prestressing force over the length of the beam is less than after the first stage.

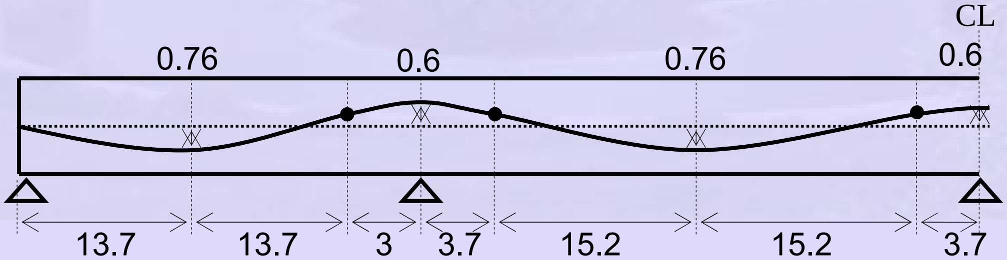
Example 2-2.2

A four span continuous bridge girder is post-tensioned with a tendon consisting of twenty strands with f_{pk} = 1860 MPa. Half of the girder is shown in the figure below. The symmetrical tendon is simultaneously stressed up to 75% f_{pk} from both ends and then anchored. The tendon properties are A_p = 2800 mm^2, E_p = 195,000 MPa, μ = 0.20, K = 0.0020/m. The anchorage slip Δ_s = 6 mm.

Calculate

a) The expected elongation of the tendon after stretching,

b) The force variation diagrams along the tendon before and after anchorage.



All dimensions are in metres

• Inflection points

Solution

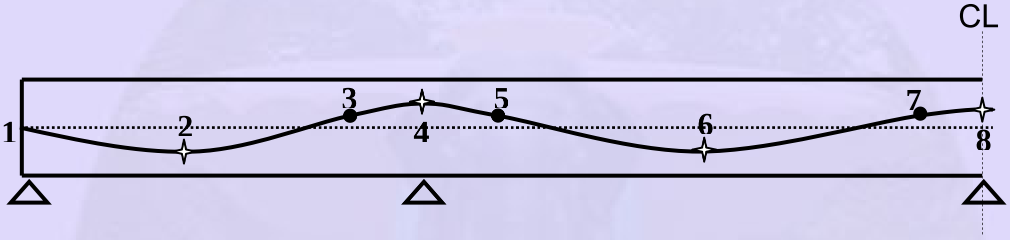
Initial force at stretching end

$0.75f_{pk}$ = 1395 MPa

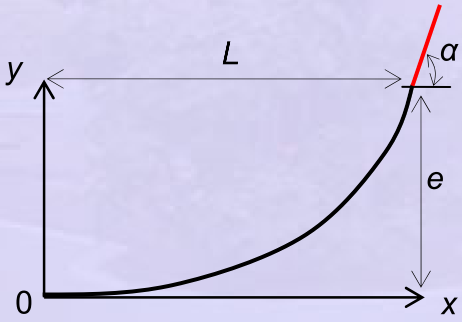
$P_0 = 0.75f_{pk}\,A_p$

= 3906 kN

The continuous tendon is analysed as segments of parabola. The segments are identified between the points of maximum eccentricity and inflection points. The inflection points are those where the curvature of the tendon reverses. The different segments are as follows: 1-2, 2-3, 3-4, 4-5, 5-6, 6-7 and 7-8.



The following properties of parabolas are used. For segment 1-2, the parabola in the sketch below is used.

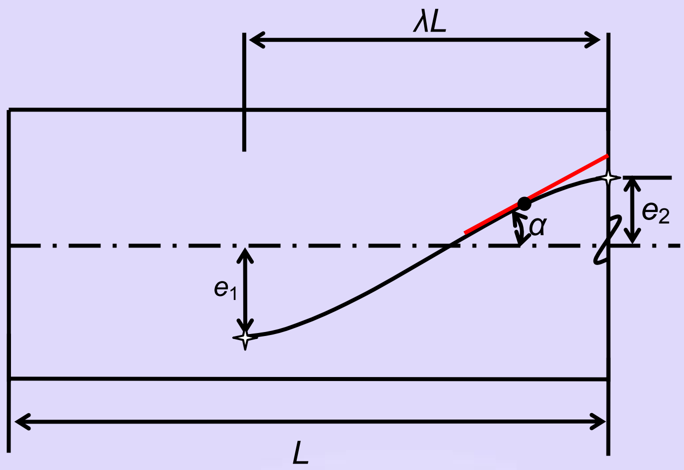


The change in slope from the origin to the end of the parabola is same as the slope at the end of the tendon which is $\alpha = 2e/L$, where

L = length of the segment

e = vertical shift from the origin.

For segments 2-3 and 3-4 and subsequent pairs of segments, the following property is used.



For the two parabolic segments joined at the inflection point as shown in the sketch above, the slope at the inflection point $\alpha = 2(e_1 + e_2)/\lambda L$.

Here,

e_1, e_2 = eccentricities of the CGS at the span and support respectively

L = length of the span

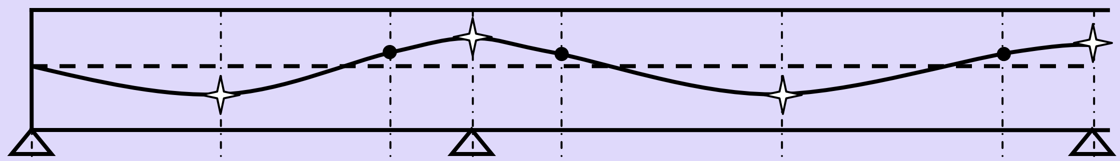
λL = fractional length between the points of maximum eccentricity

The change in slope between a point of maximum eccentricity and inflection point is also equal to α.

The change in slope (α) for each segment of the tendon is calculated using the above expressions. Next the value of $\mu\alpha + kx$ for each segment is calculated using the given values of μ, k and x, the horizontal length of the segment. Since the loss in prestress accrues with each segment, the force at a certain segment is given as follows.

$$P_x = P_0 e^{-\Sigma(\mu\alpha + kx)}$$

The summation $\sum$ is for the segments from the stretching end up to the point in the segment under consideration. Hence, the value of $\sum(\mu\alpha + kx)$ at the end of each segment is calculated to evaluate the prestressing force at that point (P_x, where x denotes the point).



	Segment 1	Segment 2	Segment 3	Segment 4	Segment 5	Segment 6	Segment 7
α	0.111	0.163	0.163	0.144	0.144	0.144	0.144
$\mu\alpha + kx$	0.050	0.060	0.039	0.036	0.059	0.059	0.036
$\Sigma(\mu\alpha + kx)$	0.050	0.110	0.149	0.185	0.244	0.303	0.339

$e^{-\Sigma(\mu\alpha + kx)}$	1.000	0.952	0.896	0.861	0.831	0.783	0.738	0.712
P_x (kN)	3906	3718	3500	3363	3246	3058	2883	2781

The force variation diagram before anchorage can be plotted with the above values of P_x. A linear variation of the force can be assumed for each segment. Since the stretching is done at both the ends simultaneously, the diagram is symmetric about the central line.

a) The expected elongation of the tendon after stretching

First the product of the average force and the length of each segment is summed up to the centre line.

$$
\begin{aligned}
P_{av}L &= \frac{1}{2}[3906+3718]\times 13.7 + \frac{1}{2}[3718+3500]\times 13.7 \\
&+ \frac{1}{2}[3500+3363]\times 3 + \frac{1}{2}[3363+3246]\times 3.7 \\
&+ \frac{1}{2}[3246+3058]\times 15.2 + \frac{1}{2}[3058+2883]\times 15.2 \\
&+ \frac{1}{2}[2883+2718]\times 3.7 \\
&= 227612.2\ \text{kN}
\end{aligned}
$$

The elongation (Δ) at each stretching end is calculated as follows.

$$\begin{aligned}\Delta &= \frac{P_{av}L}{A_P E_P} \\ &= \frac{227612\times10^3}{2800\times195000} \\ &= 0.417\ \text{m}\end{aligned}$$

b) The force variation diagrams along the tendon before and after anchorage

After anchorage, the effect of anchorage slip is present up to the setting length l_{set}. The value of l_{set} due to an anchorage slip Δ_s = 6 mm is calculated as follows.

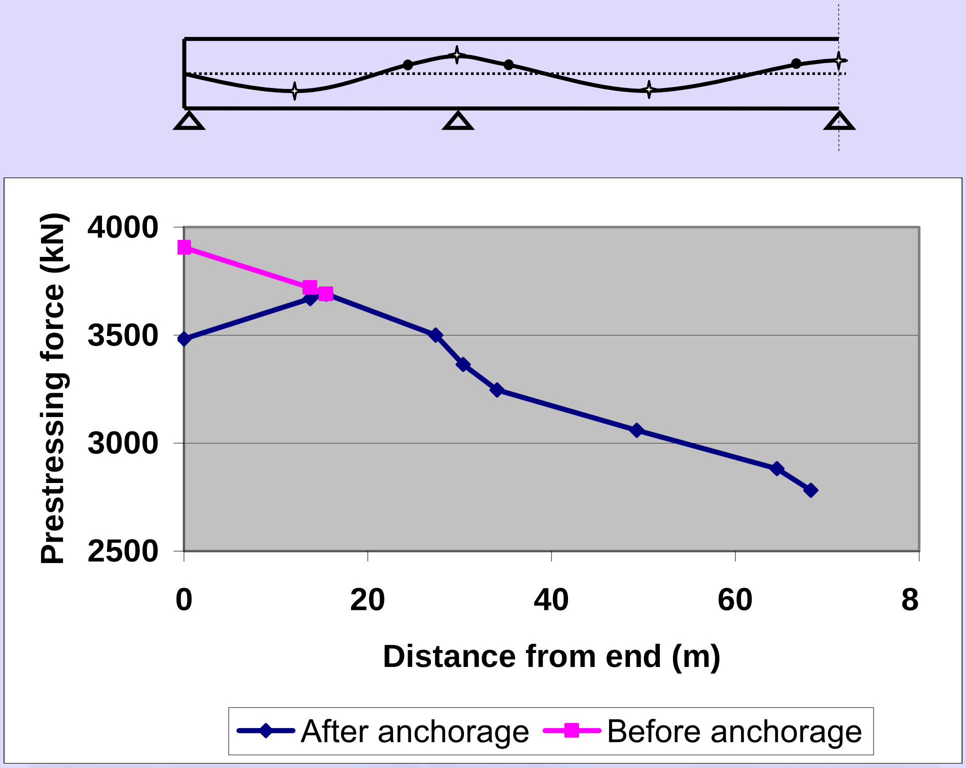
$$\begin{aligned}l_{set} &= \sqrt{\frac{\Delta_s A_P E_P}{P_0\mu}} \\ &= \sqrt{\frac{6\times2800\times195000}{13.7}} \\ &= 15.46\ \text{m}\end{aligned}$$

The quantity $P_0\mu$ is calculated from the loss of prestress per unit length in the first segment. $P_0\mu$ = (3906 – 3718) kN /13.7 m = 13.7 N/mm. The drop in the prestressing force (Δ_p) at each stretching end is calculated as follows.

$$\begin{aligned}\Delta_p &= 2P_0\mu l_{set} \\ &= 2\times13.7\times15464 \\ &= 423.7\ \text{kN}\end{aligned}$$

Thus the value of the prestressing force at each stretching end after anchorage slip is 3906 – 424 = 3482 kN. The force variation diagram for l_{set} = 15.46 m is altered to show the drop due to anchorage slip.

The force variation diagrams before and after anchorage are shown below. Note that the drop of force per unit length is more over the supports due to change in curvature over a small distance.


4000
3500
3000
2500
Prestressing force (kN)
0
20
40
60
8
Distance from end (m)
After anchorage
Before anchorage