FLOW IN OPEN CHANNELS

e-NOTES: Dr.N.Balasubramanya

Professor, Department of Civil Engineering, M.S.R.I.T., Bangalore-54

*** ***

Definition: An open channel is a passage in which liquid flows with a free surface, open channel flow has uniform atmospheric pressure exerted on its surface and is produced under the action of fluid weight. It is more difficult to analyse open channel flow due to its free surface. Flow is an open channel is essentially governed by Gravity force apart from inertia and viscous forces.

Classification: An open channel can be natural or artificial.

Natural: Open channels are streams,rivers,estuaries,etc.Such channels are irregular in shape,alignment and surface roughness.

Artificial open channels are built for some specific purpose, such as irrigation, water supply, water power development etc. Such channels are regular in shape and alignment. Surface roughness is also uniform.

Depending upon the shape, a channel is either prismatic or non-prismatic.

A channel is said to be prismatic when the cross section is uniform and the bed slope is constant. Ex. Rectangular, trapezoidal, circular, parabolic.

A channel is said to be non-prismatic when its cross section and for slope change. Ex: River,Streams & Estuary.

Depending upon the form, a channel is either exponential or non exponential.

A channel is said to be exponential when its area of cross section can be expressed in the form where, A = area of cross section

K = constant, y = depth of flow m= exponent.

Examples for exponential channel are: Rectangular, parabolic and triangular. Examples for non-exponential channels are trapezoidal & circular channels.

Depending upon the material a channel is said to be rigid boundary channel or mobile boundary channel or alluvial channel.

A channel with immovable bed and sides is known as rigid boundary channel. Ex: Concrete channel.

A channel composed of loose sedimentary particles moving under the action of flowing water is known as mobile boundary channel or alluvial channel.

Difference b/w pipe flow and open channel flow:

pipe Flow	Open Channel Flow
1. Flow occurs due to difference of pressure	1. Flow occurs due to the slope of the channel
2. Free surface is absent in a pipe flow	2. Free surface is present in an open channel flow.
3. Line joining piezometric surface (Z+p/γ) indicates the hydraulic Grade line	3. Liquid surface itself represents the hydraulic grade line (HGL)
4. There is no relation b/w the drop of the energy gradient line and slope of the pipe axis.	4. For uniform flow in an open channel, the drop in the energy gradient line is equal to the drop in the bed.

Types of flow in open channel

Flow in an open channel can be classified into different types based on different criteria.

a) Laminar and Turbulent flow: The ratio of inertia force to viscous force is known as Reynold's number Re and is written as $R_e = \frac{VL}{\neg}$

V=characteristic velocity (generally average velocity)

L=characteristic length

ν = Kinematic viscosity of the liquid.

Based on Reynold's number Re flow is said to be laminar when layers of liquid slide one over the other. This generally occurs at low Reynold's numbers. (Re is less than equal to 500)

where g=acceleration due to gravity.

Flow is critical if F=1.0

Flow is sub critical by F<1.

Flow is super critical if F>1

On the other hand flow in an open channel is also classified on the values of Froude's number and Reynold's number as

On the other hand flow in an open channel is also classified on the values of Froude number and Reynold's number as:

Subcritical laminar – F<1, $R_e \leq 500$
Supercritical laminar – F>1, $R_e \leq 500$
Subcritical laminar F<1, $R_e \geq 2000$
Supercritical turbulent F>1, $R_e \geq 2000$

c) Steady and Unsteady flow
Flow is an open channel is said to be steady when the depth, discharge mean velocity do not change with time.Ex: $\frac{\partial Q}{\partial t} = 0$

When these quantities change with time flow is know as unsteady Ex: $\frac{\partial Q}{\partial t} \neq 0$

d) Uniform and Non uniform flows
Uniform flow is one in which the depth, discharge, mean velocity etc. do not change along the channel at any given instant. $\frac{\partial Q}{\partial l} = 0$
Non uniform flow is one in which the above quantities change along the channel at any given instant. $\frac{\partial Q}{\partial l} \neq 0$
Non Uniform flow is also known as varied flow such a flow can be further divided into gradually varied and rapidly flows, depending on whether these flow variations are gradual or rapid.
In a gradually varied flow (GVF) the change occurs over a large length of the channel.
Ex: Flow behind a dam, flow over a spillway etc.,
In a rapidly varied flow (RVF) the change occurs over a short length of the channel.
Ex: Hydraulic jump
.
Geometric properties of open channels
Depth of flow (y): It is the vertical distance between the lowest point of the channel sections from the free liquid surface. It is expressed in meters.

Area of cross section or Wetted area (A) It is the area of the liquid surface when a cross section is taken normal to the direction of flow. It is expressed in meter2.
Wetted perimeter (P): It is the length of the channel boundary in contact with the flowing liquid at any section. It is expressed in meters.

Hydraulics radius or Hydraulic mean depth (R): It is the ratio of area of cross section (A) to the wetted perimeter(P). $\therefore R = \left(\frac{A}{P}\right)$ R is expressed in meters.

Top width (T): It is the width of the channel at the free surface as measured perpendicular to the direction of flow at any given section. It is expressed in meters.

Hydraulic depth (D) It is the ratio of area of cross section (A) to the top width (T). $\therefore D = \left(\frac{A}{T}\right)$ It is expressed in meters.

Section factors (Z): It is the product of the area of cross section (A) to the square root of the hydraulic depth (D).

$\therefore Z = A\sqrt{D} = \left(\frac{A^3}{T}\right)^{\frac{1}{2}}$ Z is expressed in meters.

Hydraulic Slope (S): Hydraulic slope of the total energy line is defined as the ratio of drop in total energy line (hf) to the channel length (L).

$$\therefore S = \left(\frac{h_f}{L}\right)$$

Geometric properties for different types of prismatic channels

- Rectangular Channel
-
- y=Depth of flow **(FOR FIGURES DOWNLOAD PRESENTATION)**
- B=Bed width of the channel

Area of cross section A= B x y

Wetted perimeter P = (B+2y)

Hydraulic radius $R = \frac{A}{P} = \left(\frac{By}{B+2y}\right)$

Top width T=B

Hydraulic depth $D = \frac{A}{T} = \frac{By}{B} = y$

b) Trapezoidal channel

n or ¯ is side slope of the channel.

Area of flow A = (Area of rectangular 2x Area of the half triangle)

$$By + 2\left(\frac{1}{2} ny \ x \ y\right)$$

$$A = \left(By + ny^2\right)$$

$$A = By + 2\left[\frac{1}{2} y \tan\text{¯} \ x \ y\right]$$

$$A = \left(By + y^2 \tan\text{¯}\right)$$

Wetted perimeter $P = B + 2\sqrt{n^2 y^2 + y^2}$

$$P = \left\lfloor B = 2y\sqrt{1+n^2} \right\rfloor$$

$$P = \left(B + 2y\sqrt{y^2 \tan^{2}\text{¯} + y^2}\right)$$

$$P = \left(B + 2y\sqrt{1 + \tan^{2}\text{¯}}\right)$$

Hydraulic radius

$$R = \frac{A}{P} = \left\{\frac{By + ny^2}{B + 2y\sqrt{1+n^2}}\right\} \quad \text{Or} \quad R = \left\{\frac{By + y^2 \tan\text{¯}}{B + 2y\sqrt{1+\tan^{2}\text{¯}}}\right\}$$

Top Width

$T = \left(B + 2ny\right)$ or $T = \left(B + 2y \tan\text{¯}\right)$

Hydraulic Depth

$$D = \frac{A}{T} = \left(\frac{By + ny^2}{B + 2ny}\right) \quad \text{or} \quad \frac{By + y^2 \tan\text{¯}}{B + 2y\sqrt{1+\tan^{2}\text{¯}}}$$

Triangular channel

Area of cross section

$$A = 2\left(\frac{1}{2} Base\ x\ altitude\right) \quad = 2\left(\frac{1}{2} ny\ x\ y\right)$$

$$A = ny^2 \quad \text{or} \quad A = 2\left(\frac{1}{2} y \tan\theta\ x\ y\right) \qquad A = y^2 \tan\theta$$

Wetted Perimeter

$$P = 2\ x\ \sqrt{n^2 y^2 + y^2} \qquad P = 2y\sqrt{1+n^2}$$

$$P = 2\ x\ \sqrt{y^2 \tan^2\theta + y^2} \qquad P = \left(2y\sqrt{1+\tan^2\theta}\right)$$

Top Width

$$T = 2ny \quad \text{Or} \quad 2y\tan\theta$$

Hydraulic Radius

$$R = \frac{A}{P} = \left(\frac{ny^2}{2y\sqrt{1+n^2}}\right) \quad = \left(\frac{y^2 \tan\theta}{2y\sqrt{1+\tan^2\theta}}\right)$$

Hydraulic Depth

$$D = \frac{A}{T} = \frac{ny^2}{2ny} = y/2 \quad \underline{\text{or}} \quad = \frac{y^2 \tan\theta}{2y\sqrt{1+\tan^2\theta}} = y/2$$

Circular Channel

oa=ob=oc=r (radius)

bd=y (depth of flow)

$a\hat{o}d = \theta (centralangle)$

Area of Flow A=[Area of sector oabc-Area of triangle oac

$$= \left[r^2\theta - 2\left(\frac{1}{2} r \sin\theta\ x\ r\cos\theta\right)\right] \quad = r^2\theta - \frac{1}{2} r^2 \left(2\sin\theta\ \cos\theta\right) \quad = r\left(\theta - \frac{2\sin\theta\ \cos\theta}{2}\right)$$

$$A = r^2\left(\theta - \frac{\sin 2\theta}{2}\right)$$

θ Should be in radians Π radians=180^0

Wetted Perimeter P = Arc length oabc $= r\ x\ 2\theta \qquad P = 2r\theta$

Hydraulic radius

$$R = \frac{A}{P} \quad = \left\{\frac{r^2\left(\theta - \frac{\sin 2\theta}{2}\right)}{2r\theta}\right\}$$

Uniform Flow in open channels

Flow in an open channel is said to be uniform when the parameters such as depth area of cross section, velocity discharge etc., remain constant throughout the entire length of the channel.

Features of Uniform flow

a] Depth of flow, area of cross section, velocity and discharge are constant at every section along the channel reach.

b] Total energy line, water surface and channel bottom are parallel to each other, also their slopes are

Equal or $s_0 = s_w = s_f$

$S_0 = channel\ bed\ slope,\ s_w = water\ surface\ slope,$

$s_f = energy\ line\ slope.$

CHEZY'S FORMULA

Consider uniform flow between two sections 11 and 2 2, L distant apart as shown

Various forces acting on the control volume are:

i] Hydrostatic forces

ii] Component of weight w $\sin\theta$, along the flow.

iii] Shear or resistance to flow acting along the wetted perimeter and opposite to the direction of motion

From second law of Newton

Force = Mass x acceleration

As the flow is uniform, acceleration = Zero (O) $\therefore \sum forces = 0$

Or

$forces = +F_1 - F_2 + w\sin\theta - \tau_0\ x\ contact\ area = 0$

$Again\ F_1 = F_2 \because Flow\ is\ uniform$

$\therefore w\sin\theta - \tau_0\ x\ contact\ area = 0$

$\therefore w\sin\theta = \tau_0\ x\ contact\ area - (1)$

From the definition of specific weight $\gamma = \frac{weight}{volume}$

Weight w = γ x volume

= γ x A x L

Contact area = wetted perimeter x length = P x L

Also, for small values of $\theta, \sin\theta \approx \tan\theta = S_0$

Substituting all values in eq 1 and simplifying

$AL\,S_0 = \tau_0\,PL \qquad \tau_0 = \frac{A}{P} S_0$

$But,\ \frac{A}{P} = R\ (Hydraulic\ radius)$

$\therefore \tau_0 = \gamma R S_0 - (2)$

From experiment it is established that shear stress $\tau_0 = \frac{f}{8} \rho V^2$

$\therefore \frac{f}{8} \rho V^2 = \gamma R S_0 \qquad V = \sqrt{\frac{8\gamma}{\rho f}} \sqrt{R S_0}$ or

$V = C\sqrt{RS}$

Where, $C = \sqrt{\frac{8\gamma}{\rho f}}$ C= Chezy's constant

From continuity equation Q=AV

$\therefore Q = AC\sqrt{RS_0} \quad -(3)$

It should be noted that chezy's C is not just a non – dimensional number and it has a dimension of $\left[L^{\frac{1}{2}}\ T^{-1} \right]$

Chezy's equation is used in pipe flow also. The value of Chezy's C is different for Different types of channels.

MANNING'S FORMULA

Robert Manning in 1889, proposed the formula $V = \frac{1}{N} R^{\frac{2}{3}}\ S_0^{\frac{1}{2}}$

The above formula is known as Manning's formula where N is Manning's roughness or rugosity coefficient. Similar to Chezy's C

Table 1 gives the range of value of the **Manning's constant N**

No	Surface	Recommended Value of N
1	Glass, Plastic, Brass	0.010
2	Timber	0.011 – 0.014
3	Cement plaster	0.011
4	Cast iron	0.013
5	Concrete	0.012 – 0.017
6	Drainage tile	0.013
7	Brickwork	0.014
8	Rubble masonry	0.017 – 0.025
9	Rock cut	0.035 – 0.040

<u>PROBLEMS:-</u>

1. Establish a relation between Chezy's C and Manning's N

Soln: Chezy's equation is $V = C\sqrt{RS_0}$

Manning's equation is $V = \frac{1}{n} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$

Equating the two equations $CR^{\frac{1}{2}} S_0^{\frac{1}{2}} = \frac{1}{n} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$ $\qquad C = \frac{1}{N} R^{\frac{1}{6}}$

Manning's N has dimensions. The dimensions of N being $\left[TL^{\frac{-1}{3}} \right]$

2. A rectangular channel 1.5 m wide with a bed slope of 0.0001 carries water to a depth of 1.2m. The channel has Manning's N=0.025. Calculate the rate of uniform flow in the channel.

Soln: B=1.5m, y=1.2m, N=0.025,

$$Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

$$where\ A = BY = 1.5\ x\, 1.2 = 1.8\, m^2$$

$$P = B + 2y = (1.5 + 2\ x\, 1.2) = 3.9\ m$$

$$R = \frac{A}{P} = \frac{1.8}{3.9} = 0.4615\ m$$

$$\therefore Q = \frac{1.8}{0.025}\ x\, (0.4615)^{\frac{2}{3}}\ (0.0001)^{\frac{1}{2}}$$

$$Q = 0.43\, m^3 / s$$

3. Calculate the uniform depth of flow in a rectangular channel of 3m width designed to carry 10 cumecs of water.Given Chezy's C=65 and channel bed slope= 0.025 %.

Ans. B=3m, y=?,Q=10 cumecs,

C=65,S0=0.025%

Chezy's eqn is Q=AC(RS0)0.5

A=By=3y m2

$$P = B + 2y = (3 + 2y)\, m \qquad R = \frac{A}{P} = \left(\frac{3y}{3+2y}\right)$$

Substituting all values in chezy's eqn $10 = 3y\ x\ 65\ x\ \sqrt{\left(\frac{3y}{3+2y}\right)\left(\frac{0.025}{100}\right)}$

$$1.8726 = y\ x \left(\frac{y}{3+2y}\right)^{\frac{1}{2}} \qquad 1.8726\,(3+2y)^{\frac{1}{2}} = y^{\frac{3}{2}}$$

$$3.5065\,(3+2y) = y^3 \qquad \therefore y^3 - 7.013y - 10.519 = 0$$

Solving by trial and error y=3.21m

4. Find the rate of flow of water through a triangular channel having the total angle between the sides as 60. Take the value of N=0.015 and the slope bed as 1m in 1km. The depth of flow is 1.6m

$$Q = ?, {}^{-} = 30^0, N = 0.015, S_0 = 1m \ in \ 1km = \frac{1}{1000}$$

$$y = 1.6m \ Manning's \ equation \ Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

$$A = 2\left(\frac{1}{2} x \ y \tan^{-} x \ y\right) = 1.6^2 \ \tan 30^0 = 1.478m^2$$

$$P = 2y\sqrt{1+\tan^{2-}} = 2 \ x \ 1.6 \ x \ \sqrt{1+\tan^2 30^0} = 3.695m$$

$$R = \frac{A}{P} = \frac{1.478}{3.695} = 0.40m$$

$$\therefore Q = \frac{1.478}{0.015} x \ 0.4^{\frac{3}{2}} \ x \left(\frac{1}{1000}\right)^{\frac{1}{2}} = 0.789 \ m^3/s$$

5. Water flows at a velocity of 1 m/s in a rectangular channel 1m wide. The bed slope is 2x10-3 area N=0.015. find the depth of flow under uniform flow conditions.
Soln: v=1 m/s ,B=1m,S =2x10, N=0.015, y=?

From Manning's equation $V = \frac{1}{N} R^{\frac{2}{3}} \ S_0^{\frac{1}{2}}$

If the depth of flow is constant at 1.7m calculate (a) the hydraulic mean depth (b) the velocity of flow (c) the volume rate of flow. Assume that the value of coefficient C in the Chezy's formula is 50.
Soln:

$$B = 2.5M, \ S_0 = \frac{1}{500}, \ Y = 1.7m, \ R = ?, \ V = ? \quad Q=?, C=50$$

$$A = BY = 2.5 \ X \ 1.7 = 4.25m^2$$

$$P = B + 2Y = (2.5 + 2 \ X \ 1.7) = 5.9 \ m$$

$$R = \frac{A}{P} = \frac{4.25}{5.9} = 0.72 \ m$$

$$V = C\sqrt{RS_0} = 50 \ X \ \sqrt{0.72 \ X \ \frac{1}{500}} = 1.9 \ m/s$$

$$Q = AV = 4.325 \ X \ 1.9 = 8.066 \ m^3/s$$

7. An open channel of trapezoidal section base width 1.5m and side slopes 60 to the horizontal is used to convey water at a constant depth of 1m. If the channel bed slope is 1: 400. Compute the discharge in cumecs. The Chezy's constant may be evaluated using the relation $C = \frac{87}{1+\left(0.2/\sqrt{R}\right)}$ Where R is the hydraulic radius (VTU, Aug 2005)

$$\left(By + y^2 \tan\right) = \left(1.5\ x\ 1 + 1^2\ \ x\ \tan 30^0\right) = 2.077 m^2$$

$$\text{p=}\left(B + 2y\sqrt{1+\tan^2}\right) = \left(1.5 + 2\ x\ 1\ x\sqrt{1+\tan^2 30^0}\right) = 3.81\,\text{m}$$

$$Hyd\ radius\ R = \frac{A}{P} = \frac{2.077}{3.81} = 0.545\ m$$

$$From\ \ chezy's\ \ eqn\ \ Q = AC\sqrt{RS_0}$$

$$= 2.077\ x\ 68.46\ x\ \sqrt{0.545\ x\ \frac{1}{400}}$$

$$\therefore Q = 5.25\ \ m^3/s\ \ or\ \ cumecs$$

8.A channel 5m wide at the top and 2m deep has sides sloping 2v:1H. The volume rate of flow when the depth of water is constant at 1m. Take C=53. What would be the depth of water if the flow were to be doubled.

Soln: From fig T=B+2ny

$$5 = B + 2\ x\ \frac{1}{2} x\ 2\ \ \therefore B = 3\ m$$

Given the depth of flow y=1m

$$A = By + ny^2 = 3\ x\ 1 + \frac{1}{2} x\ 1^2 = 3.5\ m^2$$

$$P = B + 2y\sqrt{1+n^2} = \left\{3 + 2\ x\ 1\ x\ \sqrt{1+\left(\frac{1}{2}\right)^2}\right\} = 5.24\ m$$

$$R = \frac{A}{P} = \frac{3.5}{5.24} = 0.669\ \ m$$

$$From\ \ Chezy's\ \ eqn\ \ Q = AC\sqrt{RS_0}$$

$$= 3.5\ x\ 53\ x\ \sqrt{\left(0.669\ x\ \frac{1}{1000}\right)} \qquad Q = 4.8\ m^3/s$$

$Now,\ Q = 2\ x\ 4.8 = 9.6\ m^3/s,\ y_1 = ?$

$From\ Chezy's\ eqn\ Q = AC\sqrt{RS_0}$

$$9.6 = \left(3\ x\ y_1 + \frac{1}{2}\ x\ y_1^2\right) x\ 53\ x \sqrt{\frac{3y_1 + \frac{1}{2}y_1^2}{3 + 2y_1\ x\sqrt{1 + \left(\frac{1}{2}\right)^2}}}$$

Solving by trial and error y_1 = 1.6m

9. A trapezoidal channel 1.8 m wide at the bottom and having sides of slope 1:1 is laid on a slope of 0.0016. If the depth of the water is 1.5m. Find the rate of uniform flow Assume N=0.014

Soln: B=1.8m, n=1, S_0 =0.0016, y=1.5m Q=? N=0.014

$A = By + ny^2 = (1.8\ x\ 1.5 + 1\ x\ 1.5^2) = 4.95m^2$

$P = (B + 2y\sqrt{1+n^2} = (1.8 + 2\ x1.5\ x\ \sqrt{1+1.5^2}) = 7.21m$

$R = \frac{A}{P} = \frac{4.95}{7.21} = 0.687m$

$From\ Manning's\ eqn\ Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$

$= \frac{4.95}{0.014}\ x\ (0.687)^{\frac{2}{3}}\ (0.0016)^{\frac{1}{2}}$

$Q = 11\ m^3/s$

10. A concrete lined trapezoidal channel with side slope 2H:IV has a base width of 3m and carries 5.5 m 3/s of water on a slope of 1m 10000. Find the depth of flow. Assume N=0.011

Soln: n=2,B=3m, Q=5.5 m 3/s,y=?, N=0.011, $S_0 = \frac{1}{10000}$

$Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$ $\quad A = By + ny^2 = 3y + 2y^2$

$P = B + 2y\sqrt{1+n^2} = B + 2y\sqrt{1+2^2} = 3 + 4.47y$

$R = \frac{A}{P} = (\frac{3y + 2y^2}{3 + 4.47y})$

$\therefore 5.5 = \left(\frac{3y + 2y^2}{0.011}\right) x \left(\frac{3y + 2y^2}{3 + 4.47y}\right)^{\frac{2}{3}} \left(\frac{1}{10{,}000}\right)^{\frac{1}{2}}$

Solving by trial and error y=1.32m

11. A trapezoidal channel is designed to convey 1.5 cumces of water at a depth of 1m if the mean velocity of flow is0.5 m/s and side slopes are1:1 find the base width and the bed slope. Take C=60
Soln: Q=1.5 cumecs y=1m, v=0.5m/s
n=1 B=? C=60

$From\ continuity\ eqn\ A = \frac{Q}{V} = \frac{1.5}{0.5} = 3m^2 \qquad A = By + ny^2$

$3 = B\ x\,1 + 1\,x\,1^2 \therefore B = 2m$

$P = B + 2y\sqrt{1 + n^2}$

$= \left(2 + 2\ x\,1\,x\ \sqrt{1 + 1^2}\right) = 4.828m$

$R = \frac{A}{P} = \frac{3}{4.828} = 0.621m$

$Now\ from\ chezy's\ eqn\ V = C\sqrt{RS_0}$

$0.5 = 60\ x\ \sqrt{0.621\,x\,S_0} \qquad S_0 = 1.118\,x\,10^{-4} = \frac{1}{8947}$

12.Water flows through a channel of circular section of 600mm diameter at the rate of 200lps the slope of the channel is 1m in 2.5km and the depth of flow is 0.45m. Calculate the mean velocity and the value of chezy's coefficient
Soln: Q=200lps=0.20m 3/s
Ac=y=0.45m
Oc=r=600/2=300mm=0.3m
Oa=(ac-oc)=(0.45-0.3)=0.15m
From triangle oab

$\frac{oa}{ob} = \cos\sim \qquad \cos\sim = \frac{0.15}{0.3} = 0.5$

$\sim\, = 60^0$ *and* $\,^{-}\, = (180 - \sim) = 120^0 = \frac{a}{180}\, x\, 120^0 = 2.094$

$= 0.3^2\left(2.094 - \frac{\sin 2\ x\ 2.094}{2}\right) \qquad A = 0.2275 m^2$

Wetted perimeter P=2rθ=2x0.3x2.094=1.2564m

$Hydraulic\ \ radius\ \ R = \frac{A}{P} = \frac{0.2275}{1.2564} = 0.181 m$

$From\ \ Chery's\ \ eqn\ \ Q = AC\sqrt{RS_0}$

$C = \left\{ \frac{0.2}{0.2275\ x\ \sqrt{\left(0.181\ x\ \frac{1}{2500}\right)}} \right\} \qquad$ C=103.3

$From\ \ continuity\ eqn\ \ V = \frac{Q}{A} = \frac{0.2}{0.2275}$

V=0.88m/s

13. An open channel has a cross section semicircular at the bottom with vertical sides and is 1.2m wide. It is laid at a bed slope of 0.375m per km. Calculate the values of chezy's C and Manning's N, if the depth of flow Is 1.2m while the discharge is 0.85 m3/s

Soln: C=? N=? Y=1.2m, Q= 0.85m 3/s, $S_0 = \frac{0.375}{1000}$

Area of flow A=(Area of rectangle A_1 + Area of semicircle A_2)

$= \left(0.6\ x\ 1.2 + \frac{®x\ 0.6^2}{2}\right) \qquad A = 1.2856\ m^2$

Wetted perimeter P=[2 x 0.6+ Π 0.6] = 3.085 m

Hydraulic radius $R = \frac{A}{P} = \frac{1.2856}{3.085} = 0.417\ \ m$

$From\ \ Chezy's\ \ eqn\ \ Q = AC\sqrt{RS_0}$

$$\therefore C = \left\{ \frac{0.85}{1.2856\ x\ \sqrt{0.417\ x\ \frac{0.375}{1000}}} \right\}$$

$A = 52.9$

Relation between Manning's N and Chezy's C is

$$C = \frac{1}{N} R^{\frac{1}{6}}$$

$$\therefore N = \left\{ \frac{R^{\frac{1}{6}}}{C} \right\} = \left\{ \frac{(0.417)^{\frac{1}{6}}}{52.9} \right\} \qquad \text{N=0.0163}$$

14. Water is conveyed in a channel of semicircular cross section with a stage of 1 in 2500. The chezy's coefficient C has a value of 56. If the radius of the channel is 0.55 m. what will be the volume rate of flow in m3 /s flowing when the depth is equal to the radius?

If the channel had been rectangular in the form with the same width of

The form width the same width of 1.1m and depth of flow of 0.55m. What would be the discharge for the same slope and value of C ?.

Soln: Case (i) Semicircular channel

C=56, r=0.55m,Q=?,y=r=0.55m i.e the channel is flowing full. $S_0 = \frac{1}{2500}$

$$A = \frac{®r^2}{2} = \frac{®\ x\ 0.55^2}{2} = 0.475\ m^2$$

$$P = ®r = ®\ x\ 0.55 = 1.728\ m$$

$$R = \frac{A}{P} = \frac{0.475}{1.728} = 0.275\ \ m$$

$$Q = AC\sqrt{RS_0} = 0.475\ x\ 56\ x\ \sqrt{\left(0.275\ x\ \frac{1}{2500}\right)}$$

$$Q = 0.279\ m^3 / S$$

Case (ii) Rectangular channel

B=1.1m,y=0.55m,C=56,Q=? $S_0 = \frac{1}{2500}$

A=By=1.1 x 0.55=0.605
P=B+2y=1.1+2x0.55=2.2m

$$R = \frac{A}{P} = \frac{0.605}{2.2} = 0.275m$$

$$Q = AC\sqrt{RS_0}$$

$$= 0.605\ x\ 56\ x\ \sqrt{\left(0.275\ x\ \frac{1}{2500}\right)}$$

$$Q = 0.3553\ m^3 / S$$

15. A rectangular channel conveys a discharge of 9.6 cumecs. If the width of channel is 6m, find the depth of flow.
Take C=55 and bed slope = 2x10-4
Soln:

$$Q = AC\sqrt{RS_0} \qquad A = By = 6ym^2; \quad P = B + 2y \ = (6+2y)m$$

$$R = \frac{A}{P} = \frac{6y}{6+2y} \qquad C = 55, S_0 = 2x10^{-4}$$

Substituting all values in eq (i)

$$9.6 = 6yx55x\sqrt{\left(\frac{6y}{6+2y}\right)x2x10^{-4}} \qquad \frac{y^3}{6=2y} = 0.705$$

Solving by trial and error $y = 1.92m$

16. A flow of 100 lps flow down in a rectangular flume of 60cm width and having adjustable bottom slope if Chezy's constant C is 56, find the bottom slope necessary for uniform flow with a depth of flow of 30cm. Also calculate the conveyance k for the flume.

Soln:
Area of flow A=By=0.6x0.3=0.18m2

Wetted perimeterP= B+2y=0.6+2x0.3
Hydraulic radius R=A/P=0.18/1.2=0.15m
From Chezy's formula $Q = AC\sqrt{RS_0}$

$$\therefore S_0 = \frac{Q^2}{C^2RA^2} = \frac{0.1^2}{56^2x0.15x0.18^2} = 6.56x10^{-4}$$

$$\therefore S_0 = \frac{1}{1524}$$

Conveyance $K = AC\sqrt{R} = 0.18x56x\sqrt{0.15} = 3.904 cumec$

17. A channel of trapezoidal section has a bottom width of 5m, one side is sloping at 400 with the vertical and the other has a slope of 1V to 2H. If the depth of flow is 1.5m, find the bed slope required to discharge 35 cumecs. Taking Manning's N=0.017.

$$A = \left(5x1.5 + \frac{1}{2}x1.26x1.5 + \frac{1}{2}x3x1.5\right) = 10.69m^2$$

$$P = \left(5 + \sqrt{3^2 + 1.5^2} + \sqrt{1.26^2 + 1.5^2}\right) = 10.313m$$

$$R = \frac{A}{P} = \frac{10.69}{10.313} = 1.0365m \qquad Q = \frac{A}{N}R^{\frac{2}{3}}S^{\frac{1}{2}}{}_0$$

$$35 = \frac{10.69}{0.017}x(1.0365)^{\frac{2}{3}}(S_0)^{\frac{1}{2}}$$

$$S = 1/338.6$$

18. An earthen canal in good condition is 16.8 wide at the bottom and has side slopes of 2H to 1V. One side slope extends to a height of 2.52m above the bottom level and the other side extends flat to a distance of 150m and rises vertically. If the slope of the canal is 69cm per 1584m estimate the discharge when the depth of water is 2.52m. Assume C=35

Wetted perimeter P=length AB+BC+CD+DE+EF.

$$= \sqrt{2.52^2 + 5.04^2} + 16.80\sqrt{1.8^2 + 3.6^2} + 150 + 0.72$$

$$\therefore P = 177.2m$$

Cross section area A=A1+A2+A3+A4

$$= 0.5X\,2.52X\,5.04 + 16.80X\,2.52 + \frac{2.52+0.72}{2}X\,3.6 + 150X\,0.72$$

$A = 160.5m^2$

Hydraulic radius R=A/P= 160.5/177.2=0.906m

From Chezy's equation $Q = AC\sqrt{RS_0}$ $= 160.5X\,35X\sqrt{\left(0.906X\,\frac{0.69}{15.84}\right)}$

$Q = 111.6m^3S^{-1}$

19. A circular sewer of 500mm internal dia, has a slope of 1in 144. Find the depth when the discharge is 0.3 cumecs.Take Chezy's C=50.

Soln:

Let 2α be the angle subtended by the free surface at the centre.

Area of flow $A = \left(\frac{d^2}{4}\sim - \frac{d^2}{8}Sin2\sim\right)$

Wetted perimeter $P = \sim d$

$= 160.5X\,35X\sqrt{\left(0.906X\,\frac{0.69}{15.84}\right)}$

$Q = 111.6m^3S^{-1}$

Now, $Q = AC\sqrt{RS_0}$ $= CA^{\frac{3}{2}}S_0^{\frac{1}{2}}/P^{\frac{1}{2}}$

Squaring both the sides

$$Q^2 = \frac{C^2A^3S_0}{P}$$

Substituting all values

$$0.3^2 = \frac{50^2x\left(\frac{d^2}{4}\sim - \frac{d^2}{8}Sin2\sim\right)^3 x1}{\sim d\ \ x\ \ 144}$$

Substituting d=0.5m

$(2\sim - Sin2\sim)^3 = 85\sim$

Solving by trial & error $\sim = 2.5radians = 143^0$

The corresponding depth of flow

$D = \frac{d}{2}\left(1-\cos143^0\right)$ $= \frac{0.5}{2}\left(1-\cos143^0\right)$

$\therefore D = 0.45m$

20. A trapezoidal channel having a cross sectional area A1, wetted perimeter P1, Manning's N is laid to a slope of S, carries a certain discharge Q1, at a depth of flow equal to d. To increase the discharge, the base width of the channel is widened by x, keeping all other parameters same. Prove that $\left(\frac{Q_2}{Q_1}\right)^3 = \left(1+\frac{x}{P_1}\right)^2 = \left(1+\frac{xd}{A_1}\right)^5$

$$Soln: Q_1 = A_1 x \frac{1}{N}\left(\frac{A_1}{P_1}\right)^{\frac{2}{3}} S^{\frac{1}{2}}$$

In the second case,

$$Q_2 = A_2 x \frac{1}{N}\left(\frac{A_2}{P_2}\right)^{\frac{2}{3}}\left(\frac{A_2}{A_1}\right)$$

$$\therefore \frac{Q_2}{Q_1} = \left(\frac{A_2}{P_2} x \frac{P_1}{P_2^2}\right)^{\frac{2}{3}}\left(\frac{A_2}{A_1}\right)$$

$$\left(\frac{Q_2}{Q_1}\right)^3 = \frac{A_2^5}{A_1^5}\ \frac{P_1^2}{P_2^2}$$

Now, $P_2 = P_1 + x \qquad A_2 = A_1 + xd$

Substituting these values & simplifying

$$\therefore \frac{Q_2}{Q_1} = \left(\frac{A_1 + xd}{A_1}\right)^5 \left(\frac{P_1}{P_1 + x}\right)^2$$

Or,

$$\therefore \left(\frac{Q_2}{Q_1}\right)^3 = \left(1+\frac{x}{P_1}\right)^2 = \left(1+\frac{xd}{A_1}\right)^5$$

21. Water is flowing through a circular open channel at the rate of 400lps. When the channel is having a bed slope of 1in 9000. Find the diameter of the channel, if the depth of flow is 1.25 times the radius of the channel. Take N=0.015.

Solution;

$$Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$$

From Figure $\cos\sim = \frac{0.25r}{r} = 0.25$

$$\sim = 75.52^0, \bar{} = 104.48^0 = 1.8235 radians$$

$$A = r^2\left(\bar{} - \frac{Sin2\bar{}}{2}\right) = r^2\left\{1.8235 - \frac{Sin2x1.8235}{2}\right\}$$

$$A = 2.0655r^2$$

$$P = 2r\bar{} = 2rx1.8235 = 3.647r$$

$$R = \frac{A}{P} = \frac{2.0655r^2}{3.647r} = 0.566r$$

$$\therefore 0.4 = \frac{2.0655r^2}{0.015}x(0.566r)^{\frac{2}{3}}\left(\frac{1}{9000}\right)^{\frac{1}{2}}$$

$$0.4 = 0.9936r^{\frac{8}{3}} \qquad r = \left(\frac{0.4}{0.9936}\right)^{\frac{3}{8}} = 0.71m$$

$$\therefore Diameter\ of\ the\ channel\ d = 2r = 1.422m$$

MOST ECONOMICAL
OR
MOST EFFICIENT
OR
BEST HYDRAULIC OPEN CHANNEL

Definition: The most efficient cross section may be defined as that offers least resistance to flow and hence passes maximum discharge for a given slope, area and roughness.

From continuity equation Q=AV, Discharge Q is maximum when the velocity V is maximum for a given area of cross section A

From Chezy's equation $V = c\sqrt{RS_0}$,Velocity V is maximum when the hydraulic radius R is maximum for given values of Chezy's C and bed slope S_0

But, by definition Hydraulic radius $R = \frac{Area(A)}{Wetted\ Perimeter(P)}$

Therefore hydraulic radius R is maximum when the wetted perimeter P is minimum for a given area of cross section A

Hence an open channel is most economical when the wetted perimeter P is least or minimum for a given area of cross section A.

MOST ECONOMICAL RECTANGULAR OPEN CHANNEL

Area A=BY (i)

B=A/Y (ii)

Wetted perimeter P=B+2Y (iii)

Substituting eq (ii) in eq (iii)

$$P = \frac{A}{Y} + 2Y$$

For the channel to be most economical wetted perimeter should be minimum. But from Eq (iv) we see that P is a function of the depth of flow Y, for a given area of cross section A. Hence for the condition is that $\frac{dP}{dY} = 0$

$$i.e \frac{d}{dy}\left\{\frac{A}{y} + 2y\right\} = 0$$

$$Differentiating \ \frac{-A}{Y^2} + 2 = 0$$

Or, A=2Y2 --(v)

Equating Eq (i) and (v) we have

$BY=2Y^2$ or B=2Y --(vi)

Also, Hydraulic radius $R = \frac{A}{P} = \left\{\frac{BY}{B+2Y}\right\} = \left\{\frac{(2Y)Y}{2Y+2Y}\right\}$

$$Therefore, R = \left\{\frac{Y}{2}\right\}$$

It may thus be concluded that for a rectangular channel to be most economical or efficient the bed width Should be twice the depth of flow or the hydraulic radius R should be half the depth of the flow.

MOST ECONOMICAL TRIANGULAR CHANNEL

2θ= Central angle

Side slopes of the channel is IV:nH

Y= Depth of the flow

Area of cross section $A = \frac{1}{2}(2ny)\,x\,Y$

$$A = ny^2 --- (i)$$

$$A = 2\left(\frac{1}{2}Y \tan^{-}\,x\,Y\right) = Y^2 \tan^{-}$$

$$\therefore Y = \sqrt{\frac{A}{\tan\theta}} \; --(ii)$$

Wetted perimeter $P = 2\sqrt{y^2 \tan^2\theta + y^2} \quad = 2y\sqrt{1+\tan^2\theta} \; ---(iii)$

Substituting eq(i) in eq (iii) and simplifying

$$P = 2\sqrt{\frac{y}{\tan\theta}} \; x \; \sqrt{1+\tan^2\theta} \; ---(iv)$$

$$P = 2\sqrt{y} \;\; \text{x} \sqrt{\frac{1+\tan^2\theta}{\tan\theta}}$$

Channel is most economical when the wetted perimeter P is minimum i.e when $\frac{dP}{d\theta} = 0$

because P is a function of θ for a given value of Y

$$\therefore \frac{\partial}{\partial\theta}\left\{\frac{1+\tan^2\theta}{\tan\theta}\right\} = 0 \; , \quad \frac{\partial}{\partial\theta}\left\{\frac{1}{\tan\theta} + \tan\theta\right\} = 0 \, , \quad \frac{\partial}{\partial\theta}\{\cot\theta + \tan\theta\} = 0$$

Differianting and simplifying $\sin\theta = \cos\theta$ or $\tan\theta = 1 \, or \, \theta = 45^0 \;\; n = 1$

Hence the triangular channel is most economical when the side slopes are IV:IH or the side slope at 45 with the vertical.

The corresponding hydraulic radius R for a most economical triangular section would be

$$R = \frac{A}{P} = \frac{y^2 \tan\theta}{2y\sqrt{1+\tan^2 0}} \; = \frac{y^2 \tan 45^0}{2y\sqrt{1+\tan 45^0}} \; = \left(\frac{y}{2\sqrt{2}}\right)$$

MOST ECONOMICAL TRAPEZODIAL CHANNEL

From the geometry of such a channel we see that it is a combination of a rectangular and a triangle. In a rectangular channel for a given bed width B the hydraulic radius R was a function of the depth of flow y. while in the case of a triangular channel it was a function of side slope

Hence, the condition for most economical trapezodial channel may be discussed under the following two headings.

In a rectangular channel for a given bed width B the hydraulic radius R was a function of the depth of flow y. while in the case of a triangular channel it was a function of side slope

a)Depth of flow y varying but side slope n constant

A=By+ny (i)

$$B = \frac{A-ny^2}{y} \;\; or \; B = \left(\frac{A}{y} - ny\right) \; ---(ii)$$

Wetted perimeter $P = \{B + 2y\sqrt{1+n^2}\} --- (iii)$

Substituting the value of B from Eq (ii) in Eq(iii)

$$P = \left\{\frac{A}{y} - ny + 2y\sqrt{1+n^2}\right\} \; --- (iv)$$

For given values of A and n, P is a function of y. therefore P is minimum when $\frac{\partial P}{\partial y} = 0$

$$\frac{\partial}{\partial y}\left\{\frac{A}{y} - ny + 2y\sqrt{1+n^2}\right\} = 0$$

$$Differentiating, \; \frac{-A}{y^2} - n + 2\sqrt{1+n^2} = 0$$

$$A = \left(2\sqrt{1+n^2} - n\right)y^2 \; --- (v)$$

Equating (i) and (v)

$$By + ny^2 = 2y^2\sqrt{1+n^2} - ny^2$$

$$2y^2\sqrt{1+n^2} = By + 2ny^2$$

$$or, \; 2y\sqrt{1+n^2} = (B + 2ny) \quad vi$$

Substiuting eq(vi) in eq(iii)

P=B+(B+2ny) or P=2B+2ny

$$Hydraulic \;\; radius \;\; R = \frac{A}{P} = \frac{By + ny^2}{2B + 2ny} \;\; = \frac{y(B+ny)}{2(B+ny)} \qquad \therefore R = \frac{y}{2}$$

Hence the trapezodial channel is most economical when the hydraulic depth is half the depth of flow for given values of area of cross section A, and side slopes n

b) Depth of flow y constant side slopes n variable

A= By+ny (i)

$$B = \frac{A}{y} - ny \qquad (ii)$$

$$P = B + 2y\sqrt{1+n^2} \quad (iii)$$

Substituting eq(ii) in eq(iii)

$$P = \frac{A}{y} - ny + 2y\sqrt{1+n^2} \quad (iv)$$

For given value of A and y, p the wetted perimeter is a function of the side slope n. P is minimum when $\frac{\partial p}{\partial n} = 0$

$$i.e,\ \frac{\partial}{\partial n}\left\{\frac{A}{y} - ny + 2y\sqrt{1+n^2}\right\} = 0$$

Differentiating and equating to zero

$$O - y + 2y\frac{\partial}{\partial n}\left(1+n^2\right)^{\frac{1}{2}} = 0 \qquad \left\{-y + 2y\ x\ \frac{1}{2}\left(1+n^2\right)^{\frac{1}{2}-1}(2n)\right\} = 0$$

$$-1 + \frac{2n}{\left(1+n^2\right)^{\frac{1}{2}}} = 0 \qquad or,\ 2n = \left(1+n^2\right)^{\frac{1}{2}}$$

Squaring both sides and simplifying $3n^2 = 1\ \ or\ \ n = \left(\frac{1}{\sqrt{3}}\right)$

$$from\ \ fig,\ \tan¯ = \frac{ny}{y} = n\ ,\ i.e,\ \tan¯ = \left(\frac{1}{\sqrt{3}}\right) \qquad \therefore\ ¯ = 30^0\ \ with\ \ the\ \ vertical$$

Or, 60^0 with the horizontal or bed of the channel.
From the geometry of a regular hexagon, we know that the external angle of 600 corresponds to a regular hexagon. Hence, we can conclude that a best hydraulic trapezodial channel corresponds to half of a regular hexagon.

MOST ECONOMICAL CIRCULAR CHANNEL

For a circular channel $A = r^2\left(¯ - \frac{\sin 2¯}{2}\right) \qquad P = 2r¯$

From the above two equations, we see that both A and P are functions of ¯ only. But for the channel to be most economical we know that P should be minimum which means that ¯ should be zero. However when $¯ = 0^0$ the channel cannot exist.

Hence the following two conditions.

a) Condition for maximum velocity

From chezy's equation V= C√RS_0
For given values of C and velocity V is maximum when the hydraulic radius R is maximum $i.e\ \frac{\partial(R)}{\partial ¯} = 0\ \because R\ is\ a\ \ function\ of\ ¯$

$$or,\ \frac{\partial}{\partial ¯}\left(\frac{A}{P}\right)=0$$

$$Differianting,\ \left\{\frac{P\dfrac{\partial A}{\partial ¯}-A\dfrac{\partial P}{\partial ¯}}{P^2}\right\}=0 \qquad or,\ \left\{P\frac{\partial A}{\partial ¯}-A\frac{\partial P}{\partial ¯}\right\}=0$$

Substituting for A and P and simplifying tan $2¯ = 2¯$

$¯ = 2.247\ \ Radians\ = 128^0 45^1$

The corresponding depth of flow is given by $y = r(1+\cos\sim)$

$where,\ \sim = (180^0 - 128^0 45^1) = 51^0 15^1 \quad \therefore y = r(1+\cos 51^0 15^1)$

Y=1.626r=0.813 x diameter also the corresponding depth of hydraulic radius R=0.608r=0.304 x diameter.

b) Condition for maximum discharge

Again from chezy's equation $Q = AC\sqrt{RS_0}$

For given values of c and S_0 discharge Q is a function of A & R Q is maximum when

$$\frac{\partial(Q)}{\partial ¯}=0 \qquad i.e,\ \frac{\partial}{\partial ¯}\left(A\sqrt{R}\right)=0\quad ie,\ \frac{\partial}{\partial ¯}\left(\frac{A^3}{P}\right)^{\frac{1}{2}}=0 \qquad or,\ \frac{\partial}{\partial ¯}\left(\frac{A^3}{P}\right)=0$$

Differentiating and simplifying, $4¯ - 6¯\cos 2¯ + \sin 2¯ = 0$

The value of $¯$ satisfying the above equation is $¯ = 2.688$ radians

The corresponding depth of flow $y = r(1+\cos\sim)$ $= r(1+\cos 26^0)$=1.899 radius=0.955X diameter. The corresponding Hydraulic radius R=A/P =0.573xradius =0.287 x diameter

Hence we see that when the velocity is maximum discharge is not maximum and vice versa.

Problems:

1. A rectangular channel carries water at the rate of 2.25 m3 /s when the slope of the channel is 0.025 % find the most economical dimensions of the channel if the manning's N=0.020

Soln: For a rectangular channel

A=BY (i)

P=B+2y (ii)

Condition for most economic channel is B=2y and R=y/2 (iii)

From manning's equation $Q = \frac{A}{N}R^{\frac{2}{3}}S_0^{\frac{1}{2}}$ $\qquad 2.25 = \left\{\frac{2y\ x\ y}{0.020}\ x\left(\frac{y}{2}\right)^{\frac{2}{3}}\left(\frac{0.025}{100}\right)^{\frac{1}{2}}\right\}$

$or,\ A = \frac{Q}{V} = \frac{12}{2} = 6m^2$

But, A=By or 6=2y+y y=1.732m

And hydraulic radius R=y/2=0.833m

From chezy's equation $V = C\sqrt{RS_0}$

$$S_0 = \left\{\frac{2^2}{70^2\ x\ 0.866}\right\} = 90426\ x\ 10^{-4} = \frac{1}{1061}$$

2. A rectangular channel is designed for maximum efficiency, if the wetted perimeter is 8m and the bed slope is 1 in 100 calculate the discharge, given manning's N=0.025

Soln: for a rectangular channel P=(B+2y) & for it to be most efficient B=2y & R=y/2

P=(2y+2y)=4y

i.e, 8=4y or y=2m B=4m R=1m

Area of cross section A=By=4x2=8m

From manning's equation , $Q = \frac{A}{N} R^{\frac{2}{3}} S_0{}^{\frac{1}{2}}$ $\quad Q = \frac{8}{0.025}\ x\ (1)^{\frac{2}{3}}\ x\left(\frac{1}{1000}\right)^{\frac{1}{2}}$

$Q = 10.12 m^3 s^{-1}$

3.A rectangular channel 4.5m wide 1.2 m deep is laid on a slope of 0.0009 and is laid with rubber masonry N=0.017, what saving in excavation and lining can be had by using the best hydraulic dimensions, but at the same time keeping the same shape, discharge and slope.

Soln: From manning's equation $Q = \frac{A}{N} R^{\frac{2}{3}} S_0{}^{\frac{1}{2}}$

Where, A=By=4.5x1.2=5.40m

P=B+2y=4.5+2X1.2=6.9m

$R = \frac{A}{P} = \frac{5.4}{6.9} = 0.7826m \quad S_0 = 0.0009 \quad Q = 8.09 m^3 / s$

Now, considering the channel to be most economical or best hydraulic section B=2y R=y/2 A=2y

Again, from manning's equation $(8.09) = \frac{(2y^2)}{0.017} x \left(\frac{y}{2}\right)^{\frac{2}{3}} x (0.0009)^{\frac{1}{2}}$

$y = 1.62m,\ R = 0.81m,\ B = 3.246m,\ A = 5.27m^2$

% Saving in excavation in terms of area=2.5%.

4. Find the maximum discharge for least excavation of a rectangular channel 3m wide, when c=65 and bed slope 1m in 1.25km

Soln: for a best hydraulic rectangular channel B=2y y=B/2 R=y/2

From chezy's equation $Q = AC\sqrt{RS_0}$

$$Q = \left(3x\frac{3}{2}\right)x65x\sqrt{\frac{3}{2x2}x\frac{1}{1250}} = 7.165\, m^3/s$$

5. It is proposed to provide a rectangular channel of best section of area 12.5m^2, find the breadth and depth. If the bed slope is 1 in 2000. find the discharge, take C=45.

Soln: for a best rectangular channel

B=2y R=y/2

A=By i.e 12.5=(2y)(y) $\therefore y = \left(\frac{12.5}{2}\right)^{\frac{1}{2}} = 2.5m \ \ (Depth)$

$and,\ B = 2y = 5.0m \ \ (breadth)$

From chezy's equation $Q = AC\sqrt{RS_0}$

$$Q = 12.5\ x\ 45\ x\ \sqrt{(1.25)\left(\frac{1}{2000}\right)} \quad = 14.06\ \ m^3/s$$

6. A triangular channel section 20m, what is the apex angle and depth for the condition of maximum discharge.

Soln: when the channel carries the maximum discharge it will be most economical or best hydraulic section. For such a channel $R = \frac{y}{2\sqrt{2}}$

And side slopes are $^{-} = 45^0$ with the vertical.

$$A = \frac{1}{2}(2y\tan^{-}\ x\ y) = y^2\tan^{-} = y^2\tan 45^0$$

$$\therefore A = y^2 \ \ or \ \ y = \sqrt{A} = \sqrt{20} = 4.47m$$

7. An open channel is to be excavated in trapezodial section with side slopes of 1:1 find the proportions for minimum excavation.

Soln: for a trapezodial channel $A = By + ny^2$ $given n = 1 (side\ slope)$

$\therefore A = By + y^2 \quad (i)$

$B = \left(\frac{A}{y} - y\right) \quad (ii)$

$P = B + 2y\sqrt{1+n^2} \quad = B + 2y\sqrt{1+1^2} \quad P = B + 2\sqrt{2}y$

Substituting eq(ii) in eq(iii)

$P = \frac{A}{y} - y + 2\sqrt{2}y \quad \therefore P = \frac{A}{y} + 1.82y \quad (iv)$

For minimum excavation the channel has to be most economical ie $\frac{\partial P}{\partial y} = 0$ (n the side slope is constant)

From eq (iv) $\frac{\partial P}{\partial y} = 0$, gives $\left(\frac{-A}{y^2} + 1.82\right) = 0 \quad \therefore A = 1.82y^2$

Equating eq(iv) and eq(v)

By+y=1.82y Or By=0.82y B=0.82y or $\frac{B}{y} = 0.82$

8. A discharge of 170 cubic meters per minute of water is to be carried in a trapezoidal channel of best hydraulic Efficiency. The bed slope is 1 in 5000 and side slopes is 1:1 compute the bottom width and depth of flow chezy's C=50

Soln: for a trapezodial channel

A=By+ny (i) B=A/y-ny (ii)

$P = B + 2y\sqrt{1+n^2}$

Substituting eq (ii) in eq(iii)

$P = \frac{A}{y} - ny + 2y\sqrt{1+n^2}$

For best hydraulic efficiency wetted perimeter P should be minimum i.e

(n the side slope is constant) $\frac{\partial P}{\partial y} = 0$

$\therefore \frac{\partial}{\partial y}\left\{\frac{A}{y} - ny + 2y\sqrt{1+n^2}\right\} = 0$

$given, \ \frac{-A}{y^2} - n + 2\sqrt{1+n^2} = 0 \quad \therefore A = \left(2\sqrt{1+n^2 - n}\right)y^2$

Substituting eq (i) in eq(v)

$By + ny^2 = 2y^2\sqrt{1+n^2} - ny^2$

$B+2ny=2y\sqrt{1+n^2}$

Substituting n=1 $B+2y=2y\sqrt{1+1^2}$

$\therefore B=\left(2\sqrt{2}-2\right)y$

B=2x0.41y or B= 0.82y

$Hydraulic\ \ radius\ \ R=\frac{A}{P}=\frac{By+ny^2}{B+2y\sqrt{1+n^2}}=\frac{y}{2}$

From chezy's equation $Q=AC\sqrt{RS_0}$

$where,\ Q=170m^{\frac{3}{\min}}=\frac{170}{60}=2.83m^3s^{-1}$

$\therefore 2.83=\left(0.82y+y^2\right)x50x\sqrt{\left(\frac{y}{2}x\frac{1}{5000}\right)}$

Solving for y=1.572m (depth)

And B=0.82x1.572=1.302 bottom width.

9. A trapezodial channel of best section carries a discharge of 13.7 cumecs at velocity of 0.9m/s the side slopes are2H:IV, find the bed width and depth of flow. Find also the bed slope if the value of manning's N=0.025.

Soln: from continuity equation Q=AV

Area of cross section $A=\frac{Q}{V}=\frac{13.7}{0.9}=15.22m^2$

$A=By+ny^2 \quad (i)$

$B=\frac{A}{y}-ny \quad (ii)$

$P=B+2y\sqrt{1+n^2}$

Substituting eq(ii) in eq(iii)

$P=\frac{A}{y}-ny+2y\sqrt{1+n^2} \quad (iv)$

For best section, $\frac{\partial P}{\partial y}=0 \qquad \frac{\partial}{\partial y}\left(\frac{A}{y}-ny+2y\sqrt{1+n^2}\right)=0 \qquad \therefore\left(\frac{-A}{y^2}-n+2\sqrt{1+n^2}\right)=0$

$\therefore A=\left(2\sqrt{1+n^2}-n\right)y^2$

Substituting n=2 the given value of the side slopes. $A = \left(2\sqrt{1+2^2} - 2\right)y^2$

$A = 2\left(\sqrt{5} - 1\right)y^2 \quad or,\ 15.22 = 2\left(\sqrt{5} - 1\right)y^2$

Y=2.48m (Depth of flow) $and,\ B = \left(\frac{15.22}{2.48} - 2x2.48\right)$

=1.174 m (Bottom width of channel)

$also,\ R = \frac{y}{2} = \frac{2.48}{2} = 1.24m$

From manning's equation $Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$

$0.9 = \frac{1}{0.025} x(1.24)^{\frac{2}{3}} (S_0)^{\frac{1}{2}}$

$S_0 = 3.8x10^{-4} = \left\{\frac{1}{2631.41}\right\}$

11. A trapezodial channel of best form has a cross sectional area of 37.2m and side slopes of 0.5H:IV, if the bed slope is 1 in 2000 and chezy's C=65. Compute the total flow in the channel.

Soln: for a best trapezodial channel $A = \left(2\sqrt{1+n^2} - n\right)y^2 \quad 37.2 = \left(2\sqrt{1+.05^2} - .05\right)y^2$
Y=4.629 m

Hydraulic radius $R = \frac{y}{2} = \frac{4.629}{2} = 2.315m$

From chezy's equation $Q = AC\sqrt{RS_0} \quad = 37.2x65x\sqrt{2.315x\frac{1}{2000}}$

$\therefore Q = 82.26m^3S^{-1}$

12.A power canal of trapezodial section has to be excavated through hard clay at the least cost. Determine the dimensions of the channel given discharge equal to 14m 3/s bed slope 1 in 2500 and manning's N=0.020. assume side slopes as 60 with the horizontal.

Soln: the cost of the channel will be least when it is economical section.

We know $A = By + ny^2 \quad (i) \quad B = \frac{A}{y} - ny \quad (ii)$

$P = B + 2y\sqrt{1+n^2} \quad (iii)$

Substituting (ii) in (iii)

$$P = \frac{A}{y} - ny + 2y\sqrt{1+n^2}$$

For most economical channel $\frac{\partial P}{\partial y} = 0$

$$\therefore \frac{-A}{y^2} - n + 2\sqrt{1+n^2} = 0 \qquad A = \left(2\sqrt{1+n^2} - n\right)y^2 \quad (iv)$$

Given side slopes as 60 with the horizontal $n = \frac{1}{\sqrt{3}}$

From eq (iv) $A = \left\{2\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2} - \frac{1}{\sqrt{3}}\right\}y^2 \qquad or,\ A = (1.7321)y^2$

For a most economical trapezodial channel R=y/2

Now from manning's equation $Q = \frac{A}{N}R^{\frac{2}{3}}S_0^{\frac{1}{2}}$

$$14 = \frac{(1.732)y^2}{0.02} x \left(\frac{y}{2}\right)^{\frac{2}{3}}\left(\frac{1}{2500}\right)^{\frac{1}{2}}$$

$Simplifying,\ y = 2.60m\ \ (Depth)$

$From\ Eq(ii),\ B = 3.00m\ \ (Bed\ Width)$

13. A trapezodial channel with side slopes of 1:1 has to be designed to carry at a velocity of so that amount of concrete lining for the bed and sides is minimum. Calculate the area of lining required for 1m length of the cannal.

Soln: from continuity equation Q=AV $A = \frac{Q}{V} = \frac{10}{2} = 5m^2$

For a most economical trapezodial channel $A = \left(2\sqrt{1+n^2} - n\right)y^2$

Substituting n=1 (side slope) and A=5m solving for y we have

$$5 = (2\sqrt{1+1^2} - 1)y^2 \qquad \therefore y = 1.66m \quad (Depth)$$

$$Also,\ B = \frac{A}{y} - ny$$

$$= \left\{\frac{5}{(1.66)} - 1x(1.66)\right\} = 1.364m\ \ (bed\ \ width)$$

now, wetted perimeter $P = B + 2y\sqrt{1+n^2} = 6.06m$

Area of lining for 1 m length of the canal Px1= 6.06m^2

14. Determine the bed width and discharge of the most economical trapezodial channel with side slopes Of IV:2H and bed slope of 1m per km and depth of flow equal to 1.25m. Roughness coefficient of channel=0.024.

Soln: for a most economical trapezodial channel $A = \left(2\sqrt{1+n^2} - n\right)y^2$

Substituting n=2 and y=1.25m

$$A = \left(2\sqrt{1+2^2} - 2\right)x\,1.25^2 \quad = 3.863m^2 \quad (Area \;\; of \;\; cross\,\text{sec}\,tion)$$

$$But, Bed \;\; Width \;\; B = \frac{A}{y} - ny$$

$$\therefore B = \frac{(3.863)}{1.25} - 2\;x\,1.25 = 0.59m$$

$$Also, Hydraulic \; radius \; R = \frac{y}{2} = \frac{1.25}{2} = 0.625m$$

Now from manning's equation $Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}} \quad = \frac{(3.863)}{0.024}\,x\,(06.25)^{\frac{2}{3}}\;\;x\left(\frac{1}{1000}\right)^{\frac{1}{2}}$

$$\therefore Q = 3.721\,m^3 S^{-1}\;(disch\,\text{arg}\,e)$$

15. Find the maximum velocity and maximum discharge through a circular sewer 0.75m radius given N=0.016 channel bed slope = 0.1 percent.

Soln: Case(a) Maximum velocity

For a maximum velocity $\overline{\;\;} = 128^0 45^1 = 2.247 \;\; radians$

$$Area \;\; A = r^2\left(\; - \frac{\sin 2\overline{\;\;}}{2}\right) \;\; = 0.75^2\left(2.247 - \frac{\sin 2x2.247}{2}\right) = 1.538m^2$$

$$wetted \;\;\; Perimeter \; P = 2r\overline{\;\;} \;\; = 2x0.75x2.247 = 3.37m$$

$$Hydraulic \; Radius, R = \frac{A}{P} = \frac{1.538}{3.37} = 0.456m$$

Now from manning's equation $Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}} \;\; = \frac{1}{0.016}x(0.456)^{\frac{2}{3}}\left(\frac{0.1}{100}\right)^{\frac{1}{2}} \;\; \therefore V = 1.17\,\frac{m}{s}$

Case (b) Maximum discharge

For maximum discharge ¯ $= 154 = 2.688$ *radians*

$A = 1.733m^2, P = 4.032m, R = 0.43m$

Now from manning's equation $Q = \frac{A}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}} = \frac{(1.733)}{(0.016)} x(0.43)^{\frac{2}{3}} \left(\frac{0.1}{100}\right)^{\frac{1}{2}} = 1.961 m^3 / s$

16. Find the depth of flow for maximum velocity in a circular sewer 1.50m diameter

Soln: for maximum velocity $\therefore \sim = (180^0 - 128^0 45^1) = 51^0 15^1$

The corresponding depth of flow y is given by $y = (r + r\cos\sim) = r(1 + \cos\sim)$

$= \frac{1.50}{2}(1 + \cos 51^0 15^1) \therefore y = 1.215m$

17. Find the depth of flow for maximum discharge in a circular sewer 1.25m diameter.

Soln: condition for maximum discharge is ¯ $= 154^0$ *and* $\sim = (180 - 154) = 26^0$

The corresponding depth of flow is $y = r(1 + \cos\sim) = \frac{1.25}{2}(1 + \cos 26^0),\ y = 1.1875m$

18. The cross section of an open channel is a square with diagonal vertical S is the side of the square and y is the portion of the water line below the apex, show that for maximum discharge, ratio is 0.127 While it is 0.414 for maximum velocity.

Soln: from figure $BE = S - y\sqrt{2}$ $EF = 2y$

$Wetted\ Perimeter, P = 2S + 2(S - y\sqrt{2})$

$Area\ of\ Flow\ A = S^2 - \frac{1}{2} x 2yxy = S^2 - y^2$

For maximum discharge, $\frac{\partial Q}{\partial y} = 0$ $\frac{\partial\left(A\frac{3}{P}\right)}{\partial y} = 0$

$\therefore 3P\frac{\partial A}{\partial y} - A\frac{\partial P}{\partial y} = 0$

Substituting for A & P solving y/s=0.127 (proved)

For maximum velocity $\frac{\partial V}{\partial y} = 0$ $\frac{\partial}{\partial y}\left(\frac{A}{P}\right) = 0$

Substituting for A & P solving $\frac{y}{S}=\sqrt{2}-1=0.414\ \ (proved)$

19. A rectangular channel 5.5 m wide and 1.25m depth has a slope of 1 in 900 determine the discharge when manning's N=0.015 if it is desired to increase the discharge to a maximum. By changing the size of the channel but keeping the same quantity of lining determine the new dimensions and percentage increase in discharge.

Soln: $A=5.5\ x\,1.25=6.875m^2$ $\qquad P=5.5+2\ x\,1.25=8m$

$$R=\frac{A}{P}=\frac{6.875}{8}=0.859m$$

Now from manning's equation $Q=\frac{A}{N}R^{\frac{2}{3}}S_0{}^{\frac{1}{2}}$

$$=\frac{6.875}{0.015}x(0.859)^{\frac{2}{3}}x\left(\frac{1}{900}\right)^{\frac{1}{2}}\quad=13.805m^3/s$$

Let B and y be the new width and depth. In order to have the same amount of lining the wetted perimeter should be the same or unchanged.

i.e., B+2y=8.0

For maximum discharge i.e the channel to be best efficient B=2y

2y+2y=8 or y=2m (depth)

B=2x2=4m (bed width)

$$A=4x2=8m^2 \qquad R=\frac{A}{P}=\frac{8}{8}=1m$$

And discharge $Q=\frac{A}{N}R^{\frac{2}{3}}S_0{}^{\frac{1}{2}}$

$$=\frac{8}{0.015}x(1)^{\frac{2}{3}}\left(\frac{1}{900}\right)^{\frac{1}{2}}=17.778m^3/s$$

Increase in discharge $=\frac{17.778-13.805}{13.805}x100 \quad =28.78\%$

20. A trapezodial channel carries a discharge of 28.5m3/s, with a mean velocity of 1.5 m/s when lined with rubble masonry N=0.017 one side is vertical and the other has a slope of 2H:IV. Determine the minimum slope and dimensions of the channel.

Sol: From continuity equation Q=AV

$A = \frac{Q}{V} = \frac{28.5}{1.5} = 19m^2$

But from figure $A = By + \frac{1}{2}ny^2$ (1)

Wetted perimeter $P = B + y + \sqrt{n^2y^2 + y^2} = B + y + y\sqrt{1+n^2}$

$P = B + y\left(1 + \sqrt{1+n^2}\right)$ (ii)

$\frac{-A}{y^2} - \frac{1}{2}n + \left(1 + \sqrt{1+n^2}\right) = 0$

Substituting A=19m, n=2 & solving y=2.92m (depth)

$B = \left\{\frac{(19)}{(2.92)} - \frac{1}{2}x2x(2.92)\right\}$ =3.60m (Bed Width)

$Hydraulic \;\; radius \; R = \frac{A}{P} = 1.456m$

Now from manning's equation $Q = \frac{A}{N}R^{\frac{2}{3}}S_0^{\frac{1}{2}}$

Substituting for V,N & R solving for S

$S_0 = 3.94x10^{-4} \;\; or \;\; \left(\frac{1}{2537.9}\right) \;\; (bed \;\; slope)$

SPECIFIC ENERGY (E)

The concept of specific energy was introduced by BORIS A BACK METEFF (1912). It is a very useful concept in the study of open channel flow problems.

Definition of Specific Energy (E)

Specific energy E is defined as the energy per unit weight of the liquid at a cross section measured above the bed level at that point.

From Bernoulli's equation $H = Z + \sim y + \frac{V^2}{2g}$ (1)

Where Z=Datum head, Y=pressure head, H= total head

$\sim = kinetic\ \ energy correction factor = 1 \quad \frac{V^2}{2g} = velocity\ \ head$

$\therefore E = y + \frac{Q^2}{2gA^2} \quad (iii)$

E is expressed in meters or N-m/N

Eq (iii) indicates that for a channel with a constant discharge Q, specific energy E is a function of the depth of flow y, because the area of flow A is also a function of y

Hence we see that eq (iii) would have three roots for y, two being positive and real, the other one will be imaginary and unreal.

Therefore a plot of depth of flow (y) with specific energy (E) results in the Specific energy diagram which will be as shown

Following are the salient features of the specific energy diagram:

(i) It is a parabola (ACB) which lies between the horizontal axis and a line at 45^0 to it.

(ii) The curve is asymptotic to both the axes

(iii) The curve has two limbs AC and BC

(iv) For any value of the specific energy E there will be two points of intersection M & N except at C which means that for any given specific energy there are two depths of flow y_1 & y_2 known as ALTERNATE DEPTHS which is possible or true except at C, where there is only depth y_c known as CRITICAL DEPTH & the velocity is the CRITICAL VELOCITY (Vc).

The specific energy E at point /c is the minimum and is known as minimum specific energy (Emin)

(v) Portion AC of the curve is always above C i.e $y_2 > y_c$ In this zone specific energy E increases with increase in the depth of flow y. hence it is known as SUBCRITICAL OR TRANQIL ZONE.

In the sub critical zone it may be noted that for a given specific energy, the depth of flow is large and the corresponding velocity is small.

In other words, in the sub critical zone, large depths of water moves with a small velocity.

(vi) Portion BC of the curve is always below C i.e $y_1 < y_c$. In this zone specific energy E increases with decrease in the depth of flow y. hence it is known as SUPER-CRITICAL OR SHOOTING OR RAPID FLOW.

In the supercritical flow zone, It may be noted that for a given specific energy the depth of flow is small and the corresponding velocity is large.

In other words, is the supercritical zone small depth of water moves with a large velocity.

Note: A uniform flow in which the depth of flow is equal to the critical depth is known as the CRITICAL FLOW.

CRITERION OF CRITICAL DEPTH (y_c)

From the foregoing discussion it is evident that the critical depth can be used as a parameter for identifying the flow is sub critical,critical or supercritical. This condition can be obtained by differentiating eq(iii) under the following two headings.

(i) Condition for minimum specific energy (Emin) for a given discharge Q

from eq (iii)

For a given discharge Q specific energy E is minimum when $\frac{\partial E}{\partial y} = 0$

$\because$ A is also a function of y

$$\therefore \frac{\partial E}{\partial y} = 0 \;\; given \;\; \frac{\partial}{\partial y}\left(y + \frac{Q^2}{2gA^2} \right) = 0$$

Differentiating and simplifying

$$\left\{ 1 + \frac{Q^2}{2g}\frac{\partial}{\partial y}\left(A^{-2}\right) \right\} = 0 \qquad \left\{ 1 + \frac{Q^2}{2g}\left(-2A^{-2-1}\right)\frac{\partial A}{\partial y} \right\} = 0 \qquad \left\{ 1 - \frac{Q^2}{gA^3}\frac{\partial A}{\partial y} \right\} = 0$$

The term $\frac{\partial A}{\partial y}$ represents the rate of increase of area with respect to the depth y

$$\therefore \frac{\partial A}{\partial y} = T(Top\;\; Width) \qquad Hence \;\; 1 - \frac{Q^2}{gA^3}T = 0 \;,\;\; \frac{Q^2 T}{gA^3} = 1, \quad \frac{Q^2}{g} = \frac{A^3}{T} \quad (iv)$$

From continuity equation Q=AV substituting the value of Q in Eq(iv) $\frac{A^2V^2}{g} = \frac{A^3}{T}$

$$\frac{V^2}{g} = \frac{A}{T} \qquad But\;,\frac{A}{T} = D \;\; (Hydraulic\;\; depth) \quad \therefore \frac{V^2}{g} = D \quad \left(\frac{V}{\sqrt{gD}}\right) = 1 \;\; or \;\; \frac{V}{\sqrt{gD}} = 1$$

But, by definition we know that $\frac{V}{\sqrt{gD}} = F$

Hence for critical flow Froude number should be unity.

(ii) Condition for max discharge (Qmax) fir a given specific energy

From Eq (iii) $E = y + \frac{Q^2}{2gA^2} \quad (iii) \qquad Q = \left\{(E-y)2gA^2\right\}^{\frac{1}{2}}$

$$or,\; Q = A\sqrt{2g} \;\; x \;\; \sqrt{E-y} \quad (iv)$$

For a given specific energy E, discharge Q is maximum when $\frac{\partial Q}{\partial y} = 0$

A is also a function of y

Differentiating Eq(vi) w.r.t. y and equating to zero $\frac{\partial Q}{\partial y} = 0$

$$i.e\ ,\ \frac{\partial}{\partial y}\left\{Ax\left(E-y\right)^{\frac{1}{2}}\right\} = 0$$

Differentiating by parts. $\left\{A\frac{1}{2}\left(E-y\right)^{\frac{1}{2}-1}\left(-1\right)+\left(E-y\right)^{\frac{1}{2}}\frac{\partial A}{\partial y}\right\} = 0$

$$\left\{\frac{-A}{2(E-y)^{\frac{1}{2}}}+(E-y)^{\frac{1}{2}}T\right\} = 0 \qquad \because \frac{\partial A}{\partial y} = T$$

As mentioned earlier

$$hence,\ \frac{A}{2(E-y)^{\frac{1}{2}}} = T(E-y)^{\frac{1}{2}} \qquad or,\ E = y+\frac{A}{2T} \quad (vii)$$

Equating Eq(iii) and (vii)

$$y+\frac{Q^2}{2gA^2} = y+\frac{A}{2T} \qquad or,\ \frac{Q^2}{gA^2} = \frac{A}{T} \qquad \therefore \frac{Q^2}{g} = \frac{A^3}{T}$$

This condition is same as Eq(iv) in the previous case i.e., for the condition of minimum specific energy.

Also, the above condition leads to Froude number F=1

It may thus be concluded that the conditions for minimum specific energy or maximum discharge, result in the same answer.

Answer, that the Froude number F=1

In other words, for critical flow to occur

a) Specific energy E is minimum for a given discharge Q
b) Discharge Q is maximum for a given specific energy E
c) Froude number F=1 (unity)

But, by definition we know that $\frac{V}{\sqrt{gD}} = F$

Hence for critical flow Froude number should be unity.

(ii) Condition for max discharge (Qmax) for a given specific energy

From Eq (iii) $E = y + \frac{Q^2}{2gA^2} \quad (iii)$

$Q = \left\{(E-y)2gA^2\right\}^{\frac{1}{2}} \qquad or, \qquad Q = A\sqrt{2g}\ \ x\ \ \sqrt{E-y} \quad (iv)$

For a given specific energy E, discharge Q is maximum when $\frac{\partial Q}{\partial y} = 0$

A is also a function of y

Differentiating Eq(vi) w.r.t. y and equating to zero. $\frac{\partial Q}{\partial y} = 0$

$i.e\ ,\ \frac{\partial}{\partial y}\left\{Ax(E-y)^{\frac{1}{2}}\right\} = 0$

Differentiating by parts.

$$\left\{A\frac{1}{2}(E-y)^{\frac{1}{2}-1}(-1) + (E-y)^{\frac{1}{2}}\frac{\partial A}{\partial y}\right\} = 0 \ \ , \ \ \left\{\frac{-A}{2(E-y)^{\frac{1}{2}}} + (E-y)^{\frac{1}{2}}T\right\} = 0$$

$\because \frac{\partial A}{\partial y} = T$, As mentioned earlier

$$hence,\ \frac{A}{2(E-y)^{\frac{1}{2}}} = T(E-y)^{\frac{1}{2}}\ , \quad \frac{A}{2T} = (E-y) \quad or,\ E = y + \frac{A}{2T} \quad (vii)$$

Equating Eq(iii) and (vii)

$$y + \frac{Q^2}{2gA^2} = y + \frac{A}{2T} \qquad or,\ \frac{Q^2}{gA^2} = \frac{A}{T} \qquad \therefore \frac{Q^2}{g} = \frac{A^3}{T}$$

This condition is same as Eq(iv) in the previous case i.e., for the condition of minimum specific energy.

Also, the above condition leads to Froude number F=1

It may thus be concluded that the conditions for minimum specific energy or maximum discharge, result in the same answer,that the Froude number F=1

In other words, for critical flow to occur

a) Specific energy E is minimum for a given discharge Q
b) Discharge Q is maximum for a given specific energy E
c) Froude number F=1 (unity)

CRITICAL FLOW IN OPEN CHANNELS

From the foregoing discussions we have seen that the critical flow in an open channel occurs when the depth of flow is the critical depth Also we have seen the other conditions resulting there of.

Critical flow in a rectangular channel

The previous discussions for minimum specific energy or maximum discharge do not account for the geometry of the channel and it holds good for all channel shapes.
For a rectangular channel
If Q is the total discharge in the channel
B is the bed width, then we can define

$\frac{Q}{B} = q$ as the discharge per unit width of the channel.

$T = B,\ A = By_c,\ E = E\min,\ y = y_c\ \ and\ \frac{Q}{B} = q$

In eq(iv) the condition for critical flow i.e $\frac{Q^2}{g} = \frac{A^3}{T}$, $\frac{Q^2}{g} = \frac{B^3 y_c^{\ 3}}{B}$, $\frac{Q^2}{B^2 g} = y_c^{\ 3}$

$\frac{q^2}{g} = y_c^{\ 3}$ Or the critical depth $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}}$ (ix)

Now from the equation for specific energy $E = y + \frac{Q^2}{2gA^2}$ (iii)

Substituting the values corresponding to the critical flow.

$$E\min = y_c + \frac{Q^2}{2gB^2 y_c^{\ 2}} = y_c + \frac{q^2}{2gy_c^{\ 2}}\ \left(\because \frac{Q}{B} = q\right) = y_c + \frac{y_c^{\ 3}}{2y_c^{\ 2}}\ \left(\because \frac{q^2}{g} = y_c^{\ 3}\right)$$

$$= y_c + \frac{y_c}{2} \qquad \therefore E\min = \frac{3}{2}(y_c) \quad (x)$$

Eq (x) is valid when the discharge Q is constant on the other hand, if specific energy is constant and the discharge is varied, the condition for maximum discharge is given by $\frac{\partial q}{\partial y} = 0$ $i.e,\ q^2 = 2g(E-y)y^2$ $or,\ q = \{2g(E-y)y^2\}$ (xi)

Differentiating Eq (xi) with respect to y and equating to zero.

$$\frac{\partial}{\partial y}\left\{ y(E-y)^{\frac{1}{2}} \right\} = 0 \quad \because \sqrt{2g} \neq 0$$

Differentiating by parts

$$\left\{ y\frac{1}{2}(E-y)^{\frac{1}{2}-1}(-1) + (E-y)^{\frac{1}{2}}1 \right\} = 0 \text{ or } \left\{ \frac{-y}{2(E-y)^{\frac{1}{2}}} + (E-y)^{\frac{1}{2}} \right\} = 0$$

$$or, \; y = 2(E-y)$$

$or, E = \frac{3}{2}y$ However, at critical conditions E=Emin and y= y_c

Emin= $\frac{3}{2}y_c$

Hence we see that for the critical flow to occur both the conditions converge to the same answer Emin= $\frac{3}{2}y_c$

PROBLEMS:

Establish a relation between the alternate depths for a horizontal rectangular open channel.

Soln: we know that the alternate depths y_1& y_2 represents the same specific energy E

$$E_1 = y_1 + \frac{Q^2}{2gA_1^{\,2}} \quad (1) \quad \text{for super critical flow}$$

$$E_2 = y_2 + \frac{Q^2}{2gA_2^{\,2}} \quad (2) \quad \text{for sub critical flow}$$

Equating the two equations in order to satisfy the definition of alternate depths we have

$$\left\{ y_1 + \frac{Q^2}{2gA_1^{\,2}} \right\} = \left\{ y_2 + \frac{Q^2}{2gA_2^2} \right\} \quad (3)$$

$$rewriting, \; (y_2 - y_1) = \frac{Q^2}{2g}\left(\frac{1}{A_1^2} - \frac{1}{A_2^2} \right)$$

Substituting $A_1 = By_{1,} \; A_2 = By_2, \frac{Q}{B} = q \text{ and } \frac{q^2}{g} = y_c^3$

$$(y_2 - y_1) = \frac{Q^2}{2g}\left\{\frac{1}{B^2 y_1^2} - \frac{1}{B^2 y_2^2}\right\} = \frac{Q^2}{2gB^2}\left\{\frac{1}{y_1^2} - \frac{1}{y_2^2}\right\} = \frac{q^2}{2g}\left\{\frac{y_2^2}{y_1^2} - \frac{y_1^2}{y_2^2}\right\}$$

$$(y_2 - y_1) = \frac{y_c^3}{2}\left\{\frac{(y_2 - y_1)(y_2 + y_1)}{y_1^2 y_2^2}\right\}$$

$$\therefore \left\{\frac{2y_1^2 y_2^2}{y_1 + y_2}\right\} = y_c^3 \quad --(4)$$

2. Water flows in a channel at a velocity 2 m/s and at a depth 2.5 m calculate the specific energy.

Soln: we know specific energy $E = y + \frac{V^2}{2g}$

$substituting,\ y = 2.5m, v = 2m/s, g = 9.81m/s^2$

$$E = \left\{2.5 + \frac{2^2}{2x9.81}\right\} = 2.71m$$

3.Water flows at 12.5 cumecs in an horizontal rectangular channel 2 m wide, a velocity of 1.25 m/s . Calculate the specific energy critical depth, critical velocity and the minimum specific energy.

Soln.

From continuity equation Q=AV $\therefore A = \frac{Q}{V} = \frac{12.5}{1.25} = 10m^2$

But, A=By $\therefore y = \frac{A}{B} = \frac{10}{2} = 5m,\ \ specific\ \ energy\ \ E = y + \frac{V^2}{2g}$

$$\therefore\ E = 5 + \frac{1.25^2}{2x9.81} = 5.08m$$

Discharge per unit width $q = \frac{Q}{B} = \frac{12.5}{2} = 6.25m^3 s^{-1} / m\ width$

Critical depth $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{6.25^2}{9.81}\right)^{\frac{1}{3}} = 1.585m$

Area corresponding to critical depth $A_c = By_c = 2x1.585 = 3.17m^2$

Critical velocity $V_c = \frac{Q}{A_c} = \frac{12.5}{3.17} = 3.943m/s$

Minimum specific energy $E\min = \frac{3}{2}y_c \quad \therefore E\min = \frac{3}{2}x(1.585) = 2.3775m$

3. A discharge of 18m3/s flow through a horizontal rectangular channel 6m wide at a depth of 1.6m. Find, (a)the specific energy, (b) the critical depth(c) minimum specific energy, (d) alternate depth corresponding to the given depth of 1.6m, (e) state of flow.

Soln: area of cross section A=By

$A = 6x1.6 = 9.6m^2$

(a) Specific energy $E = y + \frac{Q^2}{2gA^2} = \left(1.6 + \frac{18^2}{2x9.81x9.6}\right)$

(b) Discharge per unit width $q = \frac{Q}{B} = \frac{18}{6} = 3m^3/s$

Critical depth $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{3^2}{9.81}\right)^{\frac{1}{3}} = 0.972m$

(c) Minimum Specific energy $E\min = \frac{3}{2}y_c = \frac{3}{2}x(0.972) = 1.457m$

(d) Alternate depth y can be calculated from the equation

$$y_c^3 = \left\{\frac{2y_1^2y_2^2}{y_1 + y_2}\right\}$$

Where, $y_1 = 1.6m,\ y_c = 0.972m$

Substituting these values of y_1 and y_c

In the above equation we get a quadratic equation in y_2 which has two roots

$y_2 = -0.45m \quad y_2 = 0.632m$

Considering only the +ve root we have $y_2 = 0.632m$

(e) Now that $y_2 < y_c$ and referring to the specific energy diagram, we can conclude that the flow is in the super critical state.

5. 11.32 cumecs of water flows through a rectangular channel 3m wide. At what depth will the specific energy be 2.25m?. Also calculate the corresponding Froude number

Soln: Discharge per unit width $\therefore q = \frac{11.32}{3} = 3.773m^3/s/m$

Critical depth $y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left(\frac{3.773}{9.81}\right)^{\frac{1}{3}} = 1.132m$

From Continuity equation $Q = AV, \quad \therefore V = \frac{11.32}{3\,x\,y} m/s$

Substituting these values of V in the specific energy equation $E = y + \frac{V^2}{2g}$

$2.25 = y + \left(\frac{11.32}{3y}\right)^2 x \frac{1}{2x9.81}$ or $y_2^3 - 2.25y_2^2 + 0.726 = 0$

Solving by trial and error $y = 0.66m$

We know that the relation between alternate depths is given by $y_c^3 = \frac{2y_1^2 y_2^2}{y_1 + y_2}$

Substituting the values of y_1 and yc and solving we have

$y_2 = 2.17m,\ y_2 = -0.506m$

Considering only the positive value of y2 we have y2=2.17m (alternate depth)

Froude number $F = \left(\frac{V}{\sqrt{gy}}\right)$

$\therefore F_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{Q}{By_1\sqrt{gy_1}} = \left\{\frac{11.32}{3x0.66x\sqrt{9.81x0.66}}\right\} = 2.25$

$F_2 = \frac{V_2}{\sqrt{gy_2}} = \frac{Q}{By_2\sqrt{gy_2}} = \left\{\frac{11.32}{3x2.17x\sqrt{9.81x2.17}}\right\} = 0.376$

(6) The specific energy in a rectangular channel is 5 N-m/N. Calculate the critical depth if the width of the channel is 10m. Calculate the maximum discharge.

Soln: We know Emin = 3/2 yc

Considering the given value of specific energy of 5 N-m/N as the minimum specific energy, We have:

Critical depth $y_c = 2/3\,E = \frac{2}{3} x5 = 3.33m$

From the relation $\frac{q^2}{g} = y_c^3 \quad q = \left(y_c^3 g\right)2^{\frac{1}{2}} = \left(3.33^2 x9.81\right)^{\frac{1}{2}} = 19.032m^3/s/m\ \ width$

Therefore, total discharge $Q = qxB = (19.03)\,x\,10 = 190.32m^3/s$

(7) A rectangular channel is to carry a discharge of 25 cumecs at a slope of 0.006. Determine the width of the channel for the critical flow. Take N=0.016.

Soln: If B is the bed width of the channel, then by definition $Q = \frac{25}{B}$

$$V = \frac{Q}{A} = \frac{Q}{By} = \frac{25}{By}$$

For critical flow, $\frac{q^2}{g} = y_c^3 \quad or \quad y_c = \left(\frac{q^2}{g}\right)^{\frac{1}{3}} = \left[\left(\frac{25}{B}\right)^2 x \frac{1}{9.81}\right]^{\frac{1}{3}} \quad \therefore y_c = \frac{4}{B^{\frac{2}{3}}}$

From Manning's equation $V = \frac{1}{N} R^{\frac{2}{3}} S_0^{\frac{1}{2}}$, $\frac{25}{By_c} = \frac{1}{0.016}\left\{\frac{By_c}{B+2y_c}\right\}^{\frac{2}{3}}(0.006)^{\frac{1}{2}}$

Substituting $y_c = \frac{4}{B^{\frac{2}{3}}}$, in the above equation and simplifying

$$\frac{25}{4B^{\frac{1}{3}}} = \frac{1}{0.016}\left(\frac{4B^{\frac{1}{2}}}{B + \frac{8}{B^{\frac{2}{3}}}}\right)(0.0775) \quad or,\ 0.512 = \frac{B}{\left(B^{\frac{5}{3}} + 8\right)^{\frac{2}{3}}}$$

Solving by trial and error $B = 3m$

* * * * *

HYDRAULIC JUMP

Hydraulic jump is the most commonly encountered varied flow phenomenon on an open channel in which a rapid change occurs from a high velocity low depth super critical state of flow to a low velocity large depth subcritical state.

PLACES OF OCCURRENCE:

a) At the foot of an overflow spitway dam
b) Behind a dam on a steep slope
c) Below a regulating sluice
d) When a steep slope channel suddenly turns flat.

Whenever an hydraulic jump occurs There will be heavy amount of turbulence and considerable energy loss. Hence energy principle or Bernoulli's energy equation cannot be used for its analysis. Therefore the momentum equation derived from the second law of Newton is used.

USES OF HYDRUALIC JUMP
1. To dissipate excessive energy.
2. To increase the water level on the downstream side.
3. To reduce the net uplift force by increasing the weight, i.e. due to increased depth.
4. To increase the discharge from a sluice gate by increasing the effective head causing flow.
5. To Provide a control section.
6. For thorough mixing of chemicals in water.
7. For aeration of drinking water.
8. For removing air pockets in a pipe line.

Types of Hydraulic Jump (USBR classification)
Based on the initial Froude number F1, Hydraulic Jumps can be classified as follows.

a. Undular Jump Such a jump occurs when the initial Froude number F1 is between 1 and 1.7

In such a jump there will be surface undulations due to low level turbulence it would result in insignificant energy losses.
b. Weak Jump: Such a jump occurs when F1 is between 1.7 and 2.5. Head loss is low. In this type of jump, series of small rollers form on the surfaces and the loss of energy due to this type is small.
c. Oscillating jump : It occurs when F1 is between 2.5 and 4.5 . In this type the surface will be wavy, jets if water shoot from the floor to surfac. Jump moves back and forth causing some damage. Such a jump should be avoided if possible.
d. Steady Jump: It occurs when F1, is between 4.5 and 9. Such a jump 9 stable, balanced in performance, requires a stilling basin to confine the jump. Energy dissipation will be high of the order of 45 to 70%.
e. Strong Jump: It occurs when F1 is more than 9. It will be rough and violent, huge rollers are formed in the flow. Energy dissipation will be very high & is upto 85%.

Analysis of Hydraulic Jump
The equation of Hydraulic jump can derived making the following assumptions.

1. The channel bed is horizontal so that the component of the body of water weight in the direction of flow can be neglected.

2. The frictional resistance of the channel in the small length over which the jump occurs is neglected, so that the initial and final specific forces can be equated.
3. The channel is rectangular in section.
4. The portion of channel in which the hydraulic jump occurs is taken as a control volume. It is assumed that just before and just after the control volume, the flow is uniform and pressure distribution is hydrostatic.
5. The momentum correction factor (B) is unity.

Consider a hydraulic jump occuring between two section (1) (1) and (2) and (2) as shown.
The various forces acting on the control volume are:
a) Hydraulic pressures forces F1 & F2.
b) Component of weight W Sin   in the direction of flow.
c) Shear stress or Frictional resistance acting on the contact area.

From the Impulse momentum equation Algebraic sum of the forces acting = Change in momentum on the control volume.
Consider LHS of equation (1)
forces = + F1 – F2 + W Sin¯ $-{}^{2}{}_{0}x$ *contact area*

As per the assumptions made above, the slope of the channel ¯ is very small, i.e, $Sin^{-} \cong 0 \;\therefore W \sin^{-} \cong 0$

Hence it can be neglected.
The channel is smooth so that $^{-}-{}^{2}{}_{0}x$ *contact area* Can be neglected
$\therefore \sum \; forces \; = F_1 - F_2$

But F1 and F2 being Hydrostatic forces we have $F_1 = ¥A_1 \bar{y}_1 ,\; F_2 = ¥A_2 \bar{y}_2$

$or, \;\sum forces = ¥A_1 \bar{y}_1 \; - \; ¥A_2 \bar{y}_2$
Where A1 and A2 are area of cross section before and after the jump.
$\bar{y}_1$ *and* $\bar{y}_2$ are the centroidal depths F1and F2, measures from the respective liquid surfaces.
Consider RHS of Eq(1)
Change in momentum = (Momentum before the jump per unit time or Momentum after the jump)

$$= \frac{Mass}{time} x(velocity\ before \approx velocityafter) = \frac{Mass}{time} x(velocity\ before \approx velocityafter)$$

$$= \ell x \frac{volume}{time}(V_1 \approx V_2) \; = \ell \; x \; discharge(V_1 \approx V_2) \;\; = \ell \; Q\left(\frac{Q}{A_1} \approx \frac{Q}{A_2}\right) \;\; = \ell Q^2\left(\frac{1}{A_1} \approx \frac{1}{A_2}\right)$$

Change in momentum per unit time $= \frac{¥}{g} Q^2 \left(\frac{1}{A_1} \approx \frac{1}{A_2}\right) \quad ---(b)$

Substituting eq(a) and (b) in eq(1) $¥A_1 \bar{y} - ¥A_2 \bar{y} = \frac{¥}{g} Q^2 \left(\frac{1}{A_1} \approx \frac{1}{A_2}\right)$

Rewriting $A_1 \bar{y}_1 + \frac{Q^2}{gA_1} = A_2 \bar{y}_2 + \frac{Q^2}{gA_2} ---(2)$

Eq(2) is the general equation of Hydraulic jump in any type of channel.

Hydraulic jump in a Horizontal rectangular channel

For a rectangular channel $A_1 = By_1, A_2 = By_2$

$\bar{y}_1 = \frac{y_1}{2}, \bar{y}_2 = \frac{y_2}{2} \; and \; \frac{Q}{B} = q$

Substituting all these values in Eq(2) $B \bar{y}_1 \, x \frac{y_1}{2} + \frac{Q^2}{gB^2 y_1} \{=\}, \frac{y_2}{2} + \frac{Q^2}{gB^2 y_2}$

$$\frac{y_1}{2} + \frac{Q^2}{gB^2 y_1} \{=\}, \frac{y_2}{2} + \frac{Q^2}{gB^2 y_2}$$

$$\frac{y_1^2}{2} + \frac{q^2}{gy_1} = \frac{y_2^2}{2} + \frac{q^2}{gy_2} \quad \text{or} \quad \frac{y_1^2 - y_2^2}{2} = \frac{q^2}{g}\left(\frac{1}{y_1} - \frac{1}{y_2}\right)$$

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2) ---(3)$$

Eq(3) is the general equation of hydraulic jump in a rectangular channel. It can be written as $y_1^2 y_2 + y_1 y_2^2 - \frac{2q^2}{g} = 0 ---(4)$

Eq(4) can be quadratic in y_1 or y_2 . consider Eq(4) to be quadratic in y_1

$$y_2 y_1^2 + y_2^2 y_1 - \frac{2q}{g} = 0$$

$$\therefore y_1 = \frac{-y_2^2 \pm \sqrt{y_2^4 - 4y_2\left(-\frac{2q^2}{g}\right)}}{2y_2} = \left\{\frac{-y_2^2 \pm \sqrt{y_2^4 + \frac{8q^2 y_2}{g}}}{2y_2}\right\} = \left\{\frac{-y_2^2}{2y_2} + \sqrt{\frac{y_2^4 + \frac{8q^2 y_2}{g}}{4y_2^2}}\right\}$$

$$= \left\{-\frac{y_2}{2} \pm \sqrt{\frac{y_2^2}{4} + \frac{2q^2}{gy_2}}\right\} = \left\{\frac{-y_2}{2} + \frac{y_2}{2}\sqrt{\left(1 + \frac{2q^2}{gy_2} x \frac{4}{y_2^2}\right)}\right\}$$

$$y_1 = \frac{y_2}{2}\left\{-1 + \sqrt{1 + \frac{8q^2}{gy_2^3}}\right\} --- (5)$$

Similarly considering Eq(4) to be quadratic in y2, we have

$$y_2 = \frac{y_1}{2}\left\{-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}}\right\} --- (6)$$

Consider the term $\frac{q^2}{gy_2^3}$ *in Eq*(5)

For a rectangular channel $\frac{q^2}{g} = y_c^3$

Eq(5) can be written as $y_1 = \frac{y_2}{2}\left\{-1 + \sqrt{1 + \frac{8y_c^3}{y_2^3}}\right\} --- (7)$

Similarly from Eq(6) $y_2 = \frac{y_1}{2}\left\{-1 + \sqrt{1 + \frac{8y_c^3}{y_1^3}}\right\} --- (8)$

Consider the term $\frac{q^2}{gy_1^3}$ *in Eq*(4)

$$\frac{q^2}{gy_2^3} = \frac{Q^2}{B^2 gy_2^3} = \frac{A_2^2 V_2^2}{B^2 gy_2^3} = \frac{B^2 y_2^2 V_2^2}{B^2 gy_2^3} = \frac{V_2^2}{gy_2} = \left[\frac{V_2}{\sqrt{gy_2}}\right]^2$$

But, $\dfrac{V_2}{\sqrt{gy_2}} = F_2 (\textit{Froude number after the jump})$

$$\therefore \frac{q^2}{gy_2^3} = F_2^2$$

Hence, from Eq(4) $y_1 = \dfrac{y_2}{2}\left\{-1 + \sqrt{1+8F_2^2}\right\} --- (9)$

Again from Eq(5) $y_2 = \dfrac{y_1}{2}\left\{-1 + \sqrt{1+8F_1^2}\right\} --- (10)$

In Eq(10) F1 = Froude number before the jump. Eq(9) can be written as

$$\frac{y_1}{y_2} = \frac{1}{2}\left\{-1 + \sqrt{1+8F_2^2}\right\} --- (11)$$

Similarly Eq(10) as $\dfrac{y_2}{y_1} = \dfrac{1}{2}\left\{-1 + \sqrt{1+8F_1^2}\right\} --- (12)$

In Eq(11) and (12) $\dfrac{y_1}{y_2}\, or\, \dfrac{y_2}{y_1}$ Is known as the ratio of conjugate depths.

Problems:

1. Derive an equation for the loss of energy due to an hydrualic jump in a horizontal rectangular open channel.

Soln: applying Bernoull's equation between 1,1 and 2,2 with the channel bed as datum and considering head loss due to the jump

$$Z_1 + y_1 + \frac{V_1^2}{2g} = Z_2 + y_2 + \frac{V_2^2}{2g} + \Delta E$$

Since the channel is horizontal $Z_1 = Z_2$

$$\therefore y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta E \quad \text{or} \quad \Delta E = -(y_2 - y_1) + \frac{V_1^2 - V_2^2}{2g} \quad (1)$$

From continuity equation $Q = A_1V_1 = A_2V_2$

$$V_1 = \frac{Q}{By_1} = \frac{q}{y_1}, \qquad V_2 = \frac{q}{y_2}$$

Substituting these values of In eq (1) $\Delta E = \dfrac{q^2}{2g}\left(\dfrac{1}{y_1^2} - \dfrac{1}{y_2^2}\right) - (y_2 - y_1)$

$$= \frac{q^2}{2g}\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right) - (y_2 - y_1) \quad (2)$$

But for a rectangular channel the general equation of hydraulic jump is

$$\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$$

$$\frac{q^2}{g} = \frac{y_1 y_2}{2}(y_1 + y_2) \quad (3)$$

Substituting eq 3 in eq 2

$$\Delta E = \frac{y_1 y_2}{2}(y_1 + y_2)x\frac{y_2^2 - y_1^2}{2y_1^2 y_2^2} - (y_2 - y_1)$$

$$= \frac{(y_2 - y_1)(y_2 + y_1)^2}{4y_1 y_2} - (y_2 - y_1) = \frac{(y_2 - y_1)[(y_2 + y_1)^2 - 4y_1 y_2]}{4y_1 y_2}$$

$$\Delta E = \frac{(y_2 - y_1)^3}{4y_1 y_2} \quad (4)$$

In eq 4 E is expressed in meters if E is to be expressed as an energy loss interms of KW or as power lost. $P = rQ\Delta E$ (5)

Where P will be in KW

r specific weight of the liquid KN/m

Q discharge m3/s

ΔE head lost in m

2. A rectangular channel 3m wide carrying 5.65 cumecs of water at a velocity of 6m/s discharges into a channel where a hydraulic jump is obtained what is the height of the jump? Calculate the critical depth also

Soln: from the continuity eqn Q=AV

$$5.65 = 3xy_1x6$$

$$\therefore y_1 = 0.314m \ \ (initial \ \ depth)$$

$$q = \frac{Q}{B} = \frac{5.65}{3} = 1.883m3/S/m$$

Now from the equation of hydraulic jump in a rectangular channel.

$$y_2 = \frac{y_1}{2}\left\{-1 + \sqrt{1 + \frac{8q^2}{gy_1^3}}\right\} = \frac{0.314}{2}\left\{-1 + \sqrt{1 + \left(\frac{8x1.883^2}{9.81x0.314^3}\right)}\right\}$$

Substituting the values of y_1, q and g for

Height of the jump y_2 =(1.3686-0.314)=1.0547m

Critical depth y_c is calculated from the equation. $y_c^3 = \frac{q^2}{g}$

$$y_c = (\frac{q^2}{g})^{\frac{1}{3}} = \left(\frac{1.883^2}{9.81}\right)^{\frac{1}{3}} = 0.712m$$

3. In a rectangular channel 2.4 m wide the discharge is 9.1m3/s. if a hydraulic jump occurs and the depth before the jump is 0.75 m. find the height of the jump energy head loss and power lost by energy dissipation.

Soln: from the relation $q = \frac{Q}{B}$, we have $q = \frac{9.1}{2.4} = 3.792m3/s/m$

discharge per unit width.

From the equation of hydraulic jump in a rectangular channel.

$$y_2 = \frac{y_1}{2}\left\{-1+\sqrt{1+\frac{8q^2}{gy_1^3}}\right\}$$

Substituting all values and sloving for y_2

$$y_2 = \frac{0.75}{2}\left\{-1+\sqrt{1+\frac{8x3.792^2}{9.81x0.75^3}}\right\}, \quad y_2 = 1.637m$$

Height of the jump $hj = (y_2 - y_1) = (1.637 - 0.75) = 0.887m$

Energy head loss $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} = \left\{\frac{0.887^3}{4x0.75x1.637}\right\} = 0.1422m$

$Power \ \ lost, \Delta E = rQ\Delta E$ =9.81x9.1x(0.1422)=12.69 KW

4. Flow over a spill way is 3 cumces/meter width the supercritical velocity down the sillway is 12.15ms. What must be the depth of the tail water to cause on hydraulic jump at the apron? What is the energy lost per unit width? What is the total head of flow before and after the jump.

Soln: from continuity eq Q=AV $\therefore Q = By_1V_1$

$or \ q = y_1V_1 \ \ \therefore y_1 = \frac{q}{V_1} = \frac{3}{12.15} = 0.247m$

For a rectangular channel A=By

y_1 is the initial depth of flow from the equation of hydraulic jump in a rectangular channel.

$$y_2 = \frac{y_1}{2}\left\{-1+\sqrt{1+\frac{8q^2}{gy_1^3}}\right\}$$

Substituting all values and solving $y_2 = \frac{0.247}{2}\left\{-1+\sqrt{1+\frac{8x3^2}{9.81x0.247^3}}\right\}$

$y_2 = 2.606m$

y_2 Is the depth of tailwater required for the formation of hydraulic jump

Energy lost $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(2.606 - 0.247)^3}{4x0.247x2.606}$ =5.10m

Total head of flow before the jump $H_1 = y_1 + \frac{V_1^2}{2g} = (0.247 + \frac{12.15^2}{2x9.81}) = 7.771m$

From continuity eq $V_2 = \frac{q}{y_2} = \frac{3}{2.606} = 1.151m/s$

Total head of flow after the jump $H_2 = y_2 + \frac{V_2^2}{2g} = 2.606 + \frac{1.151^2}{2x9.81} = 2.673m$

5. The stream issuing from beneath a vertical sloce gate is 0.3m deep at vena contracta. Its mean velocity is 6ms a standing wave is created on the level bed below the sluice gate. Find the height of the jump the loss of head and the power dissipated per unit width of sluice.

Soln; from the continuity eq Q=AV=ByV

$or, \frac{Q}{B} = q = y_1V_1 = 0.3x6 = 1.8m^{\frac{3}{s}}/m \qquad y_1 = 0.3m$

Conjugate depth or depth after the jump y_2 is given by

$y_2 = \frac{y_1}{2}\left\{-1+\sqrt{1+\frac{8q^2}{gy_1^3}}\right\} = \frac{0.3}{2}\left\{-1+\sqrt{1+\frac{8x1.8^2}{9.80x0.3^3}}\right\}$ =1.341 m

Height of the jump $hj = (y_2 - y_1) = (1.341 - 0.3) = 1.0414m$

$\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} = \left\{\frac{1.0414^3}{4x0.3x1.341}\right\} = 0.702m$

Power dissiputed per unit width

=9.81x1.8x(0.702)=12.394KW/m.width

6. If the velocity when the water enters the channel is 4ms and Froude number is 1.4 obtain a) The depth of flow after the jump b) the loss of specific energy due to the formation of the jump.

Soln: from the definition of Froude number we have

$$F_1 = \frac{V_1}{\sqrt{gy_1}} = 1.4 = \frac{4.0}{\sqrt{9.81xy_1}} = 0.832m$$

The depth of flow after the jump is given by

$$y_2 = \frac{y_1}{2}\left\{-1+\sqrt{1+8F_1^{\,2}}\right\}, \qquad y_2 = \frac{0.832}{2}\left\{-1+\sqrt{1+8x1.4^2}\right\} = 1.283m$$

Loss of specific energy $\Delta E = \left\{\left(y_2 + \frac{V_2^2}{2g}\right) \approx \left(y_1 + \frac{V_1^2}{2g}\right)\right\}$

From continuity equation $y_1V_1 = y_2V_2$

$$\therefore V_2 = \frac{y_1V_1}{y_2} = \left(\frac{0.832x4}{1.283}\right) = 2.593m/s$$

Loss of specific energy

$$\Delta E = \left\{\left(1.283 + \frac{2.593^2}{19.62}\right) \approx \left(0.832 + \frac{4^2}{19.62}\right)\right\} = 0.0217m$$

7. In a rectangular channel 0.6m wide a jump occurs where the Froude number is 3. the depth after the jump is 0.6m estimate the total loss of head and the power dissipated by the jump.

Soln: from the eq of Hydraulic jump $\frac{y_2}{y_1} = \frac{1}{2}\left\{-1+\sqrt{1+8F_1^{\,2}}\right\}$

$$y_2 = 0.6m,\ F_1 = 3 \quad \frac{0.6}{y_1} = \frac{1}{2}\left\{-1+\sqrt{1+8x3^2}\right\} = 0.16$$

Head loss due to the jump $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$

$$\Delta E = \frac{(0.6-0.16)^3}{4x0.16x0.6} = 0.2232m$$

8. The depth and velocity of water downstream of a sluice gate in a horizontal rectangular channel is 0.4m and 6m/s respectively. Examine whether a hydraulic jump can possibly occur in the channel. If so find the depth after the jump and head loss due to the jump.

Soln: the value of initial Froude number is calculated from the relation $F_1 = \frac{V_1}{\sqrt{gy_1}}$

$$F_1 = \frac{6}{\sqrt{9.81x0.4}} = 3.029$$

Since, F1 (=3.029)>1 i.e, the flow is supercritical, an Hydraulic jump will occur.

Now, from the relation. $y_2 = \frac{y_1}{2}\left\{-1+\sqrt{1+8F_1^{\,2}}\right\}$

Substituting all values $y_2 = \frac{0.4}{2}\left\{-1+\sqrt{1+8x(3.029)^2}\right\}$=1.525m (Depth after the jump)

Loss of head due to the jump $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{(1.525-0.4)^3}{4x0.4x1.525} = 0.584m$

9. A rectangular channel 5m wide carries a discharge of 6 cumecs. If the depth on the downstream of the hydraulic jump is 1.5m, determine the depth upstream of the jump. What is the energy dissipated?

Soln: Discharge per unit width $q = \frac{Q}{B}$ $\therefore q = \frac{6}{5} = 1.2m^3/s/m\ width\ \ given\ y_2 = 1.5m$

Therefore depth before or upstream of the jump $y_1 = \frac{y_2}{2}\left\{-1+\sqrt{1+\left(\frac{8q^2}{gy_2^3}\right)}\right\}$

Substituting $y_1 = \frac{y_2}{2}\left\{-1+\sqrt{1+\left(\frac{8x1.2^2}{9.814x1.5^3}\right)}\right\}$

$\therefore y_1 = 0.1208m(Depth\ upstream\ of\ the\ jump)$

Energy dissipated $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y^2} = \frac{(1.5-0.1208)^3}{4x(0.1208)x(1.5)} = 3.62m$

10. Determine the flow rate in a horizontal rectangular channel 1.5m wide in which the depths before and after the hydraulic jumps are 0.25m and 1.0m.

Soln: From the equation of hydraulic jump. $\frac{y_2}{y_1} = \frac{1}{2}\left\{-1+\sqrt{1+\frac{8q^2}{gy_1^3}}\right\} --- (1)$

Substituting y1=0.25m, y2=1m, g=9.81m/s2 and solving for q

$$\frac{1}{0.25}=\frac{1}{2}\left\{-1+\sqrt{1+\frac{8q^2}{9.81x0.25^3}}\right\}---(1)$$

Flow rate in the rectangular channel $Q=qxB=(1.238)x(1.5)\quad \therefore Q=1.857m^3/s$

11. Water flows at the rate of 1.25 cumecs in a channel of rectangular section 1.5m wide. Calculate the critical depth, if a hydraulic jump occurs at a point where the upstream depth is 0.30m, What would be the rise of water level produced and the power lost in the jump?

Soln: Critical depth y_c is given by

$$y_c{}^3=\frac{q^2}{g}=\frac{Q^2}{B^2g}\quad \therefore y_c=\left\{\left(\frac{1.25}{1.5}\right)^2x\frac{1}{9.81}\right\}^{\frac{1}{3}}=0.414m$$

From the equation of Hydrualic jump $y_2=\frac{y_1}{2}\left\{-1+\sqrt{1+\frac{8y_c^3}{y_1^3}}\right\}$

$$=\frac{0.3}{2}\left\{-1+\sqrt{1+\frac{8x(0.414)^3}{0.3^3}}\right\}\therefore y_2=0.55m\ \ (conjugate\ \ depth)$$

Energy loss due to the jump $\Delta E=\frac{(y_2-y_1)^3}{4y_1y_2}$

$$\Delta E=\frac{(0.55-0.3)^3}{4x0.3x0.55}=0.0237m$$

Power lost due to the jump $P=rQ\Delta E$

$$P=9.81x1.25x0.0237=0.29KW$$

12. A sluice spans a channel of rectangular section 15m wide having an opening of 0.6m depth discharges water at the rate of 40 cumces. If a hydraulic jump is formed on the downstream side of the sluice, determine theProbable height of crest above the upper edge of the sluice

Soln.

From the equation of hydraulic jump $y_2=\frac{y_1}{2}\left\{-1+\sqrt{\frac{8q^2}{gy_1^3}}\right\}$

$$y_2 = \frac{0.6}{2}\left\{-1+\sqrt{\frac{8x2.67^2}{9.81x0.6^3}}\right\}$$

$$\therefore y_2 = 1.285m \ \ (conjugate \ \ depth)$$

$$(y_2 - y_1) = (1.285 - 0.60) = 0.685m$$

Loss of energy due to the jump $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2}$

$$\Delta E = \frac{(1.285 - 0.6)^3}{4x0.6x1.285} = 0.104m$$

13.In the case of a hydraulic jump in a rectangular channel, prove that

$$a^3 = \left\{\frac{(r-1)^9}{32r^4(r+1)}\right\}$$

$$where, a = \frac{\Delta E}{y_c}, \ \ r = \frac{y_2}{y_1}$$

Soln: we know $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} = \frac{y_1^3(\frac{y_2}{y_1} - 1)^3}{4y_1y_2}$

Substituting $\frac{y_2}{y_1} = r$

$$\Delta E = \frac{y_1^3(r-1)^3}{4y_1y_2} = \frac{y_1^2(r-1)^3}{4y_2}$$

Raising both sides to the power 3

$$\Delta E^3 = \frac{y_1^6(r-1)^9}{4^3 y^3{}_2}$$

Dividing both the sides by y_c^3

$$\frac{\Delta E^3}{y_c^3} = \frac{y_1^6(r-1)^9}{64y_2^3xy_c^3} \quad = \frac{(r-1)^9}{64y_c^3}x\frac{y_1}{y_2}xy_1^3$$

$$\therefore \frac{\Delta E^3}{y_c^3} = \frac{(r-1)^9}{64r^3}\left(\frac{y_1}{y_c}\right)^3 \quad -(1)$$

$$\frac{y_2}{y_1} = r = \frac{1}{2}\left\{-1+\sqrt{1+\frac{8y_c^3}{y_1^3}}\right\}$$

$$(2r+1)^2 = \left[1+8\left(\frac{y_c}{y_1}\right)^3\right] \quad or, \frac{4r^2+1^2+4r-1}{8} = \left(\frac{y_c}{y_1}\right)^3$$

$$\therefore \left(\frac{y_c}{y_1}\right)^3 = \frac{4r(r+1)}{8} \quad or, \left(\frac{y_c}{y_1}\right)^3 = \frac{r(r+1)}{2} \quad ---(2)$$

From Eq(1) and (2)

$$\left(\frac{\Delta E}{y_c}\right)^3 = \frac{(r-1)^9}{64r^3} x \frac{2}{r(r+1)}$$

$$a^3 = \left\{\frac{(r-1)^9}{32r^4(r+1)}\right\}$$

14. In a rectangular channel the discharge per unit width is 2.5 cumecs/meter. When a hydraulic jump occurs, the loss of energy is 2.68 N.m/N. Determine the depths before and after the jump.

Soln: Equation for energy loss is $\Delta E = \frac{(y_2 - y_1)^3}{4y_1y_2} --(1)$

$$y_2 = \frac{y_1}{2}\left\{-1\sqrt{1+\frac{8q^2}{gy_1^3}}\right\} \quad \therefore, \frac{y_2}{y_1} = \frac{1}{2}\left\{-1+\sqrt{1+\frac{8x2.5^2}{9.81\,x\,y_2^3}}\right\} = \frac{1}{2}\left\{-1+\sqrt{1+\frac{5.097}{y_1^3}}\right\}---(2)$$

From Eq(1) $\Delta E = \frac{y_1^3\left(\frac{y_2}{y_1}-1\right)^3}{4y_1y_2} \qquad \Delta E = \frac{y_1^2\left\{\frac{1}{2}\left(-1+\sqrt{1+\frac{5.097}{y_1^3}}\right)-1\right\}^3}{4y_2} = 2.68$

$$y_1^2\left\{0.5x\sqrt{1+\frac{5.097}{y_1^3}}-1.5\right\}^3 = -5.36y_1 + 5.36x\sqrt{1+\frac{5.026}{y_1^3}} --(3)$$

Solving Eq (3) by trial and error $y_2 = \frac{0.12}{2}\left\{-1+\sqrt{1+\left(\frac{8x2.5^2}{9.81x0.12^3}\right)}\right\}$

y_2 = 3.2m (conjugate depth)

VENTURIFLUME

It is a devise used to measure the discharge in an open channel. It is designed in such a way that the velocity at the throat is less than the critical velocity so that no standing wave will occur in the flume.

Figure shows the plan and elevation of a venturiflume.

B,H and V the width depth and velocity at the entrance to the flume b,h, and v the corresponding quantities in the throat.

The velocity V at the throat is more than the upstream velocity V. hence, there will be a drop in the water level at the throat (Bernoulli's equation).

From continuity equation

Q=BHV=bhv Also, A=BH and a= bh Q=AV=av or $V = \frac{a}{A} v$

Applying Bernoulli's equation between 1,1 and 2,2 with the channel bottom as datum and neglecting losses.

$$H + \frac{V^2}{2g} = h + \frac{v^2}{2g} \qquad v^2 - V^2 = 2g(H-h) \qquad substituting \;\; V = \frac{a}{A} v$$

$$V^2 - \frac{a^2}{A^2} v^2 = 2g(H-h) \qquad V^2 (1 - \frac{a^2}{A^2}) = 2g(H-h) \quad or, V = \frac{A}{\sqrt{A^2 - a^2}} \;\; \sqrt{2g(H-h)}$$

$$but, \; Q = av \qquad \therefore \; Q = \frac{aA}{\sqrt{A^2 - a^2}} \;\; \sqrt{2h(H-h)} \quad (1)$$

Equation 1 gives theoretical discharge $\therefore Q_{act} = C_d \frac{aA}{\sqrt{A^2 - a^2}} \;\; \sqrt{2g(h-h)}$ (2)

$C_d = coefficient\ of\ disch\arg e$

PROBLEMS:

1. A venturiflume is provided in a rectangular channel 2.5 m wide, the width of the throat being 1.5m. Find the rate of flow when the depth of flow upstream is 1.25m and that at the throat is 1m neglect losses.

Soln: B=2.5m, H=1.25m, b=1.5m,h=1.00m $A = BH = 3.75m^2 \quad a = bh = 1.5m^2$

$$disch\arg e \;\; Q = C_d \frac{Aa}{\sqrt{A^2 - a^2}} \;\; \sqrt{2g(H-h)}$$

Since there are no losses $C_d = 1 \qquad \therefore \; Q = \frac{3.75x1.5}{\sqrt{3.75^2 - 1.5^2}} \; x \;\; \sqrt{29.81(1.25-1)}$

$\therefore \; Q = 3.625 \, cumecs$

* * * * *