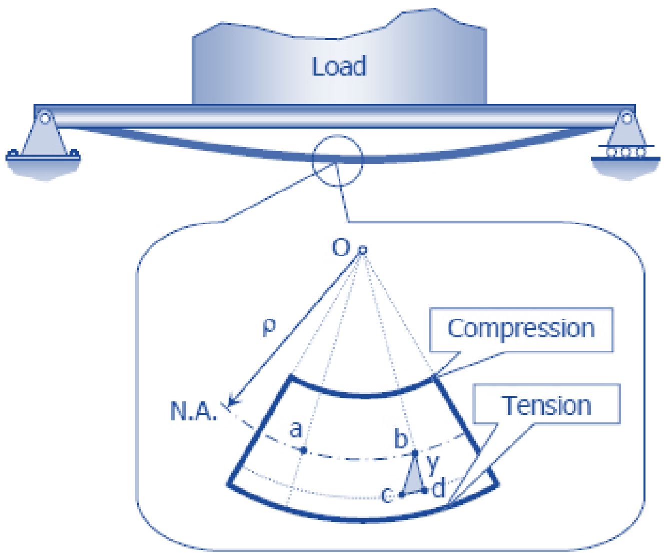
FLEXURAL STRESSES IN BEAMS

Stresses caused by the bending moment are known as flexural or bending stresses.

Consider a beam to be loaded as shown:



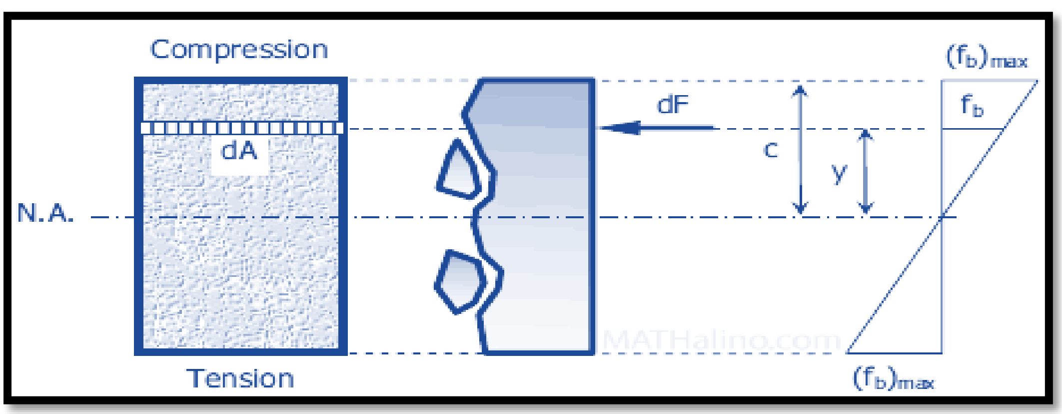
Consider a fiber at a distance y from the neutral axis, because of the beam's curvature, as the effect of bending moment, the fiber is stretched by an amount of cd. Since the curvature of the beam is very small, bcd and Oba are considered as similar triangles. The strain on this fiber is

$$\varepsilon = \frac{cd}{ab} = \frac{y}{\rho}$$

By Hooke's law, $\varepsilon = \sigma/E$, then

$$\frac{\sigma}{E} = \frac{y}{\rho};\ \sigma = \frac{y}{\rho}E$$

Which means that the stress is proportional to the distance y from the neutral axis.



Considering a differential area dA at a distance y from N.A., the force acting over the area is

$$dF = f_b\,dA = \frac{y}{\rho}E\,dA = \frac{E}{\rho}y\,dA$$

The resultant of all the elemental moment about N.A. must be equal to the bending moment on the section.

$$M = \int dM = \int y\,dF = \int y\left(\frac{E}{\rho}y\,dA\right)$$

$$M = \frac{E}{\rho}\int y^2\,dA$$

But $\int y^2\,dA = I$, then

$$M = \frac{EI}{\rho} \text{ or } \rho = \frac{EI}{M}$$

Substituting $\rho = Ey/f_b$

$$\frac{Ey}{f_b} = \frac{EI}{M}$$

Then $$f_b = \frac{My}{I}$$

And $$(f_b)_{max} = \frac{Mc}{I}$$

The bending stress in beam due to curvature is

$$f_b = \frac{Mc}{I} = \frac{\frac{EI}{\rho}c}{I}$$

$$f_b = \frac{Ec}{\rho}$$

The beam curvature is:

$$k = \frac{1}{\rho}$$

Where ρ is the radius of curvature of the beam in mm (in), M is the bending moment in N·mm (lb·in), f_b is the flexural stress in MPa (psi), I is the centroidal moment of inertia in mm4 (in^4), and c is the distance from the neutral axis to the outermost fiber in mm (in).

Section Modulus:

In the formula

$$(f_b)_{max} = \frac{Mc}{I} = \frac{M}{I/c}$$

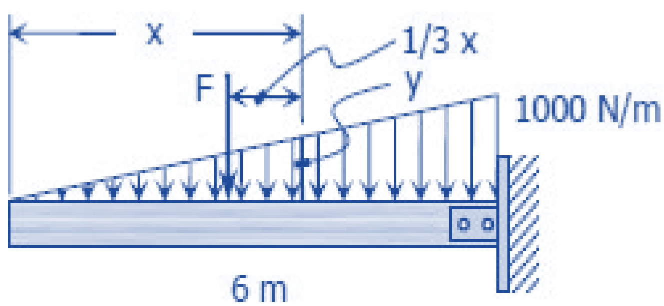
The ratio I/c is called the **section modulus** and is usually denoted by S with units of mm^3 (in^3). The maximum bending stress may then be written as

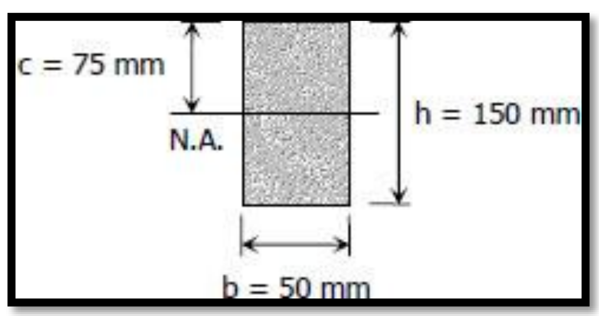
$$(f_b)_{max} = \frac{M}{S}$$

This form is convenient because the values of S are available in handbooks for a wide range of standard structural shapes.

Problem 1:

A cantilever beam, 50 mm wide by 150 mm high and 6 m long, carries a load that varies uniformly from zero at the free end to 1000 N/m at the wall. (a) Compute the magnitude and location of the maximum flexural stress. (b) Determine the type and magnitude of the stress in a fiber 20 mm from the top of the beam at a section 2 m from the free end.





$M = F(\frac{1}{3}x)$

$$\frac{y}{x} = \frac{1000}{6}$$

$y = \frac{500}{3}x$

$F = \frac{1}{2}xy$

$F = \frac{1}{2}x(\frac{500}{3}x)$

$F = \frac{250}{3}x^2$

Thus,

$M = (\frac{250}{3}x^2)(\frac{1}{3}x)$

$M = \frac{250}{9}x^3$

Part (a):

The maximum moment occurs at the support (the wall) or at x = 6 m.

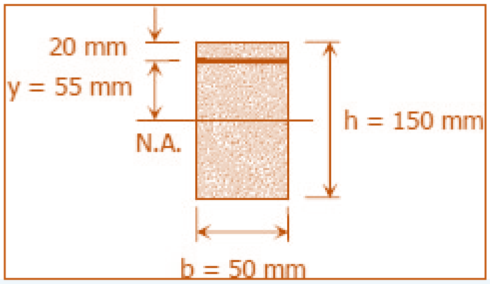
$M = \frac{250}{9}x^3 = \frac{250}{9}(6^3)$

$M = 6000 \text{ N} \cdot \text{m}$

$$(f_b)_{max} = \frac{Mc}{I} = \frac{Mc}{\frac{bh^3}{12}}$$

$$(f_b)_{max} = \frac{Mc}{I} = \frac{6000(1000)(75)}{\frac{50(150^3)}{12}}$$

$$(f_b)_{max} = 32\,\text{MPa}$$ (*Answer*)



Part (b):

At a section 2 m from the free end or at x = 2 m at fiber 20 mm from the top of the beam:

$$M = \frac{250}{9}x^3 = \frac{250}{9}(2^3)$$

$$M = \frac{2000}{9}\,\text{N}\cdot\text{m}$$

$$f_b = \frac{My}{I} = \frac{\frac{2000}{9}(1000)(55)}{\frac{50(150^3)}{12}}$$

$$f_b = 0.8691\,\text{MPa} = 869.1\,\text{kPa}$$ (*Answer*)

Problem 2:

A simply supported beam, 2 in wide by 4 in high and 12 ft long is subjected to a concentrated load of 2000 lb at a point 3 ft from one of the supports. Determine the maximum fiber stress and the stress in a fiber located 0.5 in from the top of the beam at midspan.

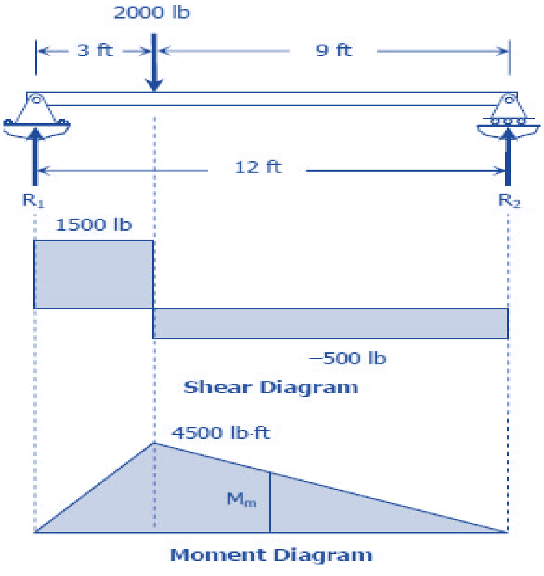
$$\Sigma M_{R2} = 0$$

$$12R_1 = 9(2000)$$

$$R_1 = 1500\,\text{lb}$$

$$\Sigma M_{R1} = 0$$

$$12R_2 = 3(2000) \Rightarrow R_2 = 500\,\text{lb}$$



Maximum fiber stress:

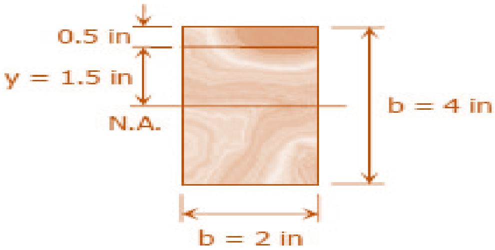
$$(f_b)_{max} = \frac{Mc}{I} = \frac{4500(12)(2)}{\frac{2(4^3)}{12}}$$

$$(f_b)_{max} = 10,125\ \text{psi}$$ (***Answer***)

Stress in a fiber located 0.5 in from the top of the beam at midspan:

$$\frac{M_m}{6} = \frac{4500}{9}$$

$$M_m = 3000\,\text{lb}\cdot\text{ft}$$



$$f_b = \frac{My}{I}$$

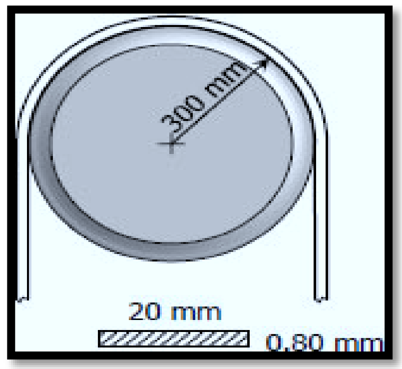
$$f_b = \frac{3000(12)(1.5)}{\frac{2(4^3)}{12}}$$

$f_b = 5,062.5$ psi **(*Answer*)**

Problem 3:

A high strength steel band saw, 20 mm wide by 0.80 mm thick, runs over pulleys 600 mm in diameter. What maximum flexural stress is developed? What minimum diameter pulleys can be used without exceeding a flexural stress of 400 MPa? Assume E = 200 GPa.

Flexural stress developed:



$$M = \frac{EI}{\rho}$$

$$f_b = \frac{Mc}{I} = \frac{(EI/\rho)c}{I}$$

$$f_b = \frac{Ec}{\rho} = \frac{200000(0.80/2)}{300}$$

$f_b = 266.67\,\text{MPa}$ **(*Answer*)**

Minimum diameter of pulley:

$$f_b = \frac{Ec}{\rho}$$

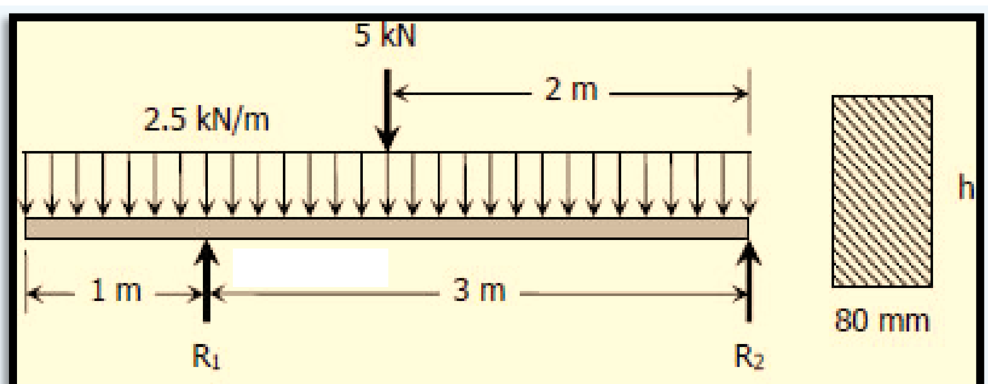
$$400 = \frac{200000(0.80/2)}{\rho}$$

$$\rho = 200\,\text{mm}$$

Diameter, d = 400 mm **(*Answer*)**

Problem 4:

Determine the minimum height h of the beam shown in Fig. if the flexural stress is not to exceed 20 MPa.



$\Sigma M_{R2} = 0$

$3R_1 = 2(5) + 2(2.5)(4)$

$R_1 = 10\,\text{kN}$

$\Sigma M_{R1} = 0$

$3R_2 = 1(5) + 1(2.5)(4)$

$R_2 = 5\,\text{kN}$

$$f_b = \frac{Mc}{I}$$

Where:

$f_b = 20\,\text{MPa}$

$M = 5\,\text{kN} \cdot \text{m} = 5(1000)^2\,\text{N} \cdot \text{mm}$

$c = \frac{1}{2}h$

$$I = \frac{bh^3}{12} = \frac{80h^3}{12} = \frac{20}{3}h^3$$

Thus,

$$20 = \frac{5(1000)^2(\frac{1}{2}h)}{\frac{20}{3}h^3}$$

$h^2 = 18\,750$

$h = 137\,\text{mm}$ **(*Answer*)**