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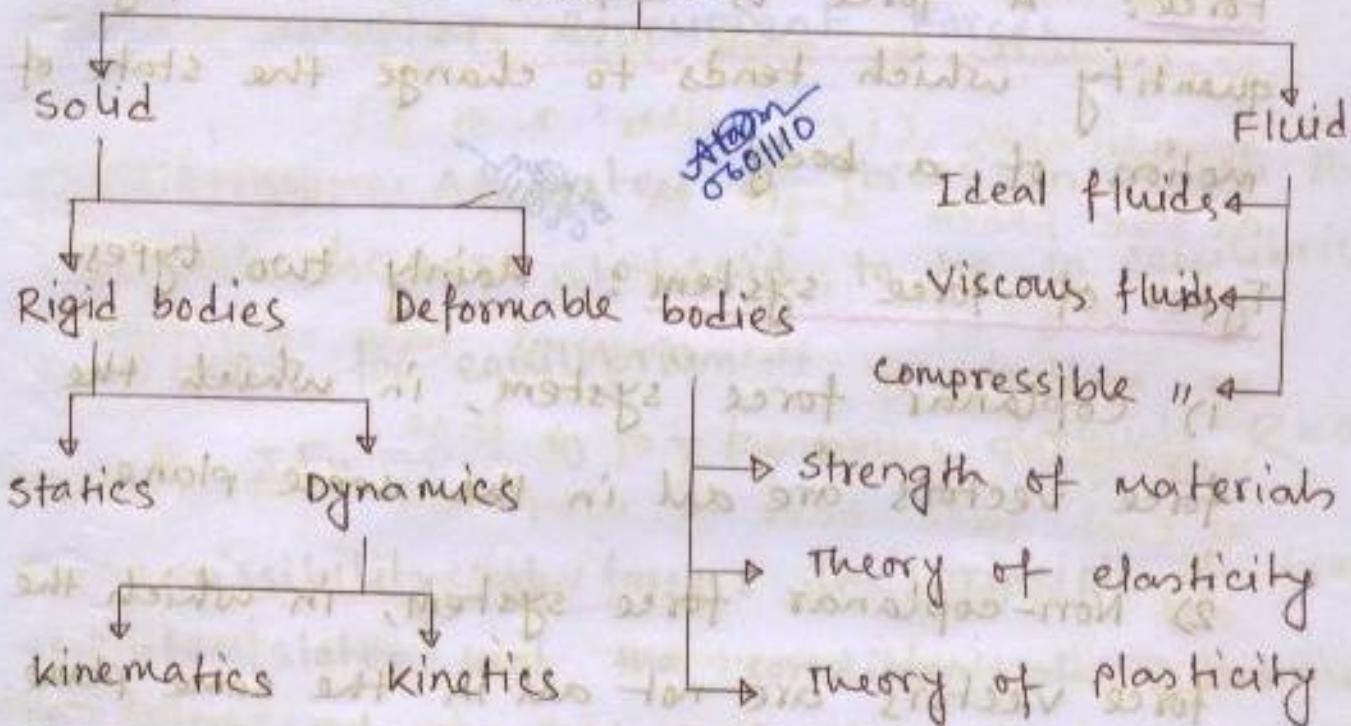
2006 Batch
Dept. of Civil Engineering
KUET, Khulna-9203

ENGINEERING MECHANICS

Resultants and components

Mechanics: Mechanics may be defined as that science which describes and predicts the conditions of rest or motion of bodies under the action of forces.

Branches of Mechanics



Statics: It is the study of equilibrium of bodies under the action of forces

Dynamics: It is the study of motion of bodies

- kinematically
- kinetically

a) **Kinematics:** It is the study of the geometry of the motion without reference to the cause of motion. It deals with position, displacement, velocity, time and acceleration.

b) Kinetics: It is the study of the relationship b/w forces and the resulting motion of bodies on which they act.

Force: A force is defined as that physical quantity which tends to change the state of motion of a body.

Types of force system: Mainly two types-

1) Coplanar force system, in which the force vectors are all in the same plane.

2) Non-coplanar force system, in which the force vectors are not all in the same plane.

Beside these classification, there are also some force system,

1) collinear force system, in which all the forces act along the same line of action. It must be coplanar.

2) concurrent force system, in which all lines of action intersect at one point.

3) Non-concurrent force system, in which the

lines of action of the force vectors do not intersect at a point.

4) Parallel force system, in which the lines of action of all force vectors are parallel.

~~Non
coplanar~~

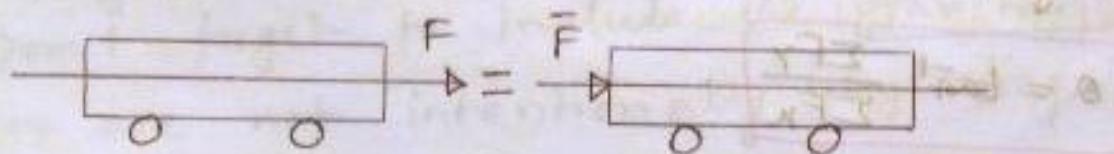
Coplanar Concurrent Forces

Equilibrium: A system of forces in which the resultant is zero is said to be in equilibrium.

conditions for equilibrium →

$$\text{i)} \sum F_x = 0 \quad \text{ii)} \sum F_y = 0 \quad \text{obviously } R = 0$$

Transmissibility of force: The principle of transmissibility states that the condition of equilibrium or of motion of a rigid body will remain unchanged if a force F acting at a given point of the rigid body is replaced by a force \bar{F} of the same magnitude and same direction, but acting at a different point, provided that the two force have same line of action.



Resultant of a force system: method to find
1) Graphical 2) Algebraical

Algebraical method:

steps: *Atom*

1) Mention the co-ordinates.

2) Evaluate the angle b/w the forces and the co-ordinates (i.e; either x or y).

3) Use cosine angle for direct component and sine angle for indirect or opposite component by multiplying the original force. It is called rectangular component of a force.

4) Each force have to component that is horizontal and vertical for 2-D bodies.

5) Find these component for every forces.

6) Use algebraic sign i.e; for horizontal component if right side directional force is positive then left side directional force will be negative and vice-versa.

7) At all the same components i.e; ΣF_x & ΣF_y .

8) Determine the resultant by using the following formula,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

where, R is the magnitude of the resultant and for each direction

$$\theta = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x}$$

■ Problem - 51, Page - 33, Faires.

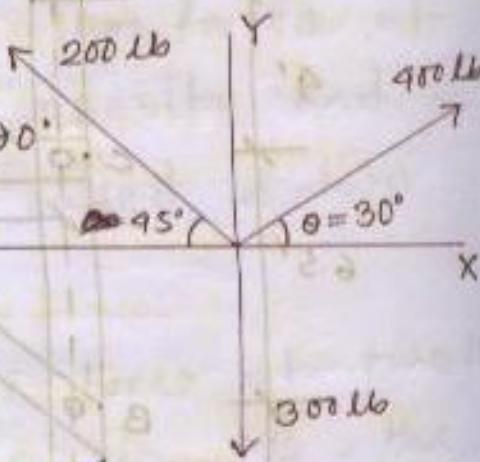
Following figure shows three con-current coplanar forces with $\theta = 30^\circ$. Determine the resultant of this force algebraically.

$$(+ve \rightarrow) \sum F_x = 400 \cos 30^\circ - 200 \cos 45^\circ + 300 \cos 90^\circ \\ = 204.99 \text{ lb}$$

$$(+ve \uparrow) \sum F_y = 400 \sin 30^\circ + 200 \sin 45^\circ - 300 \\ = 91.92 \text{ lb}$$

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = 209.13 \text{ lb}$$

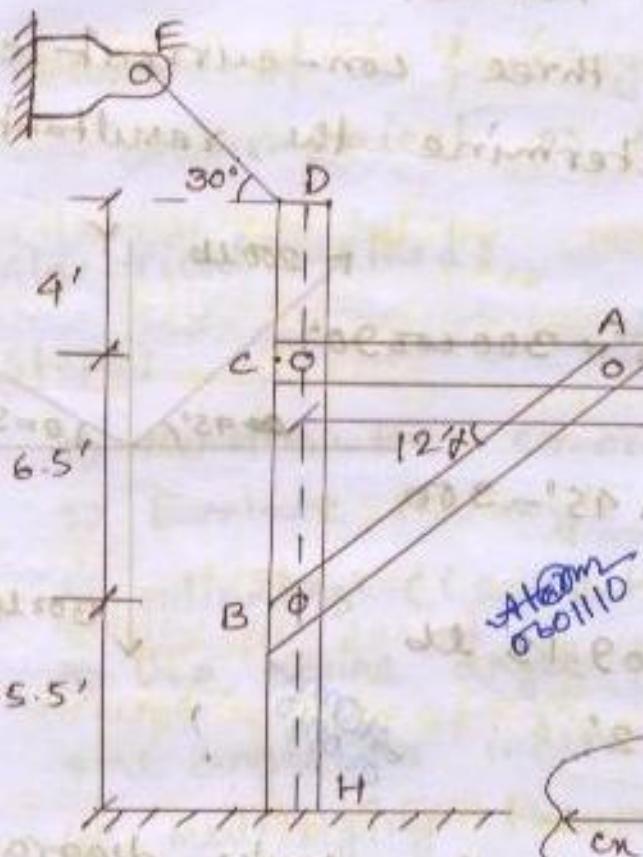
*Atom
000110*



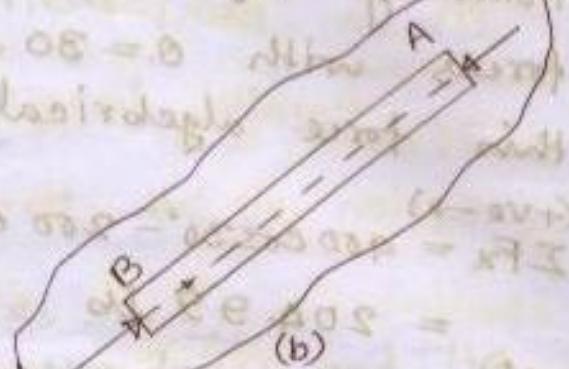
■ Free body diagram : A free body diagram is a diagram of a body or a group of bodies (or part of a body) which is isolated from its environment and on which all external forces such as weight, applied forces, constraints or reactions and friction are acting.

■ Steps :

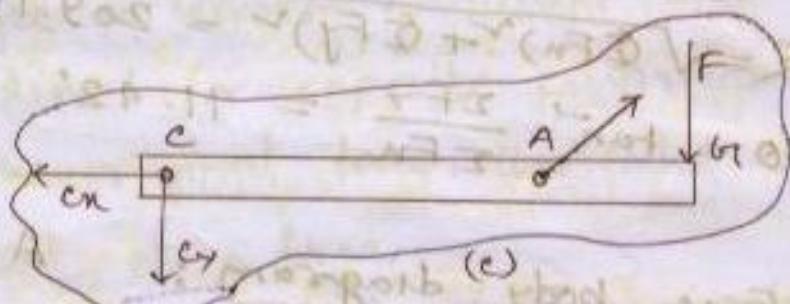
- 1) Identify the object you want to isolate.
- 2) Draw a sketch of the object isolated from its surroundings and show relevant dimensions and angles.
- 3) Don't forget to include the gravitational force if you are not intentionally neglecting it.



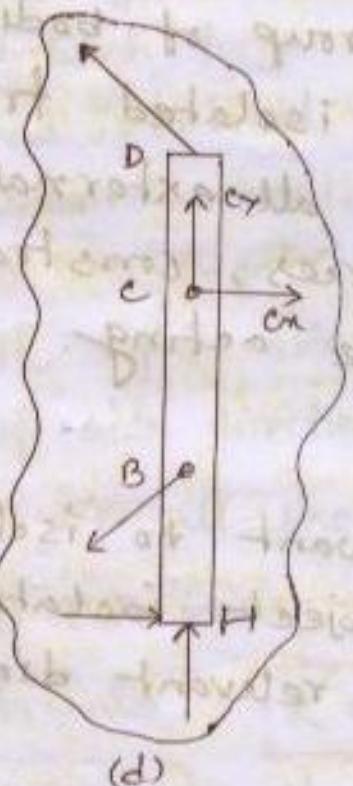
(a)



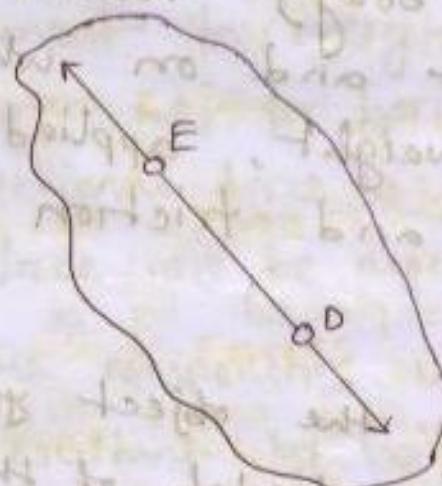
FBD of AB



FBD of CG



(d)



(e)

FBD of DE

FBD of DH

compressive force \rightarrow

Tension force \leftarrow

Two force member:

If we consider the member AB, since their member is in equilibrium, ie; ΣF (in any direction) = 0, it follows that the forces at A and B must be equal, opposite and collinear. Such a member is called a two force member.

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compression member: When the force at two compress the fibers together as in AB, the member is said to be in compression.

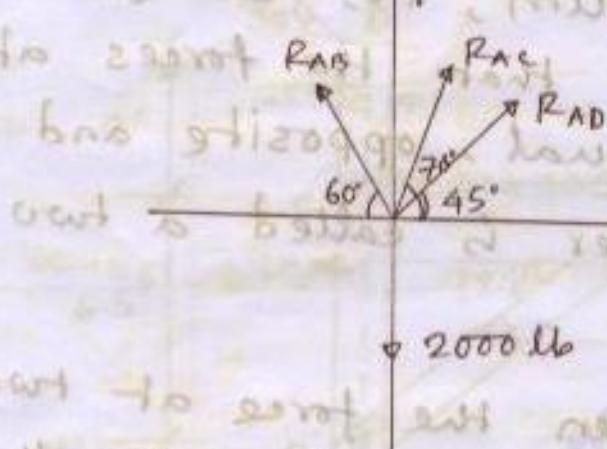
Tension member: If the forces on a two force member act away from other tending to pull apart, as in the force body of ED. The member is said to be in tension.

Flexible cord, cables rope - Tension force
magnetic force acts in magnetic direction
Gravitational force acts in downward.

problem - 2.12, page - 26, Bedford & Fowler.

The suspended weight exerts a downward 2000 lb force F at A. If you resolve F into vector component parallel to the wires AB, AC and AD, the magnitude of the component parallel to AC is 600 lb, what are the magnitudes of the components parallel to AB and AD?

consider FBD of A,



$$\sum F_x = R_{AD} \cos 45^\circ + R_{AC} \cos 70^\circ - R_{AB} \cos 60^\circ = 0$$

$$0.707 R_{AD} + 600 \times 0.342 - R_{AB} \times 0.5 = 0$$

$$\Rightarrow 0.707 R_{AD} - 0.5 R_{AB} + 205.2 = 0 \quad (1)$$

(+ve)

$$\sum F_y = R_{AD} \sin 45^\circ + R_{AC} \sin 70^\circ + R_{AB} \sin 60^\circ - 2000 = 0 \quad (2)$$

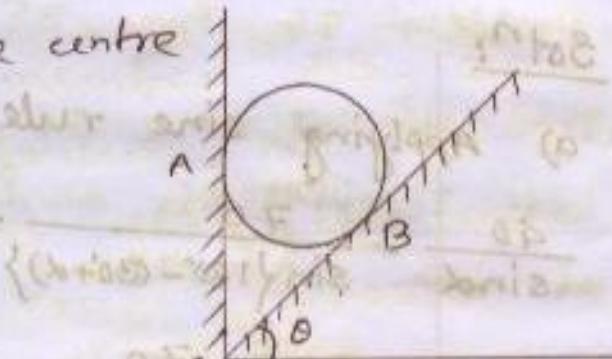
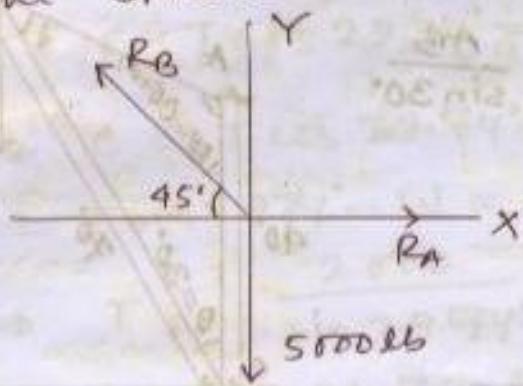
Solving the eqn (1) & (2),

$$R_{AB} = 1202 \text{ lb} \quad R_{AD} = 559 \text{ lb}$$

■ Problem-72, Page-35, Faires plane

A 5000 lb sphere rest on a smooth inclined plane at an angle $\theta = 45^\circ$ with the horizontal and against a smooth vertical wall. What are the reactions at the contact surfaces A and B?

consider the FBD of the centre of the sphere



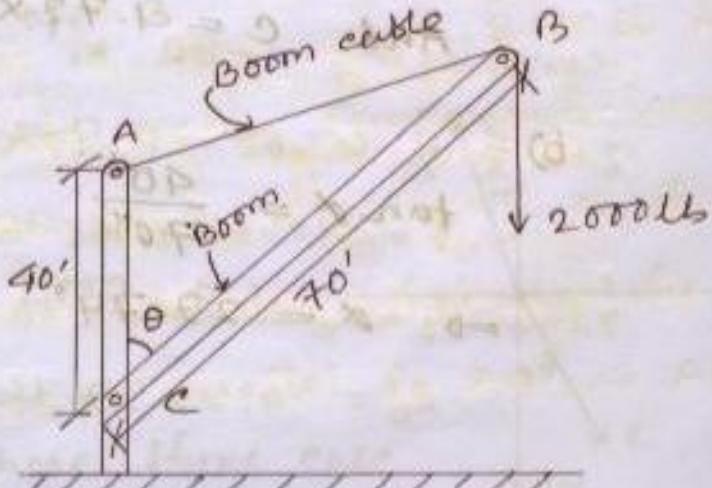
$$\sum F_x = R_A \cos 30^\circ - R_B \cos 45^\circ = 0$$

$$\Rightarrow R_A = 0.707 R_B$$

$$\sum F_y = R_B \sin 45^\circ - 5000 = 0 \Rightarrow R_B = 7071.07 \text{ lb}$$

$$\therefore R_A = 0.707 \times 7071.07 = 4999.24 \text{ lb.}$$

■ Problem - 82, Page - 35, Faires
 The derrick shown diagrammatically in following figure supports a load of $w = 2000 \text{ lb}$. Find the tension in the boom cable and the compression in the boom, when θ is a) 30° b) 90° c) 150° . which position produces the largest load on the boom?



Sol'n:

a) Applying sine rule,

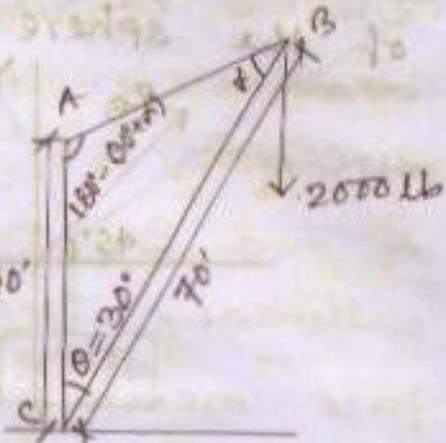
$$\frac{40}{\sin \alpha} = \frac{70}{\sin(180^\circ - (30^\circ + \alpha))} = \frac{AB}{\sin 30^\circ}$$

$$\frac{40}{\sin \alpha} = \frac{70}{\sin(30^\circ + \alpha)}$$

$$\Rightarrow \frac{4}{\sin \alpha} = \frac{7}{\sin 30^\circ \cos \alpha + \sin \alpha \cos 30^\circ}$$

$$\Rightarrow 2 \cos \alpha + 3 \cdot 4 \sin \alpha - 7 \sin \alpha = 0$$

$$\Rightarrow 2 \cos \alpha - 3.54 \sin \alpha = 0 \Rightarrow \alpha = 29.49^\circ$$



considering FBD of B

~~At A
B 0 1 1 1 0~~

$$\sum F_x = c \sin 30^\circ - T \sin 59.49^\circ = 0$$

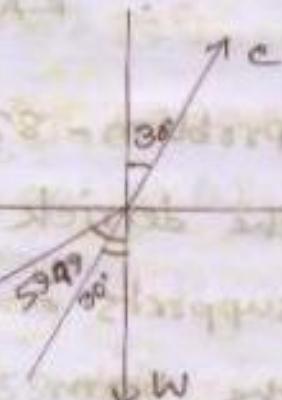
$$\Rightarrow c = 1.72 T$$

$$\sum F_y = c \cos 30^\circ - T \cos 59.49^\circ - 2000 = 0$$

$$\Rightarrow 1.49T - 0.51T = 2000$$

$$\Rightarrow T = 2036.01 \text{ lb} = 2.04 \text{ kip}$$

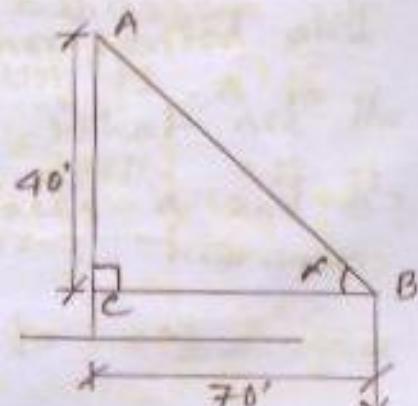
$$\text{And } c = 1.72 \times 2.04 = 3.51 \text{ kip}$$



b)

$$\tan \alpha = \frac{40'}{70'}$$

$$\Rightarrow \alpha = 29.74^\circ$$



considering FBD of B,

$$\sum F_x = c - T \cos 29.74^\circ = 0$$

$$\Rightarrow c = T \cos 29.74^\circ$$

$$\sum F_y = T \sin 29.74^\circ - w = 0$$

$$\Rightarrow T = \frac{w}{\sin 29.74^\circ}$$

$$\Rightarrow T = 4031.73 \text{ lb} = 4.03 \text{ kip}$$

$$\text{And } c = 4.03 \times \cos 29.74^\circ = 3.50 \text{ kip}$$

c) Applying sine rule,

$$\frac{40}{\sin \alpha} = \frac{70}{\sin(30^\circ - \alpha)} = \frac{AB}{\sin 150^\circ}$$

$$\frac{40}{\sin \alpha} = \frac{70}{\sin(30^\circ - \alpha)}$$

$$\Rightarrow \frac{\sin 30^\circ \cos \alpha - \cos 30^\circ \sin \alpha}{\sin \alpha} = \frac{7}{4}$$

$$\Rightarrow 2 \cos \alpha - 3.46 \sin \alpha = 7 \sin \alpha$$

$$\Rightarrow 2 \cos \alpha = 10.46 \sin \alpha$$

$$\therefore \alpha = 10.82^\circ$$

considering the FBD of B,

$$\sum F_x = c \sin 30^\circ - T \sin 19.18^\circ = 0$$

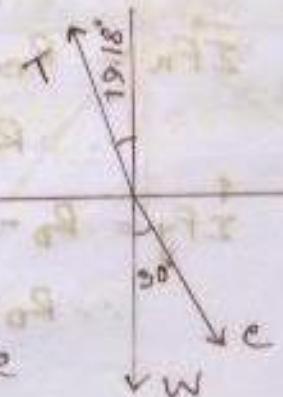
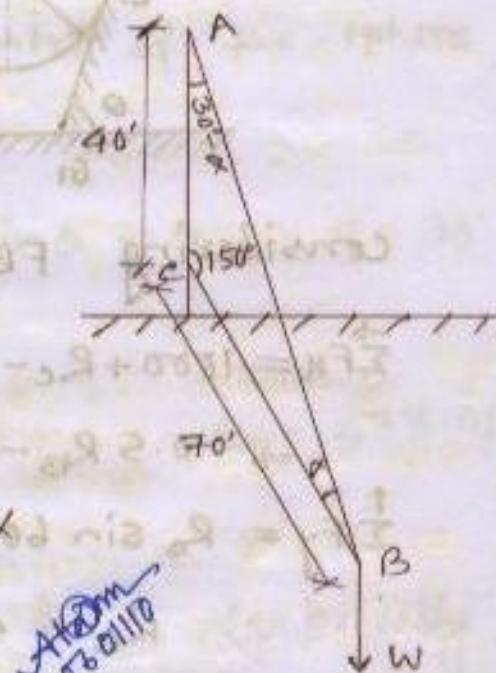
$$\Rightarrow c = 0.66 T$$

$$\sum F_y = T \cos 19.18^\circ - c \cos 30^\circ - 2000 = 0$$

$$\Rightarrow T = 5.36 \text{ kip}$$

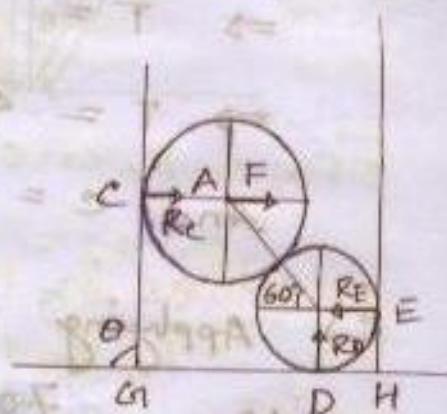
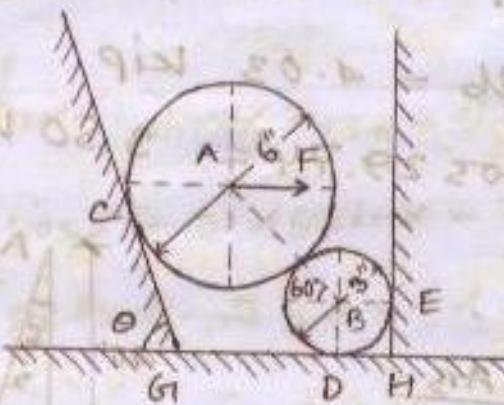
$$\text{And } c = 3.54 \text{ kip.}$$

∴ Compression of beam in these three case is always same.



Problem - 86, Page - 35, Faires.

Two spheres are at rest against smooth surfaces. Let, $F = 1000 \text{ lb}$ and $\theta = 90^\circ$. Find the reactions at the surfaces C, D and E. A weight 4000 lb and B weight 200 lb.



considering FBD of sphere A,

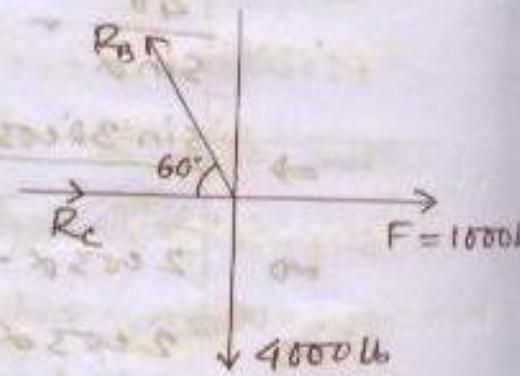
$$\sum F_x = 1000 + R_C - R_B \cos 60^\circ = 0$$

$$\Rightarrow 0.5 R_B - R_C = 1000 \quad \dots (1)$$

$$\sum F_y = R_B \sin 60^\circ - 4000 = 0$$

$$\therefore R_B = 4618.80 \text{ lb}$$

$$\text{And } R_C = 1309.4 \text{ lb}$$



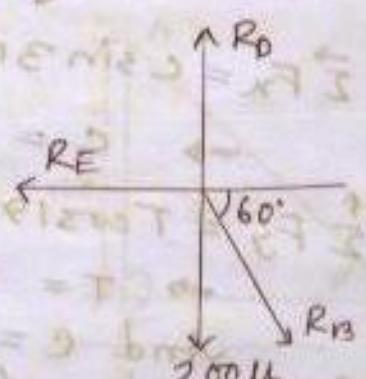
considering FBD of sphere B,

$$\sum F_x = R_B \cos 60^\circ - R_E = 0$$

$$\therefore R_B \cos 60^\circ = R_E = 2309.4 \text{ lb.}$$

$$\sum F_y = R_D - R_B \sin 60^\circ - 200 = 0$$

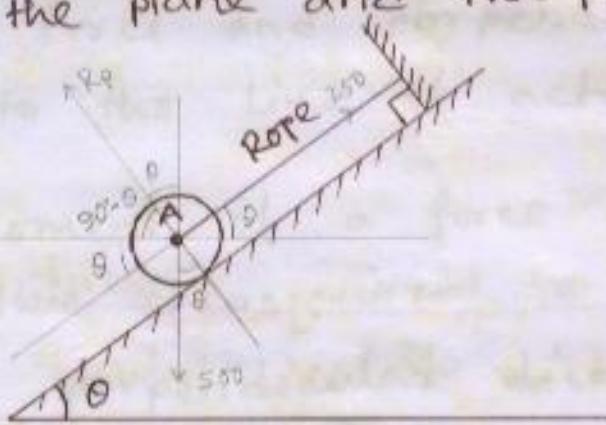
$$\therefore R_D = 4199.99 \text{ lb}$$



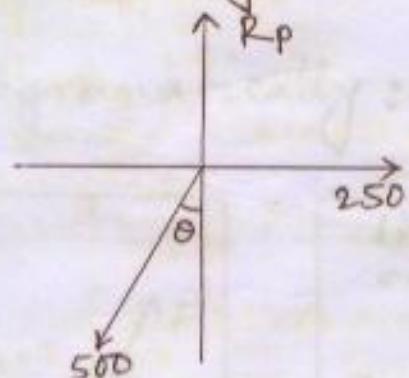
- A 500 lb cylinder A rests on a smooth inclined plane. For a tension in the rope of 250 lb. Find the inclination of the plane and the plane reaction.

spot test

Atom
600110



Considering the FBD of the centre of the sphere,



$$\sum F_x = 0$$

$$\Rightarrow 250 - 500 \sin \theta = 0 \quad \therefore \theta = 30^\circ$$

$$\sum F_y = 0$$

$$\Rightarrow R_p - 500 \cos \theta = 0 \quad \therefore R_p = 433.01 \text{ lb}$$

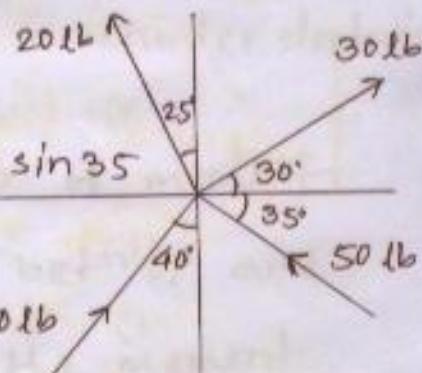
- Determine the resultant of the force algebraically.

$$\begin{aligned} \sum F_x &= 100 \sin 40 - 50 \cos 35 + 30 \cos 30 \\ &\quad - 20 \sin 25^\circ = 40.85 \text{ lb} \end{aligned}$$

$$\begin{aligned} \sum F_y &= 20 \cos 25 + 30 \sin 30 + 100 \cos 40 + 50 \sin 35 \\ &= 138.41 \text{ lb} \end{aligned}$$

$$R = \sqrt{(\sum F_x)^2 + (\sum F_y)^2} = 144.31 \text{ lb}$$

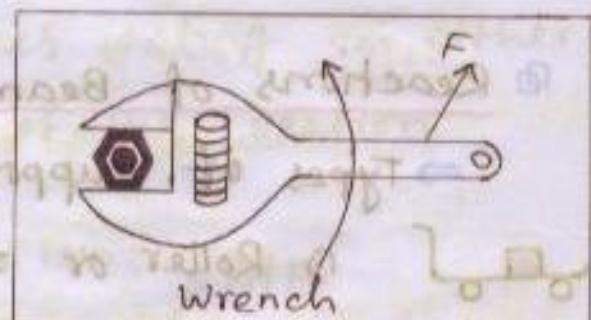
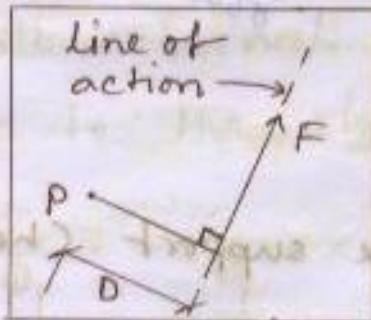
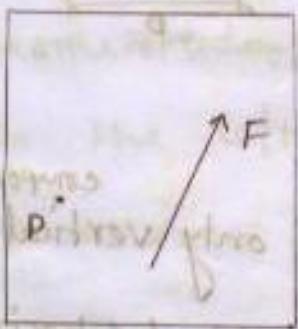
$$\theta = \tan^{-1} \frac{\sum F_y}{\sum F_x} = 73.56^\circ$$



Moments and parallel coplanar forces

- Moment of a force: This is the product of the magnitude of the force and perpendicular distance from any point to the line of action.
- Determining the moment of a force about a point p requires three steps:
 - 1) Determine the perpendicular distance
 - 2) Calculate the magnitude of the moment
 - 3) Determine the sign.

Diagrammatically:



- a) The force and the point
- b) The \perp^r distance D from point P to the line of action
- c) The sense of the moment is counter clockwise

In the above figure, P is the centre of moment.

and D is the perpendicular distance betn P and the line of action of force F . Thus the moment of the force about P is $M_p = D \times F$.

sign of moment - clockwise or anti-clockwise.

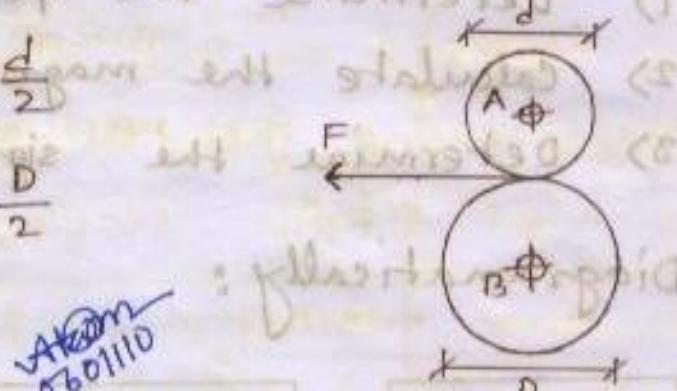
Couple: Two forces that have equal magnitudes, opposite directions and different lines of action are called a couple.



Torque or turning moment: Torque is moment of a force or forces about an axis of rotation.

$$\text{Torque on shaft, } A = F \times \frac{d}{2}$$

$$\text{Torque on shaft, } B = F \times \frac{D}{2}$$



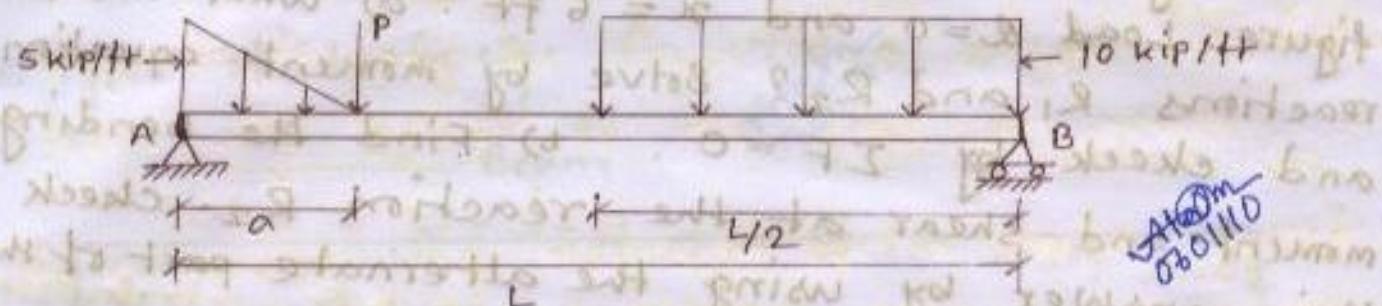
Reactions of Beam:

⇒ Types of supports:

- 1) Roller or simple support (have only vertical component).
- 2) Hinged or pinned (have only vertical & horizontal component).
- 3) Fixed " (have vertical, horizontal & moment component).

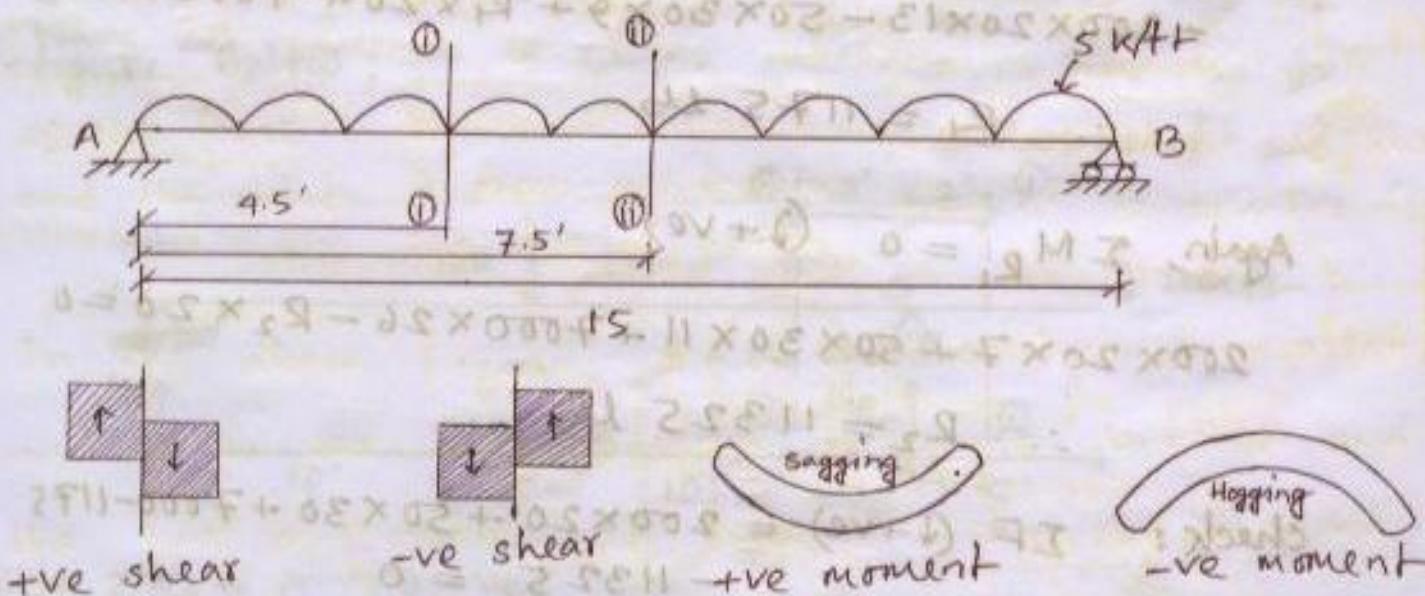
⇒ Types of loading:

- 1) Concentrated loading.
- 2) Uniformly distributed loading (U.D.L.)
- 3) Uniformly varying loading.



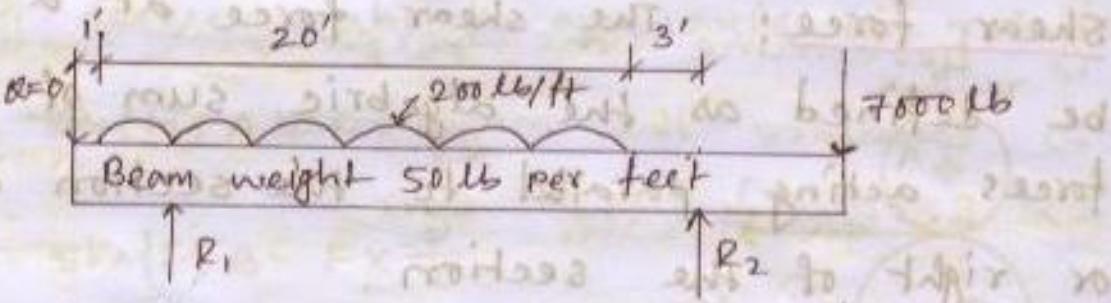
Shear force: The shear force at a section may be defined as the algebraic sum of all the external forces acting parallel to the section on either left or right of the section.

Bending moment: It is the algebraic sum of the moments of all external forces acting on either to the left or to the right of the section.



Problem-200, Page-62, Faires.

A simply supported beam is loaded as shown in figure. Load $\alpha=0$ and $x=6 \text{ ft}$. a) What are the reactions R_1 and R_2 ? Solve by moment equation and check by $\Sigma F = 0$. b) Find the bending moment and shear at the reaction R_2 . Check this answer by using the alternate part of the beam.



*Answer
0601110*

Soln: Taking summation of moment about R_2 ,

$$\sum M_{R_2} = 0 \quad (\text{Q+ve})$$

$$-200 \times 20 \times 13 - 50 \times 30 \times 9 + R_1 \times 20 + 7000 \times 6 = 0$$

$$R_1 = 1175 \text{ lb}$$

$$\text{Again, } \sum M_{R_1} = 0 \quad (\text{Q+ve})$$

$$200 \times 20 \times 7 + 50 \times 30 \times 11 + 7000 \times 26 - R_2 \times 20 = 0$$

$$\therefore R_2 = 11325 \text{ lb}$$

check: $\Sigma F \quad (\text{Q+ve}) = 200 \times 20 + 50 \times 30 + 7000 - 1175 - 11325 = 0$

Considering left side bending moment in R_2

$$= -200 \times 20 \times 13 - 50 \times 30 \times 9 + 1175 \times 20 = -42000$$

considering right side bending moment in R_2

$$= 7000 \times 6 = 42000$$

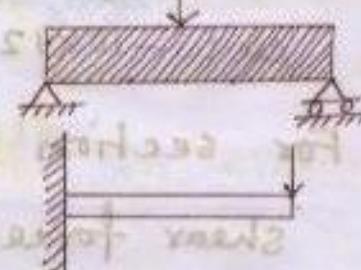
④ Types of beam:

Beam

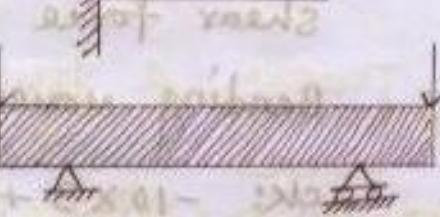
Actual
Bending

(32) General
statically determine beams ① → statically indeterminate beams

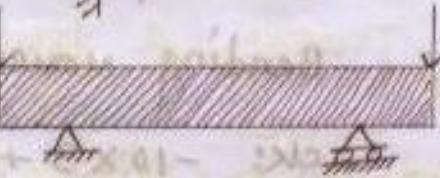
→ Simply supported beam



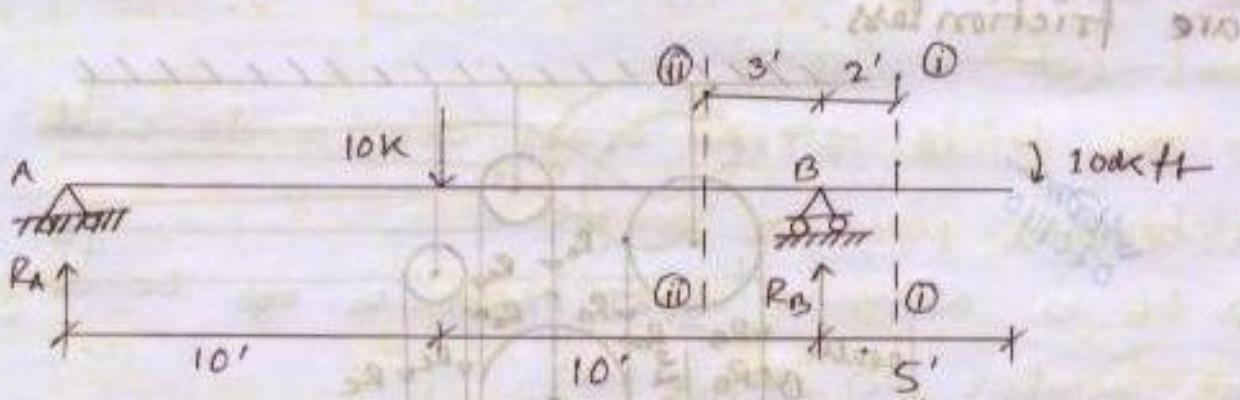
→ Cantilever beams



→ Overhanging beams



④ Determine the reactions and shear force and bending moment at section ①-① and ②-② as shown in figure below:



Taking $\sum M_B = 0$ (2+ve)

$$\Rightarrow R_A \times 20 - 10 \times 10 + 100 = 0$$

$$\therefore R_A = 0 \text{ kip}$$

Again $\sum M_A = 0$ (2+ve)

$$\Rightarrow 10 \times 10 + (R_B \times 20) + 100 = 0 \quad \text{or} \quad R_B = -\frac{100}{20} = -5 \text{ kip}$$

$$\therefore R_B = 10 \text{ kip}$$

For section ①-① :

shear force at ①-① = $10 - 0 - 10 = 0 \text{ kip}$

Bending moment at ①-① = 100 kft (clockwise)

$$\text{ck: } -10 \times 12 + 10 \times 2 = -100 \text{ kft}$$

For section ②-② :

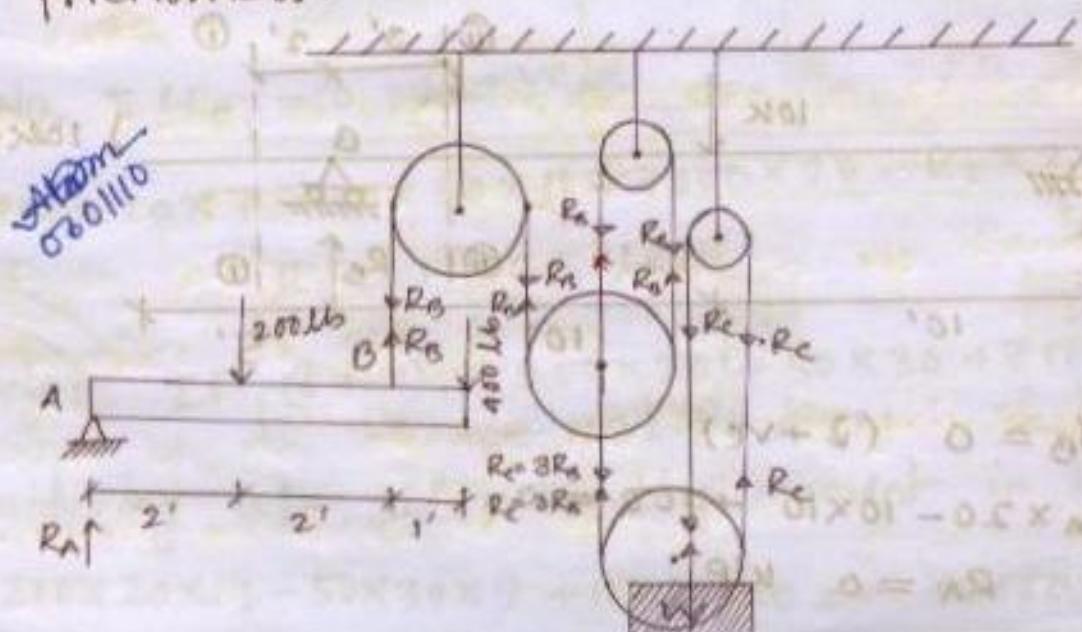
shear force at ②-② = $0 + 10 = 10 \text{ kip}$ (considering left)

Bending moment at ②-② = $-10 \times 7 = -70 \text{ kft}$

$$\text{ck: } -10 \times 3 + 100 = 70 \text{ kft}$$

■ Problem - Regular exam - 2005 (3.6).

In the following figure, what will be the value of w , if equilibrium exists? Assume the pulleys are frictionless.



Taking $\sum M_B = 0$ (↑ +ve) $0 = qM/3$ pincer

$$\Rightarrow R_A \times 4 - 200 \times 2 + 400 \times 1 = 0$$

$$\therefore R_A = 0 \quad 0 = 2 \times T - 2 \times 5 \leftarrow$$

Again $\sum M_A = 0$ (↑ +ve) $0 = 2 \times T - 2 \times 5 \leftarrow$

$$\Rightarrow 200 \times 2 - R_B \times 4 + 450 \times 5 = 0$$

$$\therefore R_B = 600 \text{ lb}$$

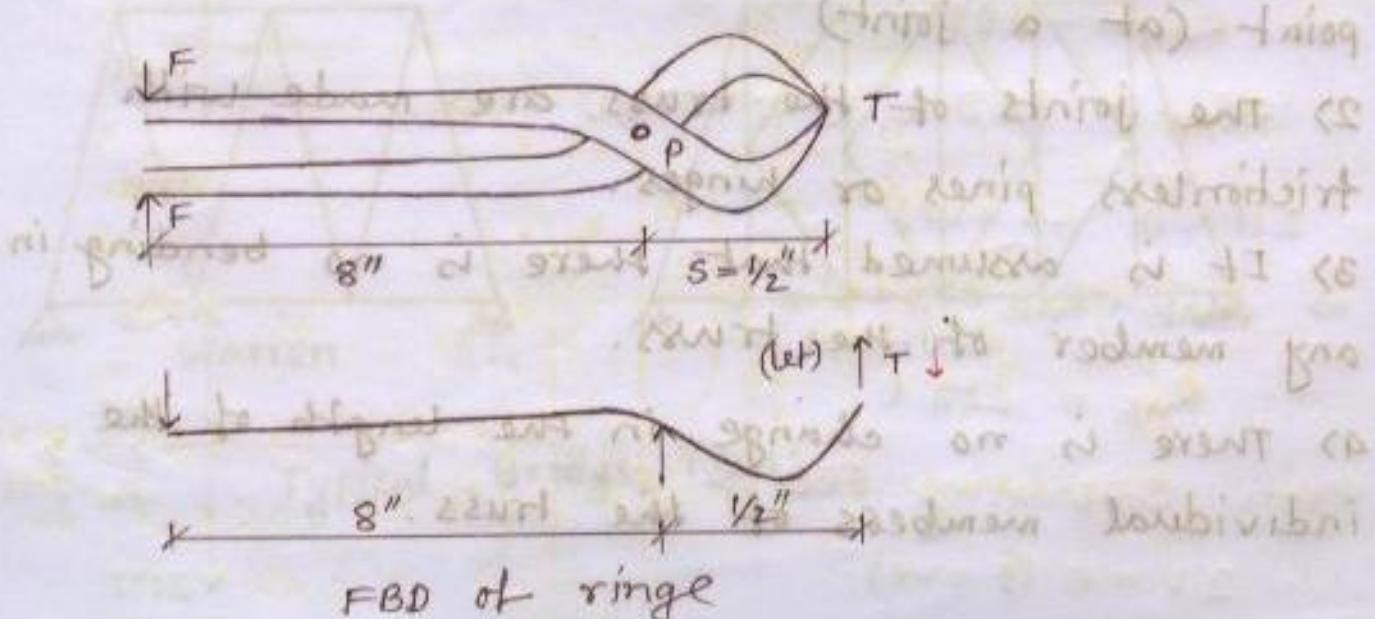
$$\therefore R_c = R_B + R_B + R_B = 1800$$

$$W = 3R_c = 3 \times 1800 = 5400 \text{ lb}$$

~~Atom
00110~~

Problem - 183, Page - 60, Faires.

on a pair of carpenter pincers, $F = 12 \text{ lb}$ and $s = \frac{1}{2} \text{ in}$. Assuming coplanar forces, find the force on the cutting edges. The pin P is in equilibrium under the action of two equal forces. What is the magnitude of these forces?



$$\text{Taking } \sum M_p = 0 \text{ (R+ve)} \quad \sum V = 0 \text{ (↑+ve)}$$

$$\Rightarrow -F \times 8 - T \times \frac{1}{2} = 0 \quad \Rightarrow P - 12 - 192 = 0$$

$$\Rightarrow -12 \times 8 - T \times \frac{1}{2} = 0 \quad \therefore P = 204 \text{ lb}$$

$$\therefore T = -192 \text{ lb}$$

Moments and parallel Coplanar Forces

Until Closed

Start

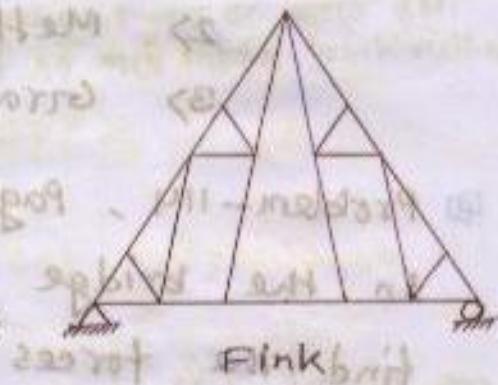
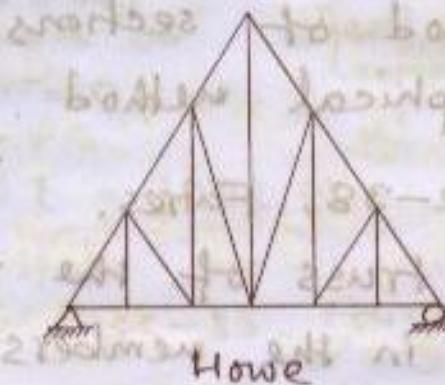
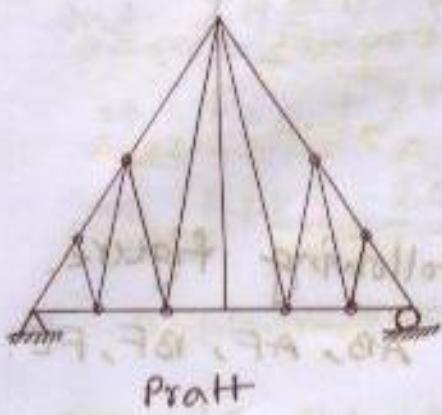
Non-concurrent, Non-parallel, Coplanar Forces

Truss: A truss is any framework of bars placed together to form triangles. The member may be riveted or pin-jointed together.

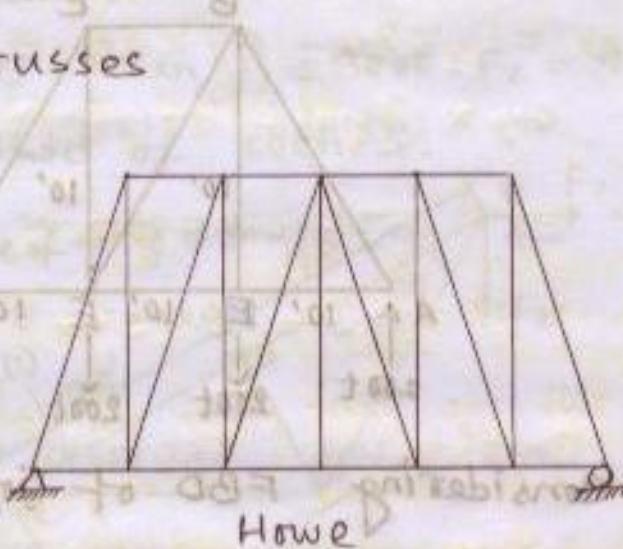
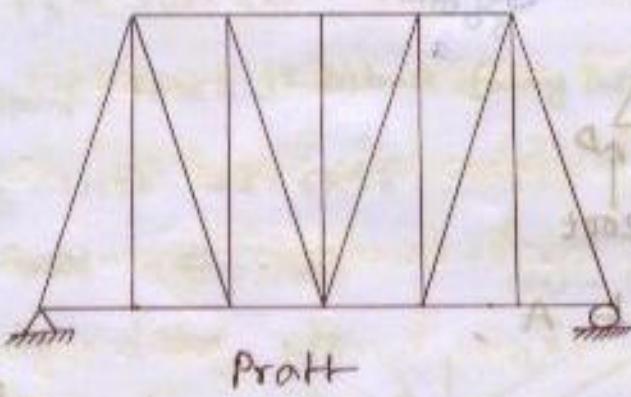
Fundamental assumptions:

- 1) The lines through the center of gravity of the section of all members meet at a single point (at a joint)
- 2) The joints of the truss are made with frictionless pins or hinges.
- 3) It is assumed that there is no bending in any member of the truss.
- 4) There is no change in the length of the individual members of the truss.

Different types of truss with figures:

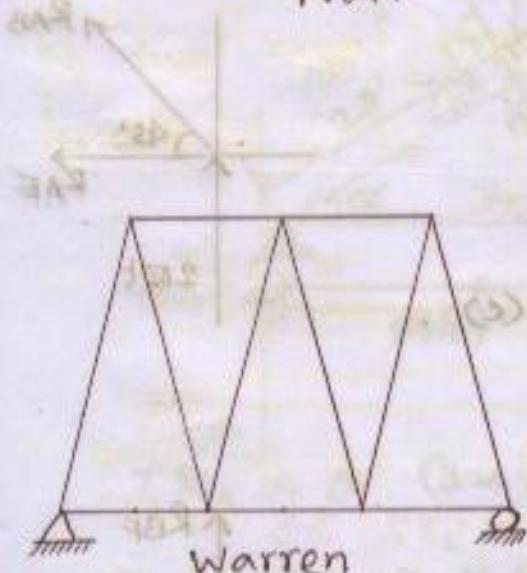


Typical roof trusses

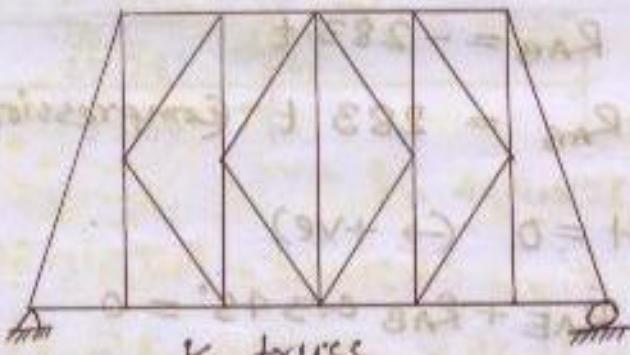


Pratt

Howe



Warren



K truss

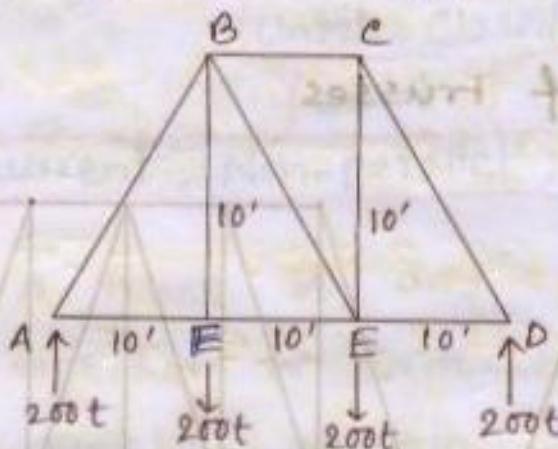
Typical Bridge Truss

Method of analysis: ~~con't~~ to 2nd part ~~for static~~

- 1) Method of joint
- 2) Method of sections
- 3) Graphical method.

Problem-114, Page-38, Faires.

In the bridge truss of the following figure, find the forces in the members AB, AF, BF, FE.



Considering FBD of joint A,

$$\sum V = 0 \quad (\uparrow +\text{ve})$$

$$\Rightarrow 200 + R_{AB} \sin 45^\circ = 0$$

$$\Rightarrow R_{AB} = -283 t$$

$$\therefore R_{AB} = 283 t \text{ (compression) or (c)}$$

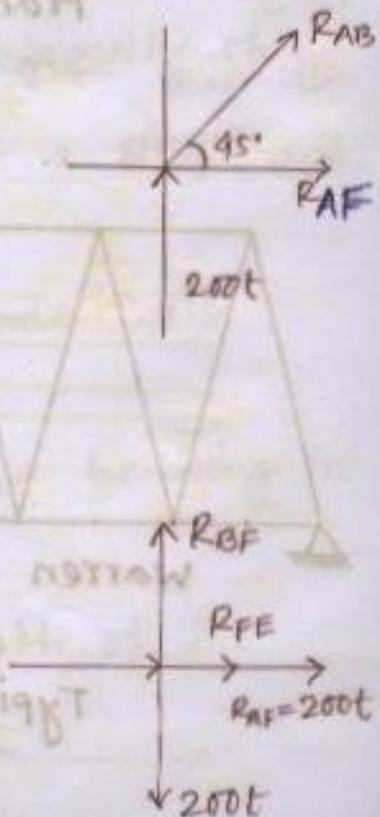
$$\sum H = 0 \quad (\rightarrow +\text{ve})$$

$$\Rightarrow R_{AE} + R_{AB} \cos 45^\circ = 0$$

$$\therefore R_{AF} = 200 t$$

Again Considering FBD of joint F,

$$\sum V = 0 \quad (\uparrow +\text{ve})$$



$$\Rightarrow R_{BF} - 200 = 0 \therefore R_{BF} = 200 \text{ t}$$

$$\text{And } \sum H = 0 \quad (\leftarrow +ve)$$

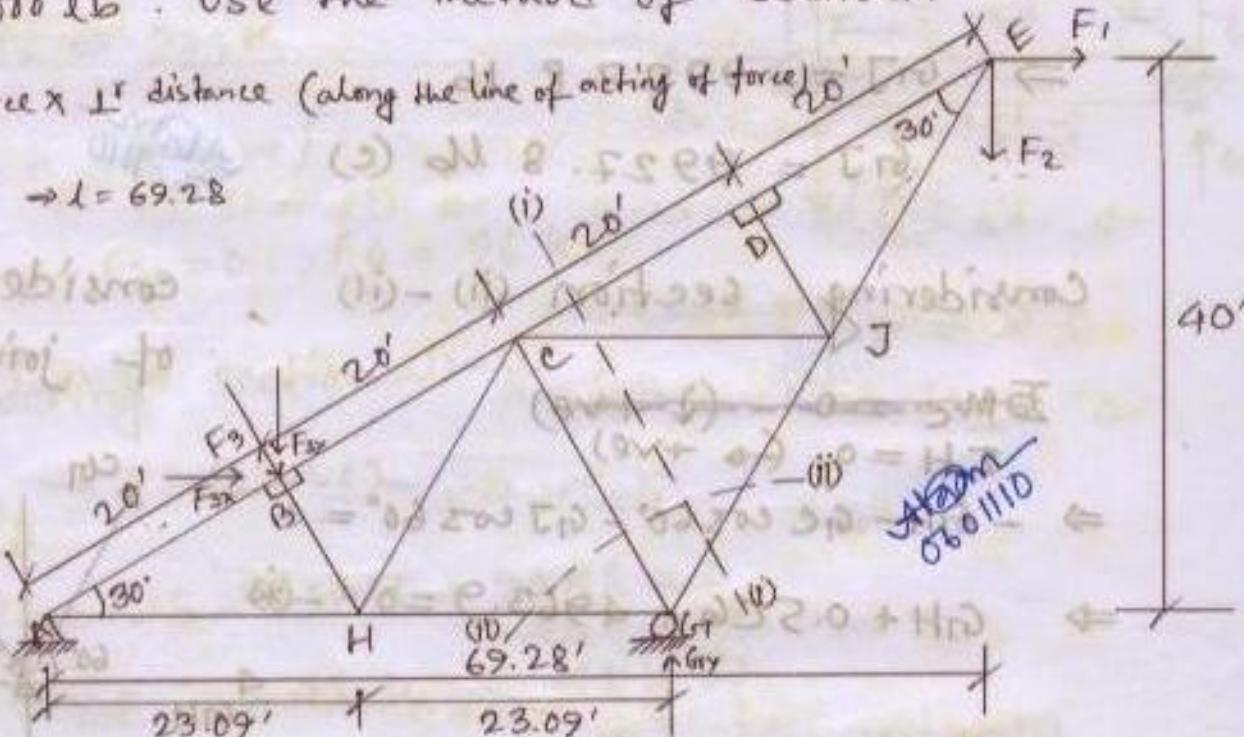
$$\Rightarrow R_{FE} + 200 = 0$$

$$\Rightarrow R_{FE} = -200 \text{ t} = 200\text{t} \quad (\text{C})$$

Problem- 295, Page- 85, Faires.

compute the external reactions and the loads on the members GH, GI and GJ when $F_1 = 3000$, $F_2 = 4000$ and $F_3 = 5000 \text{ lb}$. Use the method of section.

$$\left\{ \begin{array}{l} \text{Moment} = \text{Force} \times \perp \text{ distance (along the line of acting of force)} \\ \frac{l}{60} = \cos 30^\circ \rightarrow l = 69.28 \end{array} \right.$$



$$l = 40 / \cos 30^\circ = 46.19$$

$$\sum M_A = 0 \quad (2+ve)$$

$$\Rightarrow F_3 \times 20 + F_2 \times 69.28 + F_1 \times 40 - G_{IY} \times 46.19 = 0$$

$$\therefore G_{IY} = 10764.83 \text{ lb} = H_P$$

$$F_{3x} = F_3 \cos 60^\circ = 2500 \text{ lb}, F_{3y} = F_3 \sin 60^\circ = 4330.13 \text{ lb.}$$

$$\sum H = 0 \quad (\leftarrow +ve)$$

$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow F_{3x} + F_1 - A_x = 0$$

$$\Rightarrow A_y - F_{3y} - F_2 + G_y = 0$$

$$\therefore A_x = 5500 \text{ lb}$$

$$\therefore A_y = -2434.7 \text{ lb}$$

Considering section (i) - (i) (left bottom corner)

$$\sum M_c = 0 \quad (Q + ve)$$

$$\Rightarrow -F_3 \times 20 + A_y \times 40 \cos 30^\circ + A_x \times 40 \sin 30^\circ -$$

$$G_y \times (46.18 - 40 \cos 30^\circ) - G_J \times 40 \sin 30^\circ = 0$$

$$\Rightarrow G_J = -9927.8 \text{ lb}$$

$$\therefore G_J = 9927.8 \text{ lb (c)}$$

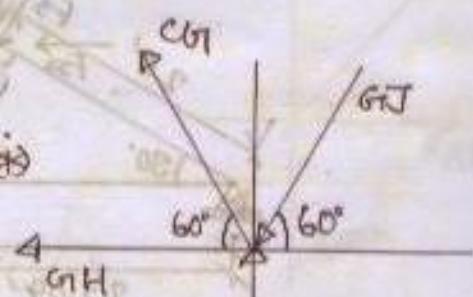
Considering section (ii) - (ii), consider the FBD of joint 6.

$$\sum M_c = 0 \quad (\downarrow +ve)$$

$$\sum H = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow -G_H - G_C \cos 60^\circ - G_J \cos 60^\circ = 0$$

$$\Rightarrow G_H + 0.5 G_C + 4963.9 = 0 \quad (iii)$$



$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow G_y + G_C \sin 60^\circ - G_J \sin 60^\circ = 0$$

$$\Rightarrow G_y = -2502.35 \text{ lb} = 2502.35 \text{ lb (c)}$$

$$\text{From eqn (ii), } G_H = -3712.73 \text{ lb}$$

$$\therefore G_H = 3712.73 \text{ lb (c)}$$

Prob - 6.30, Page - 225 Bogo & Johnston.

$$\sum M_F = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow -64y \times 15 + 30 \times 8 + 30 \times 16 + 30 \times 24 = 0$$

$$\Rightarrow 64y = 96 \text{ k}$$

$$\sum H = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow -F_H + 30 + 30 + 30 = 0$$

$$\therefore F_H = 90 \text{ k}$$

$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow G_Y - F_Y = 0 \quad . \quad F_Y = 96 \text{ k}$$

considering section (i)-(ii),

$$\sum M_D = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow -G_E \times 15 - 96 \times 15 + 90 \times 8 = 0$$

$$\therefore G_E = -48 \text{ k} \quad (C)$$

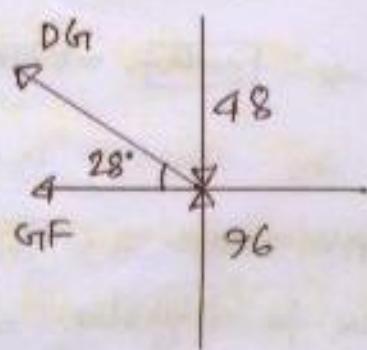
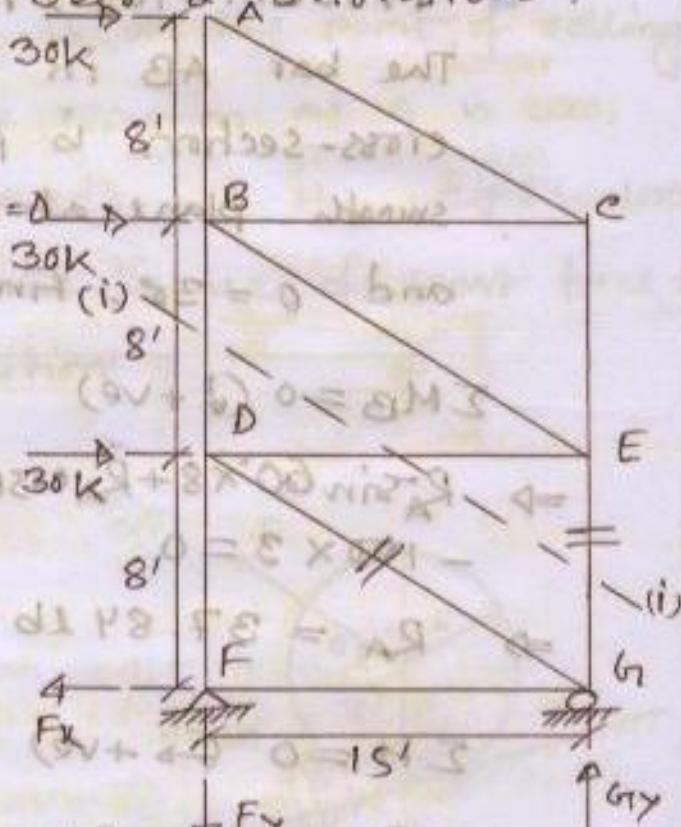
considering the FBD of point G,

$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow D_H \sin 28 - 48 + 96 = 0$$

$$D_H = -102.24 \text{ k}$$

$$= 102.24 \text{ k (C)}$$



Miscellaneous Problems (II to IV)

Q. Problem - 105, Page - 37, Faires.

The bar AB in the following figure, with uniform cross-section is pivoted at pin B and rests on a smooth plane at A. If the bar weighs $w = 100\text{lb}$ and $\theta = 30^\circ$, find the pin reaction at B.

$$\sum M_B = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow R_A \sin 60^\circ \times 8 + R_A \cos 60^\circ \times 2 - 100 \times 3 = 0$$

$$\Rightarrow R_A = 37.84 \text{ lb}$$

$$\sum H = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow R_A \cos 60^\circ - Bx = 0 \Rightarrow Bx = 18.92 \text{ lb}$$

$$\sum V = 0 \quad (\uparrow +ve)$$

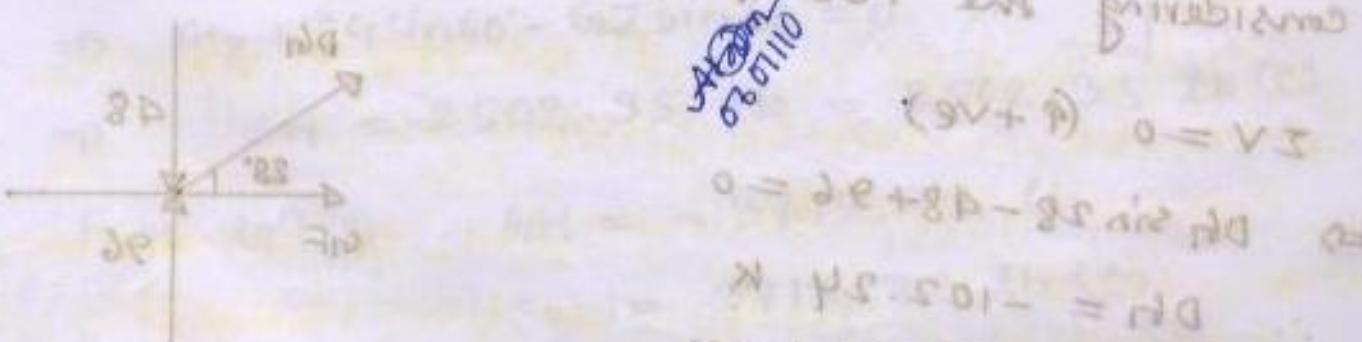
$$\Rightarrow R_A \sin 60^\circ - w + By = 0 \Rightarrow By = 67.23 \text{ lb}$$

$$R_B = \sqrt{Bx^2 + By^2} \approx 69.84 \text{ lb}$$

$$\theta = \tan^{-1} \frac{By}{Bx} = 74.28^\circ$$

Direction of $R_B = 180^\circ - 74.28^\circ = 105.72^\circ$

~~At 60°~~



Q Problem - 111, Page - 38, Faires, barbecue

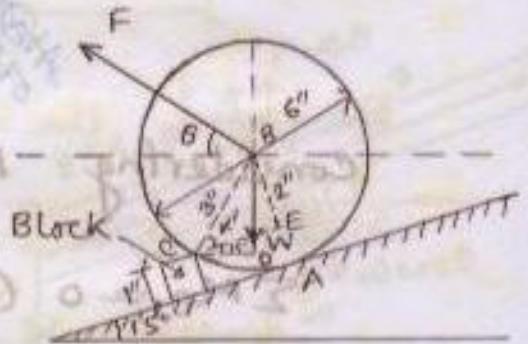
The wheel of figure below is on the point of rolling over the block (that is, the reaction at A is zero, but the wheel has not moved). If the weight $w = 1000$ what is the magnitude and sense of least force F that will produce the condition?

$$\Delta BCE, \alpha' = \sin^{-1}(\frac{2}{3}) = 41.81^\circ$$

$$\kappa = \kappa' + 15^\circ = 56.81^\circ$$

$$\Delta BCD, BD = BC \sin 56.81 = 2.51$$

$$CD = BC \cos 56.81 = 1.64$$



$$\sum M_C = 0 \quad (2 \text{ tve})$$

$$\Rightarrow -F \sin \theta \times 3 \cos 56.81 - F \cos \theta \times 3 \sin 56.81 + w \times 1.64 = 0$$

$$\therefore F = \frac{547.42}{\sin(\theta + 56.81)}$$

For least force F , the value of $\sin(\theta + 56.81)$

must be 1 i.e; $\sin(\theta + 56.81) = 1 \Rightarrow \theta = 33.19^\circ$

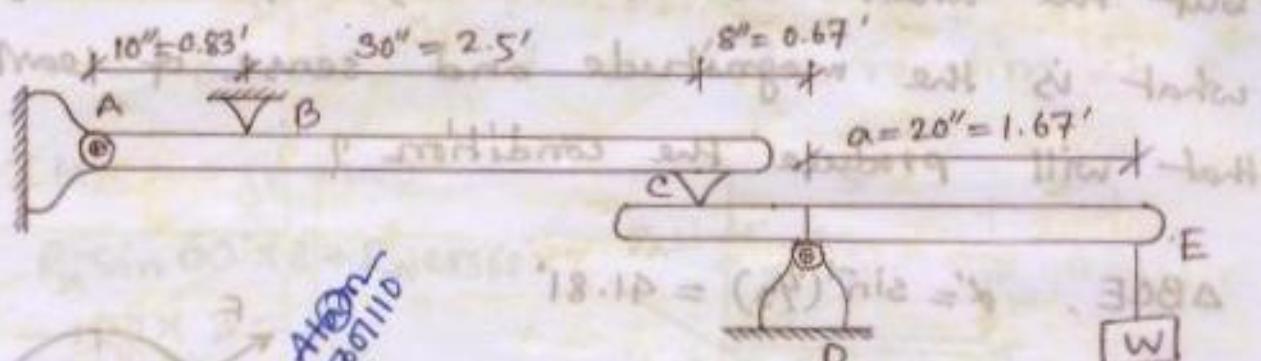
$$\therefore F = 547.42 \text{ lb and } \theta = 33.19^\circ \quad \boxed{\text{Ans}}$$

Q Problem - 228, Page - 65, Faires.

Two horizontal members AC and CE are supported as shown in figure below. If a weight of $w = 700$

Miscellaneous Problems (Ex. No. 1)

is suspended at E and if $\alpha = 20''$, what is the reaction at A? The members have a uniform weight of 5 lb per ft of length?



Considering the member CDE.

$$\sum M_D = 0 \quad (2 + ve) d.l = 12.32 \times 0.67' \quad 1.67'$$

$$R_c \downarrow \quad 5 \text{ lb/ft} \quad \uparrow R_D$$

$$\Rightarrow -R_c \times 0.67 + 7D \times 1.67 + 5 \times 2.34 \times 0.5 = 0 \quad (S=0)$$

$$\Rightarrow R_c \times 0.67 = 183.21 \text{ lb}$$

considering the member ABC

$$\sum M_B = 0 \quad (2 + ve)$$

$$(12.32 + 0) \rightarrow \uparrow A_x \quad \uparrow A_y \quad \uparrow R_c$$

$$\Rightarrow A_x \times 0.83 + 5 \times 3.33 \times 0.835 = 0$$

$$\Rightarrow A_x = 153.5 \text{ lb}$$

As there is no horizontal force in the above structure, so A_x must be zero. Resultant at A, $R_A = 535.09 \text{ lb}$ and direction, $\theta = 90^\circ$.

Problem - 261, Page - 81, Faires

A glass rod AB in figure below, weighs 5 oz and is 6" long. If it is placed in a glass tumbler C in a position of equilibrium similar to that shown. If the tumbler is 2.5 in. in diameter and if all the surfaces are smooth, what is the angle θ ?

$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow F w \cos \theta - w \sin \theta = 0$$

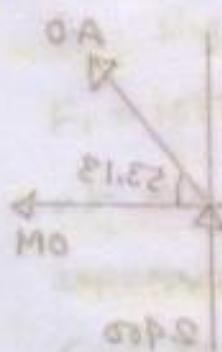
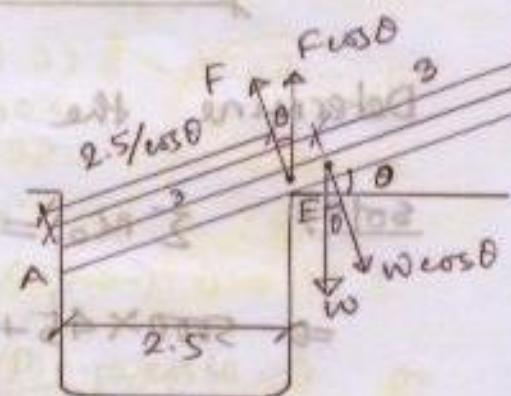
$$\Rightarrow F = \frac{w}{\cos \theta} = \frac{5}{16 \cos \theta}$$

$$\sum M_A = 0 \quad (\downarrow +ve)$$

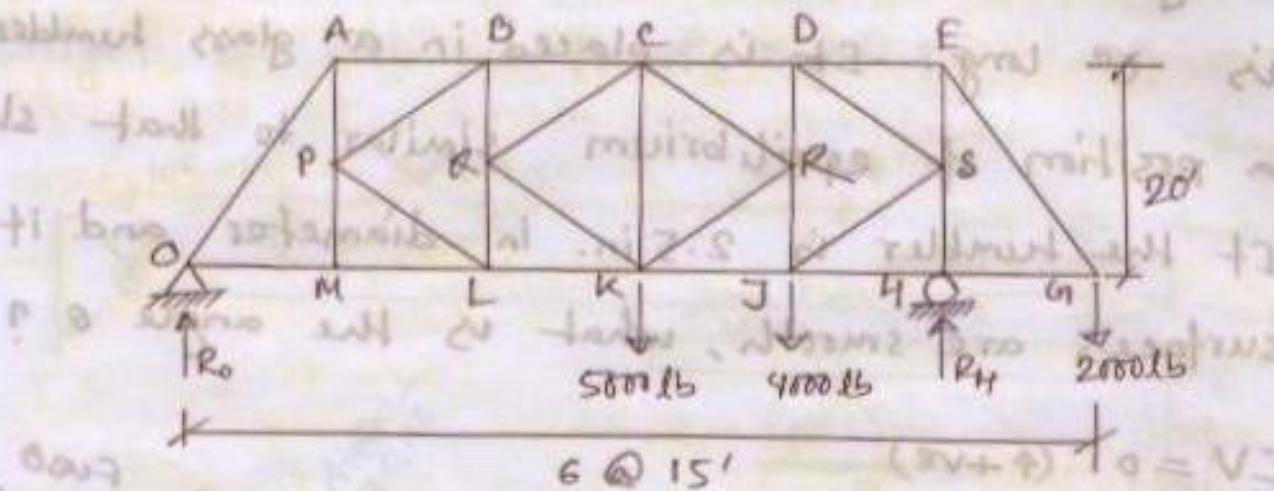
$$\Rightarrow -F X \frac{2.5}{\cos \theta} + w \cos \theta \times 3 = 0$$

$$\Rightarrow -\frac{5}{16 \cos \theta} \times \frac{2.5}{\cos \theta} + \frac{5}{16} \cos \theta \times 3 = 0 \quad \therefore \theta = \sqrt{3}$$

$$\Rightarrow \cos^3 \theta = 0.83 \Rightarrow \theta = 19.77^\circ \quad \boxed{\text{Ans}}$$



Page - 270, Prob - (4-151), TC Huang



Determine the Bar forces for the truss.

Soln: $\sum M_O = 0 \quad (\uparrow +ve)$

$$\Rightarrow 5000 \times 45 + 4000 \times 60 - R_H \times 75 + 2000 \times 90 = 0$$

$$\Rightarrow R_H = 8600 \text{ lb}$$

$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow R_o - 5000 - 4000 + 8600 - 2000 = 0$$

$$\Rightarrow R_o = 2400 \text{ lb}$$

considering the FBD of point O,

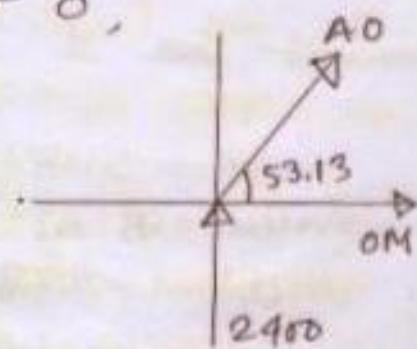
$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow A_O \sin 53.13 + 2400 = 0$$

$$\Rightarrow A_O = -3000 \text{ lb}$$

$$\sum H = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow A_O \cos 53.13 + OM = 0 \quad \Rightarrow OM = -1800 \text{ lb}$$



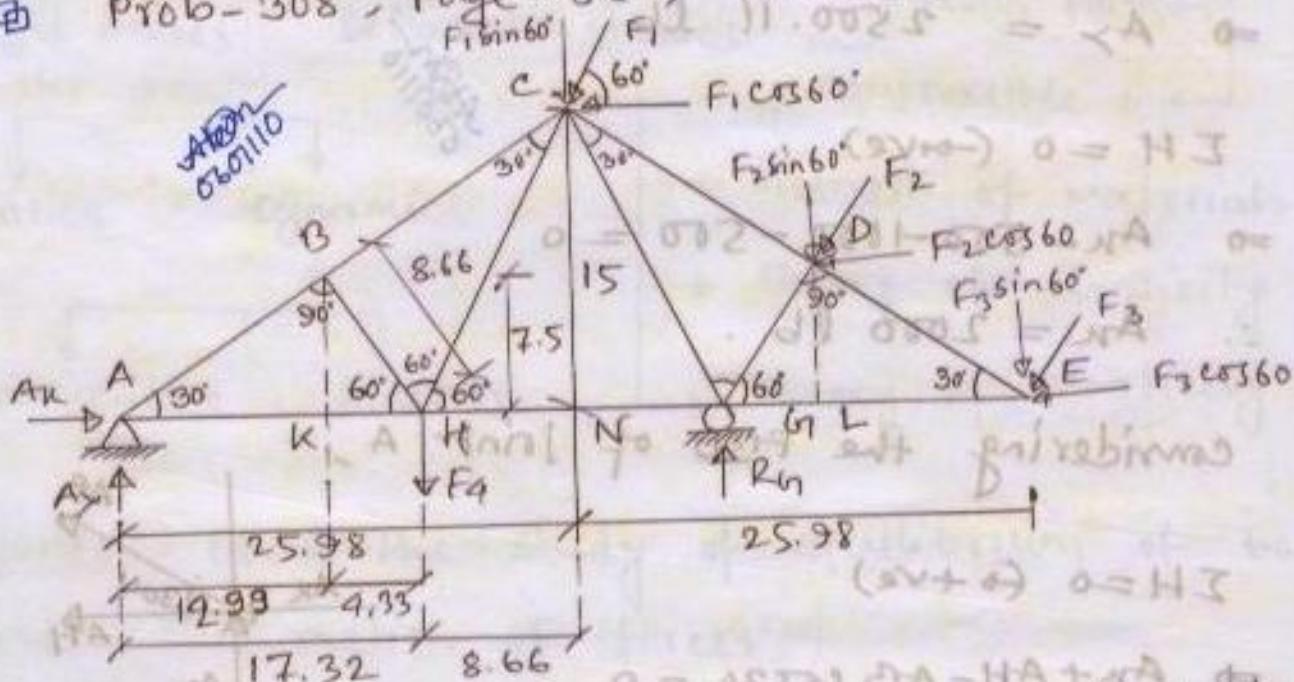
considering the FBD of joint A

$$\therefore AP = 2400 \text{ lb}$$

$$\Rightarrow A_0 \cos 53.13 + AB = 0 \Rightarrow AB = -1800 \text{ lb.}$$

(Determine the bar force similarly)

Prob-308, Page-86, Faires.



In the truss of following fig., the wind loads of $F_1 = 1000$, $F_2 = 2000$, $F_3 = 1000$ lb are assumed to act at the pin joints as shown. In order to allow for expansion, the support at G is on rollers. For $F_4 = 5000$ lb, find the external reactions (components at A) and the loads on members BC, BH and AH.

$\sum M_A = 0$ ($\uparrow +ve$) \Rightarrow DCD reaction force is zero

$$\Rightarrow F_1 \sin 60^\circ \times 25.98 - F_1 \cos 60^\circ \times 15 + F_4 \times 17.32 = 0$$

$$-R_6 \times 34.64 + F_2 \sin 60^\circ \times 47.63 - F_2 \cos 60^\circ \times 7.5 = 0$$

$$+9F_3 \sin 60^\circ \times 51.96 = 0 \quad \text{d.f. } \sin \theta = 9A$$

$$\Rightarrow R_6 = 5964.09 \text{ lb.} \quad (\text{Ans}) \quad A = 11.7$$

$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow -5000 - 866.03 + 5964 - 1732.05 - 866.03 + A_y = 0$$

$$\Rightarrow A_y = 2500.11 \text{ lb.}$$

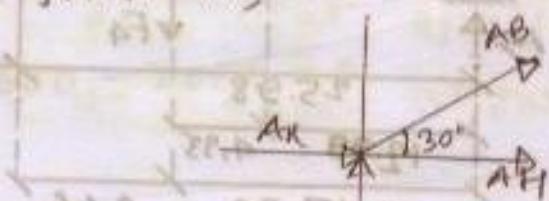
$$\sum H = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow A_h = 500 - 1500 - 500 = 0$$

$$\therefore A_h = 2000 \text{ lb.}$$

considering the FBD of joint A,

$$\sum H = 0 \quad (\rightarrow +ve)$$



$$\Rightarrow A_h + A_h - AB \cos 30 = 0$$

$$\Rightarrow A_h = 0.866 AB - 2000$$

$$\therefore A_h = 0.866 AB - 2000$$

$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow AB \sin 30 + A_y = 0 \Rightarrow AB = -5000 \text{ lb}$$

$$\therefore A_h = 3633.33 \text{ lb.}$$

Similarly determine the bar force BC and BH.

FRICITION

The frictional resistance betn two surfaces which have no relative motion is called static friction. The frictional resistance betn two sliding surfaces which do move relative to each other is called kinetic friction. The limiting frictional force F' is approximately proportional to the normal force N .

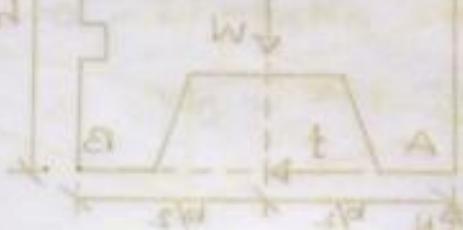
$$\text{i.e., } F' \propto N \Rightarrow F' = fN \Rightarrow f = \frac{F'}{N}$$

Case-I: If $F \leq F'$, then static condition exist

Case-II: If $F > F'$, then kinetic condition exist.

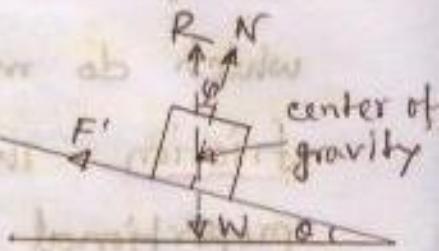
Laws of friction:

- 1) The co-efficient of the friction and the value of the frictional force are independent of the area of contact.
- 2) The limiting frictional force for two bodies in contact is directly proportional to the normal force i.e, the coefficient of friction is constant for all values of the normal force.
- 3) The coefficient of friction is independent of the velocity although static friction is greater than kinetic friction.



$$\frac{d}{dx} = d \quad d = (1)$$

The angle betn total plane reaction (R) and the line of action of normal force (N) is called the angle of friction (ϕ). The angle of inclination of plane when motion of a body under the action of gravity is impending is called the angle of repose. The angle of repose is equal to the limiting angle of static friction.



Problem - 9.2, Page - 496, Bedford

Suppose that we want to push the pull chest across the floor by applying the horizontal force. If we applied the force at two grade a height h , the chest will tip over before it slips. If the coefficient of static friction betn the floor and the chest μ_s . what is the largest value of h for which the chest will slip before its tip over.

$$\sum MA = 0 \quad (\theta + ve)$$

$$\Rightarrow W \times \frac{b}{2} - F \times h = 0 \quad \text{---(1)}$$

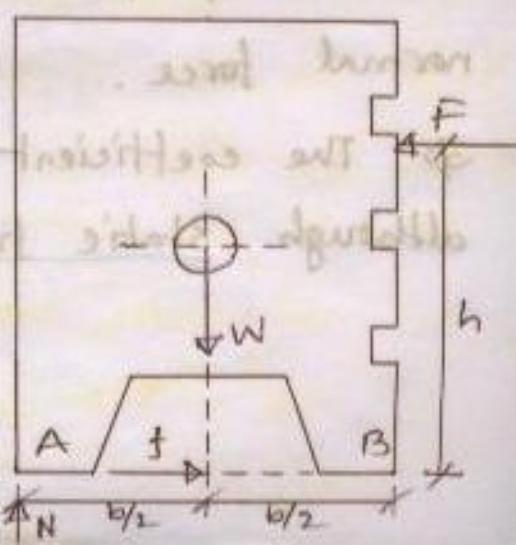
From eqn (1)

For equilibrium, $f = F$, $W = N$.

$$\therefore f = \mu_s N \Rightarrow F = \mu_s W$$

$$(1) \Rightarrow h = \frac{b}{2\mu_s}$$

If h is smaller than this value, the chest will begin sliding before it tips over.



Problem - 426, Page - 127, Faires, E2E - mechanics

In the following fig. let $w_A = 400 \text{ lb}$, $f_A = k_3$ and $f_B = \frac{1}{4}$. If the body B is on the point of moving downward, determine the tension in the cable and the weight of B.

FBD of block A,

$$\sum F_y = 0 \quad (\uparrow +\text{ve})$$

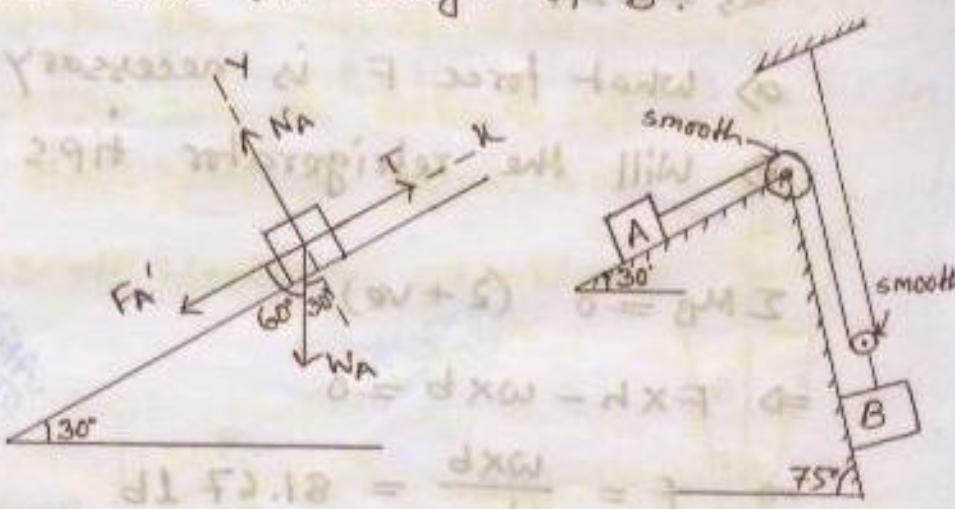
$$\Rightarrow N_A - w_A \cos 30^\circ = 0$$

$$\therefore N_A = 346.41 \text{ lb}$$

$$\sum F_x = 0 \quad (\rightarrow +\text{ve})$$

$$\Rightarrow T - F_A' - w_A \sin 30^\circ = 0$$

$$\Rightarrow T - f_A N_A - w_A \sin 30^\circ = 0 \Rightarrow T = 315.47 \text{ lb}$$



considering the FBD of block B,

$$\sum F_y = 0 \quad (\uparrow +\text{ve})$$

$$\Rightarrow N_B - w_B \sin 15^\circ = 0$$

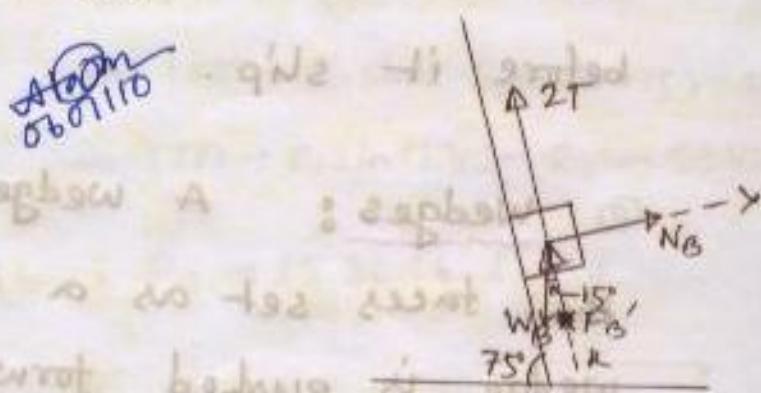
$$\Rightarrow N_B = 0.26 w_B$$

$$\sum F_x = 0 \quad (\rightarrow +\text{ve})$$

$$\Rightarrow -2T - F_B' + w_B \cos 15^\circ = 0$$

$$\Rightarrow -2T - f_B N_B + w_B \cos 15^\circ = 0$$

$$\therefore w_B = 700.32 \text{ lb}$$



Practice - 410, 411, 412, 430, 432, 434, 437, 439
439.

Faires

Problem - 9.44, Page - 456, Bedford - melder

12. The refrigerator weighs = 350 lb, the distance $h = 60"$ and $b = 14"$. The coefficient of static friction at A and B is $\mu_s = 0.24$
- what force F is necessary for impending slip?
 - Will the refrigerator tips over before it slips?

$$\sum M_B = 0 \quad (2 + ve)$$

$$\Rightarrow F \times h - w \times b = 0$$

$$\Rightarrow F = \frac{w \times b}{h} = 81.67 \text{ lb}$$

$$\text{Again, } h = \frac{b}{\mu_s} = \frac{14}{0.24} = 58.33$$

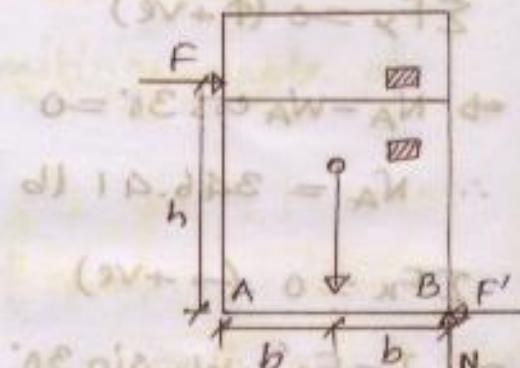
Since $h < 60$ so, the refrigerator will not tips over before it slips.

wedges: A wedge is a bi-facial pull with the faces set at a small acute angle. When a wedge is pushed forward the faces exert large lateral forces as a result of the small angle between them.

Practice - 450, 452, 453 (Problem)

- 69 (Example)

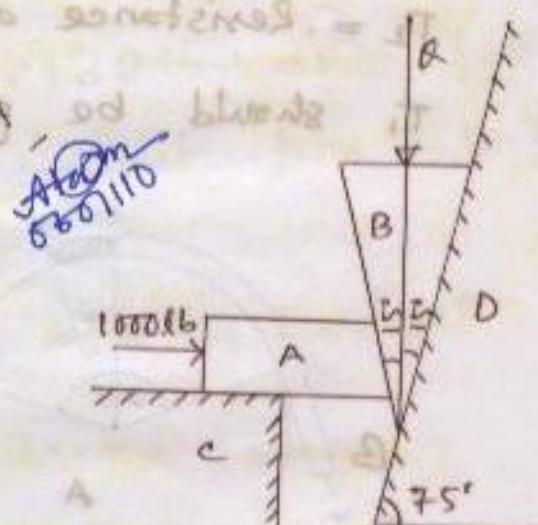
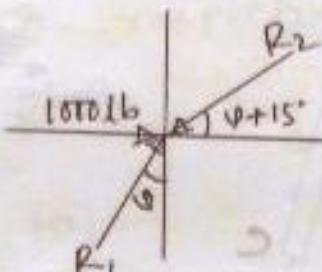
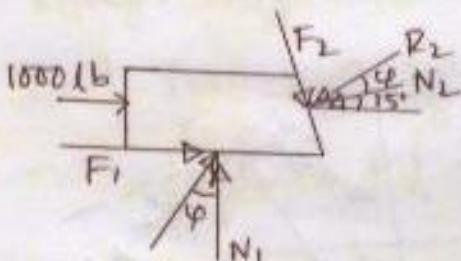
Faires



Exa-69, Page-109, Faires.

A resistance of 1000 lb is exerted on the block A, which in turn presses against the wedge B. Neglect the weight of both blocks A and B and assume that $F = \mu N$ for all surfaces. What force Q will cause the wedge B to be in impending motion downward? Considering the FBD of block A.

considering the FBD of block A



$$\sum V = 0 \quad (\uparrow +ve)$$

$$\sum H = 0 \quad (\rightarrow +ve)$$

$$\Rightarrow R_1 \cos 45^\circ - R_2 \sin(45^\circ + 15^\circ) = 0$$

$$\Rightarrow 1000 + R_1 \sin 45^\circ - R_2 \cos(45^\circ + 15^\circ) = 0$$

$$\Rightarrow R_1 \cos 18.43^\circ - R_2 \sin 33.43^\circ = 0$$

$$\Rightarrow 1000 + R_1 \sin 18.43^\circ - R_2 \cos 33.43^\circ = 0$$

Solving, $R_1 = 892.05 \text{ lb}$, $R_2 = 1536.16 \text{ lb}$

Considering the FBD of block B.

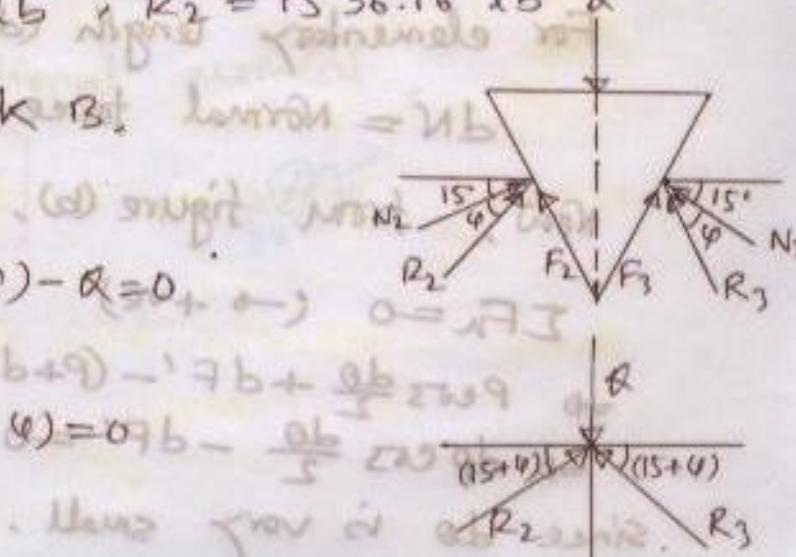
$$\sum V = 0 \quad (\uparrow +ve)$$

$$\Rightarrow R_2 \sin(15+4^\circ) + R_3 \sin(15+4^\circ) - Q = 0$$

$$\sum H = 0 \quad (\rightarrow +ve) = \frac{R_2}{2} \cos(9b+9^\circ) - \frac{R_3}{2} \cos(9b+9^\circ)$$

$$\Rightarrow R_2 \cos(15+4^\circ) - R_3 \cos(15+4^\circ) = 0.7b - \frac{0.7b}{2} \cos 9^\circ$$

Solving, $Q = 1692.60 \text{ lb}$



Belt friction:

2010-2011-2012-2013

Consider the cylinder A where as the belt BC wraps about the cylinder let

D = Diameter of the cylinder

θ = Substanding angle of BC in radians

T_1 = Pull exerted at one end = Tight tension

T_2 = Resistance at the other end = Slack tension

T_1 should be greater than T_2

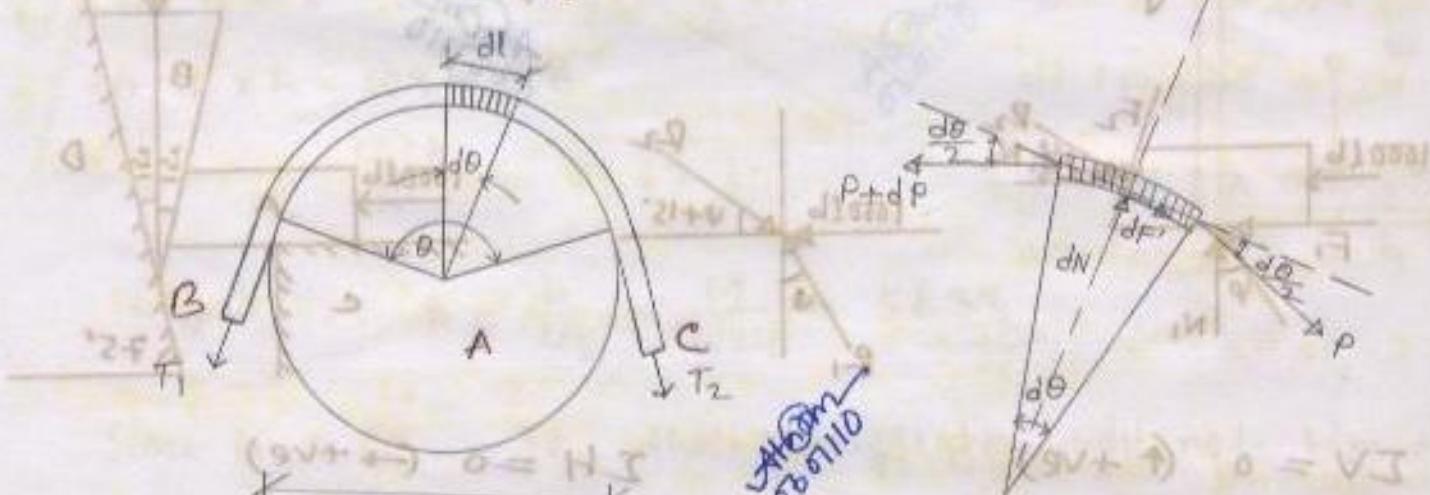


Fig. Belt friction

For elementary length (dL)

$dN = \text{Normal force}$, $dF' = \text{limiting frictional force}$

Now, from figure (b),

$$\sum F_x = 0 \quad (\rightarrow +ve) - (v+2) \sin \frac{\theta}{2} + (v+2) \cos \frac{\theta}{2} = 0$$

$$\Rightarrow P \cos \frac{\theta}{2} + dF' - (P+dP) \cos \frac{\theta}{2} = 0 \quad \Rightarrow v+2 = 0$$

$$\Rightarrow dP \cos \frac{\theta}{2} - dF' = (v+2) \cos \frac{\theta}{2} - (v+2) \cos \frac{\theta}{2} = 0$$

Since θ is very small, so $\cos \frac{\theta}{2} = 1$ and $dF' = dP$

$$\sum F_y = 0 \quad (\uparrow +ve) \quad \text{Resultant force in vertical direction is zero}$$

$$\Rightarrow dN - (P + dP) \sin \frac{d\theta}{2} - P \sin \frac{d\theta}{2} = 0 \quad \text{Normal force at A}$$

Since $d\theta$ is very small so $\sin \frac{d\theta}{2} = \frac{d\theta}{2}$

$dP \sin \frac{d\theta}{2}$ may be neglected.

$$\text{Then, } dN - 2P \frac{d\theta}{2} = 0 \Rightarrow dN = P d\theta$$

Multiplying the eqn by f (static friction)

$$f dN = P f d\theta \Rightarrow dF_f = P f d\theta \Rightarrow dP = P f d\theta$$

Integrating both sides with the limit from T_2 to T_1
and from 0 to θ respectively.

$$\int_{T_2}^{T_1} \frac{dP}{P} = f \int_0^\theta d\theta \Rightarrow \log \frac{T_1}{T_2} = f\theta \Rightarrow \frac{T_1}{T_2} = e^{f\theta}$$

$$\therefore T_1 = T_2 e^{f\theta}$$

This eqn gives the relation betw tight tension (T_1)
and slack tension (T_2).

Evidently the total frictional force betw the belt
and the cylinder is

$$F = T_1 - T_2$$

Breaking torque or frictional moment,

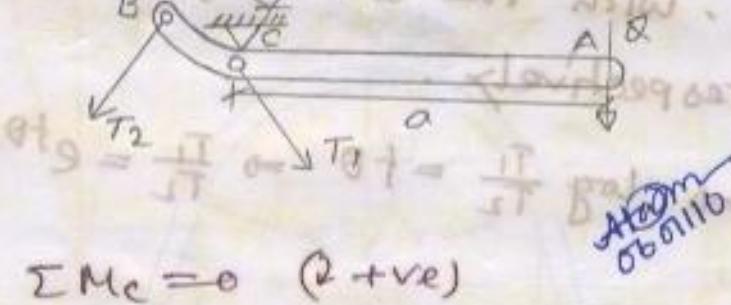
$$M_f = F \times \frac{D}{2}$$

ANS
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Example-72, Page-113, Faires (9v+*) $\theta = 73$

A band brake as shown in figure, consists of leather band on a 14" cast-iron drum, for which $f=0.3$. The data are: $a=18"$, $b=4"$, $D=14"$ and $\theta=270^\circ$. Determine the frictional torque for a clockwise rotation of the drum and a force $\alpha=100$ lb.

Considering the FBD of AB member,



$$\sum M_C = 0 \quad (\text{Q} + \text{ve})$$

$$(T) \Rightarrow \alpha \times 18 - T_2 \times 4 = 0 \Rightarrow T_2 = 450 \text{ lb}$$

$$T_1 = T_2 e^{f\theta} = 450 e^{(0.3 \times 270 \times \frac{\pi}{180})} = 1850.04 \text{ lb}$$

$$M_f = F \times D/2 = (T_1 - T_2) \frac{D}{2} = 9800.30 \text{ lb inch}$$

$$\therefore \frac{M_f}{I} = \frac{N}{L} \Rightarrow N = 816.67 \text{ lb ft}$$

Increase limit to support bearing

$$\frac{D}{5} \times 7 = fM$$

DYNAMICS

Plane motion: - law of gravitation

④ Dynamics: It is concerned with bodies that have accelerated motion.

Kinematics: It is a study of the geometry of the motion.

Kinetics: It is a study of the forces that cause the motion.

① Plane motion: When a body moves so that each particular point in the body remains in the same plane, it is said to have plane motion.

Rectilinear motion/kinematics: It refers to straight line motion. It deals with at any given instant, particle's position, displacement, velocity and acceleration.

② $v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$

$\checkmark (\frac{dv}{ds})ads = \frac{dv}{dt} ds = dv \cdot \frac{ds}{dt} = dv \cdot v = vdv$

$ads = vdv$

④ Velocity as a function of time:

Integrate $dv = \frac{ds}{dt}$

$2b^2 l - v b^2$

Assuming that initially $v=v_0$ when $t=0$

$$\text{Integrating } \int_{v_0}^v dv = \int_0^t a_c dt \text{ with initial condition } v=v_0 \text{ at } t=0$$

$$\Rightarrow v - v_0 = a_c t$$

$$\Rightarrow v = v_0 + a_c t$$

∴ position as a function of time:

$$\text{Integrate, } v = \frac{ds}{dt} = v_0 + a_c t$$

Assuming that initially $s=s_0$ when $t=0$.

$$\text{Integrating } \int_0^t v dt = \int_{s_0}^s ds \text{ with initial condition } s=s_0 \text{ at } t=0$$

$$\Rightarrow \int_0^t (v_0 + a_c t) dt = \int_{s_0}^s ds$$

$$\Rightarrow v_0 t + \frac{a_c t^2}{2} = s - s_0$$

$$\Rightarrow s = s_0 + v_0 t + \frac{1}{2} a_c t^2 \quad (2)$$

Velocity as a function of position:

$$\frac{vb}{\sqrt{b}} = \frac{vb}{\sqrt{b}} = \infty \quad \frac{vb}{\sqrt{b}} = v$$

$$\text{From (1), } t = \frac{v - v_0}{a_c}$$

$$vbv = v \cdot vb = \frac{vb}{a_c}$$

$$\text{From (2), } s = s_0 + v_0 \left(\frac{v - v_0}{a_c} \right) + \frac{1}{2} a_c \left(\frac{v - v_0}{a_c} \right)^2$$

$$\Rightarrow v^2 = v_0^2 + 2a_c(s - s_0)$$

OR, Integrate, $v dv = a_c ds$

Assuming that initially $v=v_0$ when $s=s_0$.

$$\int_{v_0}^v v dv = \int_{s_0}^s a_c ds$$

$$\Rightarrow \frac{v^2 - v_0^2}{2} = a_c s - a_s s$$

$$\Rightarrow v^2 - v_0^2 = 2a_c(s - s_0)$$

$$\Rightarrow v = v_0 + 2a_c(s - s_0)$$

Example-174, Page-247, FAIRES.

Example-175, Page-248, FAIRES.

A particle is moving in a straight line so that $\ddot{s} = a = 2s$. If it starts from rest, what is its speed after it has moved 10 ft? What time has elapsed?

Solution:

$$\int v dv = \int_0^{10} a ds \quad \Rightarrow \quad v = \frac{ds}{dt}$$

$$\Rightarrow \left[\frac{v^2}{2} \right]_0^v = \int_0^{10} 2s ds \quad \Rightarrow \quad \int_0^{10} \frac{ds}{v} = \int_0^t dt$$

$$\Rightarrow \frac{v^2}{2} = [s^2]_0^{10} = 100 \quad \Rightarrow \quad \int_0^{10} \frac{ds}{s\sqrt{2}} = \int_0^t dt$$

$$\Rightarrow v = 200 = 10\sqrt{2} = 5\sqrt{2} \quad \Rightarrow \quad \left[\frac{1}{\sqrt{2}} \ln s \right]_0^{10} = [t]_0^t$$

$$\Rightarrow \frac{1}{\sqrt{2}} \ln 10 = t$$

$$t = 1.63 \text{ s.}$$

Example-181, Page-252, FAIRES.

A particle whose acceleration is $a = 3t - 12 \text{ fps}^2$ is moving at a certain instant in a straight line with an initial velocity of 15 fps in the same sense as the initial acceleration.

a) At the end of 10 sec, what is the velocity of the particle ? b) At the end of 14 sec, what is its displacement from the origin ?

Soln:

$$a = v = \frac{dv}{dt}$$

$$a = \frac{dv}{dt} = \frac{ds}{dt^2} = 3t - 12$$

$$\int_{-15}^v dv = \int_0^{10} a dt$$

$$\Rightarrow \frac{ds}{dt} = v = \frac{3}{2}t^2 - 12t + C$$

$$\Rightarrow v = 15 = \int_0^{10} (3t - 12) dt \quad \text{when } v = -15, t = 0$$

$$\Rightarrow v + 15 = 3\left[\frac{t^2}{2}\right]_0^{10} - [12t]_0^{10} \quad C = -15$$

$$\Rightarrow v = 150 - 120 - 15 = 15 \text{ f/s.} \quad \frac{ds}{dt} = v = \frac{3}{2}t^2 - 12t - 15$$

$$\text{Now, } \frac{ds}{dt} = \frac{3}{2}t^2 - 12t - 15$$

$$\Rightarrow \int_0^s ds = \int_0^{14} (\frac{3}{2}t^2 - 12t - 15) dt$$

$$\Rightarrow s = \frac{3}{2}\left[\frac{t^3}{3}\right]_0^{14} - 12\left[\frac{t^2}{2}\right]_0^{14} - 15(t)_0^{14}$$

$$= -14 \text{ ft}$$

But, s is always +ve.

so, The displacement is to the left from the origin.

④ Angular velocity: If a body is rotating about either a fixed or a moving axis, it is said to have angular velocity. The angular velocity is measured by the time rate of change of angular displacement of a line in the body.

$$\Delta \theta = \frac{\Delta s}{r} : \text{r. midline} \rightarrow \text{initial } \Delta s$$

$$\therefore \Delta s = r \Delta \theta \quad (\text{radius}) \quad \text{vec. eq - sign}$$

④ straight (~~long~~) Angular

$$s = r \theta \quad \text{midline}$$

$$v = \omega r$$

$$a = \alpha r$$

$$v = \frac{ds}{dt} = \frac{r d\theta}{dt} = \omega r$$

$$a = \frac{dv}{dt} = \frac{d(\omega r)}{dt} = r \alpha$$

$$\alpha d\theta = \omega d\omega$$

$$v = \frac{ds}{dt} = \frac{r d\theta}{dt} = \omega r$$

~~Experiments~~ - 186, Page (258), FADRES.

Prob - 891, 893, 898, 909, 913, 914, 921, 935,
937, 939, 983, v.v.g.

Impulse and Momentum

Definition:

Principle Impulse and momentum;

Law of conservation of momentum:

Impact:

- 1) Direct central impact
- 2) Oblique impact

Coefficient of restitution, e:

Example - 303, 304 (momentum)

- 315 (Impact)

Problem - 1560, 1563, 1628, 1627, 1630.

ω

v

x

o

$$\frac{v}{r} = \omega$$

$$\frac{\Delta v}{\Delta t} = v$$

$$\frac{ab}{rb} = \lambda$$

$$\frac{vb}{rb} = \mu$$

$$ab\omega = ab\lambda$$

$$\omega r = \frac{ab\lambda}{rb} = \frac{ab}{rb} = \mu$$

23RD FEB. (20) 2009. 281 - chapter 12

see also, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219

env (E8E), PEE, FEE

CENTROIDS

④ center of gravity:

Taking moment about Y-axis.

$$dw_1 \times x_1 + dw_2 \times x_2 + dw_3 \times x_3 + \dots + dw_n \times x_n = W \times \bar{x}$$

$$\Rightarrow W \times \bar{x} = \int x \, dw$$

$$\therefore \bar{x} = \frac{\int x \, dw}{W}$$

similarly taking moment about Y

$$\bar{y} = \frac{\int y \, dw}{W}$$

④ center of mass:

$$\text{But, } w = mg \quad \therefore dw = g \, dm$$

$$\therefore \bar{x} = \frac{\int x \, dw}{W} = \frac{\int x g \, dm}{mg} = \frac{\int x \, dm}{m}$$

$$\text{similarly, } \bar{y} = \frac{\int y \, dm}{m}$$

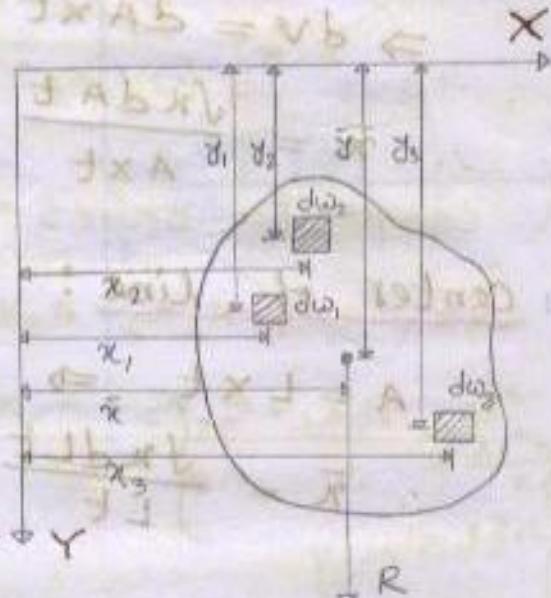
④ center of volume:

$$w = \text{wt. density}, \quad \rho = \text{mass density}$$

$$M = \rho V \Rightarrow dm = \rho dV$$

$$\therefore \bar{x} = \frac{\int x \, dm}{m} = \frac{\int x \rho \, dV}{\rho V} = \frac{\int x \, dV}{V}$$

$$\text{similarly, } \bar{y} = \frac{\int y \, dV}{V}$$



④ Center of area:

$$V = A \times t$$

t - thickness

$$\Rightarrow dV = dA \times t$$

$$\therefore \bar{x} = \frac{\int x dA \cdot t}{A \cdot t} = \frac{\int x dA}{A} \quad \text{and} \quad \bar{y} = \frac{\int y dA}{A}$$

④ Center of line:

$$A = L \times t \Rightarrow dA = dL \times t$$

$$\therefore \bar{x} = \frac{\int x dL \cdot t}{L \cdot t} = \frac{\int x dL}{L} \quad \text{similarly, } \bar{y} = \frac{\int y dL}{L}$$

④ Arc of circle:

x-axis symmetry, $\bar{y} = 0$

$$\bar{x} = \frac{\int x dL}{L}$$

$$\{L = r\theta\}$$

$$\Rightarrow dL = r d\theta$$

$$\Rightarrow L = \int_{-\beta}^{\beta} dL = \int_{-\beta}^{\beta} r d\theta = r [\theta]_{-\beta}^{\beta} = r (\beta + \beta) = 2r\beta$$

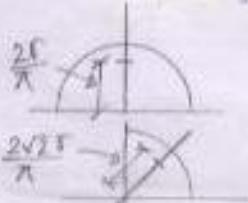
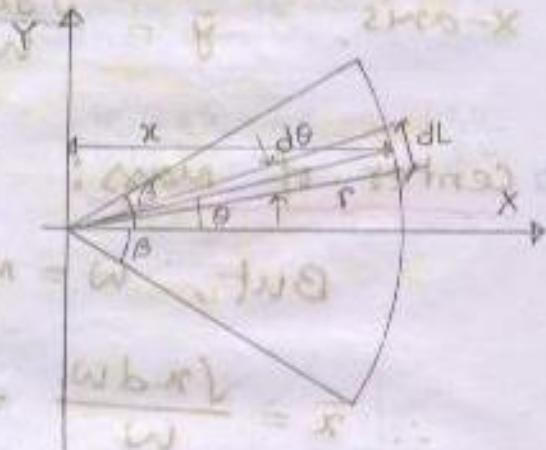
$$\text{But, } x = r \cos \theta$$

$$\therefore \int x dL = \int_{-\beta}^{\beta} r \cos \theta \cdot r d\theta = 2r^2 \sin \beta$$

$$\bar{x} = \frac{2r^2 \sin \beta}{2r\beta} = \frac{r \sin \beta}{\beta} \quad \text{when } \beta \text{ is radian}$$

$$\bar{x} = \frac{r \sin 90^\circ}{90^\circ} = \frac{r \times 1}{90^\circ \times \frac{180}{\pi}} = \frac{2r}{\pi}$$

$$\bar{x} = \frac{r \sin 45^\circ}{45^\circ} = \frac{r \times 1}{\sqrt{2} \times 45^\circ \times \frac{180}{\pi}} = \frac{2\sqrt{2}r}{\pi}$$



■ Sector of circle:

x-axis symmetry, $\bar{y} = 0$

$$\bar{x} = \frac{\int x dA}{A}$$

$$dA = \frac{1}{2} \times r \times dL$$

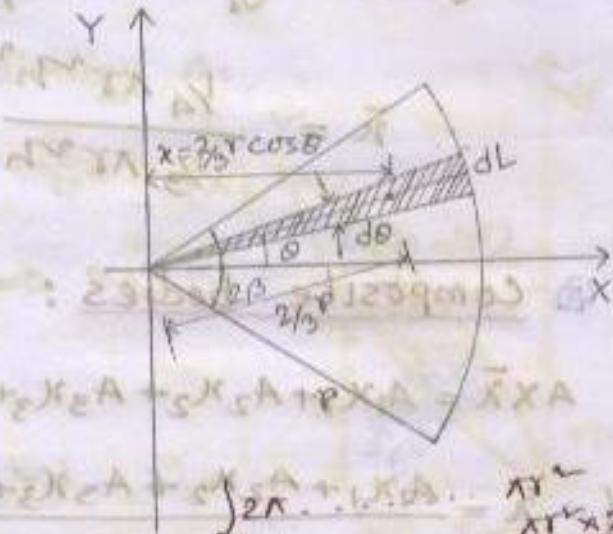
$$= \frac{1}{2} r d\theta r = \frac{1}{2} r^2 d\theta$$

$$\therefore A = \int_{-\beta}^{+\beta} \frac{1}{2} r^2 d\theta = r^2 \beta$$

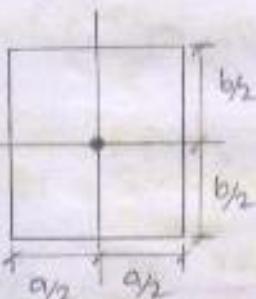
$$x = \frac{2}{3} r \cos \theta$$

$$\therefore \int x dA = \int_{-\beta}^{+\beta} \frac{2}{3} r \cos \theta \cdot \frac{1}{2} r^2 d\theta = \frac{2}{3} r^3 \sin \beta$$

$$\therefore \bar{x} = \frac{\frac{2}{3} r^3 \sin \beta}{r^2 \beta} = \frac{2r \sin \beta}{3\beta}$$



$$\begin{cases} 2\pi + 1.5\pi \\ 2\beta \end{cases} = \frac{\pi r^2 \times 2\beta}{2\pi}$$



$$\frac{2r \sin \beta}{3\beta} = \frac{2r \sin \beta / \beta}{3} = \frac{2r}{3}$$

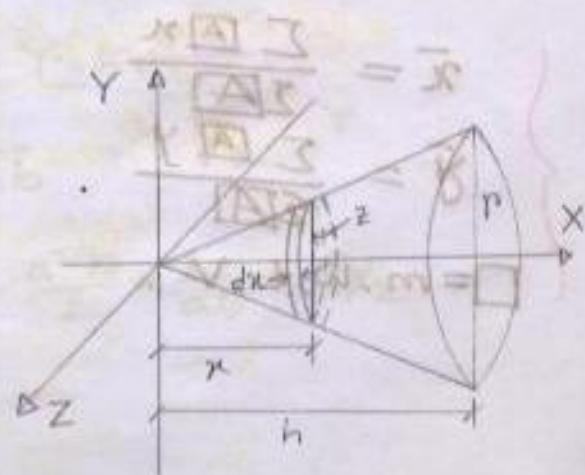
■ Circular cone:

$$dV = \pi z^2 dx$$

$$\Rightarrow V = \int dV = \int_0^h \pi z^2 dx$$

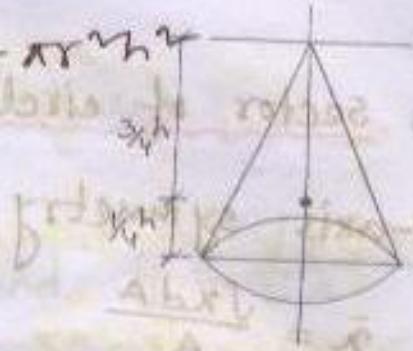
$$= \int_0^h \pi \frac{r^2 x^2}{h^2} dx = \frac{1}{3} \pi r^2 h$$

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$$\int x \, dv = \int_0^h x \pi \times \frac{\pi r^2 x^2}{h^2} \, dx = \frac{1}{4} \pi r^2 h^2$$

$$\therefore \bar{x} = \frac{\frac{1}{4} \pi r^2 h^2}{\frac{1}{4} \pi r^2 h} = \boxed{\frac{3}{4} h}$$



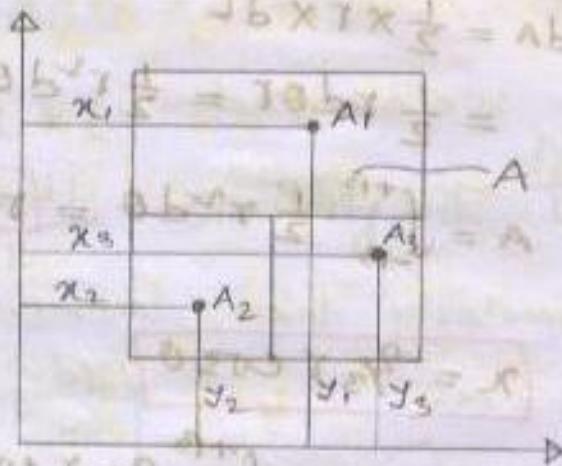
Composite bodies:

$$A \times \bar{x} = A_1 x_1 + A_2 x_2 + A_3 x_3 + \dots$$

$$\therefore \bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3 + \dots}{A}$$

Area } similarly,

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3 + \dots}{A}$$



$$\text{line } \left\{ \begin{array}{l} \bar{x} = \frac{L_1 x_1 + L_2 x_2 + L_3 x_3 + \dots}{L} \\ \bar{y} = \frac{L_1 y_1 + L_2 y_2 + L_3 y_3 + \dots}{L} \end{array} \right.$$

Area
of
each
rectangle
is
 $A_i = A_i x_i$

$$\text{Volume } \left\{ \begin{array}{l} \bar{x} = \frac{V_1 x_1 + V_2 x_2 + V_3 x_3 + \dots}{V} \\ \bar{y} = \frac{V_1 y_1 + V_2 y_2 + V_3 y_3 + \dots}{V} \end{array} \right.$$

$$\bar{x} = \frac{\sum A_i x_i}{\sum A_i}$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i}$$

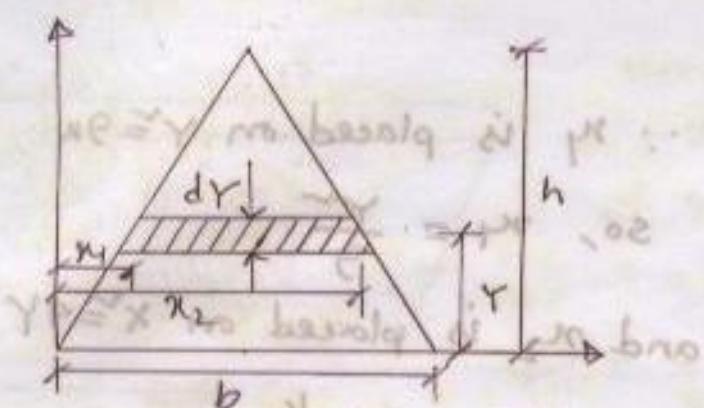
$$\square = m, w, L, V$$

$$x_b \cdot s_R = v_b$$

$$x_b \cdot s_R = v_b \cdot l_b = v$$

$$F \cdot m \cdot \frac{l}{c} = x_b \cdot \frac{s \cdot t}{c} \cdot l =$$

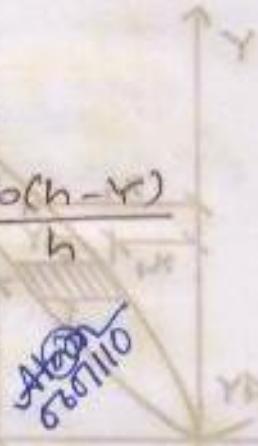
Q Find the location of centroid from X-axis.



similar As,

$$\frac{h}{b} = \frac{h-y}{(x_2 - x_1)}$$

$$\Rightarrow x_2 - x_1 = \frac{b(h-y)}{h}$$



$$dA = (x_2 - x_1) dy = \frac{b(h-y)}{h} \cdot dy$$

$$\int Y dA = \int_0^h Y \cdot \frac{b(h-y)}{h} dy = \int_0^h bY dy - \int_0^h \frac{bY^2}{h} dy$$

$$Yb\left(\frac{Y}{h} - \frac{Y^2}{h}\right)_0^h = b\left[\frac{Y^2}{2}\right]_0^h - b\left[\frac{Y^3}{3h}\right]_0^h = \frac{bh^3}{2} - \frac{bh^3}{3h} = \frac{bh^3}{6}$$

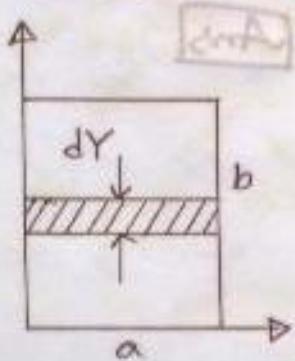
$$Y_c = \frac{\int Y dA}{\int dA} = \frac{\frac{bh^3}{6}}{bh^2} \times \frac{2}{bh} = \boxed{\frac{h}{3}}$$

similarly $\bar{x}_c = \frac{b}{3}$

$$Yb\left(\frac{Y}{h} - \frac{Y^2}{h}\right)_0^h = Ab(h-y_c)h = Abh = A$$

$$A = \left[\frac{bY}{h} \right]_0^h - \left[\frac{bY^2}{2h} \cdot h \right]_0^h$$

Q For rectangle:



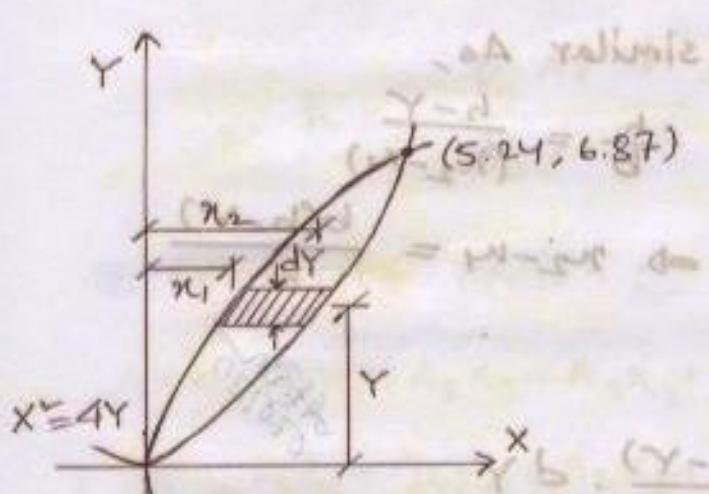
$$dA = ady$$

$$\int Y dA = \int_0^b Y ady = \frac{ab^2}{2}$$

$$\therefore \bar{Y} = \frac{ab^2}{2} \times \frac{1}{ab} = \boxed{\frac{b}{2}}$$

$$\text{similarly, } \bar{x} = \frac{ab^2}{2} \times \frac{1}{ab} = \boxed{a/2}$$

Q Find the centroid about x-axis



$\therefore k_1$ is placed on $y = 9x$
 $\therefore k_1 = \frac{y}{9}$
 $\text{and } k_2 \text{ is placed on } x = 4y$

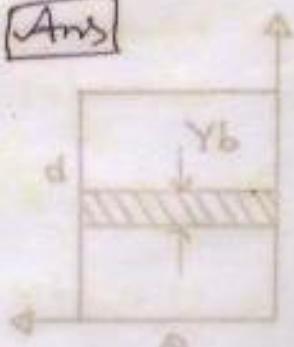
$$\text{so, } k_2 = 2y^{1/2}$$

$$\begin{aligned} dA &= (k_2 - k_1) dy \\ \int y dA &= \int_0^{6.87} y (k_2 - k_1) dy = \int_0^{6.87} y (2y^{1/2} - \frac{y}{9}) dy \\ &= \int_0^{6.87} 2 \frac{y^{3/2}}{3} - \frac{y^2}{9} dy = \int_0^{6.87} 2y^{3/2} dy - \int_0^{6.87} \frac{y^3}{9} dy \\ &= \frac{4}{5}(6.87)^{5/2} - \frac{1}{36}(6.87)^4 = 37.09 \end{aligned}$$

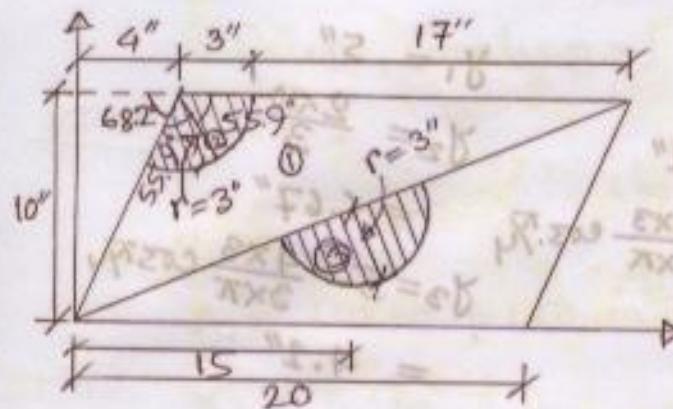
$$\begin{aligned} A &= \int dA = \int (k_2 - k_1) dy = \int_0^{6.87} (2y^{1/2} - \frac{y}{9}) dy \\ &= \left[2 \cdot \frac{y^{3/2}}{3/2} \right]_0^{6.87} - \frac{1}{9} \left[\frac{y^3}{3} \right]_0^{6.87} = 12 \end{aligned}$$

$$\therefore \bar{Y} = \frac{\int y dA}{A} = \frac{37.09}{12} = 3.09 \quad \boxed{\text{Ans}}$$

$$\bar{x} = \frac{1}{A} \times \frac{dy}{2} = \bar{x} \quad \text{in cm}^2$$



Find the centroid about the unshaded area.



Here, $\theta = 68.2^\circ$

$$x_1 = 15$$

$$x_2 = 4 + \frac{3 \sin 55.9}{55.9 \times \frac{\pi}{180}} \cos 55.9 \times \frac{2}{3}$$

$$x_2 = 4 + \frac{3 \sin 55.9}{55.9 \times \frac{\pi}{180}} \cos 55.9 \times \frac{2}{3}$$

$$x_2 = 5.43 = 4.95$$

$$x_3 = 15 + \frac{4 \times 3}{3 \pi} \sin 22.62$$

$$x_3 = 15 + \frac{4 \times 3}{3 \pi} \sin 22.62 = 15.49$$

$$y_1 = 5$$

$$y_2 = 10 - \frac{2}{3} \frac{3 \sin 55.9}{55.9 \times \frac{\pi}{180}} \sin 55.9$$

$$y_2 = 10 - \frac{2}{3} \frac{3 \sin 55.9}{55.9 \times \frac{\pi}{180}} \sin 55.9 = 8.6$$

$$y_3 = 5 - \frac{4}{3} \cos 22.62$$

$$y_3 = 5 - \frac{4}{3} \cos 22.62 = 3.82$$

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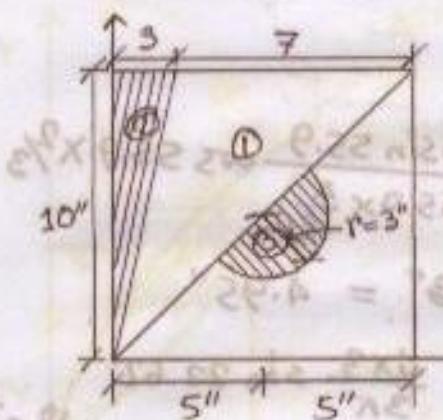
$$y_3 = 5 - \frac{4}{3} \cos 22.62 = 3.82$$

$$y_3 = 5 - \frac{4}{3} \cos 22.62$$

$$y_3 = 5 - \frac{4}{3} \cos 22.62 = 3.82$$

$$y_3 = 5 - \frac{4}{3} \cos 22.62$$

Q Find the centroid about unshaded area.



Here,

$$x_1 = 5"$$

$$x_2 = \frac{3}{3} = 1"$$

$$x_3 = 5 + \frac{4 \times 3}{3 \times \pi} \cos 74^\circ = 5.9"$$

$$y_1 = 5"$$

$$y_2 = \frac{2 \times 10}{3} = 6.67"$$

$$y_3 = 5 - \frac{4 \times 3}{3 \times \pi} \cos 74^\circ = 4.1"$$

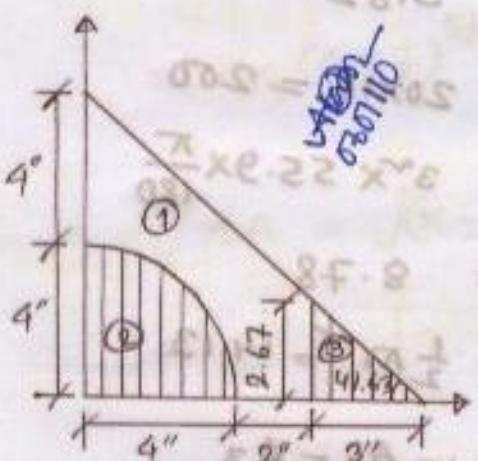
$$\bar{x} = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3}{A} = 5.67" \quad A_1 = 10 \times 10 = 100$$

$$A_2 = \frac{1}{2} \times 3 \times 10 = 15$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2 - A_3 y_3}{A} = 4.83" \quad A_3 = \frac{1}{2} \times \pi \times 3^2 = 14.13$$

$$\therefore \bar{x} = 5.67", \bar{y} = 4.83" \quad \text{Ans}$$

Q Find the centroid about unshaded area.



Here,

$$x_1 = x_3 = 3"$$

$$x_2 = \frac{4 \times 4}{3 \pi} = 1.7"$$

$$x_3 = 6 + \frac{4}{3} = 7"$$

$$A_1 = \frac{1}{2} \times 9 \times 8 = 36$$

$$A_2 = \frac{\pi \times 4^2}{4} = 12.56$$

$$A_3 = \frac{1}{2} \times 3 \times 2.67$$

$$= 4.0$$

$$A = A_1 - A_2 - A_3$$

$$= 19.44$$

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3}{A} = 3.02"$$

$$\bar{y} = \frac{A_1 y_1 - A_2 y_2 - A_3 y_3}{A} = 3.66"$$

$$\therefore \bar{x} = 3.02", \bar{y} = 3.66" \quad \text{Ans}$$

Find centroid
about unshaded
area.

Here,

$$A_1 = 10 \times 10 = 100 \text{ in}^2$$

$$x_1 = 5 \text{ in}$$

$$A_2 = \frac{\pi r^2}{2} = 14.14 \text{ in}^2$$

$$x_2 = \frac{4r}{3\pi} = 1.27 \text{ in}$$

$$A_3 = \frac{1}{2} \times 4 \times 6 = 12 \text{ in}^2$$

$$x_3 = 4 + 2/3 \times 6 = 8 \text{ in}$$

$$A_4 = \frac{\pi r^2}{4} = 12.5 \text{ in}^2$$

$$x_4 = 6 + (1 - \frac{4r}{3\pi}) = 8.30 \text{ in}$$

$$A = A_1 - A_2 - A_3 - A_4$$

$$= 61.36 \text{ in}^2$$

Answer
660/110

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2 - A_3 x_3 - A_4 x_4}{A}$$

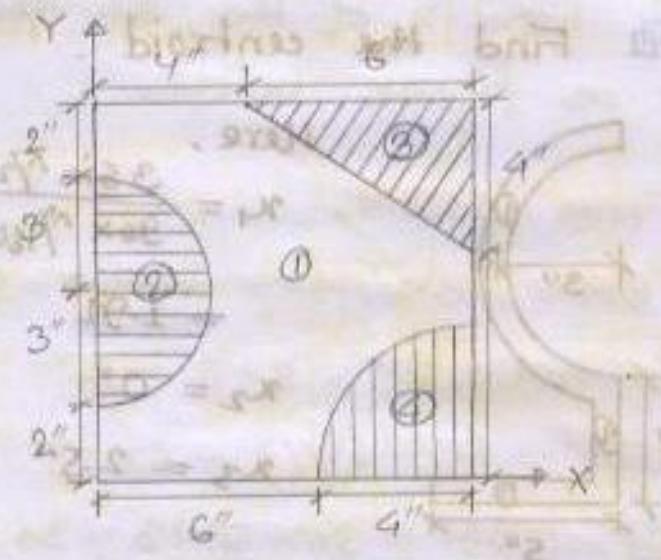
$$= \frac{100 \times 5 - 14.14 \times 1.27 - 12 \times 8 - 12.5 \times 8.30}{61.36}$$

$$= \frac{282.29}{61.36} = 4.60 \text{ in.}$$

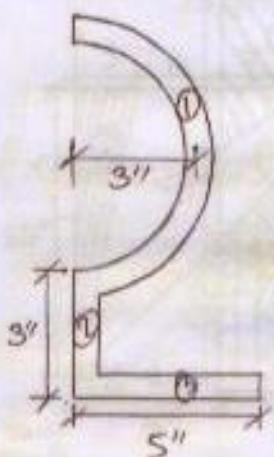
$$\bar{y} = \frac{A_1 y_1 - A_2 y_2 - A_3 y_3 - A_4 y_4}{A}$$

$$= \frac{100 \times 5 - 14.14 \times 5 - 12 \times 8.7 - 12.5 \times 1.69}{61.36}$$

$$= 4.95 \text{ in.}$$



Q Find the centroid.



Here,

$$x_1 = \frac{3 \sin 90}{90 \times \pi / 180} = 1.91''$$

$$x_2 = 0''$$

$$x_3 = 2.5''$$

breadth b =

$$L_1 = \frac{2\pi \times 3}{2} = 3\pi''$$

$$y_2 = 1.5''$$

$$L_2 = 3''$$

$$y_3 = 0''$$

$$L_3 = 5''$$

$$L = L_1 + L_2 + L_3 = 17.42''$$

$$\therefore \bar{x} = \frac{L_1 x_1 + L_2 x_2 + L_3 x_3}{L} = 1.75''$$

$$\therefore \bar{y} = \frac{L_1 y_1 + L_2 y_2 + L_3 y_3}{L} = 3.50''$$

Q Location of centroid of half circular cone:

CJ

- ④ Find X of the shaded area ABCD as shown in figure below.

From $\triangle OBM$,

$$BM = \sqrt{OB^2 - OM^2} = \sqrt{10^2 - 6^2} = 8''$$

$$\text{So, } AM = DP = 8 - 3 = 5''$$

$$\angle BOM = \tan^{-1}(8/6) = 53.13^\circ$$

$$\angle BOP = \angle OBM = 90^\circ - 53.13 = 36.87^\circ$$

$$AD = 5 \tan 36.87 = 2.25''$$

$$PK = 6 - 2.25 = 3.75''$$

$$\angle PON = \frac{36.87 + 60}{2} = 53.13^\circ$$

$$PD = 5 \tan 60^\circ = 8.66''$$

$$A_1 = \pi r^2 = 48.44 \times \frac{\pi}{180} \times 10^2 = 84.53$$

$$n_1 = \frac{2\pi \sin \beta}{3\pi} \cos 11.56 = 5.78$$

$$A_2 = \frac{1}{2} \times 3 \times 2.25 = 3.38$$

$$n_2 = 5 + \frac{3}{3} = 6$$

$$A_3 = \frac{1}{2} \times 3.75 \times 5 = 9.38$$

$$n_3 = \frac{2}{3} \times 5 = 3.33$$

$$A_4 = \frac{1}{2} \times 8.66 \times 5 = 21.65$$

$$n_4 = \frac{2}{3} \times 5 = 3.33$$

$$\bar{n} = \frac{A_1 n_1 + A_2 n_2 + A_3 n_3 + A_4 n_4}{A_1 + A_2 + A_3 + A_4}$$

$$= \frac{84.53 \times 5.78 + 3.38 \times 6 - 9.38 \times 3.33 - 21.65 \times 3.33}{84.53 + 3.38 - 9.38 - 21.65}$$

$$= 7.1296 = 7.13'' \quad [\text{Ans}]$$

- ④ Prob - 672, 676, 683, 687, 688, 717, 718, 721, 722, 723, 724, 725. > V.M. FAIRES.

Moment of inertia of area

First moment of area — Centroid

Second moment of area — Moment of inertia.

$$MI = \int ardA$$

a^2 is always +ve so, $-MI$ is always +ve.

Unit — \square^4 such as — $m^4, \text{inch}^4, \text{ft}^4$.

Rectangular moment of inertia: Axis is \perp to the area

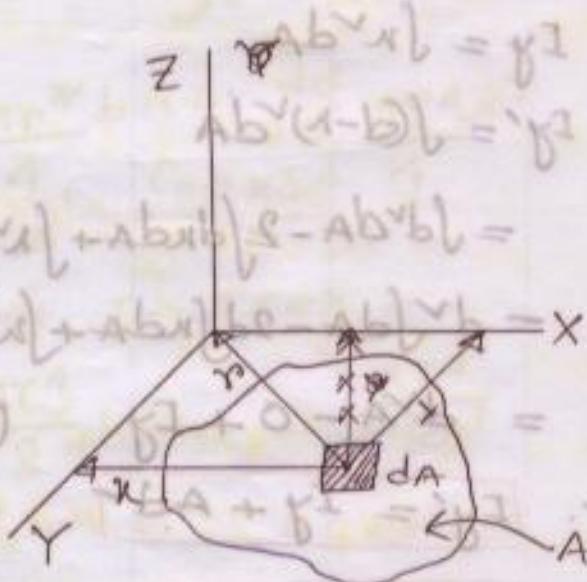
Polar moment of inertia: Axis is \perp to the area

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

$$\begin{aligned} J &= \int r^2 dA = \int (x^2 + y^2) dA \\ &= \int x^2 dA + \int y^2 dA \end{aligned}$$

$$\therefore J = I_x + I_y$$



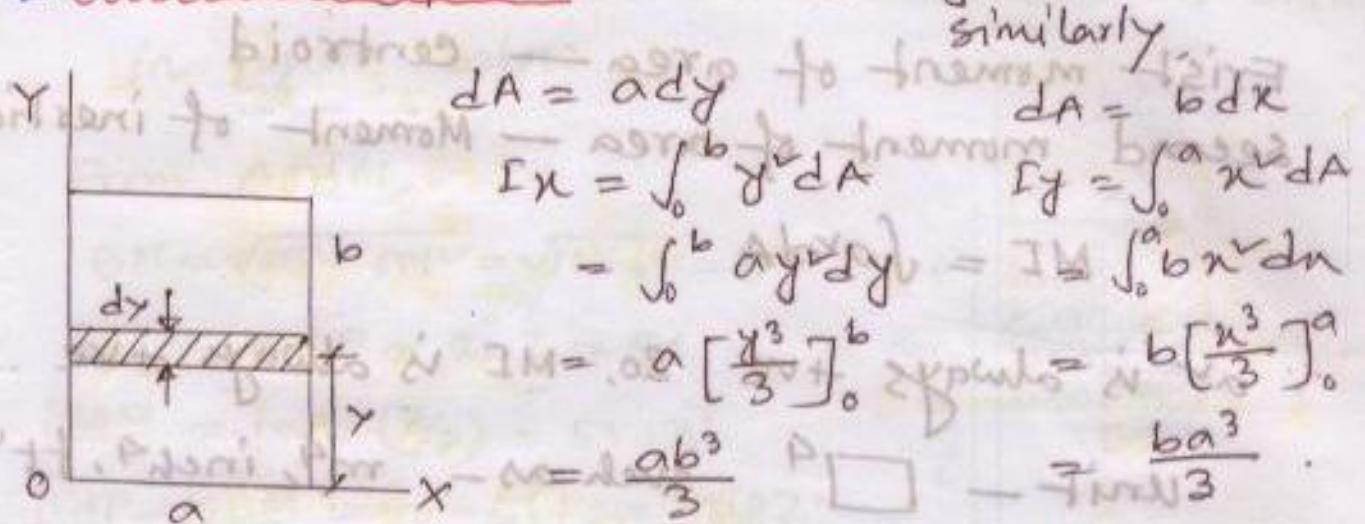
Radius of gyration: $I = bA + \bar{I} = I$

$$K = \left(\frac{I}{A}\right)^{1/2} \quad \text{or} \quad K = \left(\frac{J}{A}\right)^{1/2}$$

where, I and J — rectangular and polar moment of inertia. So, $I = K^2 A$ and $J = K^2 A$.

Note (To calculate MI we have to calculate the centroid at first if it calculates about centroidal axis)

Calculation of MI: For Rectangle,



Transfer formula:

$$I_y = \int y^2 da$$

$$I_{y'} = \int (d-y)^2 da$$

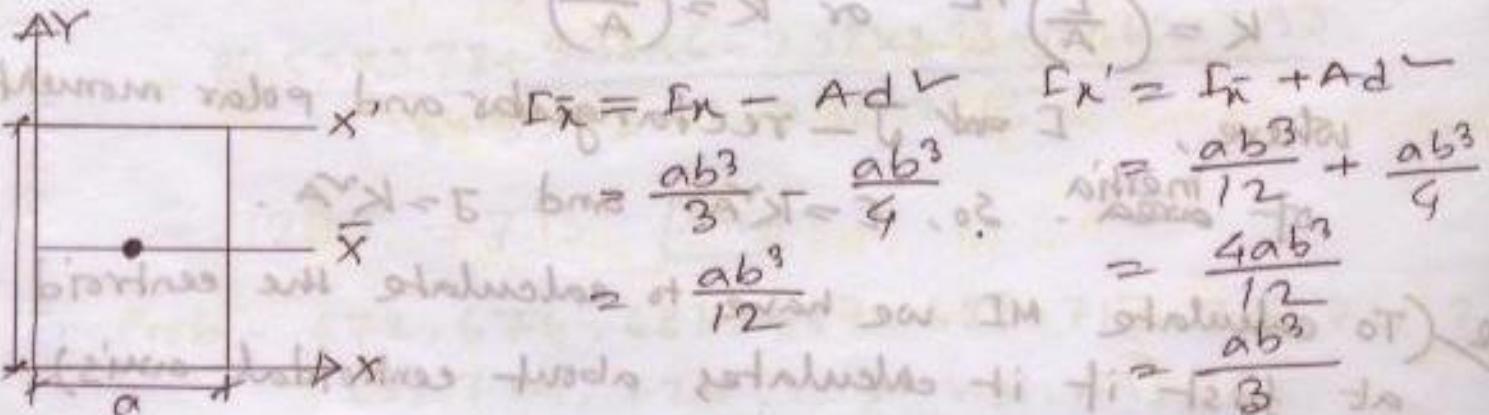
$$= \int d^2 da - 2 \int dy da + \int y^2 da$$

$$= d^2 A - 2d \int y da + \int y^2 da$$

$$= d^2 A - 0 + I_y \quad [\int y da = 0 \text{ for centroidal axis}]$$

$$\therefore I_{y'} = I_y + Ad^2 \quad (\text{parallel axis theorem})$$

$$I = \bar{I} + Ad^2 \quad (\text{General terms})$$



4

$$P = \pi r^2, P = \pi r^2$$

Figure	I_x	$I_{\bar{x}}$	I_y
	$\frac{bh^3}{3}$	$\frac{bh^3}{12}$	$\frac{bh^3}{3}$
	$\frac{bh^3}{12}$	$\frac{bh^3}{36}$	$\frac{bh^3}{12}$
	$\frac{\pi r^4}{4}$	$\frac{\pi r^4}{4}$	$\frac{\pi r^4}{4}$
	$\frac{\pi r^4}{8}$	$\frac{\pi r^4}{8}$	$\frac{\pi r^4}{8}$
	$\frac{\pi r^4}{16}$	$\frac{\pi r^4}{8}$	$\frac{\pi r^4}{16}$

$$W18.1821 = (f0.2) \times \frac{\pi r^2}{3} + 29.1P1 = sbA + pr^2 = \pi r^2$$

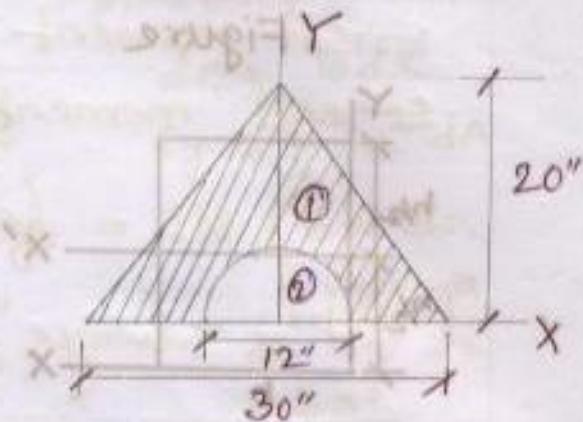
$$\text{In 12.5232} = \pi r^2 - \pi r^2 = \pi r^2$$

X A
 $\sqrt{bA} +$
 $\sqrt{bA} -$
 $\frac{b}{2}$
 $\frac{b}{2}$

$$E_n = ? , E_{\bar{n}} = ?$$

$$\begin{aligned} I_n &= I_{n1} - I_{n2} \\ &= \frac{bh^3}{12} - \frac{\pi r^4}{8} \\ &= 19491.06 \text{ in}^4. \end{aligned}$$

$$\begin{aligned} Y &= \frac{A_1 Y_1 - A_2 Y_2}{A_1 - A_2} \\ &= \frac{300 \times \frac{20}{3} - \frac{\pi \times 6^2}{2} \times \frac{4 \times 6}{3\pi}}{300 - \frac{\pi \times 6^2}{2}} \end{aligned}$$



$$E_{\bar{n}} = E_{\bar{n}1} - E_{\bar{n}2}$$

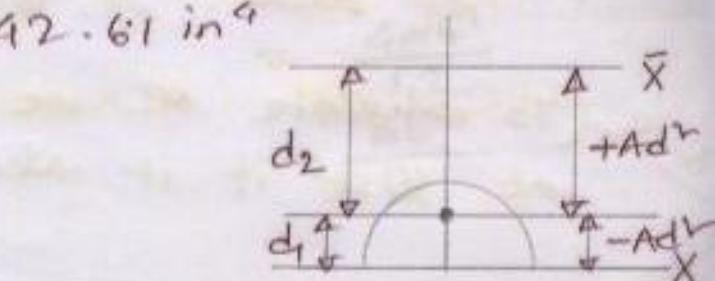
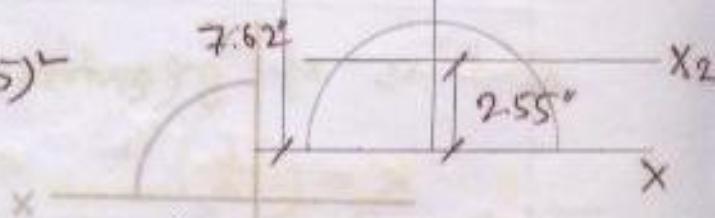
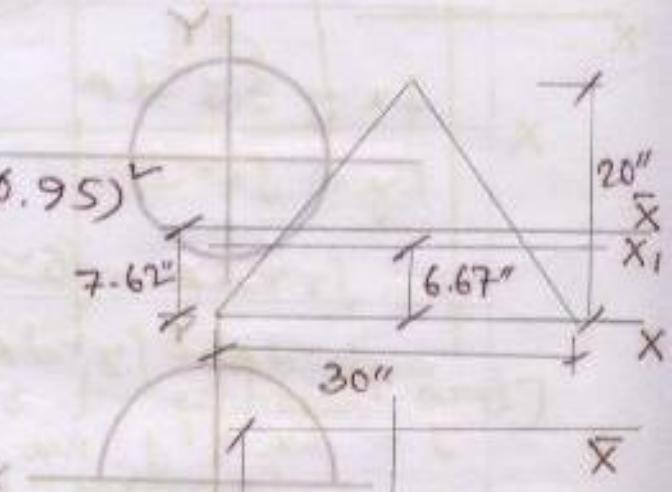
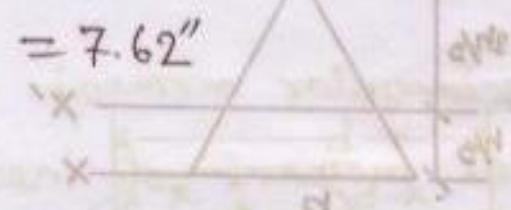
$$\begin{aligned} E_{\bar{n}1} &= \frac{bh^3}{36} + Ad^2 \\ &= \frac{30 \times 20^3}{36} + \frac{1}{2} \times 30 \times 20 \times (0.95) \\ &= 6937.42 \text{ in}^4 \end{aligned}$$

$$E_{\bar{n}2} = E_{\bar{n}2} + Ad_2^2$$

$$\begin{aligned} E_{\bar{n}2} &= E_{\bar{n}2} + Ad_2^2 \\ &= \frac{\pi r^4}{8} - A(2.55)^2 \\ &= 141.23 \text{ in}^4. \end{aligned}$$

$$E_{\bar{n}2} = E_{\bar{n}2} + Ad_2^2 = 141.23 + \frac{\pi r^2}{2} \times (5.07)^2 = 1594.81 \text{ in}^4$$

$$\therefore E_{\bar{n}} = E_{\bar{n}1} - E_{\bar{n}2} = 5342.61 \text{ in}^4$$



centroidal axis - of eccentricity Ad² (-ve)
centroidal axis eccentric about axis of CSZC Ad² (+ve)

$I = \bar{I} + Ad^2$

題 $I_{\bar{x}} = ?$

$$\bar{Y} = \frac{A_1 Y_1 + A_2 Y_2}{A_1 + A_2}$$

$$= \frac{10 \times 1 \times 7.5 + 7 \times 1 \times 3.5}{10 \times 1 + 7 \times 1}$$

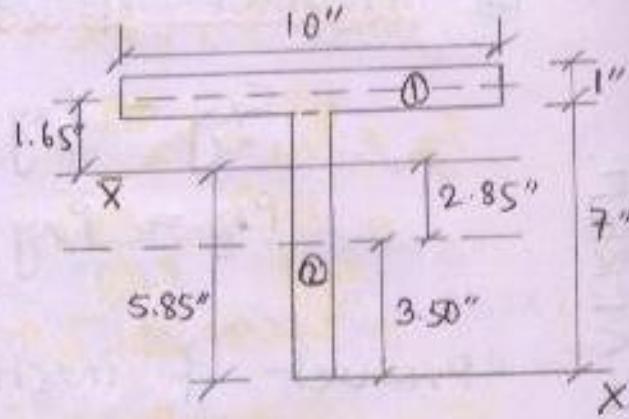
$$= 5.85 \text{ in}$$

$$I_{\bar{x}} = I_{x_1} + I_{x_2}$$

$$I_{x_1} = \frac{bh^3}{12} + Ad^2$$

$$= \frac{10 \times 1^3}{12} + 10 \times (1.65)^2$$

$$= 28.06 \text{ in}^4$$



$$I_{x_2} = \frac{bh^3}{12} + Ad^2$$

$$= \frac{1 \times 7^3}{12} + 7 \times (2.35)^2$$

$$= 67.24 \text{ in}^4$$

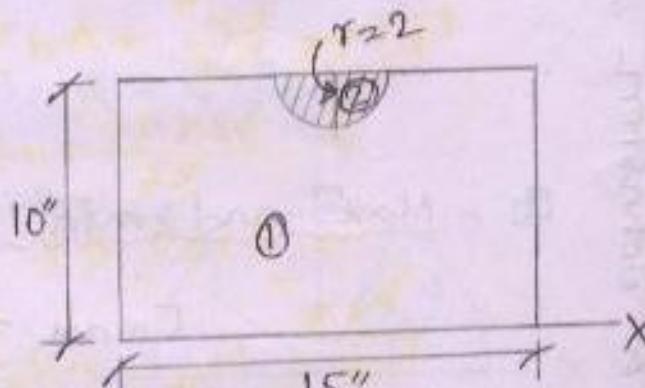
$$\therefore I_{\bar{x}} = I_{x_1} + I_{x_2} = 95.30 \text{ in}^4$$

題 $I_x = ?$, $I_{\bar{x}} = ?$

$$I_x = I_{x_1} - I_{x_2}$$

$$I_{x_1} = \frac{bh^3}{12^2} = 1250.5000$$

$$I_{x_2} = \left[\frac{\pi r^4}{8} - \frac{\pi r^2}{2} \times \left(\frac{4r}{3\pi} \right)^2 \right] + \frac{\pi r^2}{2} \left(10 - \frac{4r}{3\pi} \right)^2 = 527.94$$



$$\therefore I_x = I_{x_1} - I_{x_2} = 722.06 \text{ in}^4$$

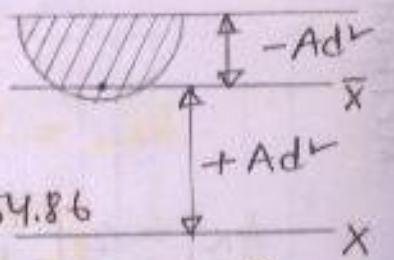
$$\bar{y} = \frac{A_1 x_1 - A_2 x_2}{A_1 + A_2} = 4.82 \text{ in}$$

$$I_{\bar{x}} = I_{x_1} - I_{x_2}$$

$$I_{x_1} = \frac{bh^3}{12} + Ad^2 = \frac{15 \times 10^3}{12} + 15 \times 10 \times 0.18^2 = 1254.86$$

$$I_{x_2} = \left[\frac{\pi r^4}{8} - \frac{\pi r^2}{2} \times \left(\frac{9r}{3\pi} \right)^2 \right] + \frac{\pi r^2}{2} \left[5 - \frac{4r}{3\pi} + 0.18 \right]^2 = 119.6$$

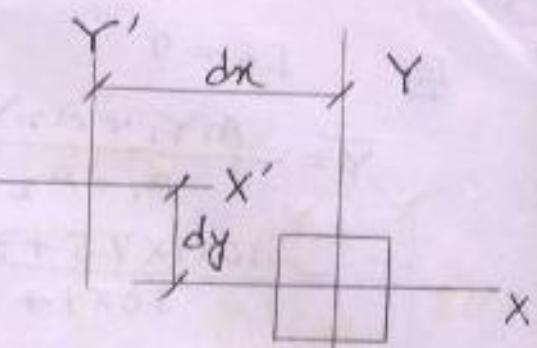
$$I_{\bar{x}} = I_{x_1} - I_{x_2} = 1135.24 \text{ in}^4$$



④ Product of Inertia:

$$I_{xy} = P_{xy} = \int xy \, dA$$

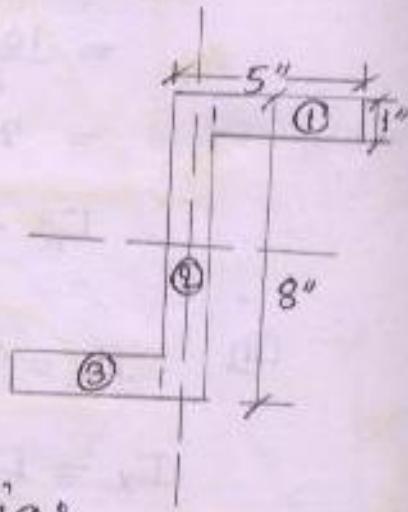
$$P_{xy} = \bar{P}_{xy} + Adx dy$$



Product of inertia about centroidal axis is zero.

④ Find the product of inertia of Z section of 8" x 5" x 1"

$$\begin{aligned} P_{xy} &= [0 + 4 \times 1 \times 2.5 \times 3.5] \times 2 + 0 \\ &= 70 \text{ in}^4 \end{aligned}$$



④ Max^m and min^m moment of inertia:

$$I_{\max} = I_{av} + R$$

$$I_{\min} = I_{av} - R$$

$$I_{av} = \frac{I_x + I_y}{2}$$

$$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + P_{xy}^2}$$

$$K_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

$$I_{x_1} = 2 \left[\frac{bh^3}{12} + Ad^2 \right] = 2 \left[\frac{4 \times 1^3}{12} + 4 \times 1 \times (3.5)^2 \right] = 98.67$$

$$I_{x_2} = \frac{bh^3}{12} = \frac{1 \times 8^3}{12} = 42.67$$

$$I_x = I_{x_1} + I_{x_2} = 141.33 \text{ in}^4$$

$$I_y = \left[\frac{1 \times 9^3}{12} + 1 \times 4 \times 2.5^2 \right] \times 2 + \frac{8 \times 1^3}{12} = 61.33 \text{ in}^4$$

$$I_{av} = \frac{I_x + I_y}{2} = 101.33 \text{ in}^4$$

$$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + P_{xy}^2} = 80.62$$

$$A = 4 \times 1 \times 2 + 8 \\ = 16 \text{ in}^2$$

$$I_{min} = I_{av} - R = 20.70$$

$$K_{min} = \sqrt{\frac{I_{min}}{A}} = 1.138 \text{ in}$$

Moment of Inertia of Masses

Defn: $I = \int r^2 dm$

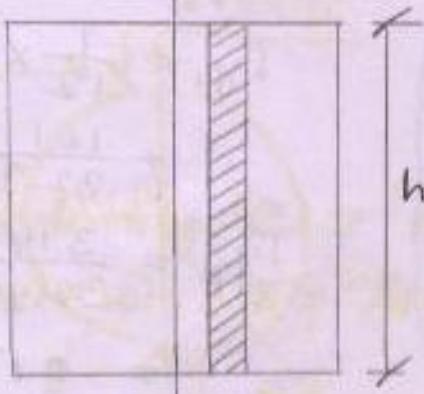
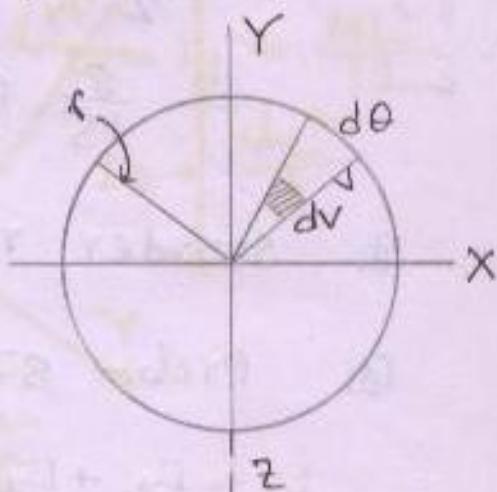
Radius of gyration, $K = \sqrt{\frac{I}{M}}$

cylinder: $I_z = ?$

$$dV = dV \times v d\theta \times h$$

$$dM = \rho V h d\theta dv$$

$$\begin{aligned} I_z &= \int r^2 dm \\ &= \int_0^r \int_0^{2\pi} v r \rho v h d\theta dv \\ &= \frac{r^4}{4} \times 2\pi \times \rho h \\ &= \frac{\pi \rho h r^4}{2} = \boxed{\frac{MR^2}{2}} \end{aligned}$$



■ Sphere: $I_x = ?$

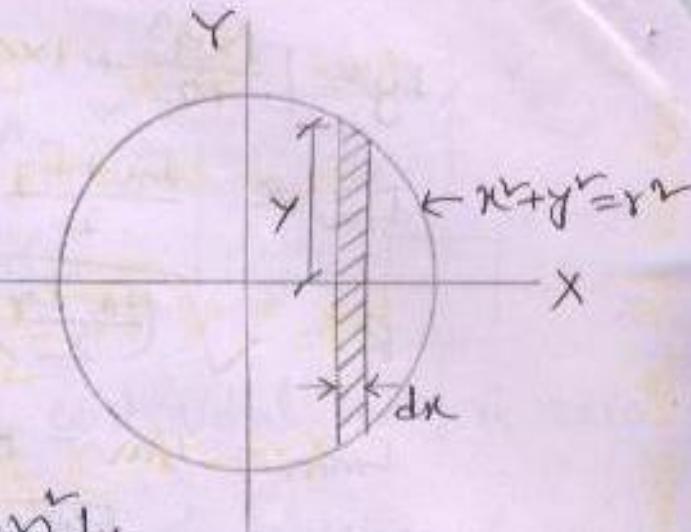
$$dV = \pi y^2 x dx$$

$$dM = \rho \pi y^2 dx$$

$$= \rho \pi (r^2 - x^2) dx$$

$$dI_x = \frac{dm \times r^2}{2}$$

$$= \frac{dm \times r^2}{2} = \frac{\rho \pi (r^2 - x^2) dx}{2}$$



$$I_x = \int_{-r}^{+r} dI_x$$

$$= \int_{-r}^{+r} \frac{\rho \pi (r^2 - x^2)^2 dx}{2} = \frac{\rho \pi}{2} \int_{-r}^{+r} (r^2 - x^2)^2 dx$$

~~$$\frac{\rho \pi r^2}{2} \int_0^{2\pi} (r^2 - x^2)^2 dx$$~~

$$= \rho \pi \int_0^r (r^2 - x^2)^2 dx$$

$$= \frac{8}{15} \rho \pi r^5 = \frac{2}{5} m r^2$$

■ Slender rod, cone, thin disk — Beer & Johnston

■ Prob - 878, page - 239, VM FAIRIES.

$$I_y = I_{y1} + I_{y2}$$

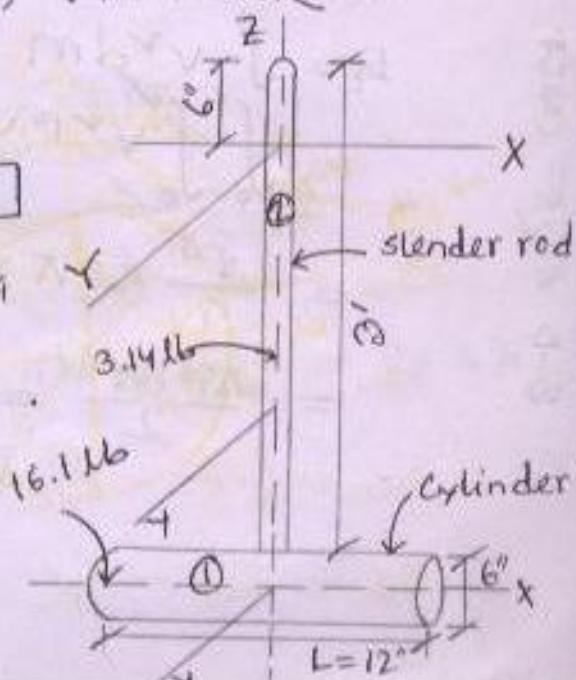
$$I_{y1} = \frac{1}{12} \times \frac{16.1}{32.2} [3 \times (0.25)^2 + 1^2]$$

$$+ \frac{16.1}{32.2} \times (2.75)^2 = 3.83 \text{ in}^4$$

$$I_{y2} = \frac{3.14}{32.2} \times \frac{3^2}{12} + \frac{3.14}{32.2} \times 1^2$$

$$= 0.17 \text{ in}^4$$

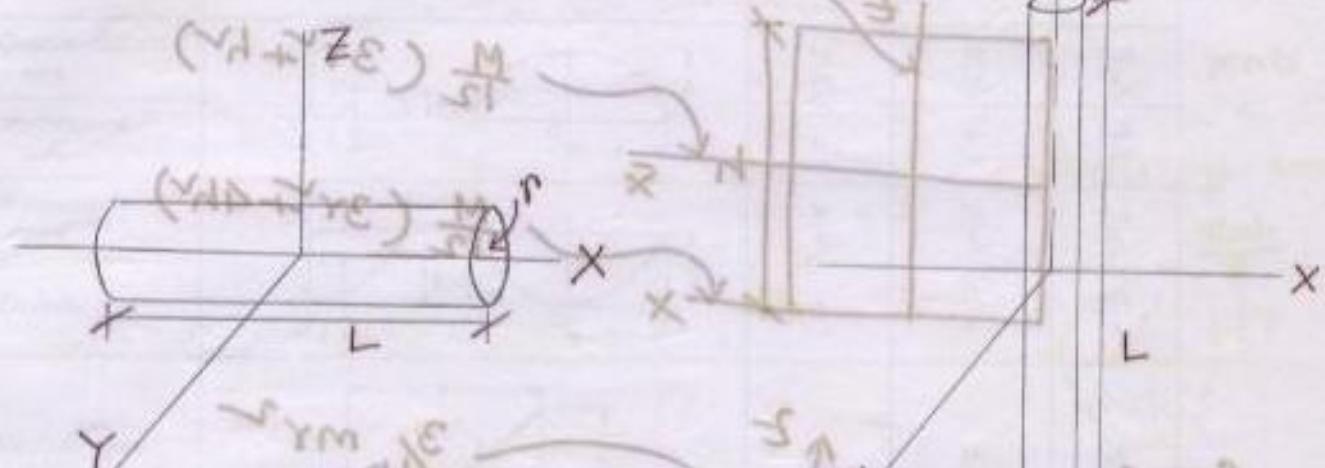
$$I_y = I_{y1} + I_{y2} = 4.0 \text{ in}^4$$



$$M_1 = \frac{16.1}{32.2} \text{ slug}, M_2 = \frac{3.14}{32.2} \text{ slug}$$

$$\therefore M = M_1 + M_2 = 0.6 \text{ slug}$$

$$k_y = \sqrt{\frac{I_y}{M}} = 2.59 \text{ ft.}$$



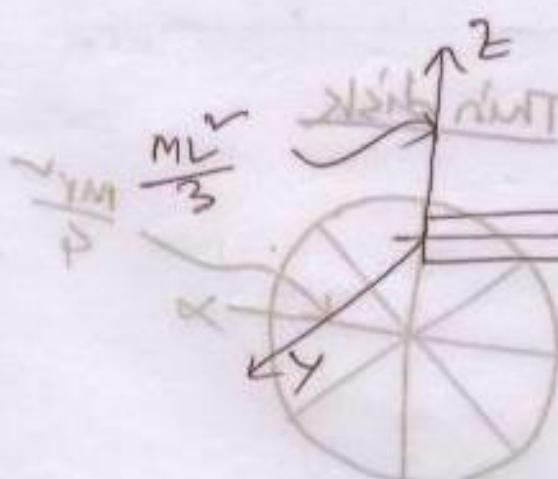
$$(I_x = \frac{M r^2}{2})$$

$$I_y = I_z = \frac{M}{12} (3r^2 + L^2)$$

$$I_x = I_y = \frac{ML^2}{12}$$

$$I_z = \frac{ML^2}{3}$$

④ slender rod



$$\frac{ML^2}{12}$$

~~error~~

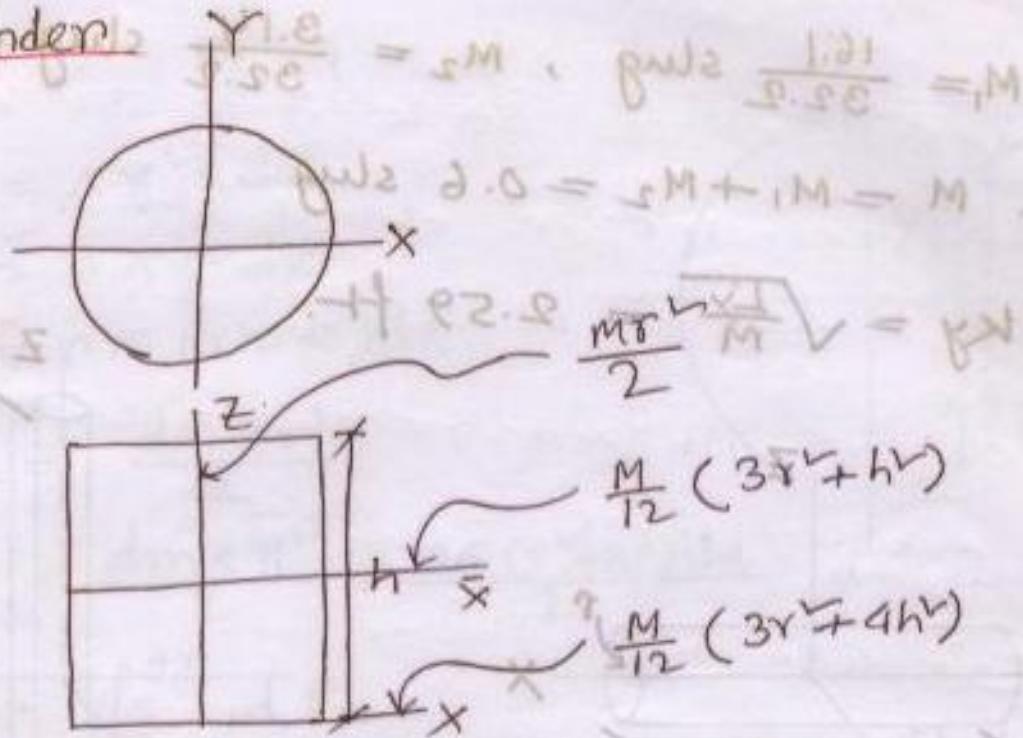
$$\text{min density, } (\rho) = \frac{w}{g}$$

$$\text{unit wt. } \gamma = \frac{w}{g}$$

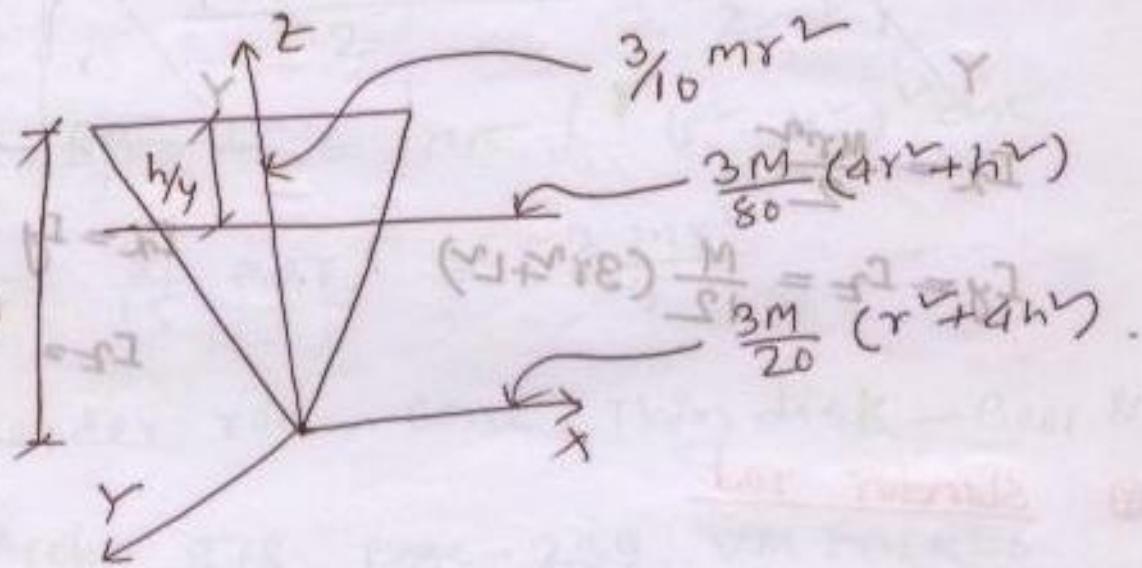
$$\gamma = \rho g$$

$$m = \frac{w}{g} (\text{slug})$$

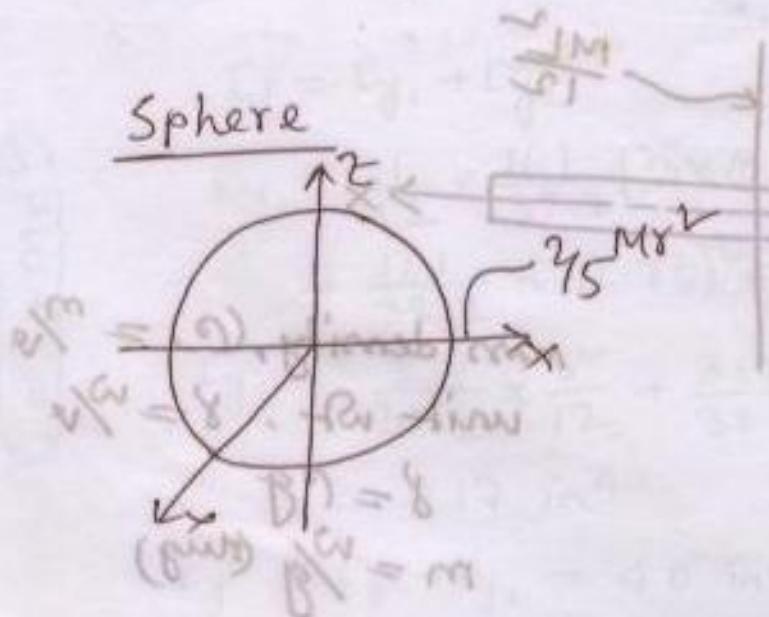
* cylinder



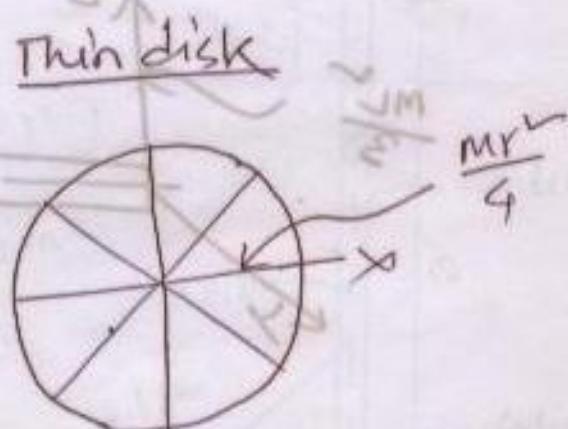
Cone



Sphere



Thin disk



CENTROIDS

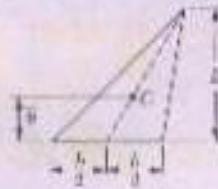
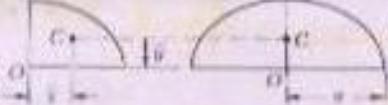
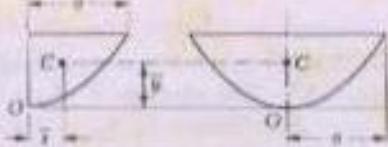
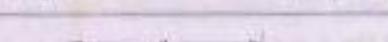
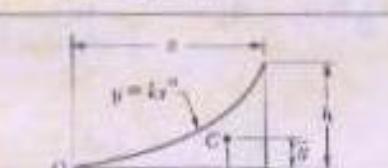
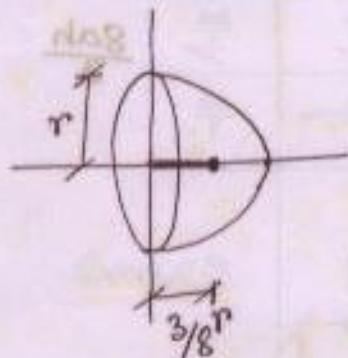
Shape		\bar{x}	\bar{y}	Area
Triangular area		$\frac{b}{3}$	$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{4a}{3}$	$\frac{ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$
General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r \sin \alpha}{3a}$	0	αr^2

Fig. 5.8A

Shape	\bar{x}	\bar{y}	Length
Quarter-circular arc	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2} \cdot 2\pi r$
Semicircular arc	0	$\frac{2r}{\pi}$	πr
Arc of circle	$\frac{r \sin \alpha}{\alpha}$	0	$2\pi r$

Fig. 5.8B

Hemi spherical -



$$V = \frac{4}{3} \pi r^3$$

$$\text{Here, } x = \frac{3}{8} r$$

$$V = \frac{1}{2} \times \frac{4}{3} \pi x^2 r^3$$

5.5. Composite Plates and Wires. In many instances, a flat plate may be divided into rectangles, triangles, or other common shapes shown in Fig. 5.8A. The abscissa X of its center of gravity G may be determined from the abscissas $\bar{x}_1, \bar{x}_2, \dots$ of the centers of gravity of the various parts by expressing that the moment of the weight of the whole plate about the y axis is equal to the sum of the moments of the weights of the various parts about the same axis (Fig. 5.9). The ordinate \bar{Y} of the center of gravity of the plate is found in a similar way by equating moments about the x axis. We write

$$\begin{aligned} \sum M_y: \quad \bar{X}(W_1 + W_2 + \dots + W_n) &= \bar{x}_1 W_1 + \bar{x}_2 W_2 + \dots + \bar{x}_n W_n \\ \sum M_x: \quad \bar{Y}(W_1 + W_2 + \dots + W_n) &= \bar{y}_1 W_1 + \bar{y}_2 W_2 + \dots + \bar{y}_n W_n \end{aligned}$$

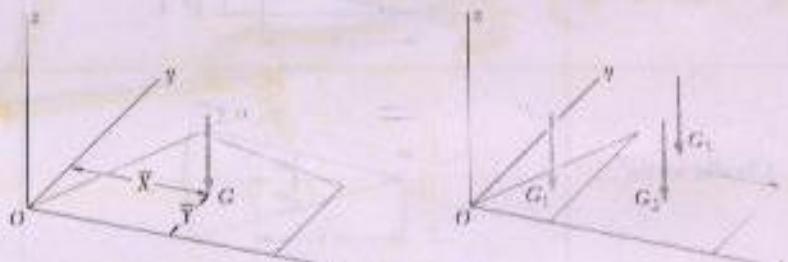


Fig. 5.9