DIRECT DESIGN METHOD
"DDM"

Load Transfer Path For Gravity Loads

- All gravity loads are basically “Volume Loads” generated due to mass contained in a volume
- Mechanism and path must be found to transfer these loads to the “Supports” through a Medium
- All type of Gravity Loads can be represented as:
  – Point Loads
  – Line Loads
  – Area Loads
  – Volume Loads

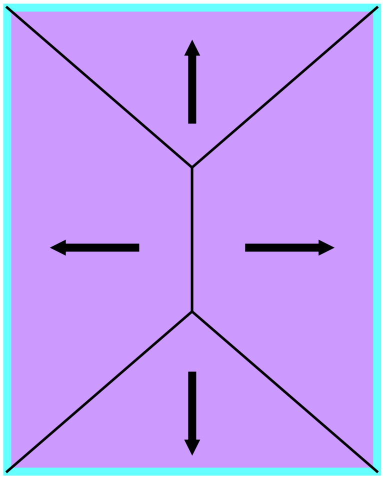
Conventional Approach

- **For Wall Supported Slabs**
  - Assume load transfer in One-Way or Two-Way manner
  - Uniform, Triangular or Trapezoidal Load on Walls

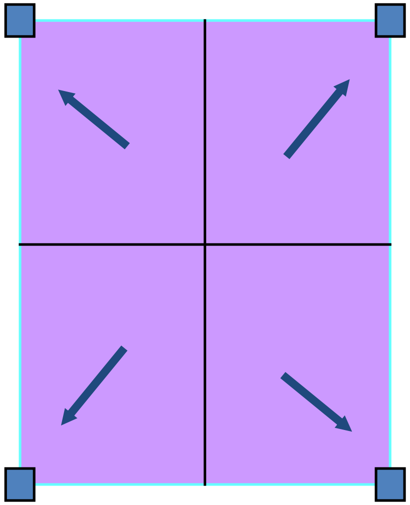
- **For Beam Supported Slabs**
  - Assume beams to support the slabs in similar ways as walls
  - Design slabs as edge supported on beams
  - Transfer load to beams and design beams for slab load

- **For Flat-Slabs or Columns Supported Slabs**
  - Assume load transfer in strips directly to columns

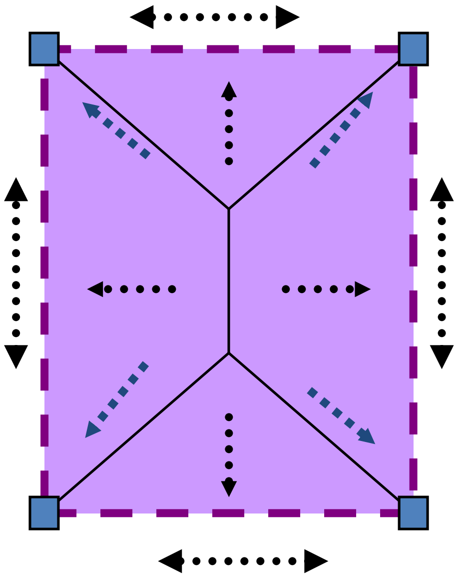
Simplified Load Transfer



To Lines



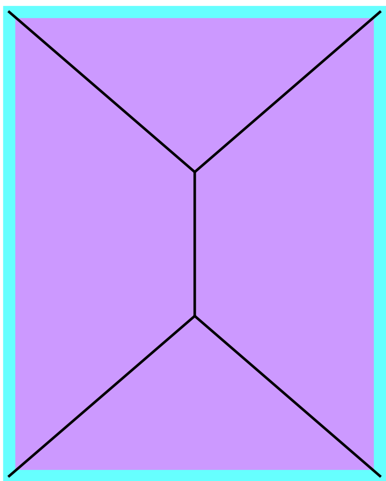
To Points



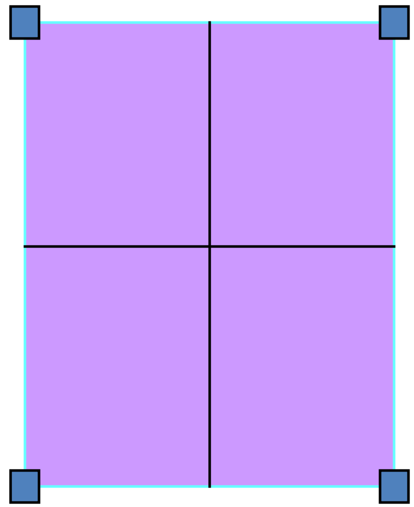
To Lines and Points

Transfer of Area Load

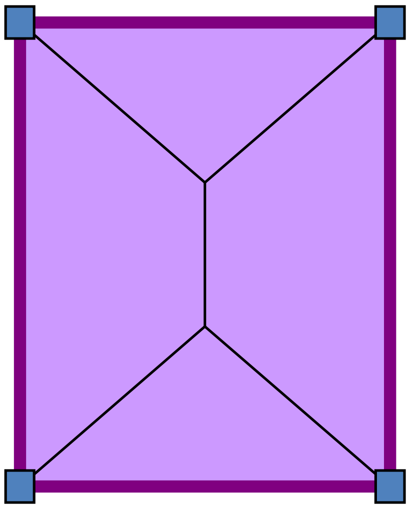
Gravity Load Transfer Paths



Single Path
Slab On Walls

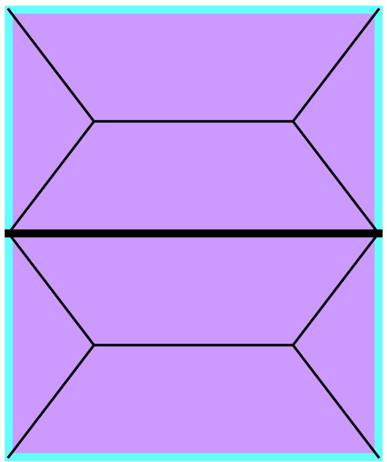


Single Path
Slab on Columns

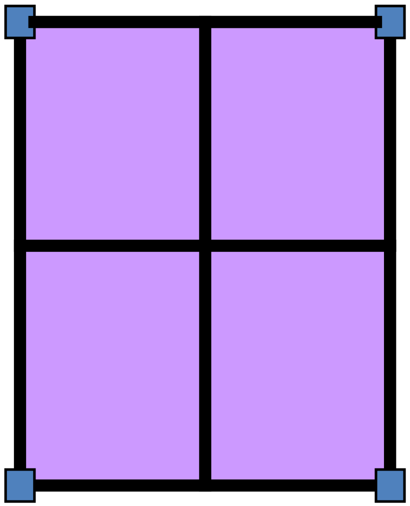


Dual Path
Slab On Beams,
Beams on Columns

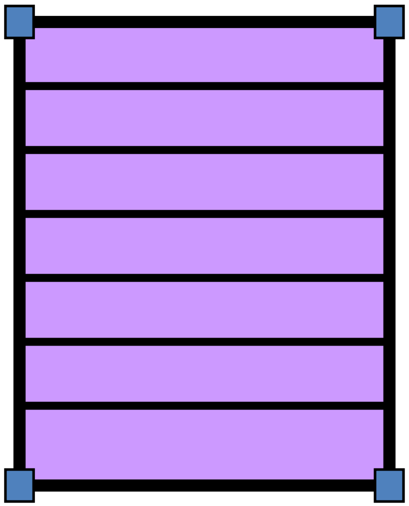
Gravity Load Transfer Paths



Mixed Path
Slab On Walls
Slab On Beams
Beams on Walls

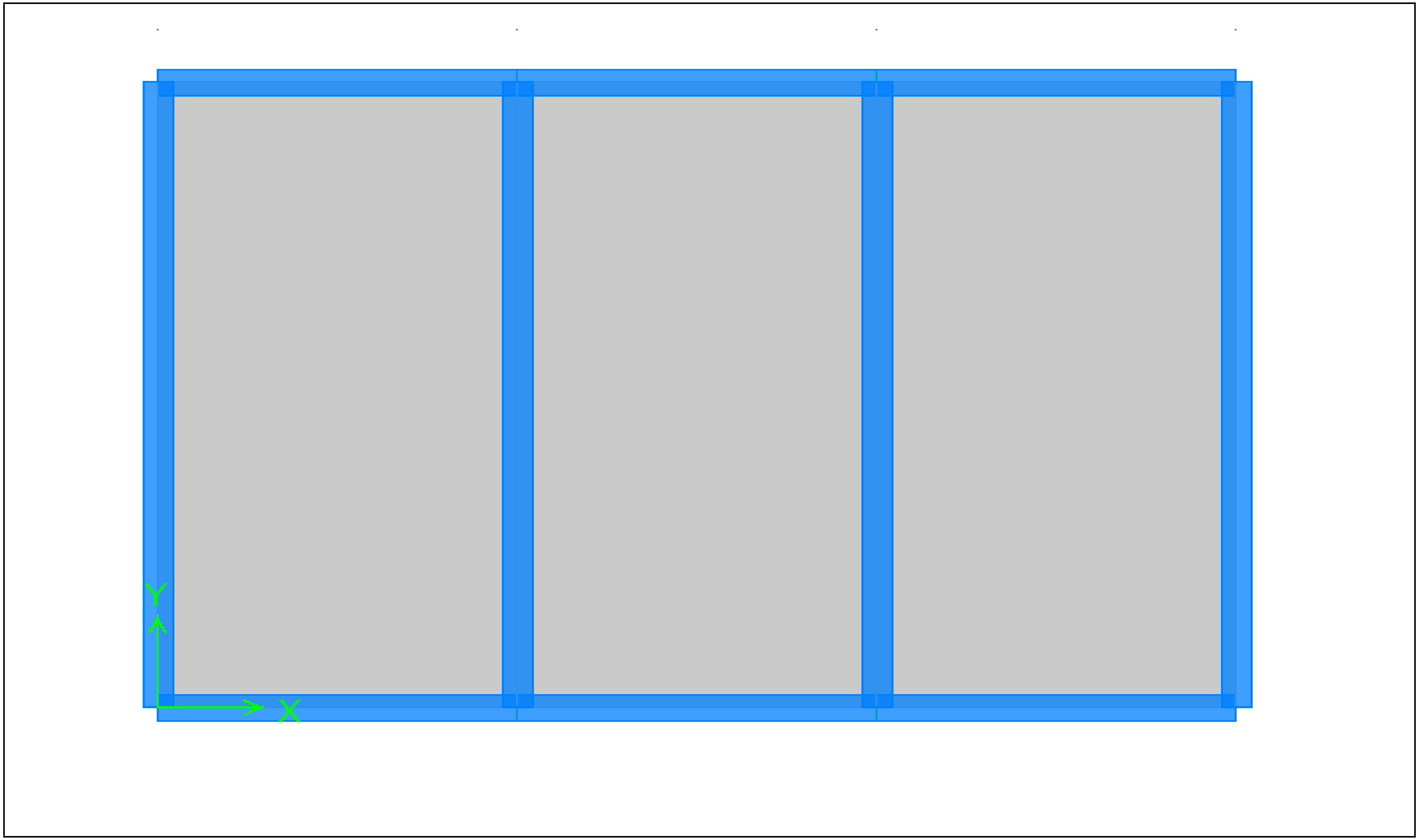


Complex Path
Slab on Beams
Slab on Walls
Beams on Beams
Beams on Columns

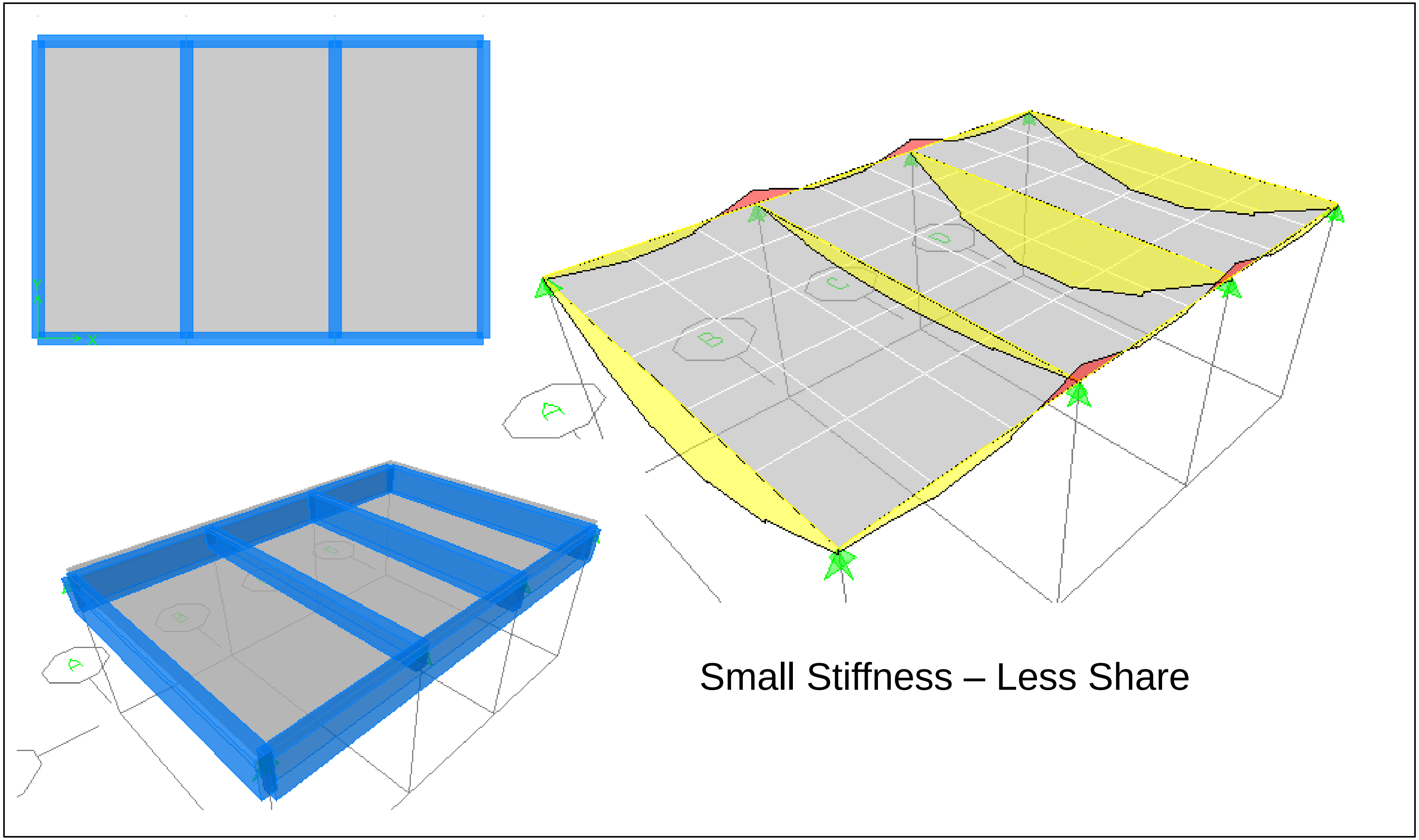


Three Step Path
Slab On Ribs
Ribs On Beams
Beams on Columns

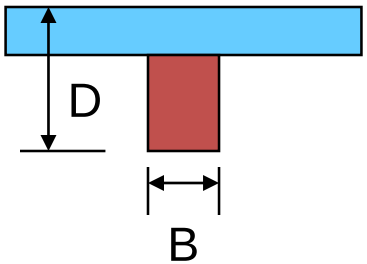
Stiffness Based Load Sharing

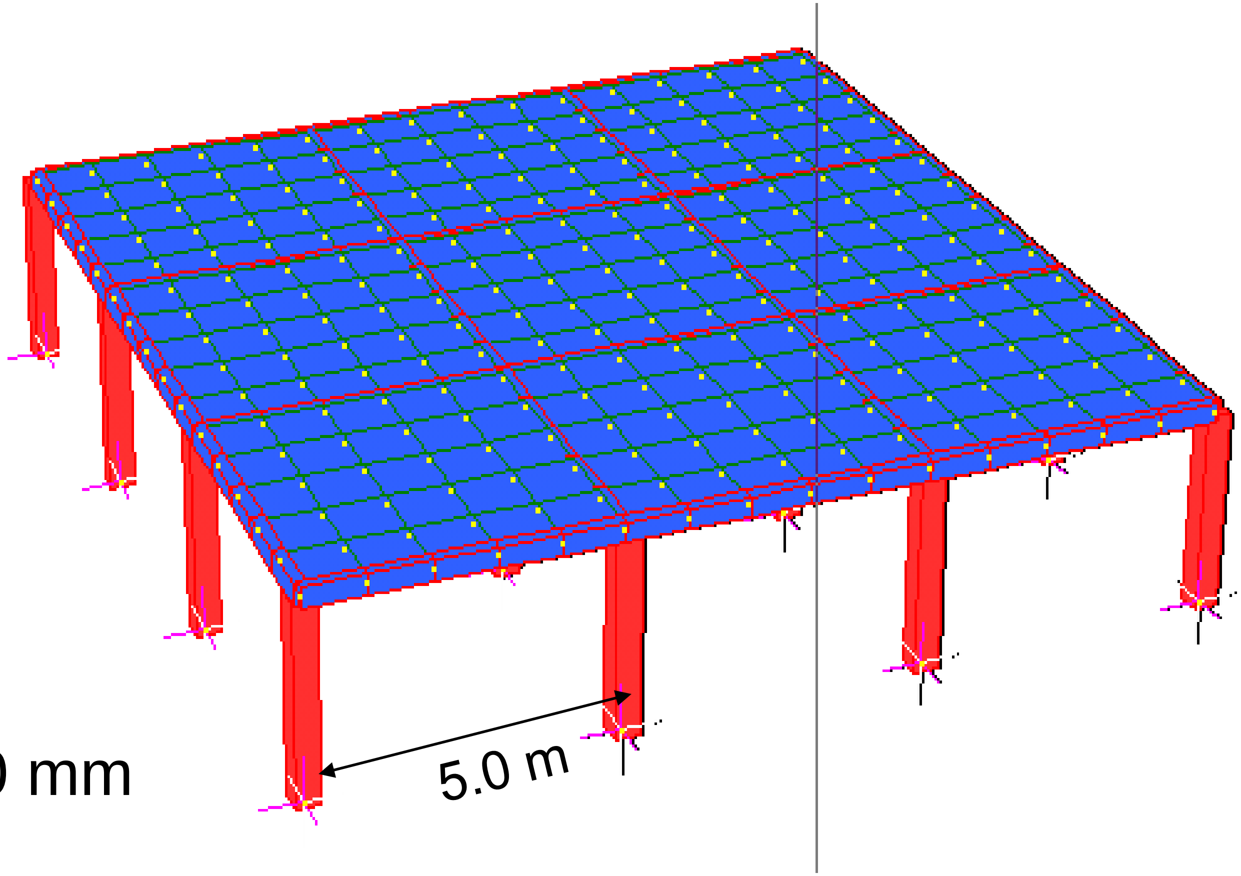


Stiffness Based Load Sharing



Stiffness Based Load Sharing



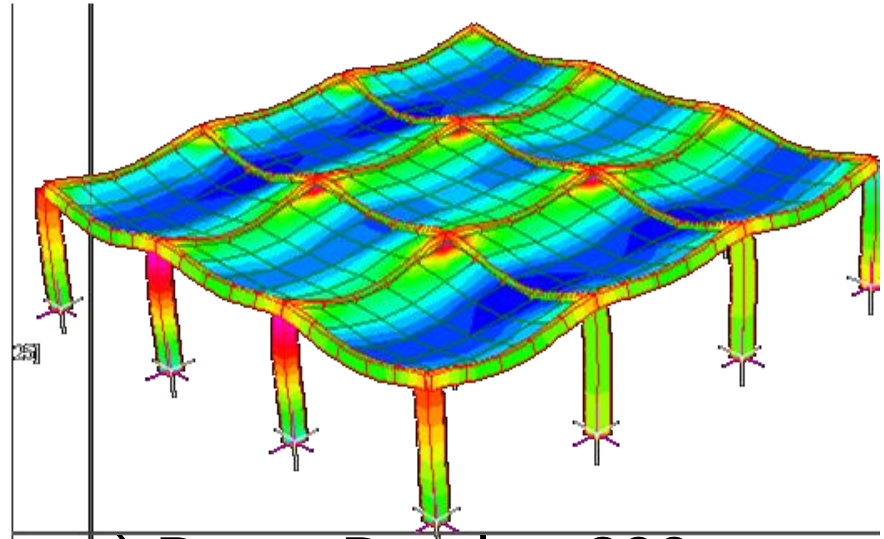


Slab T = 200 mm
Beam Width, B = 300 mm
Beam Depth, D

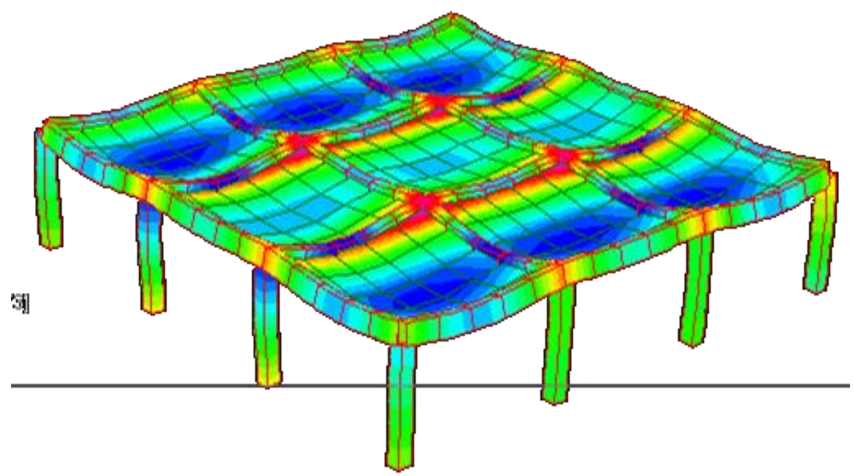
a) 300 mm
b) 500 mm
c) 1000 mm

Moment Distribution in Beam-Slab

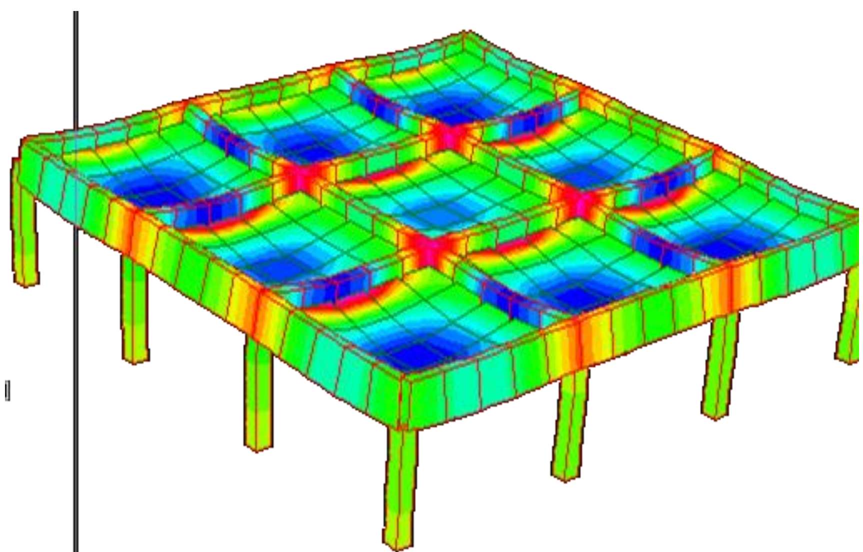
Effect of Beam Size on
Moment Distribution



a) Beam Depth = 300 mm



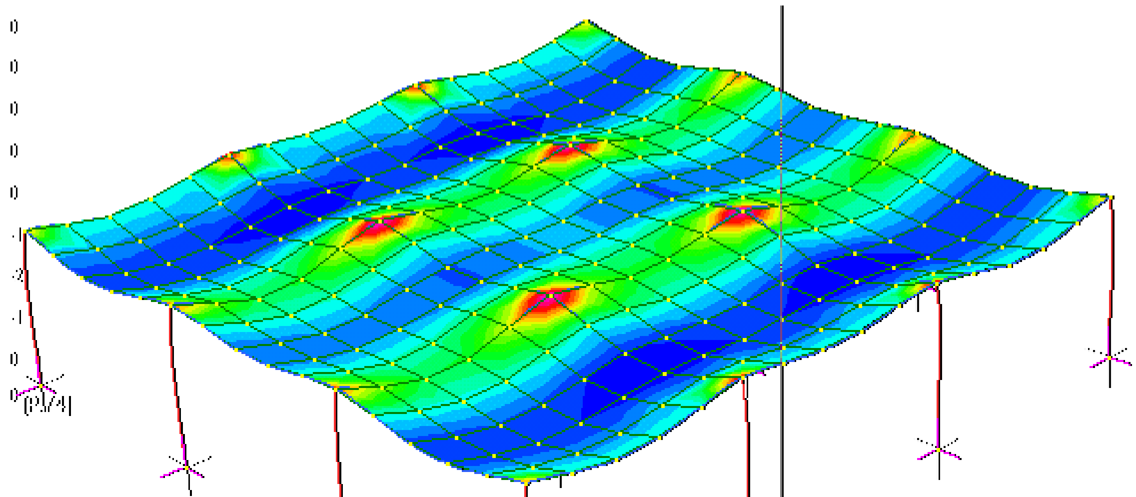
b) Beam Depth = 500 mm



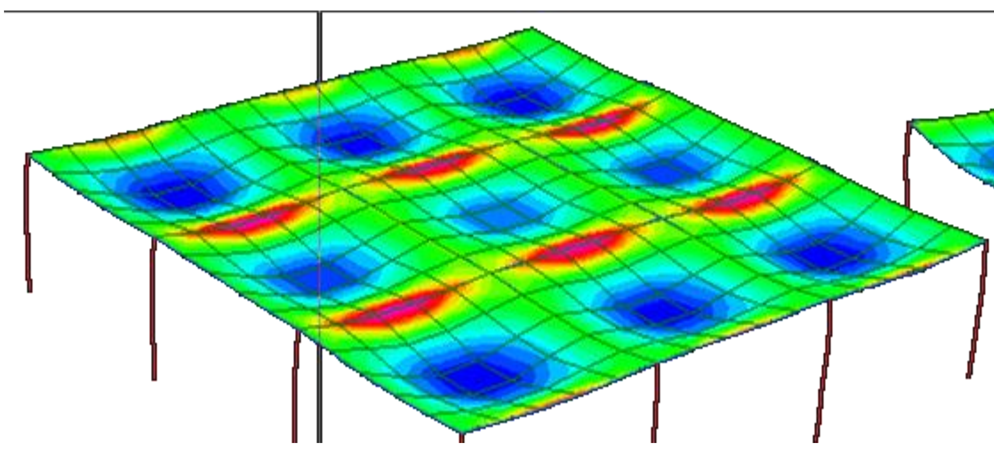
c) Beam Depth = 1000 mm

Moment Distribution in Slab Only

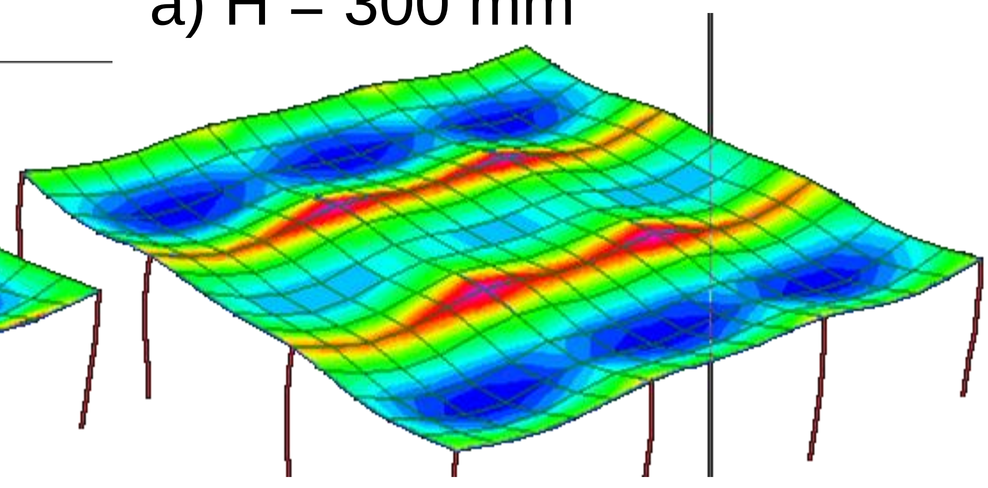
Effect of Beam Size on Moment Distribution



a) H = 300 mm



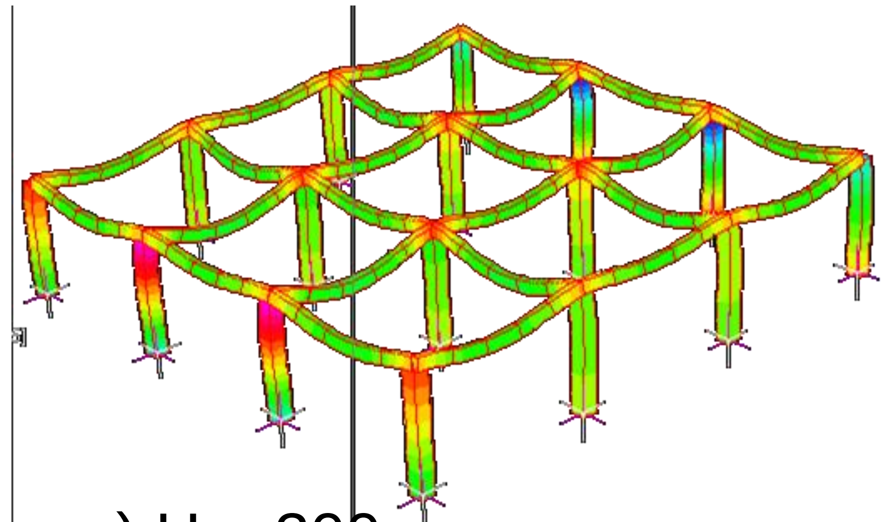
c) H = 1000 mm



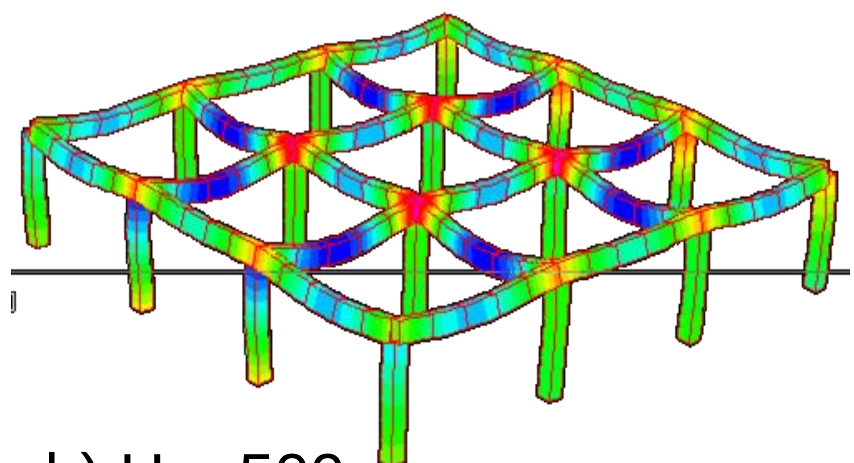
b) H = 500 mm

Moment Distribution in Beams Only

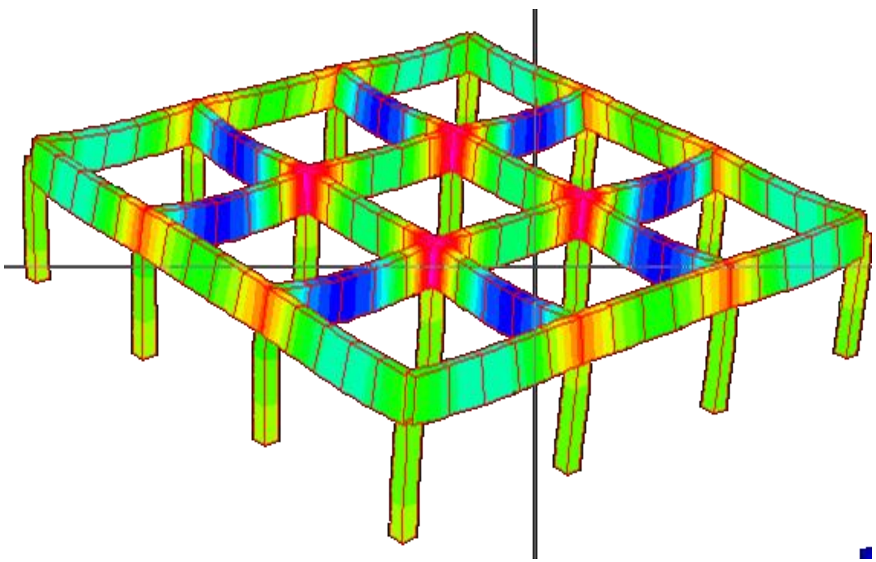
Effect of Beam Size on
Moment Distribution



a) H = 300 mm



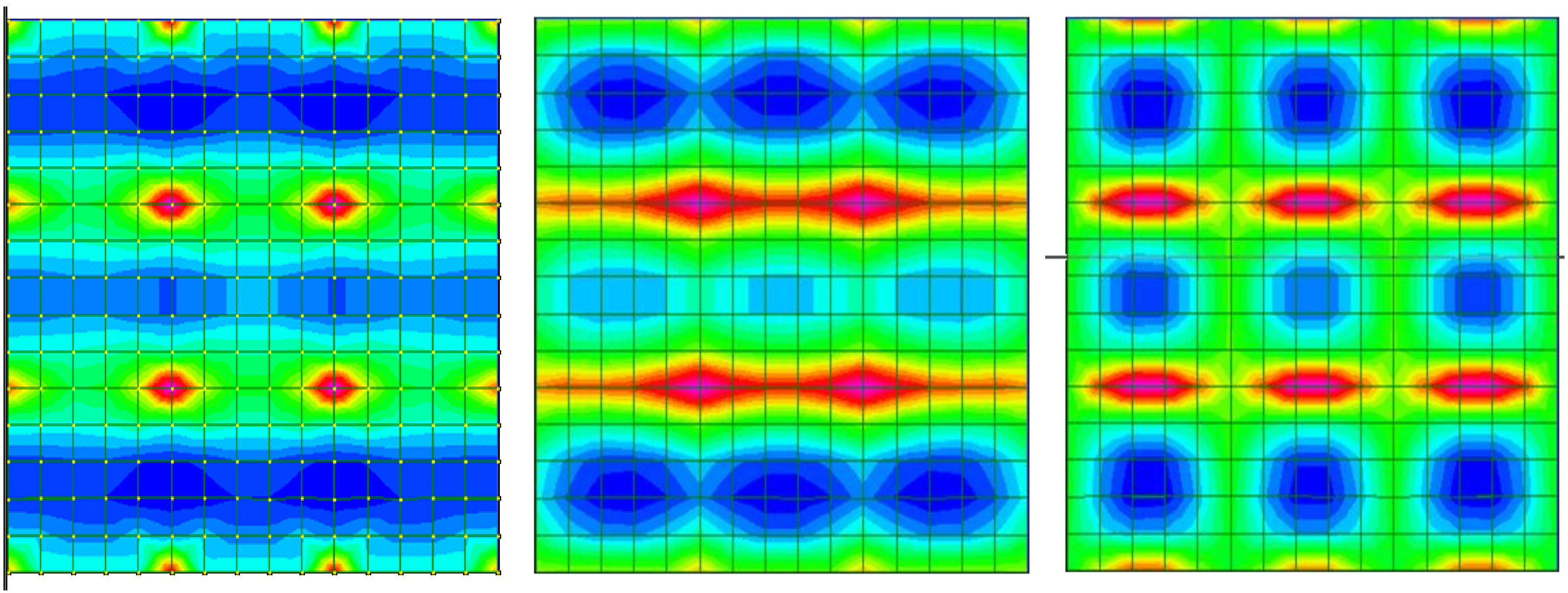
b) H = 500 mm



c) H = 1000 mm

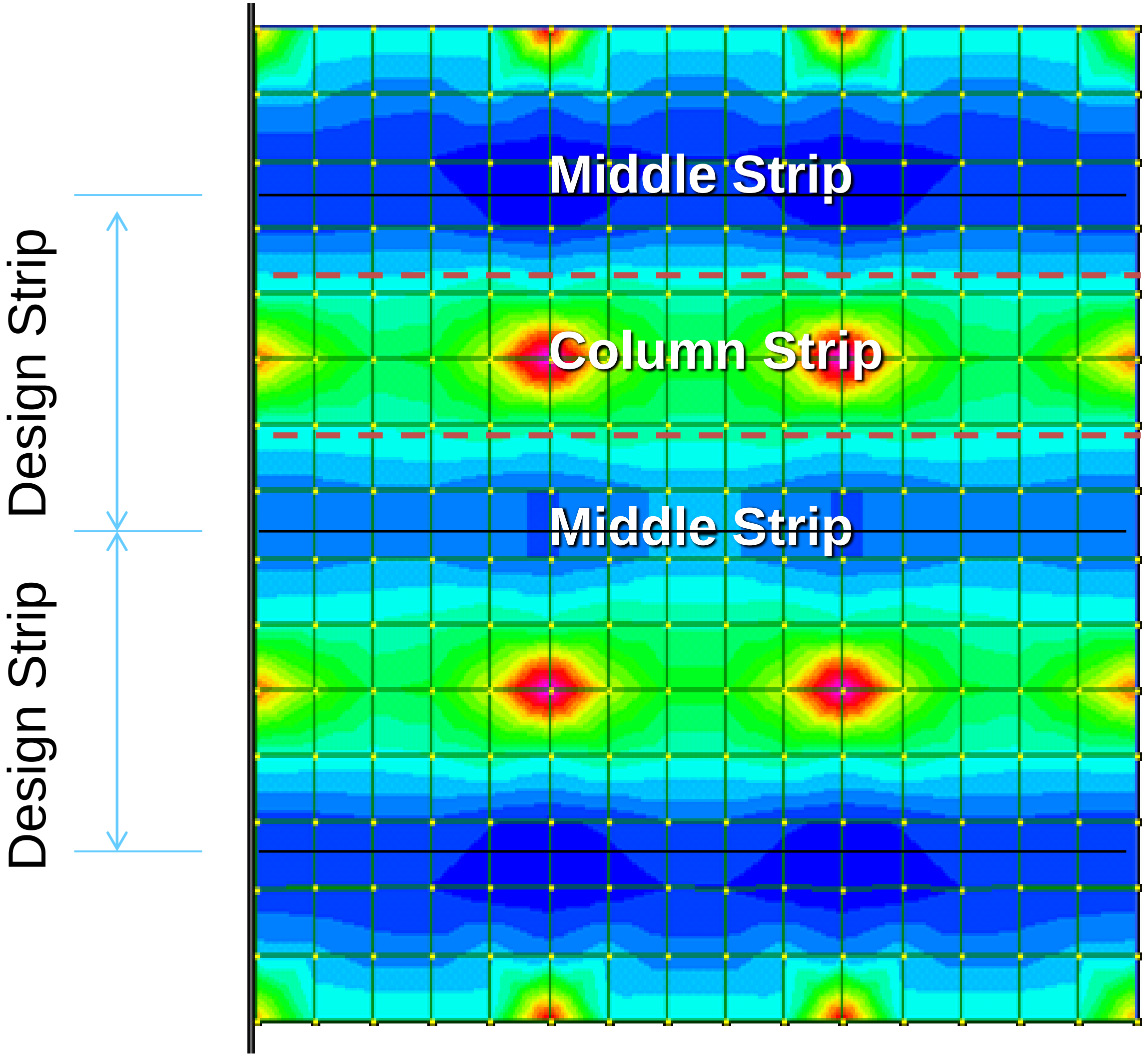
Moment Distribution in Slabs Only

Effect of Beam Size on Moment Distribution



a) H = 300 mm b) H = 500 mm c) H = 1000 mm

The Design Strip Concept



(1) General Description

In reinforced concrete buildings, a basic and common type of floor is the slab-beam-girder construction. As shown in Fig.1(a), the shaded slab area is bounded by the two adjacent beams on the sides and portions of the two girders at the ends.

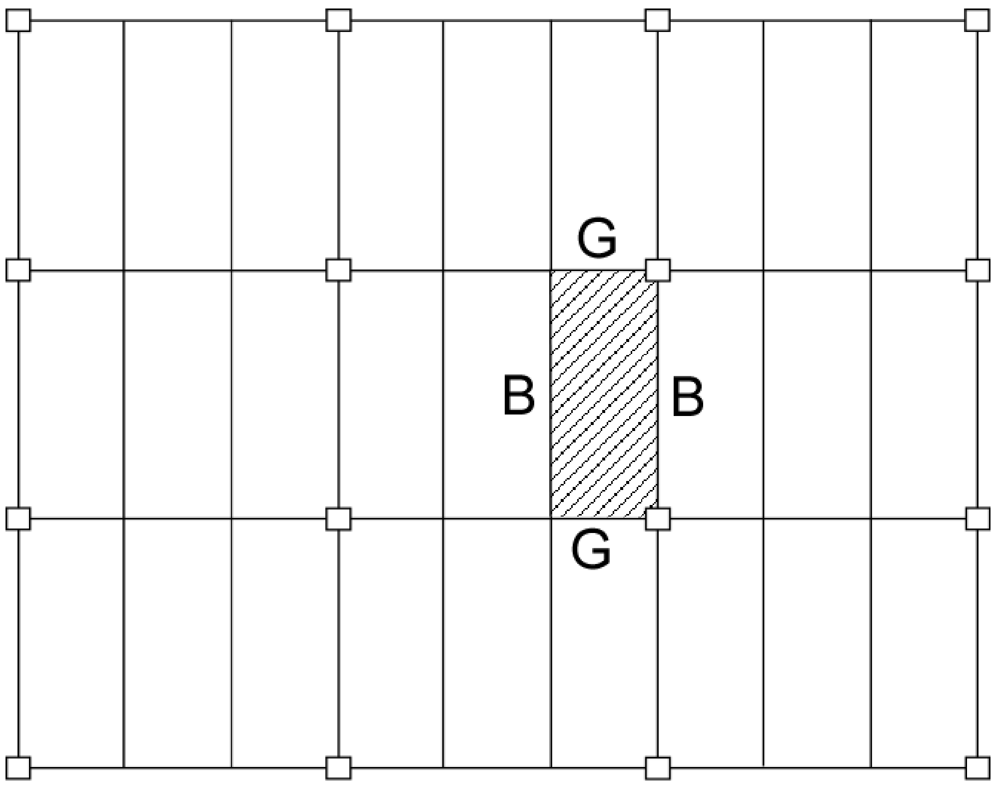


Fig.1(a)

(1) General Description

When the length of this area is two or more times its width, almost all of the floor load goes to the beams, and very little, except some near the edge of the girders, goes directly to the girders. Thus the slab may be designed as a one-way slab, with the main reinforcement parallel to the girder and the shrinkage and temperature reinforcement parallel to the beams. The deflected surface of a oneway slab is primarily one of single curvature.

(1) General Description

When the ratio of the long span L to the short span S as shown in Fig.1(b) is less than about 2, the deflected surface of the shaded area becomes one of double curvature.

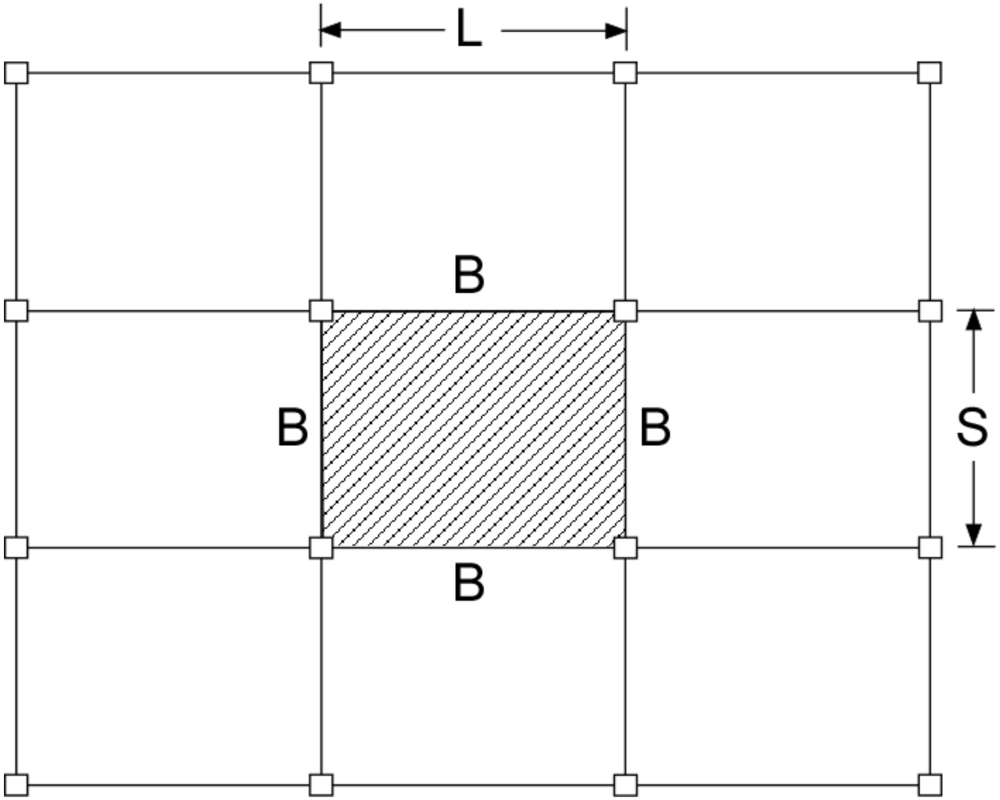


Fig.1(b)

The floor load is carried in both directions to the four supporting beams around the panel; hence the panel is a two-way slab. Obviously, when S is equal to L, the four beams around a typical interior panel should be identical; for other cases the long beams take more load than the short beams.

(1) General Description

Both the flat slab and flat plate floors shown in Figs.2(b) and 2(a) are characterized by the absence of beams along the interior column lines, but edge beams may or may not he used at the exterior edges of the f

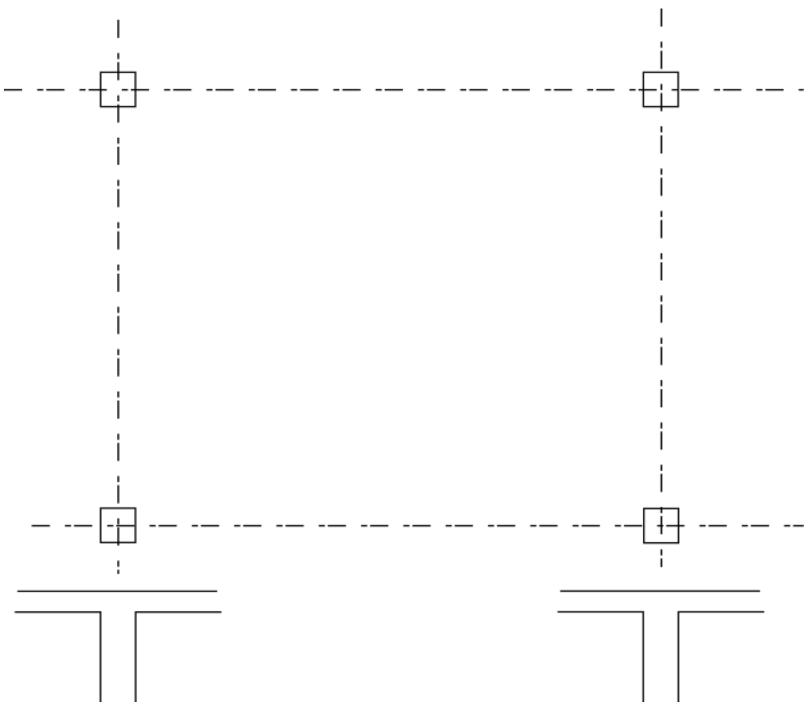


Fig.2(a)

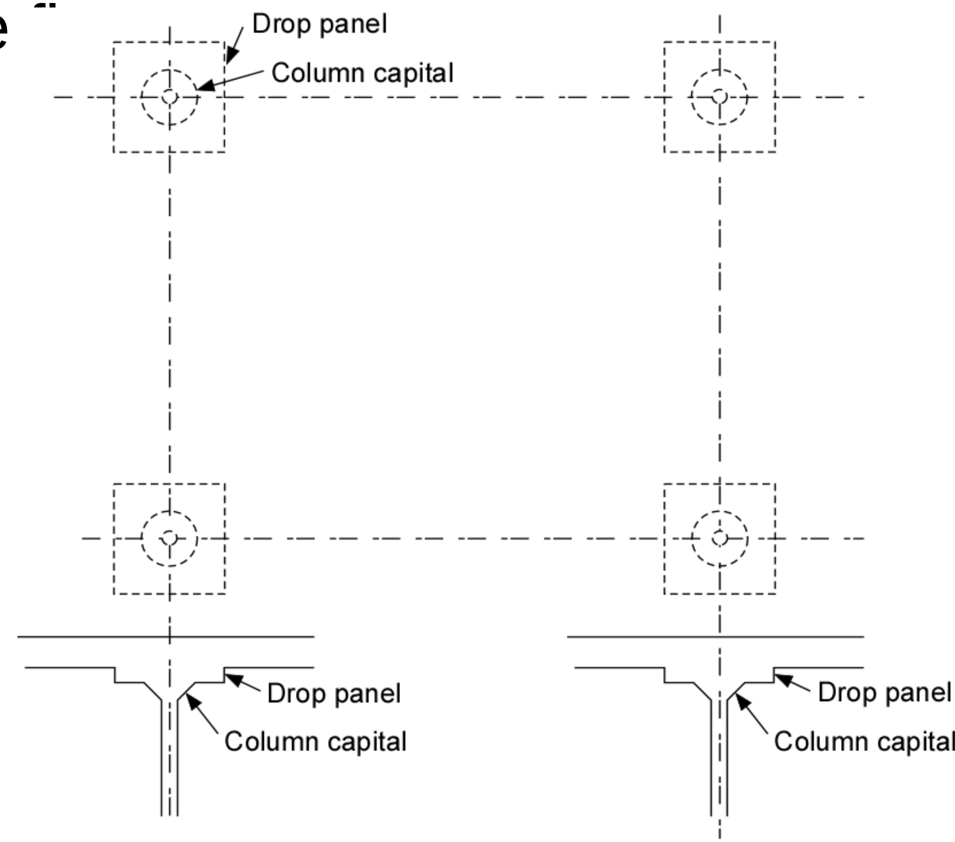


Fig.2(b)

(1) General Description

Flat slab floors differ from flat plate floors in that flat slab floors provide adequate shear strength by having either or both of the following: (a) drop panels (i.e., increased thickness of slab) in the region of the columns; or (b) column capitals (i.e., tapered enlargement of the upper ends of columns).

In flat plate floors a uniform slab thickness is used and the shear strength is obtained by the embedment of multiple-U stirrups or structural steel devices known as shearhead reinforcement within the slab of uniform thickness. Relatively speaking, flat slabs are more suitable for larger panel size or heavier loading than flat plates.

(2) General Design Concept of ACI Code

The basic approach to the design of two-way floor systems involves imagining that vertical cuts are made through the entire building along lines midway between the columns. The cutting creates a series of frames whose width lies between the centerlines of the two adjacent panels as shown in Fig.3.

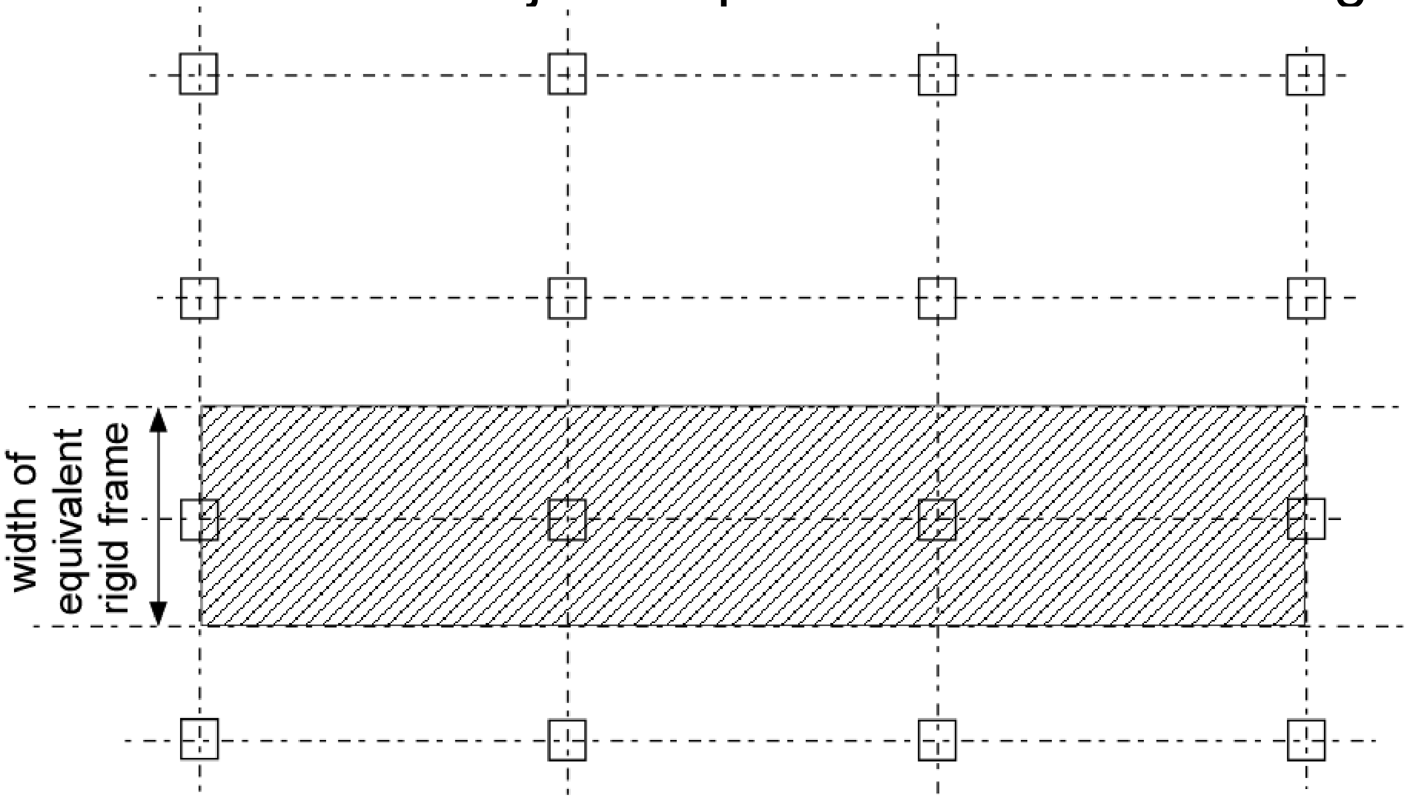


Fig.3

(2) General Design Concept of ACI Code

The resulting series of rigid frames, taken separately in the longitudinal and transverse directions of the building, may be treated for gravity loading floor by floor as would generally be acceptable for a rigid frame structure consisting of beams and columns, in accordance with ACI.

A typical rigid frame would consist of (1) the columns above and below the floor, and (2) the floor system, with or without beams, bounded laterally between the centerlines of the two panels (one panel for an exterior line of columns) adjacent to the line of columns.

(2) General Design Concept of ACI Code

Thus the design of a two-way floor system (including two-way slab, flat slab, and flat plate) is reduced to that of a rigid frame; hence the name "equivalent frame method."

As in the case of design of actual rigid frames consisting of beams and columns, approximate methods of analysis may be suitable for many usual floor systems, spans, and story heights. For gravity load only and for floor systems within the specified limitations, the moments and shears on these equivalent frames may be determined (a) approximately using moment and shear coefficients prescribed by the "direct design method", or

(2) General Design Concept Of ACI Code

(b) by structural analysis in a manner similar to that for actual frames using the special provisions of the "equivalent frame method”. An elastic analysis (such as by the equivalent frame method) must be used for lateral load even if the floor system meets the limitations of the direct design method for gravity load.

The equivalent rigid frame is the structure being dealt with whether the moments are determined by the "direct design method (DDM)" or by the "equivalent frame method (EFM)." These two ACI Code terms describe two ways of obtaining the longitudinal variation of bending moments and shears.

(2) General Design Concept of ACI Code

When the "equivalent frame method" is used for obtaining the longitudinal variation of moments and shears, the relative stiffness of the columns, as well as that of the floor system, can be assumed in the preliminary analysis and then reviewed, as is the case for the design of any statically indeterminate structure. Design moment envelopes may be obtained for dead load in combination with various patterns of live load, In lateral load analysis, moment magnification in columns due to sidesway of vertical loads must be taken into account as prescribed in ACI.

(2) General Design Concept of ACI Code

Once the longitudinal variation in factored moments and shears has been obtained, whether by ACI "DDM" or "EFM," the moment across the entire width of the floor system being considered is distributed laterally to the beam, if used, and to the slab. The lateral distribution procedure and the remainder of the design is essentially the same whether "DDM" or "EFM" has been used.

(3) Total Factored Static Moment

Consider two typical interior panels ABCD and CDEF in a two-way floor system, as shown in Fig.4. Let L_1 and L_2 be the panel size in the longitudinal and transverse directions, respectively.

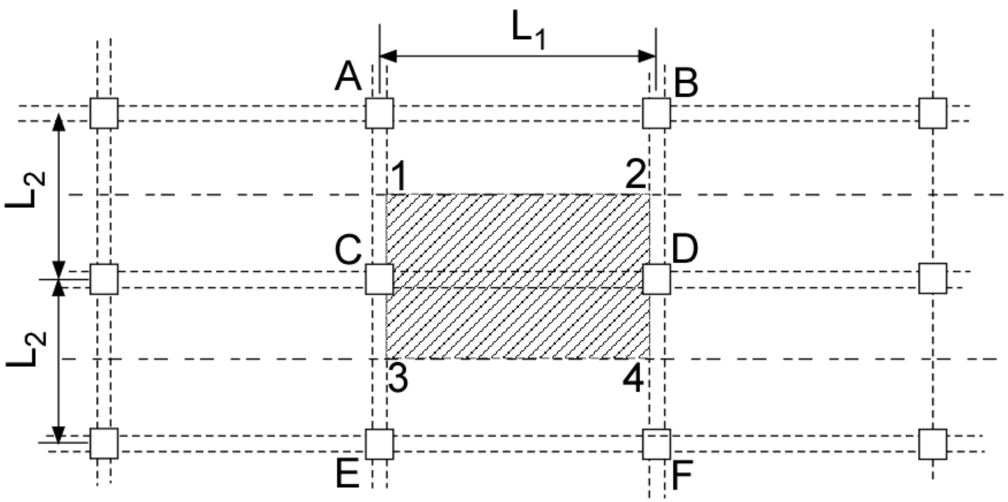


Fig.4

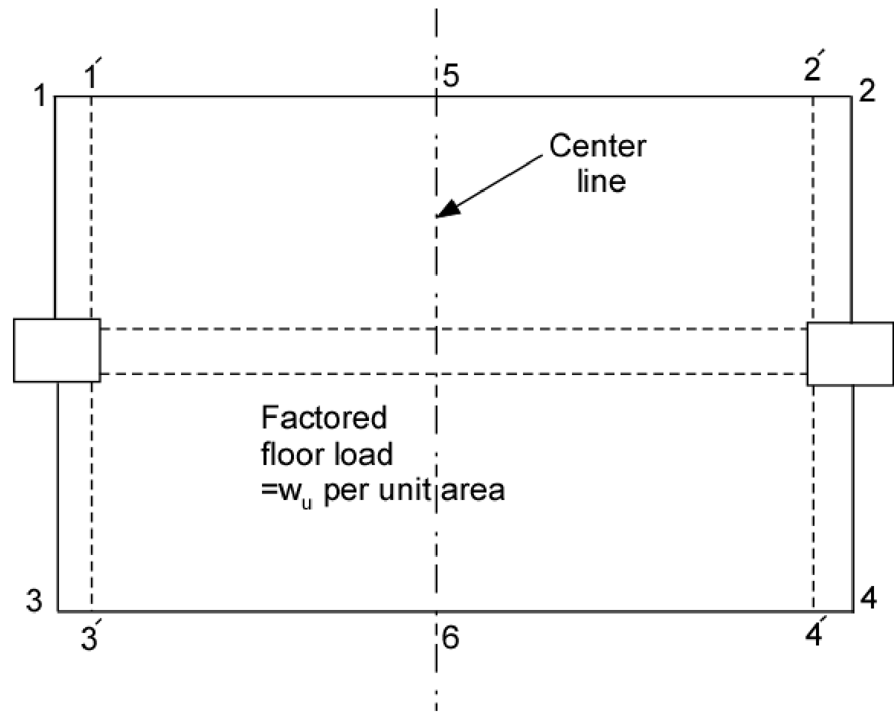


Fig.5

(3) Total Factored Static Moment

Let lines 1-2 and 3-4 be centerlines of panels ABCD and CDEF, both parallel to the longitudinal direction. Isolate as a free body [see Fig.5] the floor slab and the included beam bounded by the lines 1-2 and 3-4 in the longitudinal direction and the transverse lines 1’-3’ and 2’-4’ at the faces of the columns in the transverse direction. The load acting on this free body [see Fig.6] is $w_u L_2$ per unit distance in the longitudinal direction. The total upward force acting on lines 1’-3' or 2'-4' is $\frac{w_u L_2 L_n}{2}$, where w_u is the factored load per unit area and L_n is the clear span in the longitudinal direction between faces of supports.

(3) Total Factored Static Moment

If M_{neg} and M_{pos} are the numerical values of the total negative and positive bending moments along lines 1'-3' and 5-6, then moment equilibrium of the free body of Fig.7 requires

$$M_{neg} + M_{pos} = \frac{w_u L_2 L_n^2}{8} \qquad (1)$$

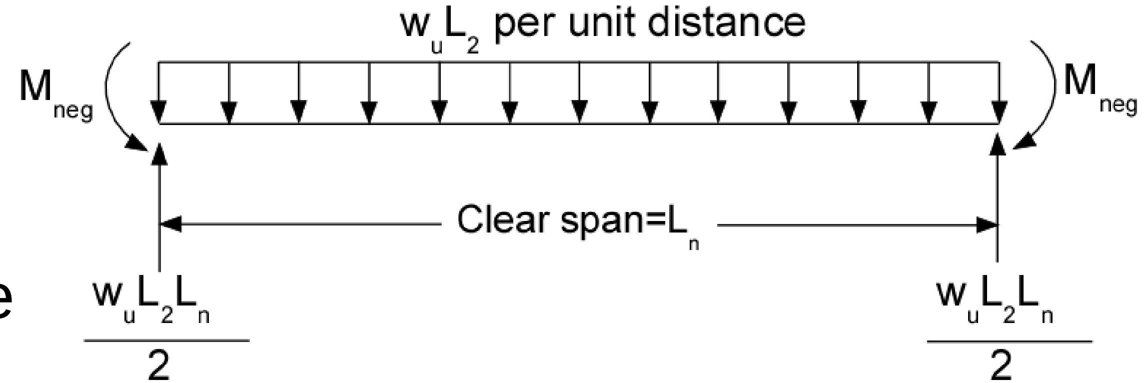


Fig.6

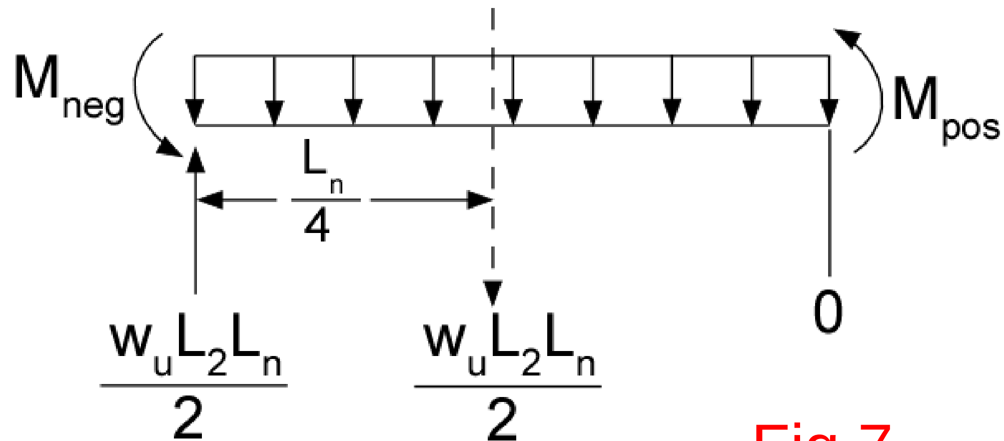


Fig.7

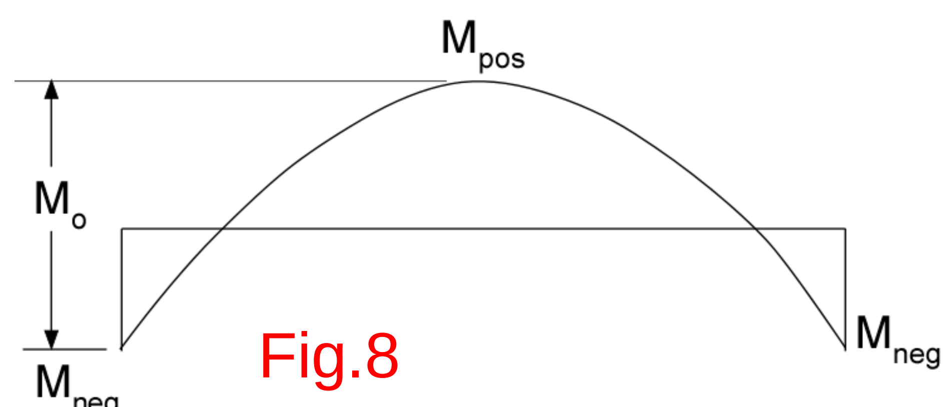


Fig.8

(3) Total Factored Static Moment

For a typical exterior panel, the negative moment at the interior support would be larger than that at the exterior support. The maximum positive moment would occur at a section to the left of the mid-span, as shown in Fi

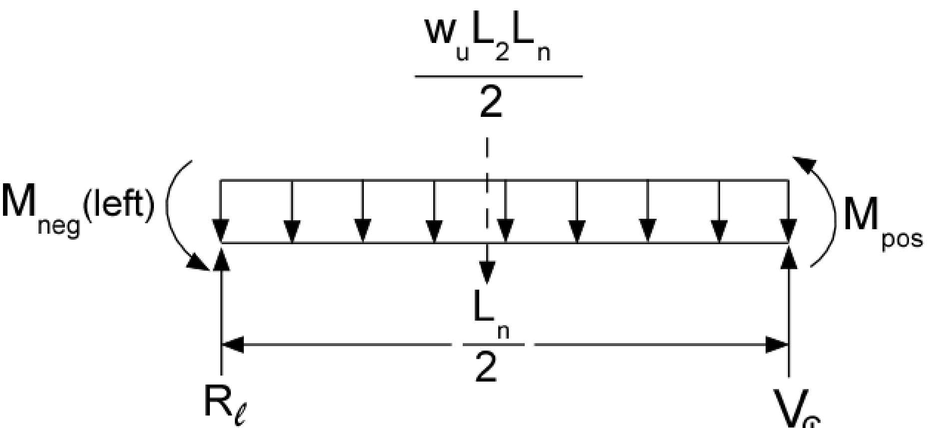


Fig.9

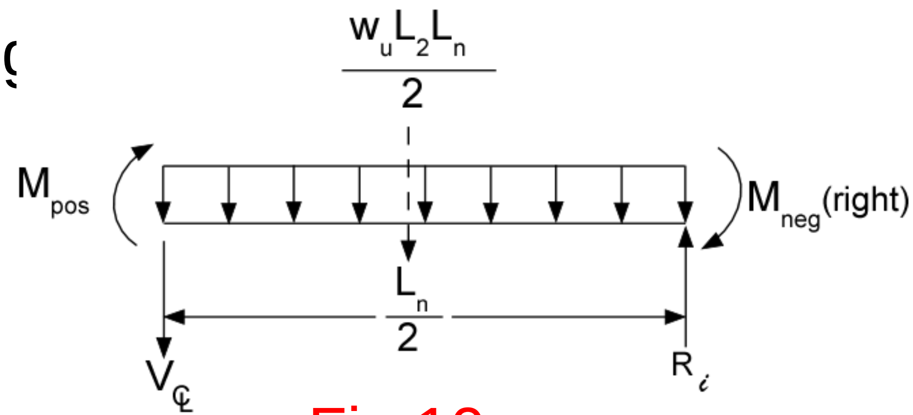


Fig.10

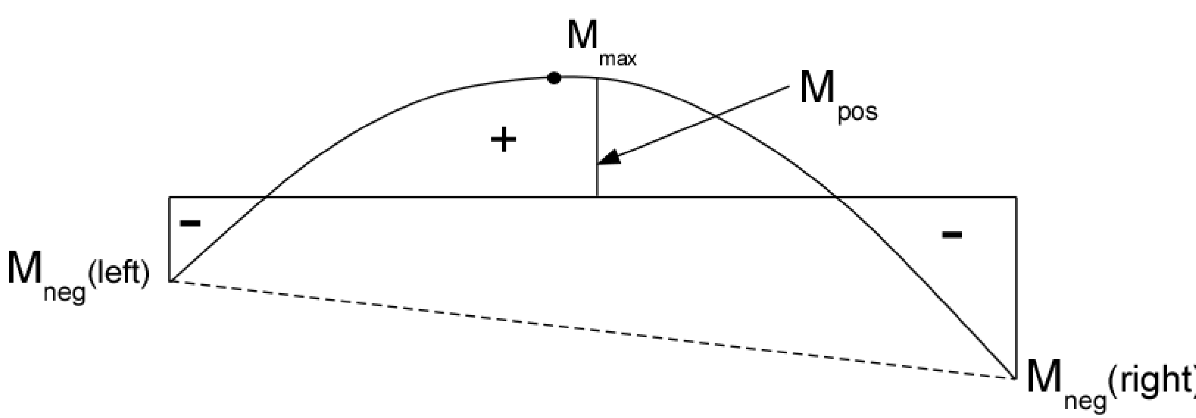


Fig.11

(3) Total Factored Static Moment

In practical design, it is customary to use M_{pos} at midspan for determining the required positive moment reinforcement.

For this case,

$$\frac{M_{neg}(left) + M_{neg}(right)}{2} + M_{pos} = \frac{w_u L_2 L_n^2}{8} \qquad (2)$$

A proof for Eq.(2) can be obtained by writing the moment equilibrium equation about the left end of the free body shown in Fig.9

$$M_{neg}(left) + M_{pos} = \frac{W_u L_2 L_n}{2}\left(\frac{L_n}{4}\right) - V_{midspan}\left(\frac{L_n}{2}\right) \qquad (3)$$

(3) Total Factored Static Moment

and, by writing the moment equilibrium equation about the right end of the free body shown in Fig.10,

$$M_{neg}(right) + M_{pos} = \frac{W_u L_2 L_n}{2}\left(\frac{L_n}{4}\right) + V_{midspan}\left(\frac{L_n}{2}\right) \qquad (4)$$

Equation(2) is arrived at by adding equations (3) and (4) and dividing by 2 on each side. Eq.(2) may also be obtained, as shown in Fig.11, by the superposition of the simple span uniform loading parabolic positive moment diagram over the trapezoidal negative moment diagram due to end moments.

(3) Total Factored Static Moment

ACI- uses the symbol M_0 to mean $\frac{w_u L_2 L_n^2}{8}$ and calls M_0 the total factored static moment. It states, Absolute sum of positive and average negative factored moments in each direction shall not be less than M_0"; or

$$\frac{M_{neg}(\text{left}) + M_{pos}(\text{right})}{2} + M_{pos} \geq \left[M_0 = \frac{w_u L_2 L_n^2}{8} \right] \quad (5)$$

in which

w_u = factored load per unit area

L_n = clear span in the direction moments are being determined, measured face to face' of supports (ACI), but not less than $0.65L_1$

L_1 = span length in the direction moments are being determined, measured center to center of supports

L_2 = transverse span length, measured center to center of supports

(3) Total Factored Static Moment

Equations (1) and (2) are theoretically derived on the basis that M_{neg}(left), M_{pos}, and M_{neg}(right) occur simultaneously for the same live load pattern on the adjacent panels of the equivalent rigid frame defined in Fig.3.

If the live load is relatively heavy compared with dead load, then different live load patterns should be used to obtain the critical positive moment at midspan and the critical negative moments at the supports. In such a case, the "equal" sign in Eqs.(1) and (2) becomes the "greater" sign. This is the reason why ACI states "absolute sum . . . shall not be less than M_o" as the design requirement.

(3) Total Factored Static Moment

To avoid the use of excessively small values of M_o in the case of short spans and large columns or column capitals, the clear span L_n to be used in Eq.(5) is not to be less than $0.65L_1$ (ACI).

When the limitations for using the direct design method are met, it is customary to divide the value of M_o into M_{neg} into M_{pos}, if the restraints at each end of the span are identical (Fig.4 to 8); or into $[M_{neg}(left) + M_{neg}(right)]/2$ and M_{pos} if the span end restraints are different (Fig.9 to 11). Then the moments M_{neg}(left), M_{neg}(right), and M_{pos} must be distributed transversely along the lines 1‘-3', 2‘-4', and 5-6, respectively. This last distribution is a function of the relative flexural stiffness between the slab and

Total Factored Static Moment in Flat Slabs.

Consider the typical interior panel of a flat slab floor subjected to a factored load of w_u per unit area, as shown in Fig.12. The total load on the panel area (rectangle minus four quadrantal areas) is supported by the vertical shears at the four quadrantal arcs. Let M_{neg} and M_{pos} be the total negative and positive moments about a horizontal axis in the L_2 direction along the edges of ABCD and EF, respectively. Then

Load on area ABCDEF= sum of reactions at arcs AB and CD

$$= w_u \left(\frac{L_1 L_2}{2} - \frac{\pi c^2}{8} \right) \qquad (6)$$

Total Factored Static Moment in Flat Slabs.

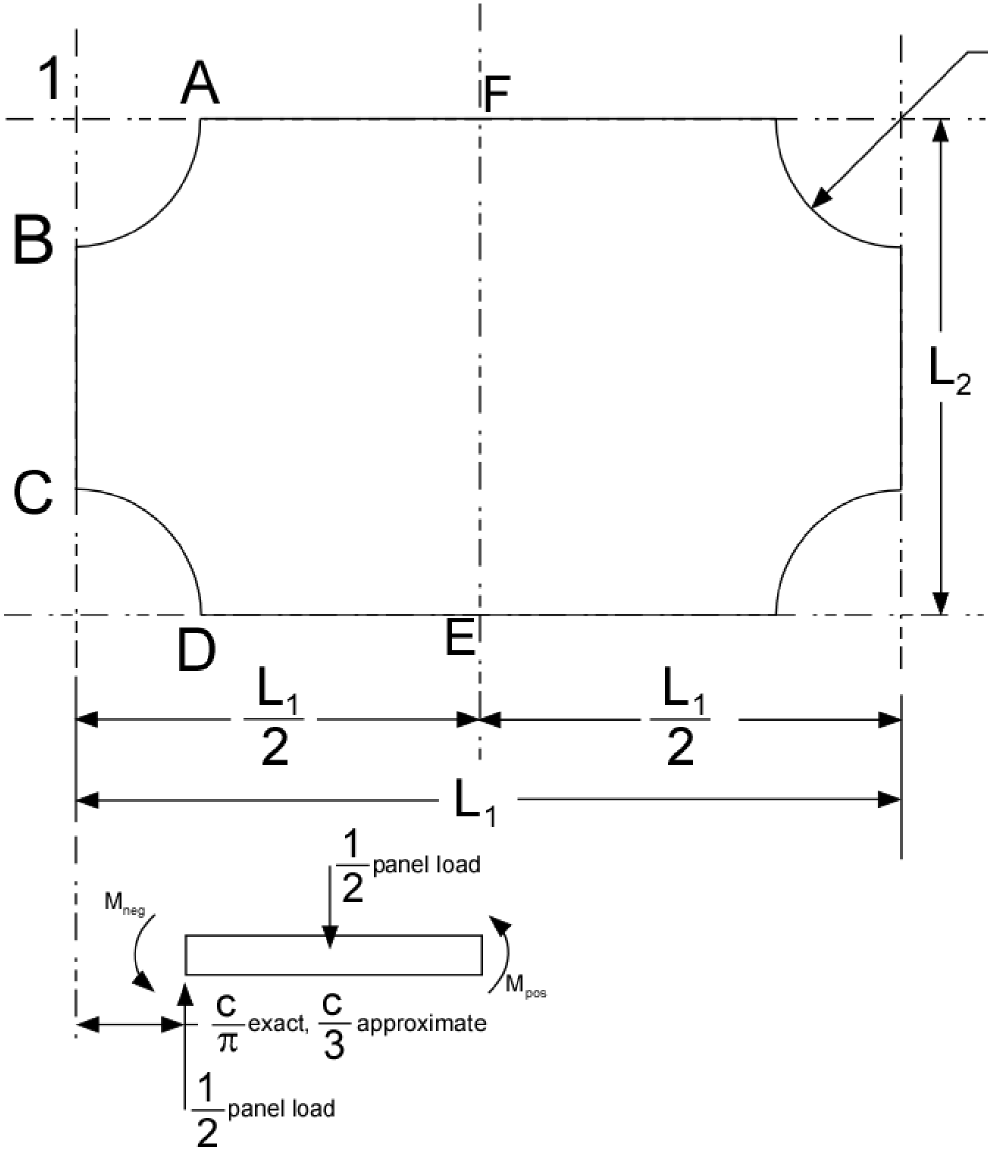


Fig.12

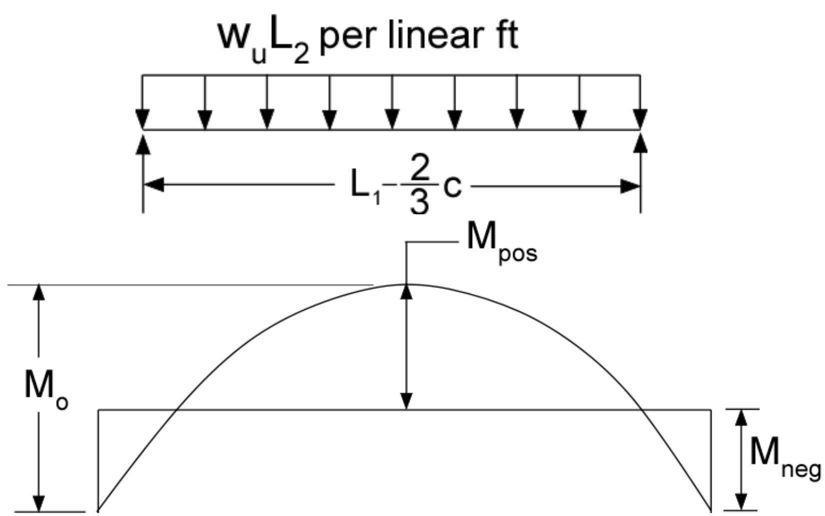


Fig.13

Total Factored Static Moment in Flat Slabs.

Considering the half-panel ABCDEF as a free body, recognizing that there is no shear at the edges BC, DE, EF, and FA, and taking moments about axis 1-1,

$$M_{neg} + M_{pos} + w_u\left(\frac{L_1L_2}{2} - \frac{\pi c^2}{8}\right)\left(\frac{c}{\pi}\right) - \frac{w_uL_1L_2}{2}\left(\frac{L_1}{4}\right) + \frac{w_u\pi c^2}{8}\left(\frac{2c}{3\pi}\right) = 0 \qquad (7)$$

Letting $M_o = M_{neg} + M_{pos}$

$$M_o = \frac{1}{8}w_uL_2L_1^2\left(1 - \frac{4c}{\pi L_1} + \frac{c^3}{3L_2L_1^2}\right) \approx \frac{1}{8}w_uL_2L_1^2\left(1 - \frac{2c}{3L_1}\right)^2 \qquad (8)$$

Actually, Eq.(8) may be more easily visualized by inspecting the equivalent interior span as shown in Fig.13.

Total Factored Static Moment in Flat Slabs.

ACI states that circular or regular polygon shaped supports shall be treated as square supports having the same area. For flat slabs, particularly with column capitals, the clear span L_n computed from using equivalent square supports should be compared with that indicated by Eq.(8), which is L_1 minus 2c/3. In some cases, the latter value is larger and should be used, consistent with the fact that ACI does express its intent in an inequality.

(4) Ratio of Flexural Stiffnesses of Longitudinal Beam to Slab

When beams are used along the column lines in a two-way floor system, an important parameter affecting the design is the relative size of the beam to the thickness of the slab. This parameter can best be measured by the ratio α of the flexural rigidity (called flexural stiffness by the AC1 Code) $E_{cb}I_b$ of the beam to the flexural rigidity $E_{CS}I_S$, of the slab in the transverse cross-section of the equivalent frame shown in Fig.14,15&16. The separate moduli of elasticity E_{cb} and E_{cs}, referring to the beam and slab, provide for different strength concrete (and thus different E_c values) for the beam and slab.

(4) Ratio of Flexural Stiffnesses of Longitudinal Beam to Slab

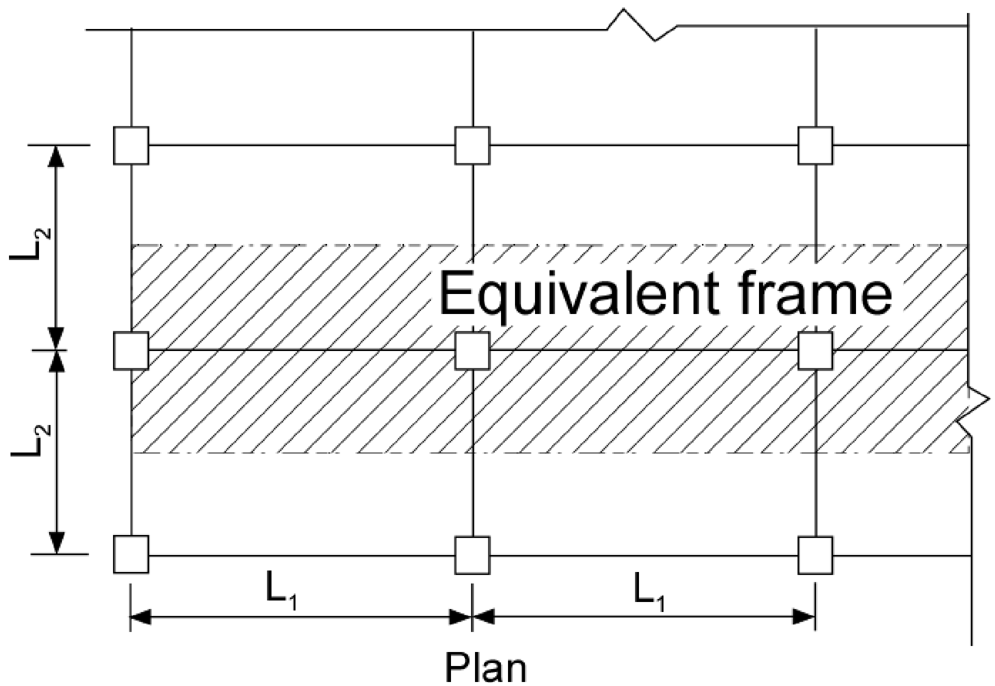


Fig.14

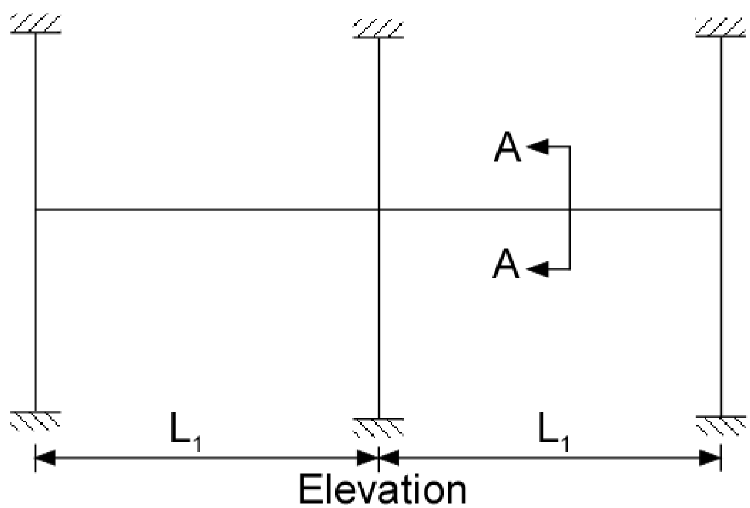


Fig.15

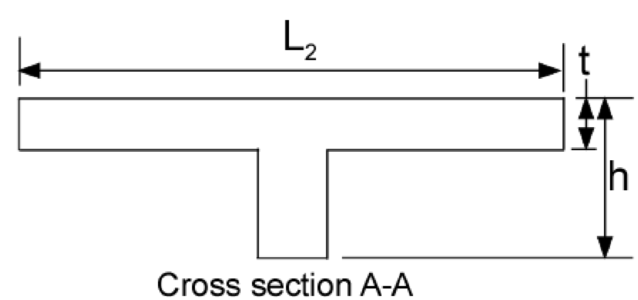


Fig.16

(4) Ratio of Flexural Stiffnesses of Longitudinal Beam to Slab

The moments of inertia I_b and I_s refer to the gross sections of the beam and slab within the cross-section of Fig.16. ACI permits the slab on each side of the beam web to act as a part of the beam, this slab portion being limited to a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness, as shown

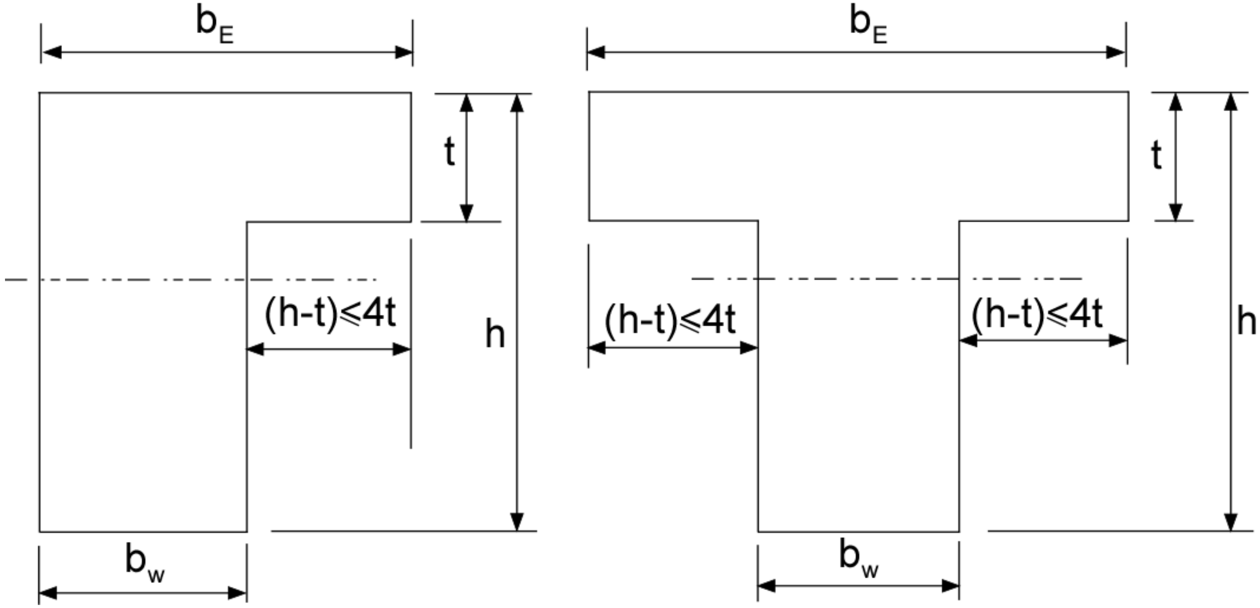


Fig.17

(4) Ratio of Flexural Stiffnesses of Longitudinal Beam to Slab

More accurately, the small portion of the slab already counted in the beam should not be used in I_s, but ACI permits the use of the total width of the equivalent frame in computing I_s. Thus,

$$\alpha = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad (9)$$

The moment of inertia of a flanged beam section about its own centroidal axis (Fig.17) may be shown to be

$$I_b = k \frac{b_w h^3}{12} \qquad (10)$$

(4) Ratio of Flexural Stiffnesses of Longitudinal Beam to Slab

In which

$$k = \frac{1+\left(\frac{b_E}{b_w}-1\right)\left(\frac{t}{h}\right)\left[4-6\left(\frac{t}{h}\right)+4\left(\frac{t}{h}\right)^2+\left(\frac{b_E}{b_w}-1\right)\left(\frac{t}{h}\right)^3\right]}{1+\left(\frac{b_E}{b_w}-1\right)\left(\frac{t}{h}\right)} \qquad (11)$$

where

h = overall beam depth

t =overall slab thickness

b_E =effective width of flange

b_w = width of web

(5) Minimum slab thickness for deflection control

To aid the designer, ACI provides a minimum thickness table for slabs without interior beams, though there can be exterior boundary beams. For slabs with beams spanning between the supports on all sides, ACI provides minimum thickness equations. If the designer wishes to use lesser thickness, ACI permits "if shown by computation that the deflection will not exceed the limits stipulated in Table." Computation of deflections must "take into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges”.

(5) Minimum slab thickness for deflection control

Slabs without interior beams spanning between supports.

The minimum thickness, with the requirement that the ratio of long to short span be not greater than 2, shall be that given by Table-1, but not less than:

For slabs without drop panels	5 in.
For slabs with drop panels	4 in.

In the flat slab and flat plate two-way systems, there may or may not be edge beams but there are definitely no interior beams in such systems.

Table-1 Minimum thickness of slab without interior beams

f^*_y (ksi)	WITHOUT DROP PANELS			WITH DROP PANELS		
	EXTERIOR PANELS		INTERIOR PANELS	EXTERIOR PANELS		INTERIOR PANELS
	$\alpha = 0$	$\alpha \geq 0.8$		$\alpha = 0$	$\alpha \geq 0.8$	
40	$\frac{L_n}{33}$	$\frac{L_n}{36}$	$\frac{L_n}{36}$	$\frac{L_n}{36}$	$\frac{L_n}{40}$	$\frac{L_n}{40}$
60	$\frac{L_n}{30}$	$\frac{L_n}{33}$	$\frac{L_n}{33}$	$\frac{L_n}{33}$	$\frac{L_n}{36}$	$\frac{L_n}{36}$
75	$\frac{L_n}{28}$	$\frac{L_n}{31}$	$\frac{L_n}{31}$	$\frac{L_n}{31}$	$\frac{L_n}{34}$	$\frac{L_n}{34}$

*For f_y between 40 and 60 ksi, min. t is to be obtained by linear interpolation.

(5) Minimum slab thickness for deflection control

Slabs Supported on Beams.

Four parameters affect the equations of ACI for slabs supported on beams on all sides; they are (1) the longer clear span L_n of the slab panel; (2) the ratio β of the longer clear span L_n to the shorter clear span S_n; (3) the yield strength f_y of the steel reinforcement; and (4) the average α_m for the four α values for relative stiffness of a panel perimeter beam compared to the slab

In terms of these parameters, ACI requires the following for "slabs with beams spanning between the supports on all sides.”

(5) Minimum slab thickness for deflection control

Slabs Supported on Beams.

Slabs supported on shallow beams where $\alpha_m \leq 0.2$.

The minimum slab thickness requirements are the same as for slabs without interior beams.

Slabs supported on medium stiff beams where $0.2 < \alpha_m \leq 2.0$.

For this case, $$\text{Min } t = \frac{L_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 5\beta(\alpha_m - 0.2)} \qquad (12)$$

The minimum is not be less than 5 in.

Slabs supported on very stiff beams where $\alpha_m > 2.0$.

For this case, $$\text{Min } t = \frac{L_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 9\beta} \qquad (13)$$

The minimum is not to be less than 3.5 in.

(5) Minimum slab thickness for deflection control

Slabs Supported on Beams.

Edge beams at discontinuous edges.

For all slabs supported on beams, there must be an edge beam at discontinuous edges having a stiffness ratio α not less than 0.80, or the minimum thickness required by Eqs.(12) or (13) "shall be increased by at least 10 percent in the panel with the discontinuous edge."

(6) Nominal requirements for slab thickness and size of edge beams, column capital, and drop panel

Whether the ACI "direct design method" or the "equivalent frame method" is used for determining the longitudinal distribution of bending moments, certain nominal requirements for slab thickness and size of edge beams, column capital, and drop panel must be fulfilled. These requirements are termed "nominal" because they are code-prescribed. It should be realized, of course, that the code provisions are based on a combination of experience, judgment, tests, and theoretical analyses.

(6) Nominal requirements for slab thickness and size of edge beams, column capital, and drop panel

Slab Thickness. As discussed in Section-5, ACI Formulas [Eqs.12&13], along with ACI-Table [Table-1] set minimum slab thickness for two-way floor systems. In addition, ACI set lower limits for the minimum value based on experience and practical requirements. These lower limits for two-way slab systems are summarized:

Flat plates and flat slabs without drop panels	5 in.
Slabs on shallow interior beams having $\alpha_m < 0.2$	5 in.
Slabs without interior beams but having drop panels	4 in.
Slabs with stiff interior beams having $\alpha_m \geq 2.0$	3.5 in.

(6) Nominal requirements for slab thickness and size of edge beams, column capital, and drop panel

Edge Beams. For slabs supported by interior beams, the minimum thickness requirements assume an edge beam having a stiffness ratio α not less than 0.80. If such an edge beam is not provided, the minimum thickness as required ACI Formulas [Eqs.12&13] must be increased by 10% in the panel having the discontinuous edge. For slabs not having interior support beams, the increased minimum thickness in the exterior panel having the discontinuous edge is given by ACI-Table [Table-1].

(6) Nominal requirements for slab thickness and size of edge beams, column capital, and drop panel

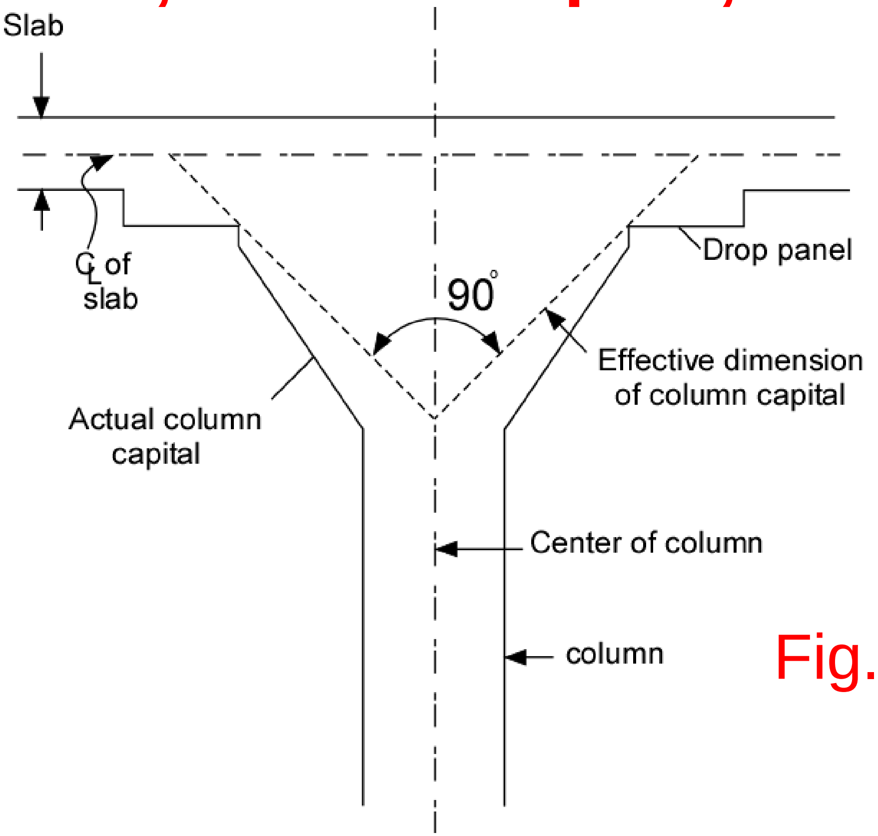


Fig.18

Column Capital. Used in flat slab construction, the column capital (Fig.18) is an enlargement of the top of the column as it meets the floor slab or drop panel.

(6) Nominal requirements for slab thickness and size of edge beams, column capital, and drop panel

Since no beams are used, the purpose of the capital is to gain increased perimeter around the column to transmit shear from the floor loading and to provide increasing thickness as the perimeter decreases near the column. Assuming a maximum 45° line for distribution of the shear into the column, ACI requires that the effective column capital for strength considerations be within the largest circular cone, right pyramid, or tapered wedge with a 90° vertex that can be included within the outlines of the actual supporting element (see fig. 18). The diameter of the column capital is usually about 20 to 25% of the average span length between column.

(6) Nominal requirements for slab thickness and size of edge beams, column capital, and drop panel

Drop Panel. The drop panel (Fig.2b) is often used in flat slab and flat plate construction as a means of increasing the shear strength around a column or reducing the negative moment reinforcement over a column. It is an increased slab thickness in the region surrounding a column. A drop panel must comply with the dimensional limitations of ACI. The panel must extend from the centerline of supports a minimum distance of one-sixth of the span length measured from center-to-center in each direction; the projection of the panel below the slab must be at least one-fourth of the slab thickness outside of the drop.

(6) Nominal requirements for slab thickness and size of edge beams, column capital, and drop panel

Drop Panel. When a qualifying drop is used, the minimum thickness given by ACI-Table-1 has been reduced by 10% from the minimum when a drop is not used.

For determining the reinforcement requirement, ACI stipulates that the thickness of the drop below the slab be assumed no larger than one-quarter of the distance between the edge of the drop panel and the edge of the column or column capital. Because or this limitation, there is little reason to use a drop panel of greater plan dimensions or thickness than enough to satisfy using the reduced thickness for the slab outside the drop panel.

(7) Limitations of Direct Design Method

Over the years the use of two-way floor systems has been extended from one-story or low-rise to medium or high-rise buildings. For the common cases of one-story or low-rise buildings, lateral load (wind or earthquake) is of lesser concern; thus most of the ACI Code refers only to gravity load (dead and live uniform load). In particular, when the dimensions of the floor system are quite regular and when the live load is not excessively large compared to the dead load, the use of a set of prescribed coefficients to distribute longitudinally the total factored static moment M_o seems reasonable.

(7) Limitations of Direct Design Method

As shown in Figs.4 to 11, for each clear span in the equivalent rigid frame, the equation

$$\frac{M_{neg}(\text{left}) + M_{neg}(\text{right})}{2} + M_{pos} \geq \left[M_0 = \frac{W_u L_2 L_n^2}{8} \right] \qquad (14)$$

is to be satisfied.

In order that the designer may use the direct design method, in which a set of prescribed coefficients give the negative end moments and the positive moment within the span of the equivalent rigid frame, ACI imposes the following limitations:

(7) Limitations of Direct Design Method

(1) There is a minimum of three continuous spans in each direction.

(2) Panels must be rectangular with the ratio of longer to shorter span center-to-center of supports within a panel not greater than 2.0.

(3) The successive span lengths center-to-center of supports in each direction do not differ by more than one-third of the longer span.

(4) Columns are not offset more than 10% of the span in the direction of the offset.

(7) Limitations of Direct Design Method

(5) The load is due to gravity only and is uniformly distributed over an entire panel, and the service live load does not exceed two times the service dead load.

(6) The relative stiffness ratio of L^2_1/α_1 to L^2_2/α_2 must lie between 0.2 and 5.0, where α is the ratio of the flexural stiffness of the included beam to that of the slab.

(8) Longitudinal distribution of moments

Fig.19 shows the longitudinal moment diagram for the typical interior span of the equivalent rigid frame in a two-way floor system, as prescribed by ACI.

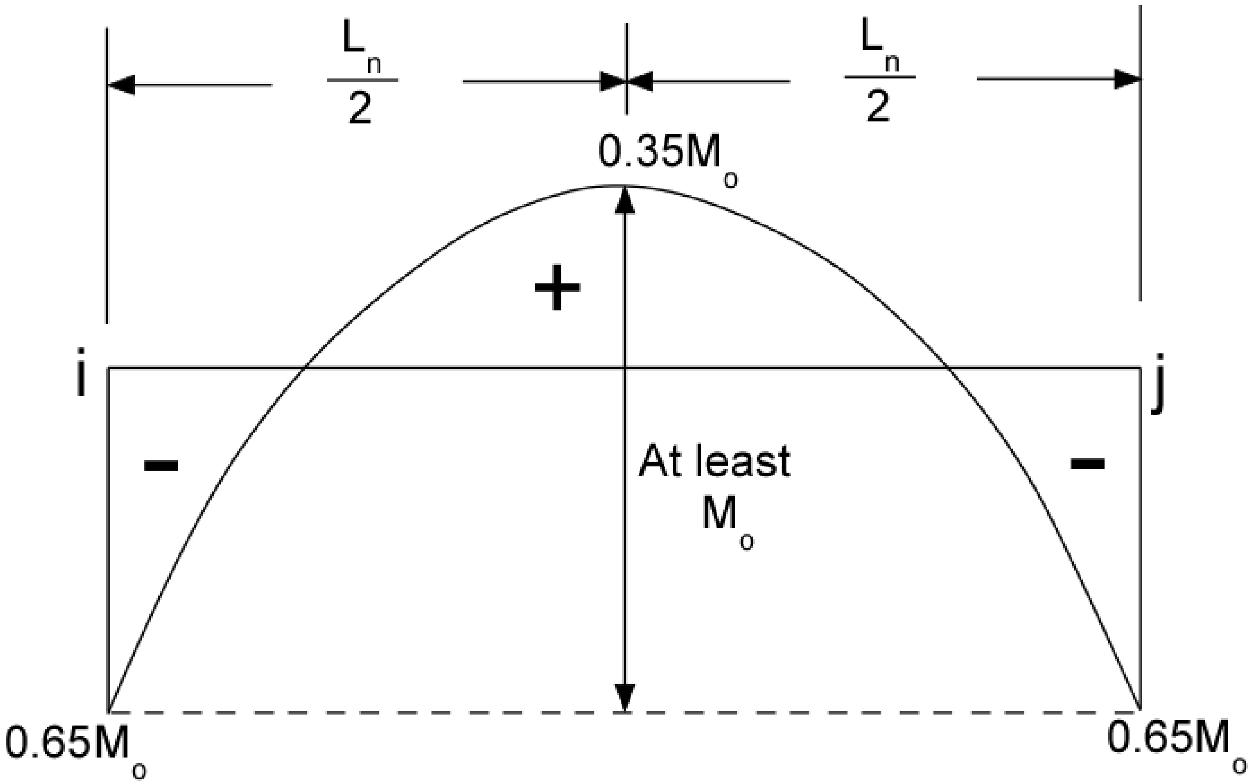


Fig.19

where $$M_o = \frac{1}{8} w_u L_2 L_n^2 \qquad (15)$$

(8) Longitudinal distribution of moments

For a span that is completely fixed at both ends, the negative moment at the fixed end is twice as large as the positive moment at midspan.

For a typical interior span satisfying the limitations for the direct design method, the specified negative moment of $0.65M_o$ is a little less than twice the specified positive moment of $0.35M_o$, which is fairly reasonable because the restraining effect of the columns and adjacent panels is definitely less than that of a completely fixed-ended beam.

(8) Longitudinal distribution of moments

For the exterior span, ACI provides the longitudinal moment diagram for each of the five categories as described in Fig.20 to 24.

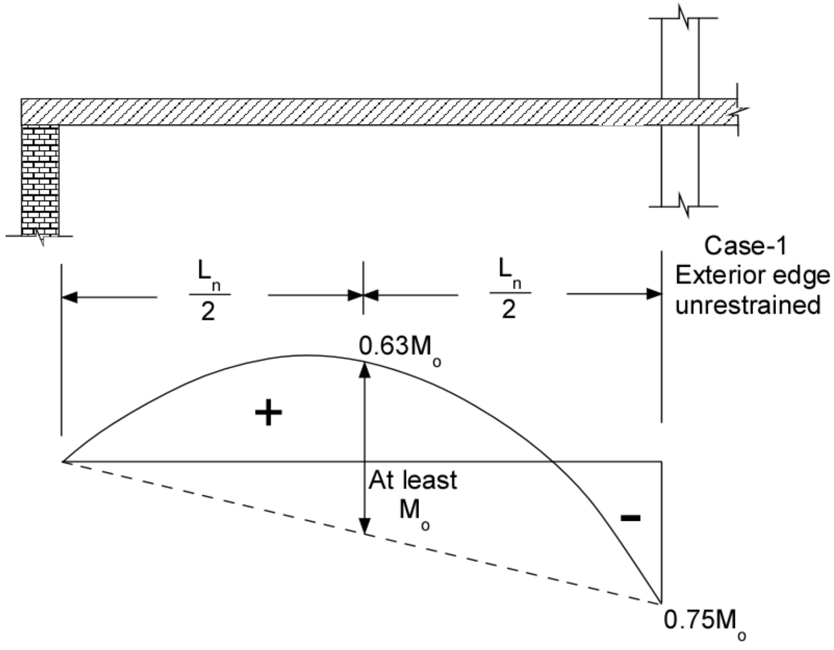


Fig.20

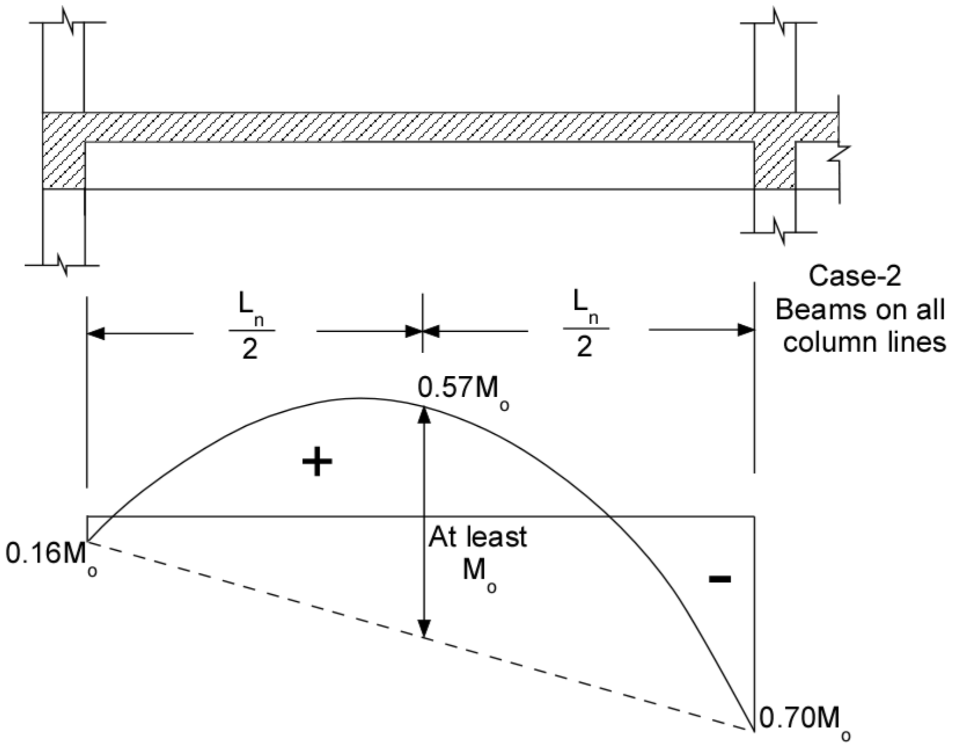


Fig.21

(8) Longitudinal distribution of moments

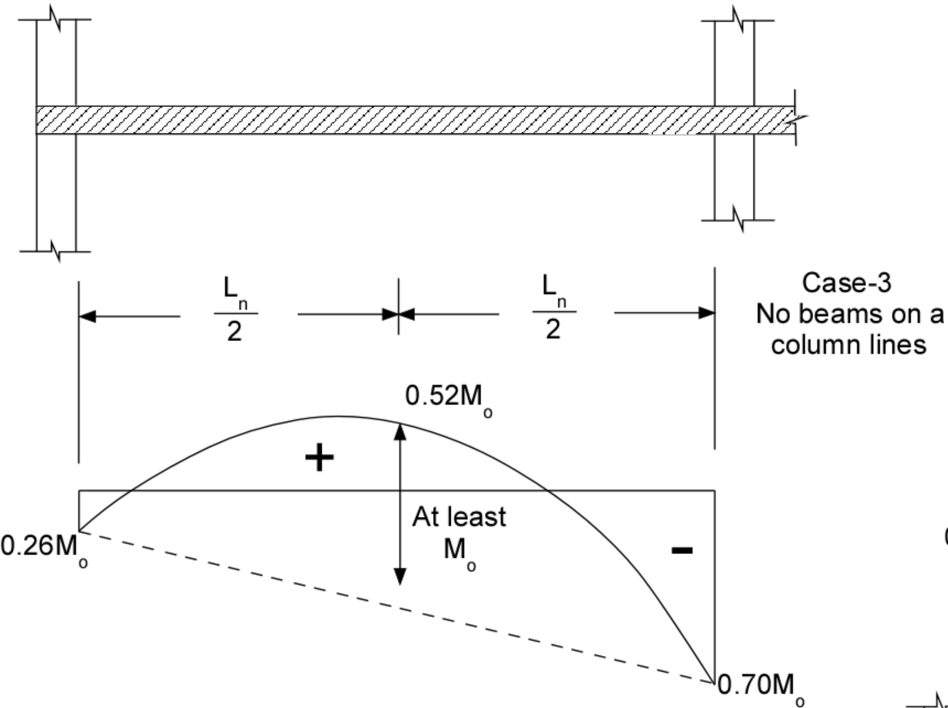


Fig.22

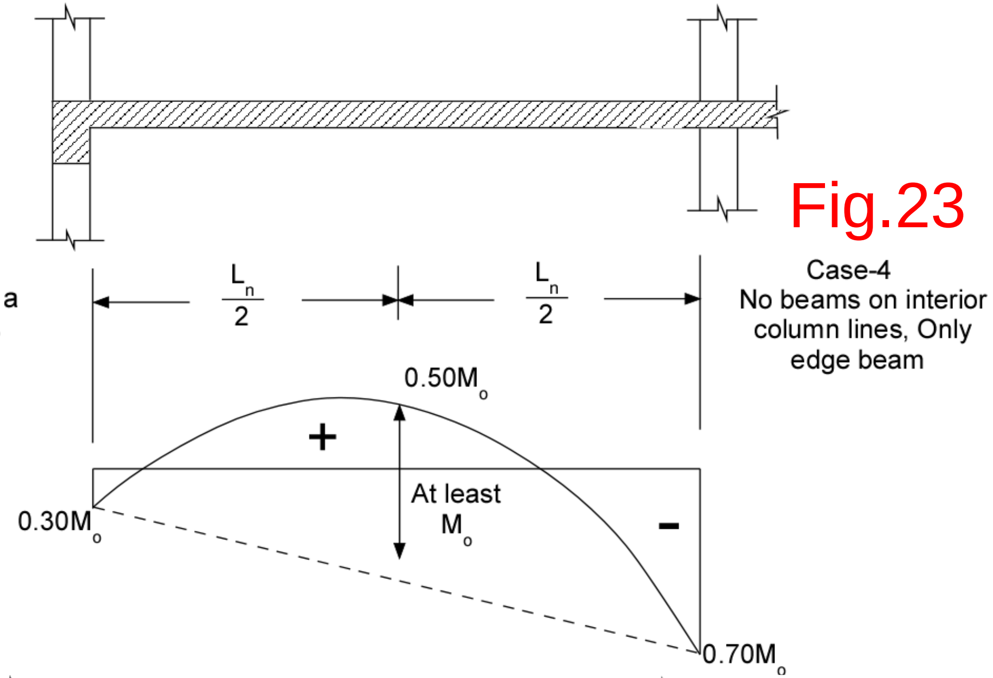


Fig.23

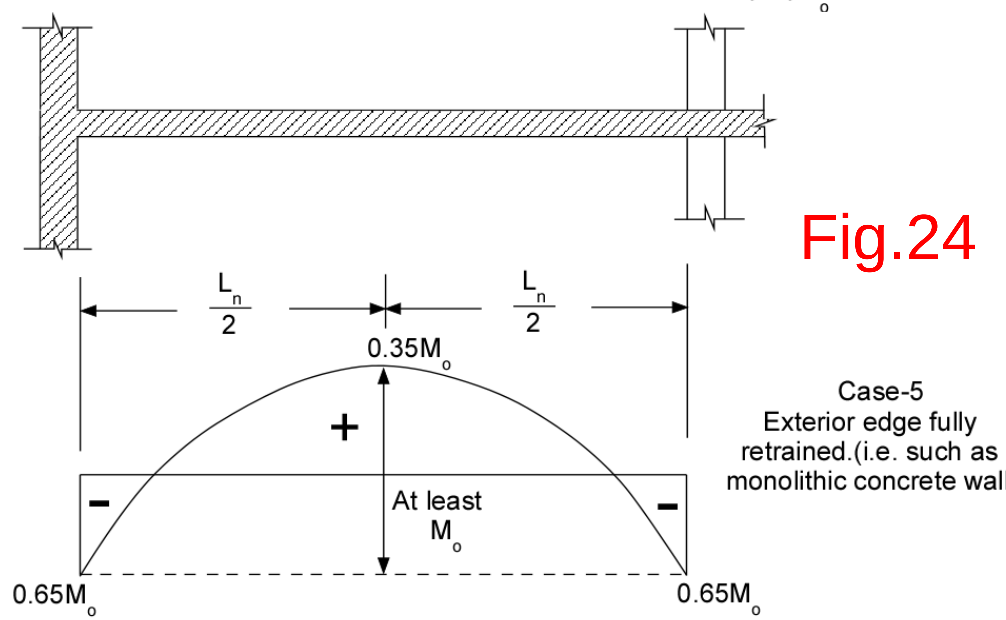


Fig.24

(8) Longitudinal distribution of moments

The negative moment at the exterior support increases from 0 to $0.65M_o$, the positive moment within the span decreases from $0.63M_o$ to $0.35M_o$, and the negative moment at the interior support decreases from $0.75M_o$ to $0.65M_o$ all gradually as the restraint at the exterior support increases from the case of a masonry wall support to that of a reinforced concrete wall built monolithically with the slab. ACI Commentary states that high positive moments are purposely assigned into the span since design for exterior negative moment will be governed by minimum reinforcement to control cracking.

(8) Longitudinal distribution of moments

Regarding the ACI Code suggested moment diagrams of Figs.19 to 24, ACI permits these moments to be modified by 10% provided the total factored static moment M_o for the panel is statically accommodated.

(9) Effect of pattern loadings on positive moment

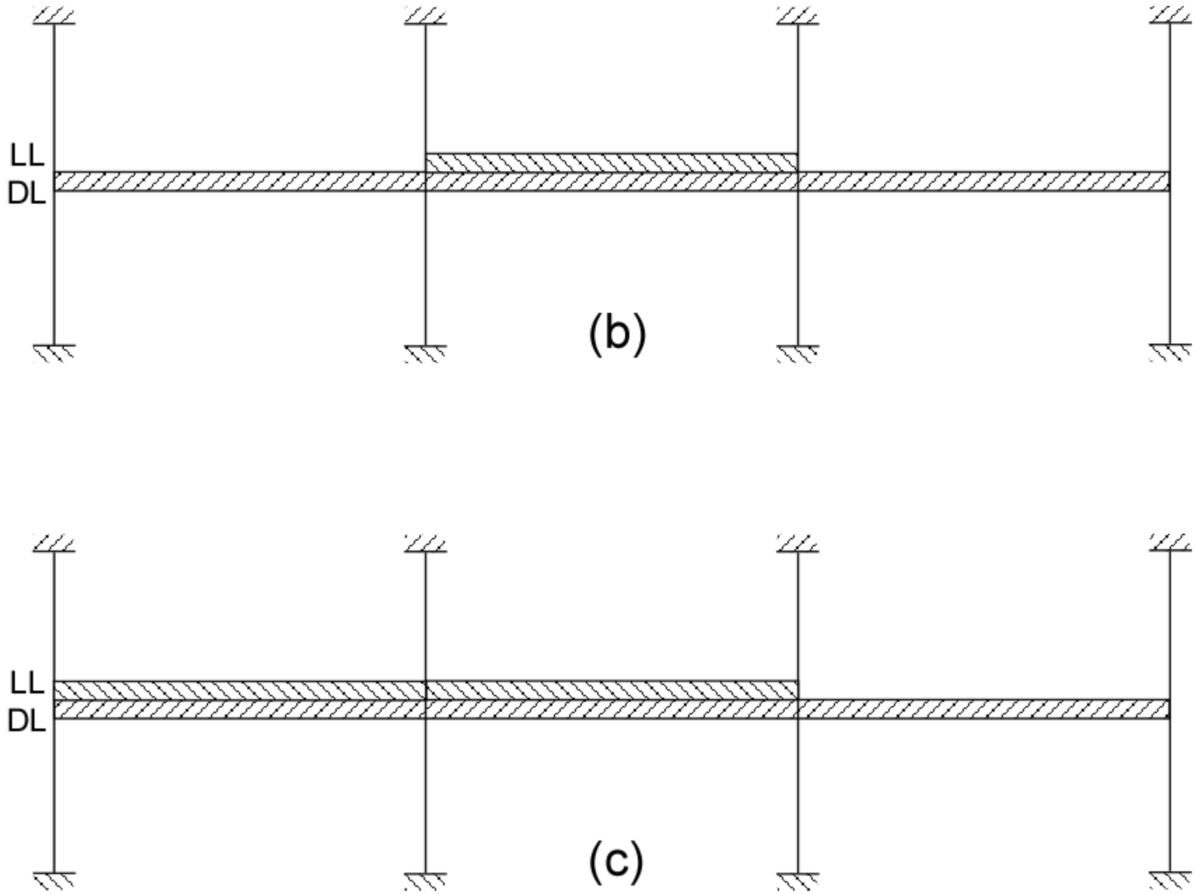


Fig.25

(9) Effect of pattern loadings on positive moment

Some of the findings, which might be visualized by continuous frame analysis in influence lines and maximum moment envelopes due to dead and live load combinations, are as follows:

(1) the higher the ratio of column stiffness to beam stiffness, the smaller the effect of pattern loadings, because the ends of the span are closer to the fixed condition and less effect is exerted on the span by loading patterns on adjacent spans

(2) the lower the ratio of dead load to live load, the larger the effect of pattern loadings, because dead load exists constantly on all spans and the pattern is related to live load only

(3) maximum negative moments at supports are less affected by pattern loadings than maximum positive moments within the span.

(9) Effect of pattern loadings on positive moment

It was recommended that in order to limit the increase in positive moment within the two-way floor system caused by pattern loading to a maximum of about 30% such that the moment values in Fig.20 to 24 could be used, the ratio of column stiffness to slab stiffness must exceed specified minimum values. When the actual ratio is lower than the minimum, the positive moment must be increased in accordance with a formula involving the ratio of service dead load to service live load, the ratio α_c of column stiffness to the sum of slab and beam stiffness, and a minimum ratio α_{min} given in tabular form.

(9) Effect of pattern loadings on positive moment

The 1995 ACI Code restricts the uses of the direct design method to cases where the service live load does not exceed two times the service dead load. With this lower maximum ratio for live load to dead load, the ACI Code committee concluded the number of cases where pattern loading would have a significant effect would be small; thus, adjustment for pattern loading does not appear in the 1995 ACI Code.

(10) Procedure for computation of longitudinal moments

The procedure for computing the longitudinal moments by the "direct design method" may be summarized:

(1) Check limitations 1 through 5 for the "direct design method listed in Section-7. If they comply, and the slab is supported on beams, follow Steps 2 through 6 given below. For slabs not supported on beams, proceed to Step 6.

(2) Compute the slab moment of inertia I_s

$$I_s = \sum L_2 \left(\frac{t_3}{12} \right) \qquad (15)$$

(10) Procedure for computation of longitudinal moments

(3) Compute the longitudinal beam (if any) moment of inertia I_b

(4) Compute the ratio α of the flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by centerlines of adjacent panels (if any) on each side of the beam

$$\alpha = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad (16)$$

(5) Check that the ratio $\frac{L_1^2}{\alpha_1}$ to $\frac{L_2^2}{\alpha_2}$ lies between 0.2 and 5.0 for the cases where the slab is supported by beams.

(10) Procedure for computation of longitudinal moments

(6) Compute the total static moment $M_o = \dfrac{w_u L_2 L_n^2}{8}$ as stated by Eq.2. L_n is not to be taken less than $0.65L_1$. For flat slabs, Eq.8 should preferably by used for computing M_o.

(7) Obtain the three critical ordinates on the longitudinal moment diagrams for the exterior and interior spans using Figs.19 to 24.

(11) Torsion constant ‘C’ of the transverse beam

One important parameter useful for the transverse distribution of the longitudinal moment is the torsional constant C of the transverse beam spanning from column to column. Even if there is no such beam (as defined by projection above or below the slab) actually visible, for the present use one still should imagine that there is a beam made of a portion of the slab having a width equal to that of the column, bracket, or capital in the direction of the span for which moments are being determined.

(11) Torsion constant ‘C’ of the transverse beam

When there is actually a transverse beam web above or below the slab, the cross-section of the transverse beam should include the portion of slab within the width of column, bracket, or capital described in Fig.26 plus the projection of beam web above or below the slab. As a third possibility, the transverse beam may include that portion of slab on each side of the beam web equal to its projection above or below the slab, whichever is greater, but not greater than four times the slab thickness. The largest of the three definitions as shown in Fig.26 may be used.

(11) Torsion constant 'C' of the transverse beam

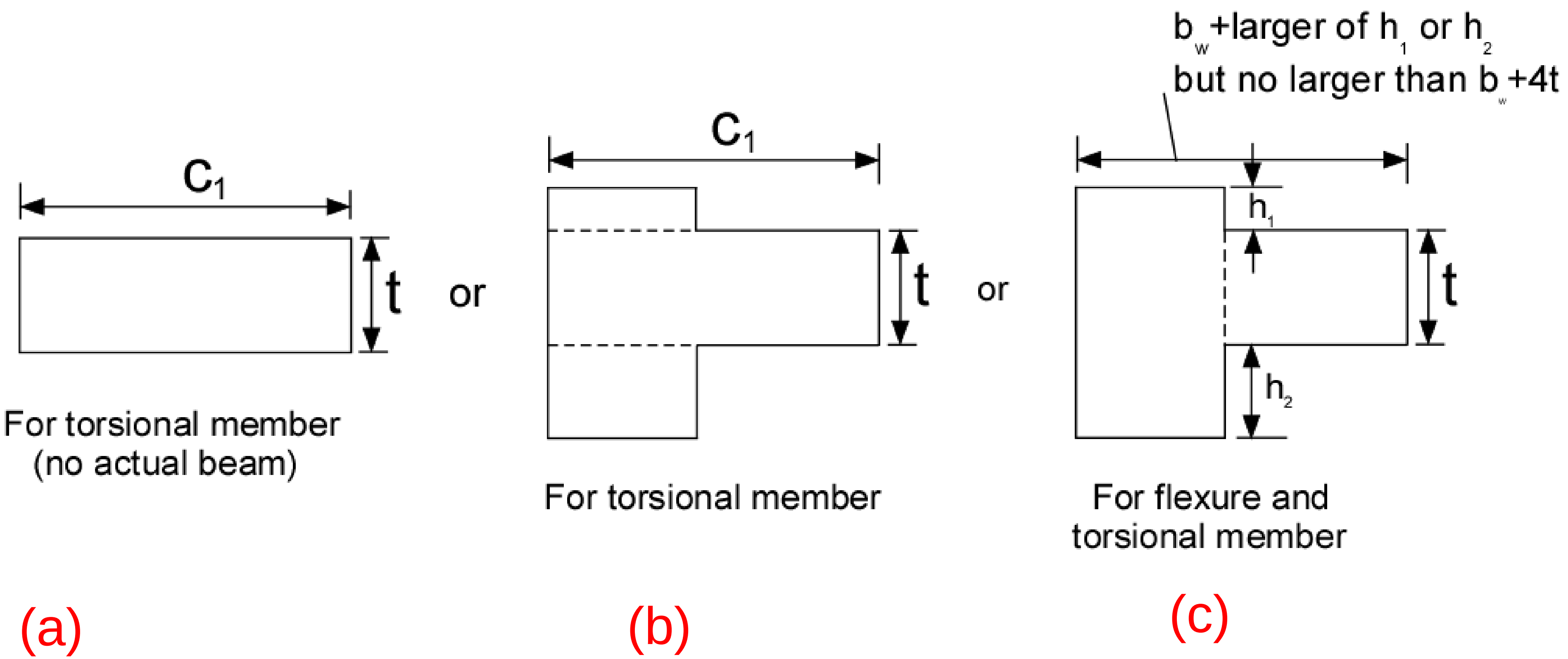


Fig.26: Definition of cross-sections for transverse beams in torsion[Projection of slab beyond beam in case (c) is allowed on each side for interior beam

(11) Torsion constant ‘C’ of the transverse beam

The torsional constant C of the transverse beam equals,

$$C = \sum \left(1 - 0.63\frac{x}{y}\right)\left(\frac{x^3 y}{3}\right) \qquad (17)$$

where

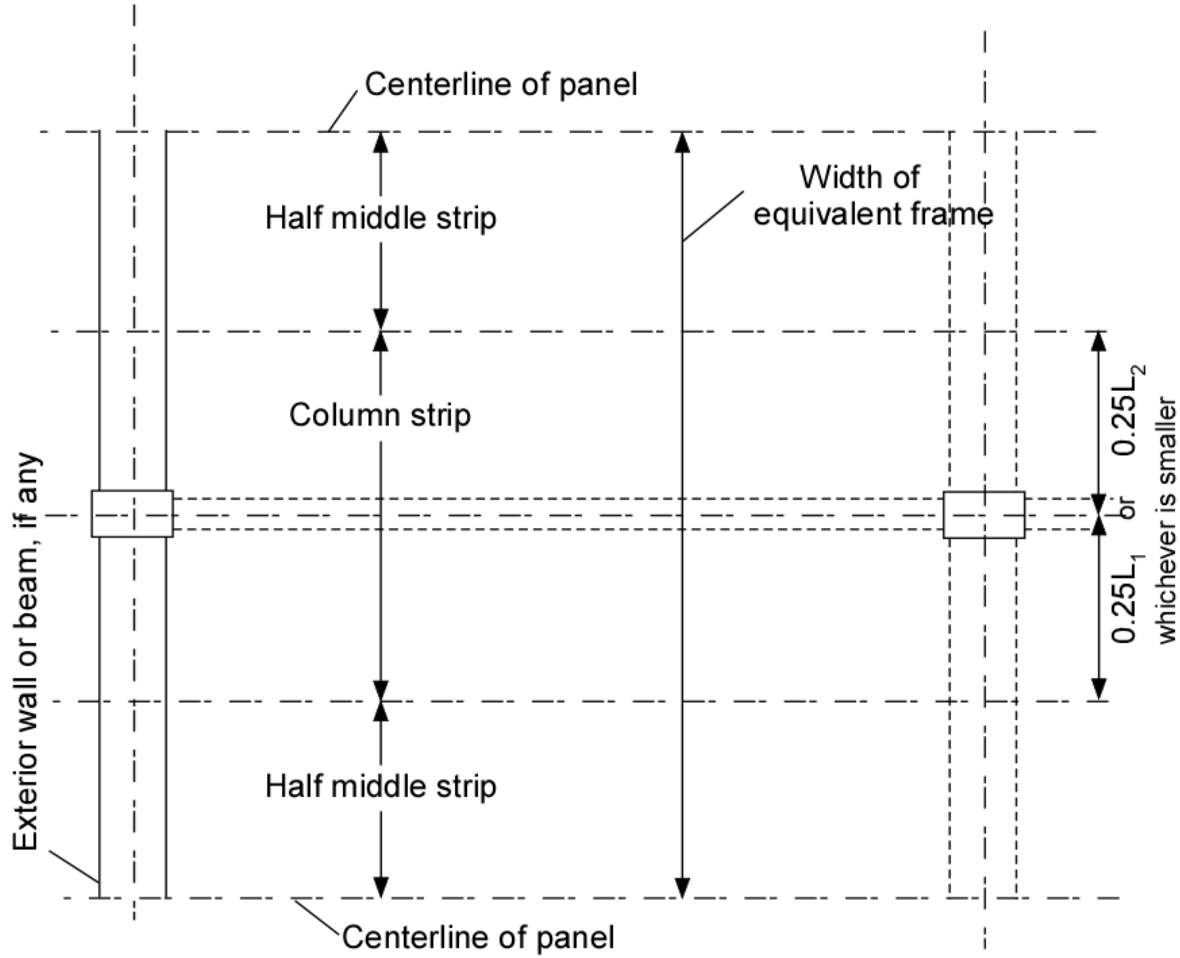
x = shorter dimension of a component rectangle

y = longer dimension of a component rectangle

and the component rectangles should be taken in such a way that the largest value of C is obtained.

(12) Transverse distribution of longitudinal moment

The longitudinal moment values shown in Figs.19 to 24 are for the entire width (sum of the two half panel widths in the transverse direction, for an interior column line) of the equivalent rigid frame.



Definition of column and middle strip

Fig.27

Each of these moments is to be divided between the column strip and the two half middle strips as defined in Fig.27.

(12) Transverse distribution of longitudinal moment

If the two adjacent transverse spans are each equal to L_2, the width of the column strip is then equal to one-half of L_2, or one-half of the longitudinal span L_1, whichever is smaller. This seems reasonable, since when the longitudinal span is shorter than the transverse span, a larger portion of the moment across the width of the equivalent frame might be expected to concentrate near the column centerline.

(12) Transverse distribution of longitudinal moment

The transverse distribution of the longitudinal moment to column and middle strips is a function of three parameters, using L_1 and L_2 for the longitudinal and transverse spans, respectively:

(1) the aspect ratio L_2/L_1

(2) the ratio $\alpha_1 = \dfrac{E_{cb}I_b}{E_{cs}I_s}$ of the longitudinal beam stiffness to slab stiffness;

(3) the ratio $\beta_t = \dfrac{E_{cb}C}{2E_{cs}I_s}$ of the torsional rigidity of edge beam section to the flexural rigidity of a width of slab equal to the span length of the edge beam.

(12) Transverse distribution of longitudinal moment

According to ACI, the column strip is to take the percentage of the longitudinal moment as shown in Table-2. As may be seen from Table-2, only the first two parameters affect the transverse distribution of the negative moments at the first and typical interior supports as well as the positive moments in exterior and interior spans, but all three parameters are involved in the transverse distribution of the negative moment at the exterior support.

Table-2: Percentage of longitudinal moment in column strip

ASPECT RATIO L_2/L_1				0.5	1.0	2.0
Negative moment at exterior support	$\alpha_1 L_2/L_1$	$= 0$	$\beta_t = 0$	100	100	100
			$\beta_t \geq 2.5$	75	75	75
	$\alpha_1 L_2/L_1$	≥ 1.0	$\beta_t = 0$	100	100	100
			$\beta_t \geq 2.5$	90	75	45
Positive moment	$\alpha_1 L_2/L_1$	$= 0$		60	60	60
	$\alpha_1 L_2/L_1$	≥ 1.0		90	75	45
Negative moment at interior support	$\alpha_1 L_2/L_1$	$= 0$		75	75	75
	$\alpha_1 L_2/L_1$	≥ 1.0		90	75	45

(12) Transverse distribution of longitudinal moment

Regarding the distributing percentages shown in Table-2, the following observations may be made:

(1) In general, the column strip takes more than 50% of the longitudinal moment.

(2) The column strip takes a larger share of the negative longitudinal moment than the positive longitudinal moment.

(3) When no longitudinal beams are present, the column strip takes the same share of the longitudinal moment, irrespective of the aspect ratio. The column strip width is a fraction of L_1 or L_2 ($0.25L_1$ or $0.25L_2$ on each side of column line), whichever is smaller.

(12) Transverse distribution of longitudinal moment

(4) In the presence of longitudinal beams, the larger the aspect ratio, the smaller the distribution to the column strip. This seems consistent because the same reduction in the portion of moment going into the slab is achieved by restricting the column strip width to a fraction of L_1 when L_2/L_1 is greater than one.

(5) The column strip takes a smaller share of the exterior moment as the torsional rigidity of the edge beam section increases.

When the exterior support consists of a column or wall extending for a distance equal to or greater than three-fourths of the transverse width, the exterior negative moment is to be uniformly distributed over the transverse width.

(12) Transverse distribution of longitudinal moment

The procedure for distributing the longitudinal moment across a transverse width to the column and middle strips may be summarized as follows:

(1) Divide the total transverse width applicable to the longitudinal moment into a column strip width and two half middle strip widths, one adjacent to each side of the column strip. For an exterior column line, the column strip width is $\frac{1}{4} L_1$, or $\frac{1}{4} L_2$, whichever is smaller; for an interior column line, the column strip width is $\Sigma(\frac{1}{4} L_1$ or $\frac{1}{4} L_2$, whichever is smaller, of the panels on both sides).

(12) Transverse distribution of longitudinal moment

(2) Determine the ratio $\beta_t = \dfrac{E_{cb}C}{2E_{cs}I_s}$ of edge beam torsional rigidity to slab flexural rigidity. (The 2 arises from approximating the shear modulus of elasticity in the numerator as $\dfrac{E_{cb}}{2}$)

(3) Determine the ratio $\alpha_1 = \dfrac{E_{cb}I_b}{E_{cs}I_s}$ of longitudinal beam flexural stiffness to slab flexural stiffness.

(4) Divide the longitudinal moment at each critical section into two parts according to the percentage shown in Table-2: one part to the column strip width; and the remainder to the half middle strip for an exterior column line, or to the half middle strips on each side of an interior column line.

(12) Transverse distribution of longitudinal moment

(5) If there is an exterior wall instead of an exterior column line, the strip ordinarily called the exterior column strip will not deflect and therefore no moments act. In this case there can be no longitudinal distribution of moments; thus there is no computed moment to distribute laterally to the half middle strip adjacent to the wall. This half middle strip should be combined with the next adjacent half middle strip, which itself receives a lateral distribution in the frame of the first interior column line. The total middle strip in this situation is designed for twice the moment in the half middle strip from the first interior column line.

Distribution of moment in column strip to beam and slab

When a longitudinal beam exists in the column strip along the column centerline, the column strip moment as determined by the percentages in Table-2 should be divided between the beam and the slab. ACI states that 85% of this moment be taken by the beam if $\alpha L_2/L_1$ is equal to or greater than 1.0, and for values of $\alpha L_2/L_1$ between 1.0 and 0, the proportion of moment to be resisted by the beam is to be obtained by linear interpolation between 85 and 0%. In addition, any beam must be designed to carry its own weight (projection above and below the slab), and any concentrated or linear loads applied directly on it

(13) Design of slab thickness and reinforcement

Slab Thickness.

Ordinarily the minimum thickness specified in ACI controls the thickness for design. Of course, reinforcement for bending moment must be provided, but the reinforcement ratio ρ required is usually well below $0.5\rho_{max}$; thus, it does not dictate slab thickness. For flat slabs, flexural strength must be provided both within the drop panel and outside its limits. In evaluating the strength within a drop panel, the drop width should be used as the transverse width of the compression area, because the drop is usually narrower than the width of the column strip.

(13) Design of slab thickness and reinforcement

Slab Thickness.

Also, the effective depth to be used should not be taken greater than what would be furnished by a drop thickness below the slab equal to one-fourth the distance from the edge of drop to the edge of column capital.

The shear requirement for two-way slabs (with beams) may be investigated by observing strips 1-1 and 2-2 in Fig.28. Beams with $\alpha_1 L_2/L_1$ values larger than 1.0 are assumed to carry the loads acting on the tributary floor areas bounded by 45° lines drawn from the corners of the panel and the centerline of the panel parallel to the long side.

(13) Design of slab thickness and reinforcement

Slab Thickness.

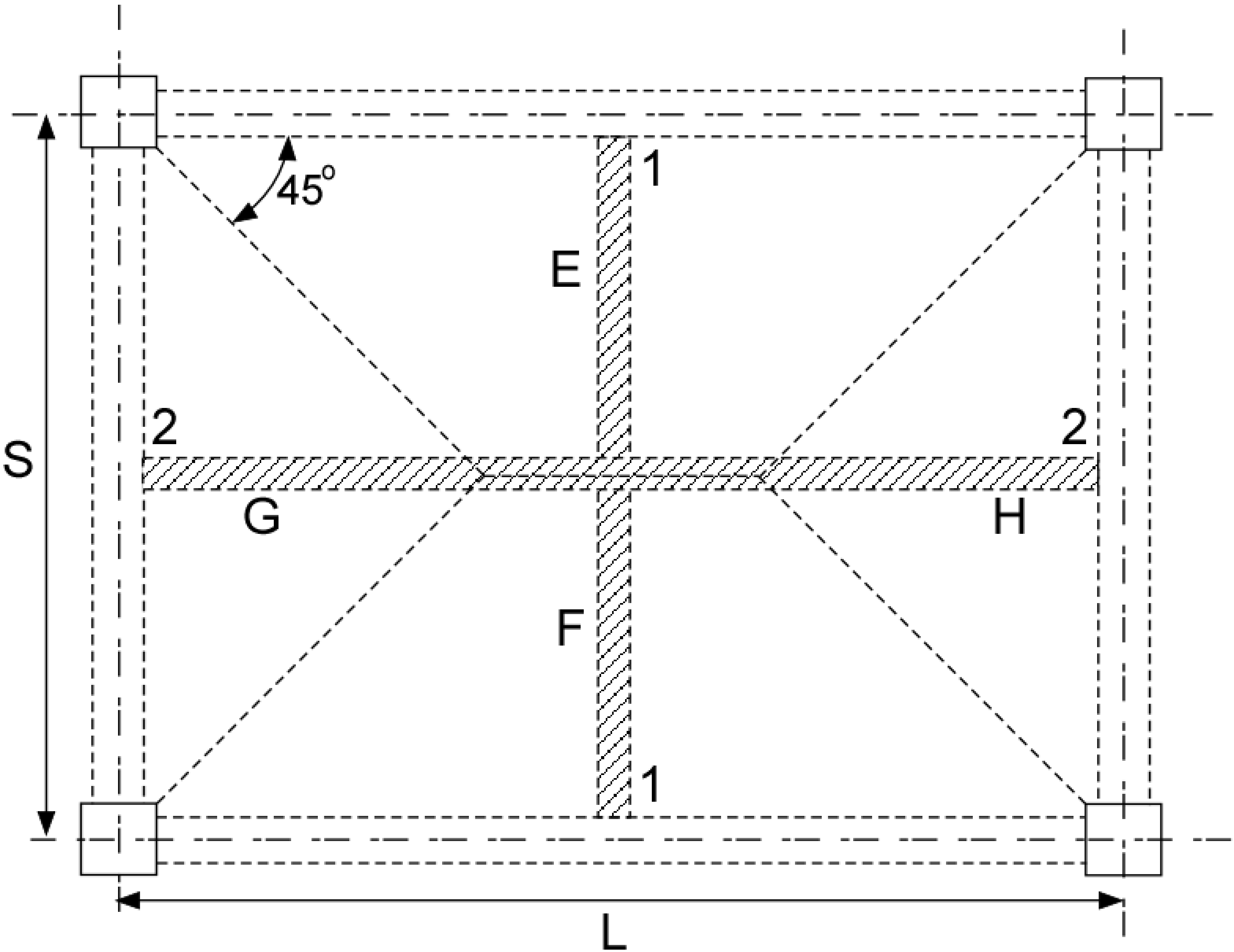


Fig.28

(13) Design of slab thickness and reinforcement

Slab Thickness.

If this is the case, the loads on the trapezoidal areas E and F of Fig.28 go to the long beams, and those on the triangular areas G and H go to the short beams. The shear per unit width of slab along the beam is highest at the ends of slab strips 1-1 and 2-2, which, considering the increased shear at the exterior face of the first interior support, is approximately equal to

$$V_u = 1.15\left(\frac{w_u S}{2}\right) \qquad (18)$$

(13) Design of slab thickness and reinforcement

Slab Thickness.

If $\alpha_1 L_2/L_1$ is equal to zero, there is, of course, no load on the beams (because there are no beams). When the value of $\alpha_1 L_2/L_1$ is between 0 and 1.0, the percentage of the floor load going to the beams should be obtained by linear interpolation. In such a case, the shear expressed by Eq.18 would be reduced, but the shear around the column due to the portion of the floor load going directly to the columns by two-way action should be investigated as for flat plate floors.

The shear strength requirement for flat slab and flat plate systems (without beams) is treated separately in sections 15,16 and 18.

(13) Design of slab thickness and reinforcement

Reinforcement

When the nominal requirements for slab thickness as discussed in section 6 are satisfied, no compression reinforcement will likely be required. The tension steel area required within the strip being considered can then be obtained by the following steps:

(1) $$\text{required } M_n = \frac{\text{factored moment } M_u \text{ in the strip}}{(\phi = 0.90)}$$

(2) $$m = \frac{f_y}{0.85 f'_c} \quad , \quad R_n = \frac{M_n}{bd^2}$$

$$\rho = \frac{1}{m}\left(1 - \sqrt{1 - \frac{2mR_n}{f_y}}\right) \quad , \quad A_s = \rho b d$$

(13) Design of slab thickness and reinforcement

Reinforcement

The values of b and d to be used in Step-2 for negative moment in a column strip with drop are the drop width for b, and for d the smaller of the actual effective depth through the drop and that provided by a drop thickness below the slab at no more than one-fourth the distance between the edges of the column capital and the drop. For positive moment computation, the full column strip width should be used for b, and the effective depth in the slab for d.

(13) Design of slab thickness and reinforcement

Reinforcement

After obtaining the steel area A_s required within the strip, a number of bars may be chosen so that they provide either the area required for strength or the area required for shrinkage and temperature reinforcement, which is 0.002bt for grades 40 and 50 steel and 0.0018bt for grade 60. The spacing of reinforcing bars must not exceed 2 times the slab thickness, except in slabs of cellular or ribbed construction where the requirement for shrinkage and temperature reinforcement governs (i.e., 5 times the slab thickness but not greater than 18 in.).

(13) Design of slab thickness and reinforcement

Reinforcement details in slabs without beams

Fig.29 provides detailed dimensions for minimum extensions required for each portion of the total number of bars in the column and middle strips.

For unbraced frames, reinforcement lengths must be determined by analysis but not less than those prescribed in Fig.29. ACI requires that the use of “integrity steel” which consists of a minimum of two of the column strip bottom bars passing continuously (or spliced with class A splice or anchored within support) through the column core in each direction. The purpose of this integrity steel is to provide some residual strength following a single punching shear failure.

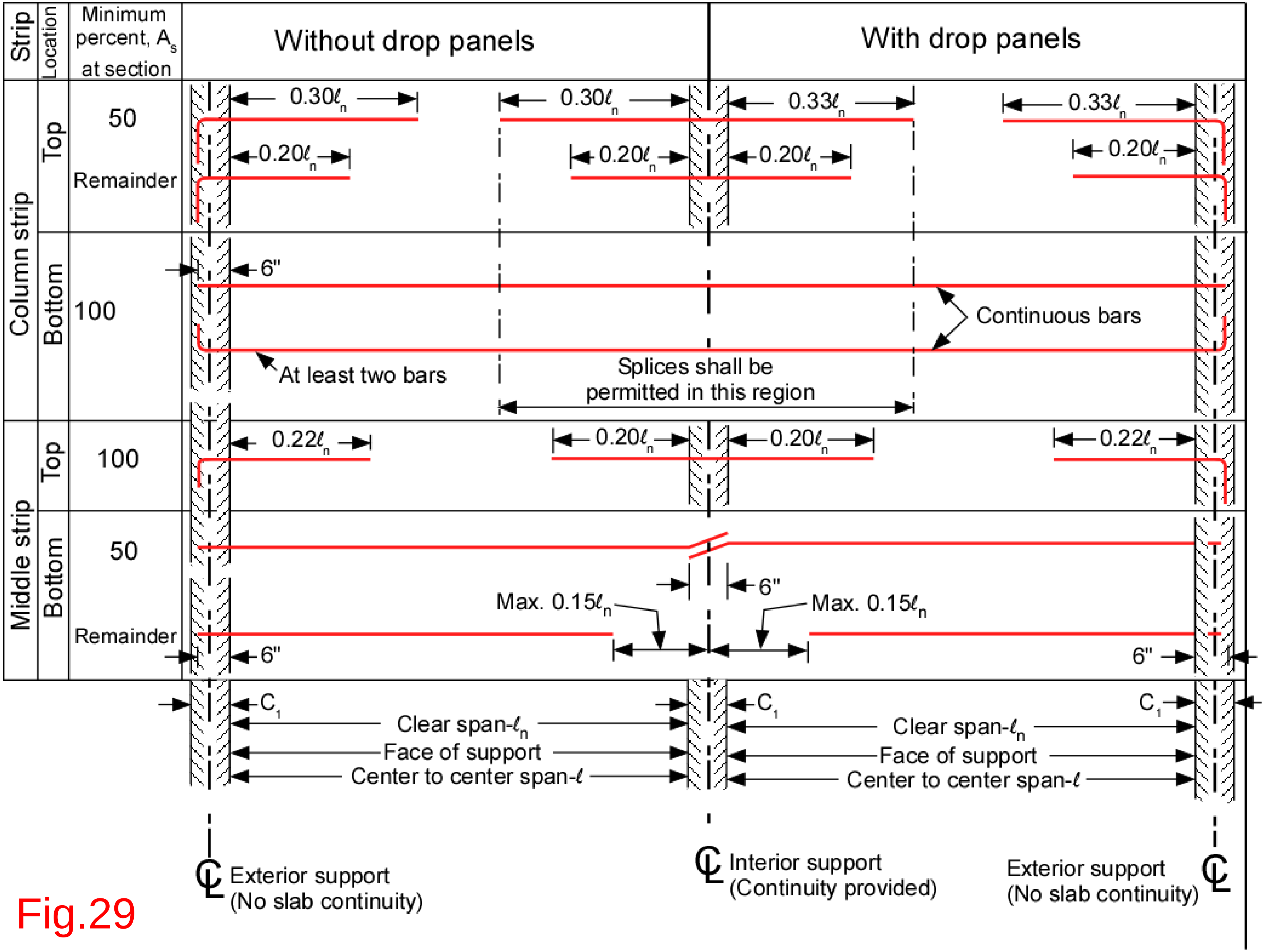

Strip
Location
Minimum percent, A_s at section
Without drop panels
With drop panels
Column strip
Top
50
Remainder
Bottom
100
Middle strip
Top
100
Bottom
50
Remainder
$0.30\ell_n$
$0.33\ell_n$
$0.20\ell_n$
$0.22\ell_n$
6"
Continuous bars
At least two bars
Splices shall be permitted in this region
Max. $0.15\ell_n$
C_1
Clear span-ℓ_n
Face of support
Center to center span-ℓ
Exterior support (No slab continuity)
Interior support (Continuity provided)

Fig.29

(13) Design of slab thickness and reinforcement

Corner reinforcement for two-way slab(with beams).

It is well known from plate bending theory that a transversely loaded slab simply supported along four edges will tend to develop corner reactions as shown in Fig.30, for which reinforcement must be provided. Thus in slabs supported on beams having a value of α greater than 1.0, special reinforcement (Fig.31) shall be provided at exterior corners in both the bottom and top *of* the slab. This reinforcement (ACI) is to be provided for a distance in each direction from the corner equal to one-fifth the longer span.

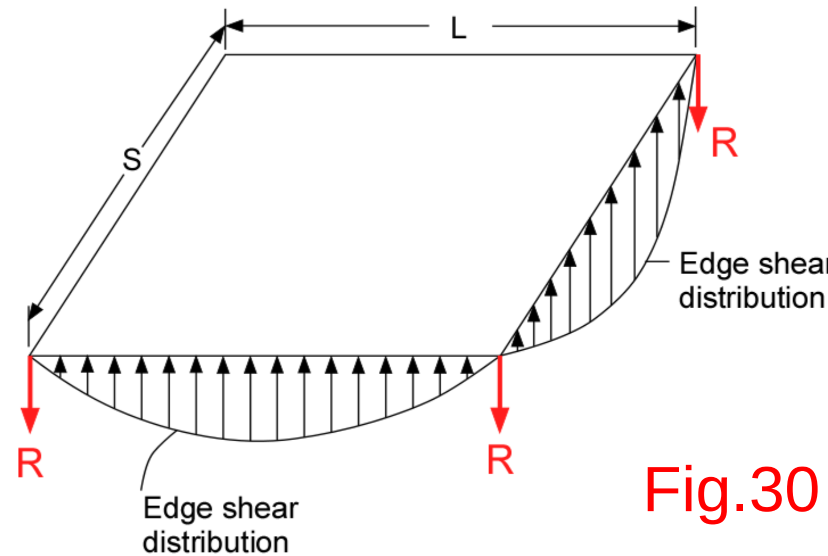


Fig.30

(13) Design of slab thickness and reinforcement

Corner reinforcement for two-way slab(with beams).

The reinforcement in both the top and bottom of the slab must be sufficient to resist a moment equal to the maximum positive moment per foot of width in the slab, and it may be placed in a single band parallel to the diagonal in the top of the slab and perpendicular to the diagonal in the bottom of the slab, or in two bands parallel to the sides of the slab.

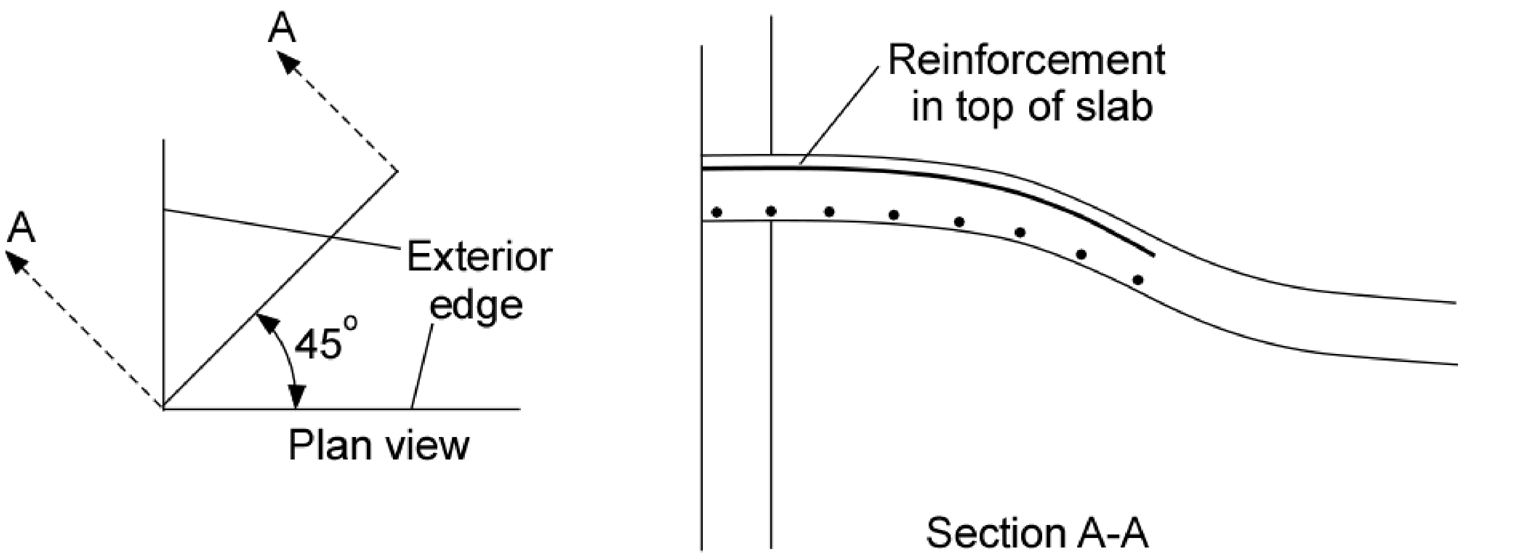


Fig.31

(13) Design of slab thickness and reinforcement

Crack Control

In addition to deflection control, crack control is the other major serviceability requirement usually considered in the design of flexural members. ACI gives criteria for beams and one-way slabs to ensure distribution of flexural reinforcement to minimize crack width under service loads.

(13) Design of slab thickness and reinforcement

Crack Control

No ACI Code provisions are given for two-way floor systems; however, ACI Committee has suggested a formula to predict the possible crack width in two-way acting slabs, flat slabs, and flat plates. When the predicted crack width is considered excessive (there are no ACI Code limits for slabs), the distribution (size and spacing) of flexural reinforcement may be adjusted to decrease predicted crack width. Ordinarily crack width is not a problem on two-way acting slabs, but when steel with f_y equal to 60,000 psi or higher is used, crack control should be considered.

(14) Beam size requirement in flexure and shear

- The size of the beams along the column centerlines in a two-way slab (with beams) should be sufficient to provide the bending moment and shear strengths at the critical sections.
- For approximately equal spans, the largest bending moment should occur at the exterior face of the first interior column where the available section for strength computation is rectangular in nature because the effective slab projection is on the tension side. Then with the preliminary beam size the required reinforcement ratio ρ may be determined and compared with $0.75\rho_b$, the maximum value permitted under ACI. There is no explicit limit for design according to ACI.

(14) Beam size requirement in flexure and shear

- ❖ Deflection is unlikely to be a problem with T-sections, but must be investigated if excessive deflection may cause difficulty.

The maximum shear in the beam should also occur at the exterior face of the first interior column. The shear diagram for the exterior span may be obtained by placing the negative moments already computed for the beam at the face of the column at each end and loading the span with the percentage of floor load interpolated (ACI) between $\alpha_1 L_2/L_1 = 0.$ and $\alpha_1 L_2/L_1 \geq 1.0$. The stem (web) $b_w d$ should for practicality be sized such that nominal shear stress

$v_n = V_u/(\phi b_w d)$ does not exceed about $6\sqrt{f'_c}$ at the critical section 'd' from the face of support.

(15) Shear strength in two-way floor systems

The shear strength of a flat slab or flat plate floor around a typical interior column under dead and full live loads is analogous to that of a square or rectangular spread footing subjected to a concentrated column load, except each is an inverted situation of the other. The area enclosed between the parallel pairs of centerlines of the adjacent panels of the floor is like the area of the footing, because there is no shear force along the panel centerline of a typical interior panel in a floor system

(15) Shear strength in two-way floor systems

Wide-beam action

The shear strength of the flat slab or flat plate should be first investigated for wide-beam action (one-way action) and then for two-way action. In the wide-beam action, the critical section is parallel to the panel centerline in the transverse direction and extends across the full distance between two adjacent longitudinal panel centerlines. As in beams, this critical section of width b_w times the effective depth ‘d’ is located at a distance ‘d’ from the face of the equivalent square column capital or from the face of the drop panel, if any.

(15) Shear strength in two-way floor systems

Wide-beam action

The nominal strength in usual cases where no shear reinforcement is used is

$$V_n = V_c = 2\sqrt{f'_c}\,b_w d \qquad (19)$$

according to the simplified method of ACI code.

Alternatively, V_c may be determined using the more detailed expression involving $\rho V_u d/M_u$.

$$V_n = V_c = \left(1.9\sqrt{f'_c} + 2500\rho\frac{V_u d}{M_u}\right) b_w d \le 3.5\sqrt{f'_c}\,b_w d \qquad (20)$$

(15) Shear strength in two-way floor systems
Two-way action.

Potential diagonal cracking may occur along a truncated cone or pyramid around columns, concentrated loads, or reactions. The critical section is located so that its periphery b_o is at a distance d/2 outside a column, concentrated load, or reaction.

When shear reinforcement is not used, the nominal shear strength $V_n = V_c$, which is given by ACI as the smallest of

$$V_c = \left(2 + \frac{4}{\beta_c}\right)\sqrt{f_c'}\,b_o d \qquad (21)$$

$$V_c = \left(\frac{\alpha_s}{\frac{b_o}{d}} + 2\right)\sqrt{f_c'}\,b_o d \qquad (22)$$

$$V_c = 4\sqrt{f_c'}\,b_o d \qquad (23)$$

where

b_o = perimeter of critical section

β_c = ratio of long side to short side of column.

α_s = 40 for interior columns, 30 for edge columns, and 20 for corner columns

(15) Shear strength in two-way floor systems

Two-way action.

Eq.21 recognizes that there should be a transition between, say, a square column (β_c = 1) where V_c might be based on $4\sqrt{f'_c}$ ’: for two-way action, and a wall ($\beta_c = \infty$) where V_c should be based on the $2\sqrt{f'_c}$ used for one-way action as for beams. However, unless is larger than 2.0, eq.21 does not control.

The two way action strength may reduce, even for a square concentrated load increases, both (1) as the distance to the critical section from the concentrated load increases, such as for drop panels, and (2) as the perimeter becomes large compared to slab thickness, such as for example a 6 in. slab supported by a 10ft square column ($b_o/d \approx 80$). The eq.22 accounts for this reduced strength.

(15) Shear strength in two-way floor systems

Two-way action.

In the application of Eqs.(21,22 & 23), b_o is the perimeter of the critical section at a distance d/2 from the edge of column capital or drop panel. For Eq.(22), α_s for an "interior column" applies when the perimeter is four-sided, for an "edge column" when the perimeter is three-sided, and for a "corner column" when the perimeter is two-sided.

(15) Shear strength in two-way floor systems

Two-way action.

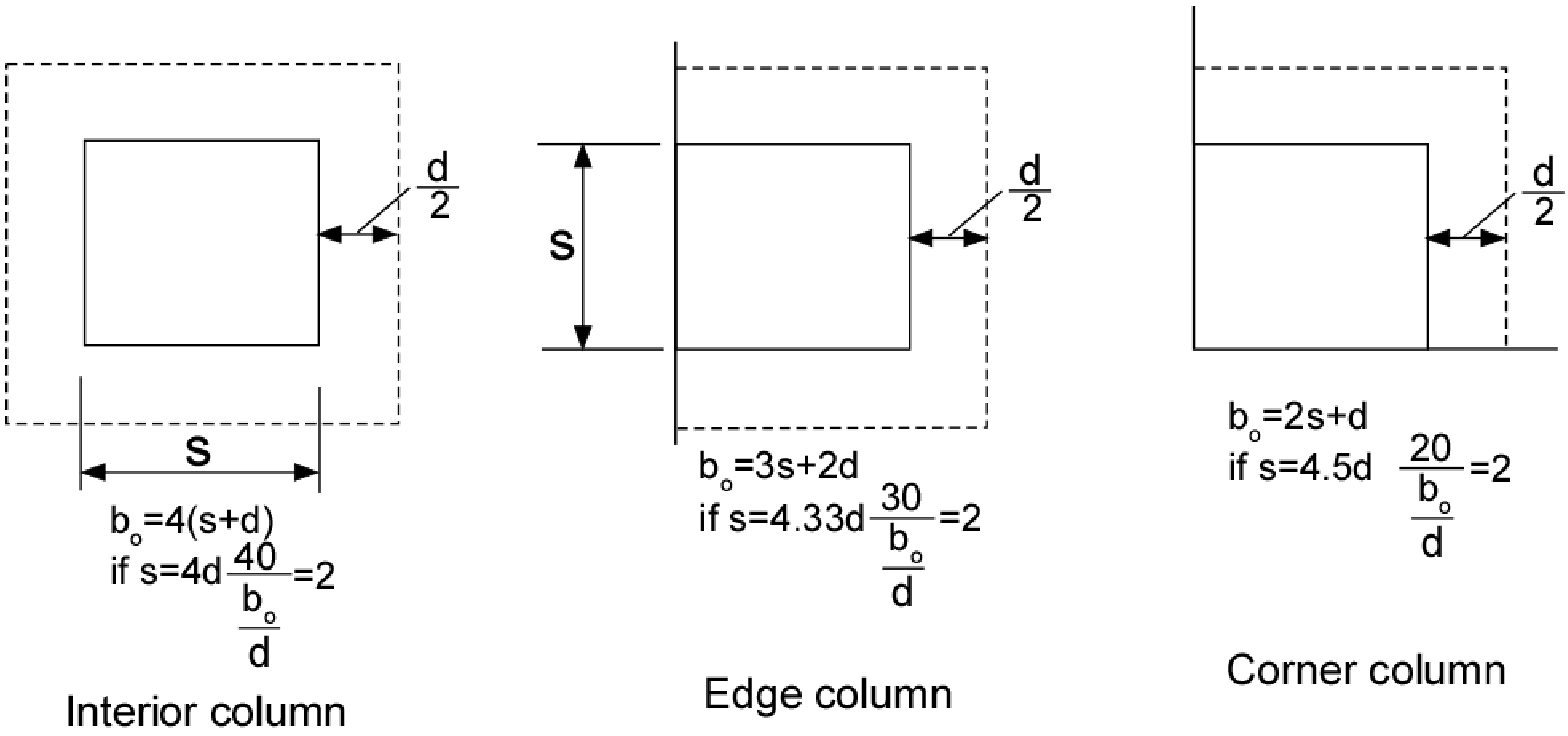


Fig.32

(15) Shear strength in two-way floor systems

Two-way action.

As shown by Fig.32, Eq.22 will give a V_c smaller than $4\sqrt{f_c'}b_o d$ for columns (or very thin slabs), such as a square interior column having side larger than 4d, a square edge column having side larger than 4.33d, and a square corner column having side larger than 4.5d. Thus, the nominal shear strength v_c in a two way system is generally set by Eq.(23), that is $v_c = 4\sqrt{f_c'}b_o d$ unless either of Eq.21 or Eq.22 gives a lesser value.

(15) Shear strength in two-way floor systems

Shear Reinforcement.

Even when shear reinforcement is used (ACI), the nominal strength is limited to a maximum of

$$V_n = V_c + V_s \leq 6\sqrt{f_c'}b_o d \qquad (24)$$

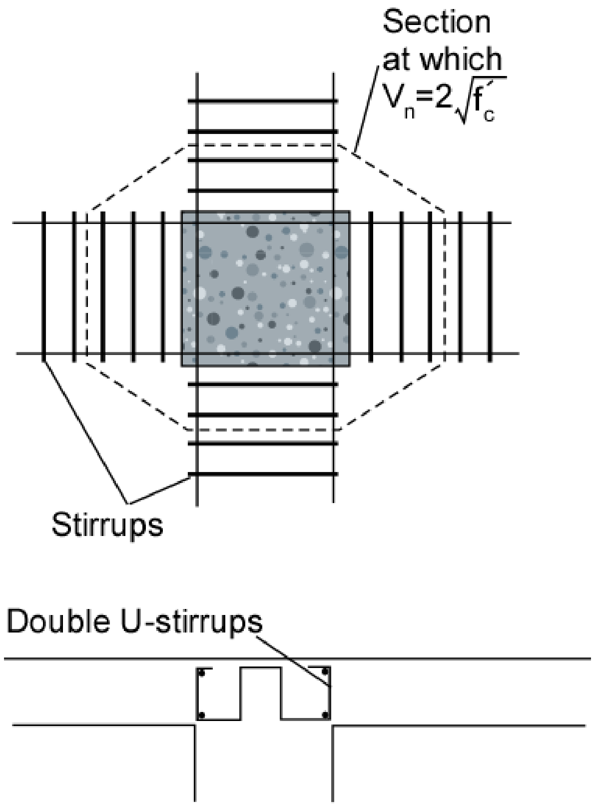
Further, in the design of any shear reinforcement, the portion of the strength V_c may not exceed $2\sqrt{f_c'}b_o d$. If shearhead reinforcement such as described in section 12 is used, the maximum V_n in Eq.(24) is $7\sqrt{f_c'}b_o d$.

When there must be transfer of both shear and moment from the slab to the column, ACI applies as will be discussed in section18.

(16) Shear reinforcement in flat plate floors

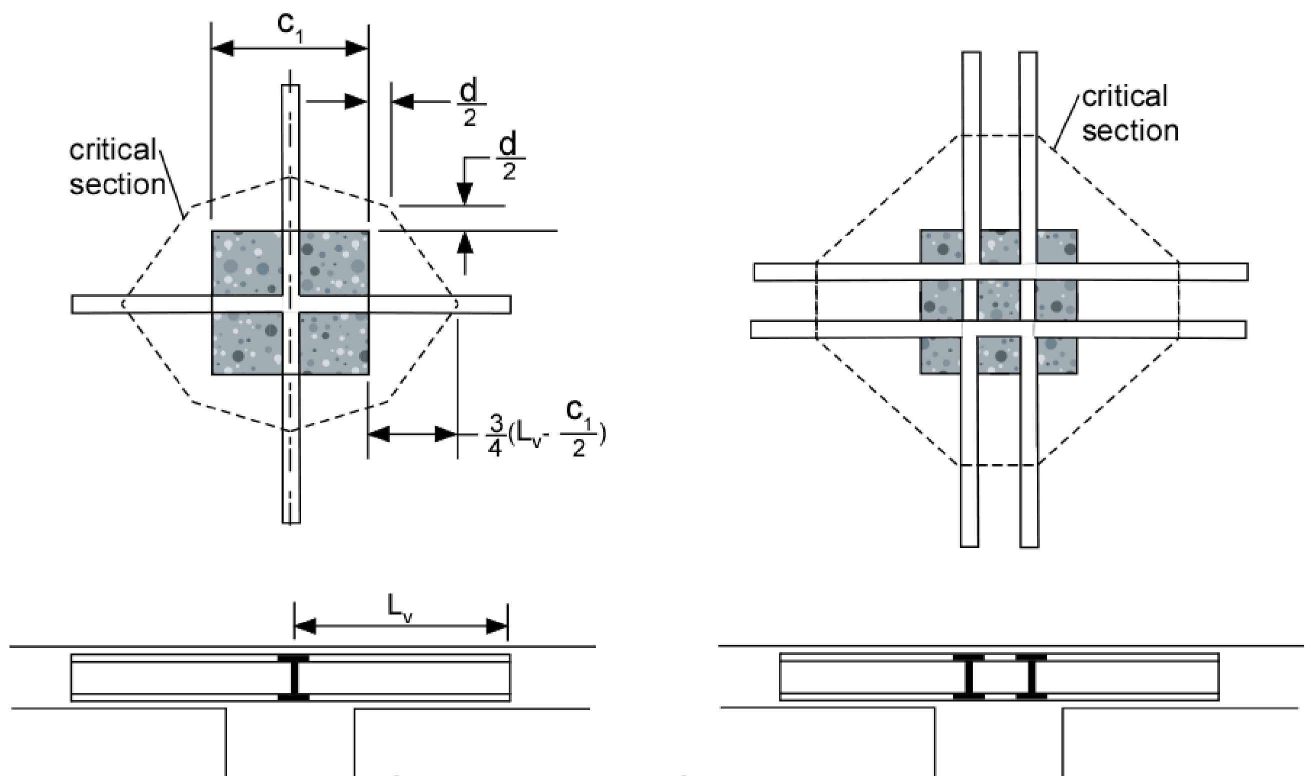
In flat plate floors where neither column capitals nor drop panels are used, shear reinforcement is frequently necessary. In such cases, two-way action usually controls. The shear reinforcement may take the form of properly anchored bars or wires placed in vertical sections around the column [Fig.33], or consist of shearheads, which are steel-I- or channel-shaped sections fabricated by welding into four (or three for an exterior column) identical arms at right angles and uninterrupted within the column section[Fig.34].

(16) Shear reinforcement in flat plate floors



Bar reinforcement

Fig.33



Shearhead reinforcement

Fig.34

(16) Shear reinforcement in flat plate floors

When bar or wire shear reinforcement is used, the nominal strength is

$$V_n = V_c + V_s = 2\sqrt{f_c'}\,b_o d + \frac{A_v f_y d}{s} \qquad (25)$$

where

b_o = periphery around the critical section

A_v = total stirrup bar area around b_o

Such bar or wire reinforcement is required wherever V_u exceeds ϕV_c based on V_c of Eqs.(21,22 & 23). However, in the design of shear reinforcement, V_c for Eq.(25) may not be taken greater than $2\sqrt{f_c'}\,b_o d$, and the maximum nominal strength V_n (i.e. V_c+V_s) when shearreinforcement is used may not exceed $6\sqrt{f_c'}\,b_o d$.

(16) Shear reinforcement in flat plate floors

Shear strength may be provided by shearheads under ACI whenever V_u/ϕ at the critical section is between that permitted by Eqs. (21,22 & 23) and $7\sqrt{f'_c}b_od$. These provisions apply only where shear alone (i.e., no bending moment) is transferred at an interior column. When there is moment transfer to columns, ACI applies, as is discussed in Section18.

With regard to the size of the shearhead, it must furnish a ratio α_v of 0.15 or larger (ACI) between the stiffness for each shearhead arm (E_sI_x) and that for the surrounding composite cracked slab section of width (c_2 + d),

$$\min \alpha_v = \frac{E_sI_x}{E_c(\text{composite } I_s)} = 0.15 \qquad (26)$$

(16) Shear reinforcement in flat plate floors

The steel shape used must not be deeper than 70 times its web thickness, and the compression flange must be located within 0.3d of the compression surface of the slab (ACI). In addition, the plastic moment capacity M_p of the shearhead arm must be at least (ACI).

$$\min M_p = \frac{V_u}{2\eta\phi}\left[h_v + \alpha_v\left(L_v - \frac{c_1}{2}\right)\right] \qquad (27)$$

where

$\eta =$ number (usually 4) of identical shear head arms

$V_u =$ factored shear around the periphery of column face

$h_v =$ depth of shear head

$L_v =$ length of shear head measured from column centerline

$\phi = 0.90$ the strength reduction factor for flexure

(16) Shear reinforcement in flat plate floors

Eq.(27) is to ensure that the required shear strength of the slab is reached before the flexural strength of the shearhead is exceeded.

The length of the shearhead should be such that the nominal shear strength V_n will not exceed $4\sqrt{f'_c}b_o d$ computed at a peripheral section located at $\frac{3}{4}(L_v - c_1/2)$ along the shearhead but no closer elsewhere than d/2 from the column face (ACI). This length requirement is shown in Fig.33&34.

(16) Shear reinforcement in flat plate floors

When a shearhead is used, it may be considered to contribute a resisting moment (ACI).

$$M_v = \frac{\phi \alpha_v V_u}{2\eta}\left(L_v - \frac{c_1}{2}\right) \qquad (28)$$

to each column strip, but not more than 30% of the total moment resistance required in the column strip, nor the change in column strip moment over the length L_v, nor the required M_p given by Eq.(27).

(17) Moments in Columns

The moments in columns due to unbalanced loads on adjacent panels are readily available when an elastic analysis is performed on the equivalent rigid frame for the various pattern loadings. In "DDM", wherein the six limitations listed in section 7 are satisfied, the longitudinal moments in the slab are prescribed by the provisions of the ACI code. In a similar manner, the Code prescribes the unbalanced moment at an interior column as follows:

$$M = 0.07\left[\left(w_D + \frac{1}{2}w_L\right)L_2 L_n^2 - w'_D L'_2 \left(L'_n\right)^2\right] \qquad (29)$$

where

w_D = factored dead load per unit area

w_L = factored live load per unit area

w'_D, L'_2, L'_n = quantities referring to the shorter span

(17) Moments in Columns

The moment is yet to be distributed between the two ends of the upper and lower columns meeting at the joint.

The rationale for Eq.(29) may be observed from the stiffness ratios at a typical interior joint shown in Fig.35, wherein the distribution factor for the sum of the column end moments is taken as 7/8 and the unbalanced moment in the column strip is taken to be 0.080/0.125 times the difference in the total static moments due to dead plus half live load on the longer span and dead load only on the shorter span.

(17) Moments in Columns

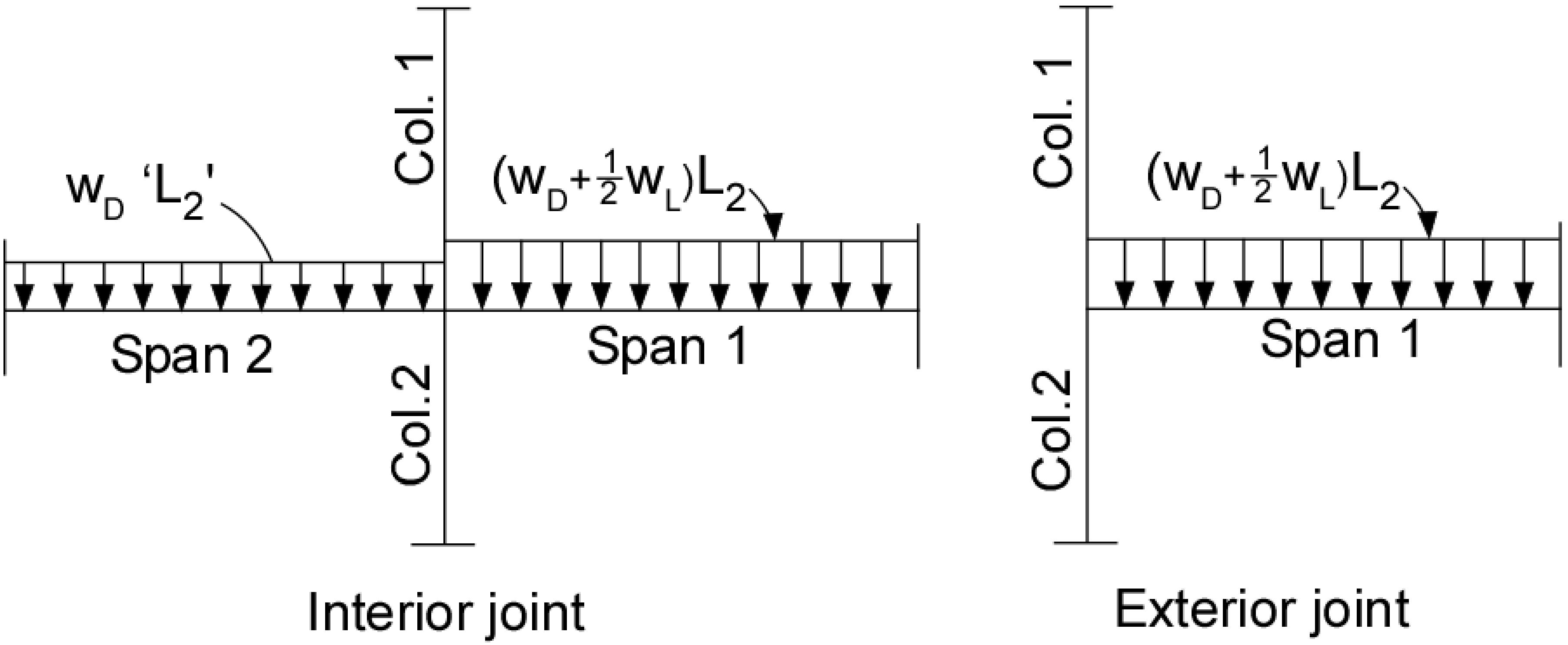


Fig.35

For the edge column, ACI requires using $0.3M_o$ as the moment to be transferred between the slab and an edge column.

(18) Transfer of moment and shear at junction of slab and column

In as much as the columns meet the slab at monolithic joints, there must be moment as well as shear transfer between the slab and the column ends. The moments may arise out of lateral loads due to wind or earthquake effects acting on the multistory frame, or they may be due to unbalanced gravity loads as considered in Section-17. In addition, the shear forces at the column ends and throughout the columns must be considered in the design of lateral reinforcement (ties or spiral) in the columns (ACI). The transfer of moment and shear at the slab-column interface is extremely important in the design of flat plates.

(18) Transfer of moment and shear at junction of slab and column

Let M_u be the total factored moment that is to be transferred to both ends of the columns meeting at an exterior or an interior joint. The ACI Code requires the total factored moment M_u to be divided into M_{ub} "transferred by flexure" (ACI) and M_{uv} "transferred by shear" (ACI) such that

$$M_{ub} = \gamma_f M_u = \left(\frac{1}{1 + \frac{2}{3}\sqrt{\frac{b_1}{b_2}}} \right) M_u \qquad (30)$$

where

$$b_1 = \text{critical section dimension in the longitudinal direction}$$

$$= c_1 + \frac{d}{2} \quad \text{for exterior column Fig.36}$$

$$= c_1 + d \quad \text{for interior column Fig.37}$$

$b_2 =$ critical section dimension in the transverse direction

$= c_2 + d$

Fig.36 & 37

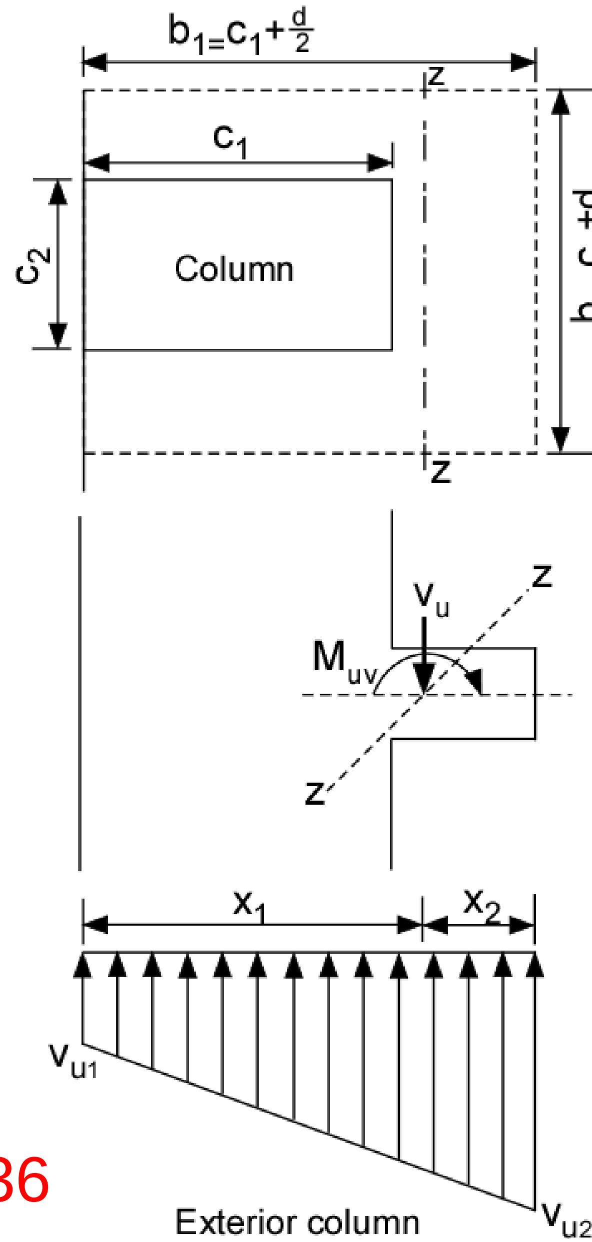


Fig.36

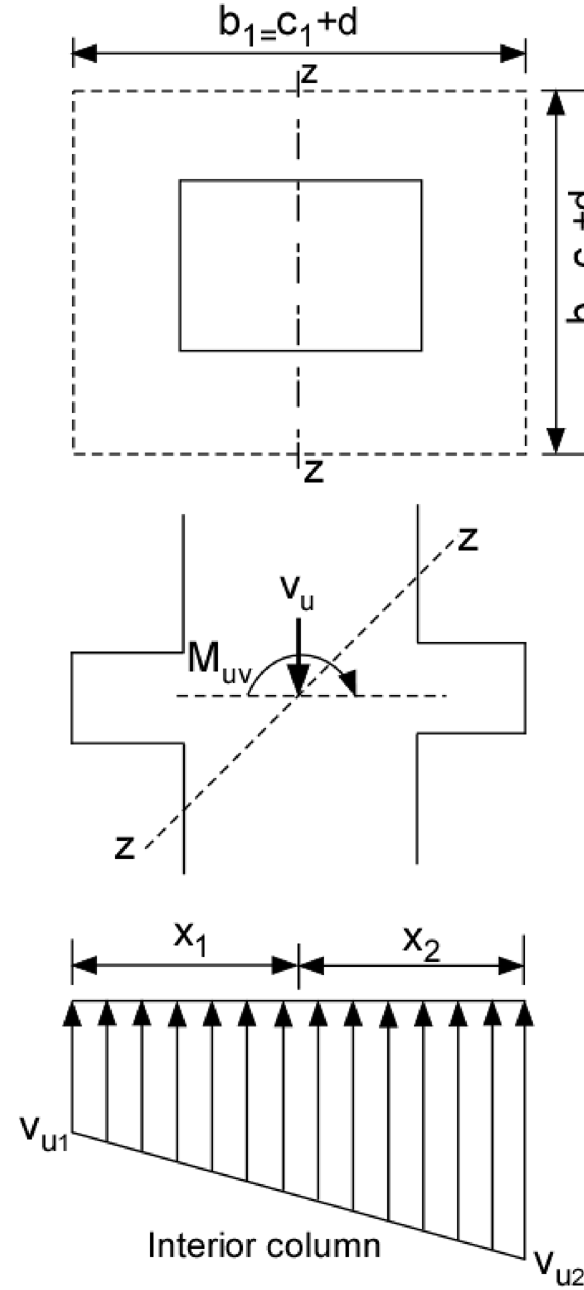


Fig.37

and

$$M_{uv} = M_u - M_{ub} = M_u(1 - \gamma_f) \qquad (31)$$

(18) Transfer of moment and shear at junction of slab and column

The moment M_{ub} is considered to be transferred within an effective slab width equal to $(c_2 + 3t)$ at the column (ACI), where t is the slab or drop panel thickness. The moment strength for M_{ub} is achieved by using additional reinforcement and closer spacing within the width $(c_2 + 3t)$.

If $b_2 = b_1$, Eq.(30) becomes $M_{ub} = O.60M_u$

If $b_2 = 1.5b_1$, Eq.(30) becomes $M_{ub} = O.648M_u$

(18) Transfer of moment and shear at junction of slab and column

- It appears reasonable that when b_2 in the transverse direction is larger than b_1 in the longitudinal direction, the moment transferred by flexure is greater because the effective slab width (c_2 + 3t) resisting the moment is larger.
- Because the aspect ratio b_2/b_1 affects only slightly the proportion of the exterior support moment "transferred by flexure, the 1995 ACI Code has simplified the procedure for many situations.

(18) Transfer of moment and shear at junction of slab and column

1995 simplified procedure.

- For unbalanced moments about an axis parallel to the edge at exterior supports, where the factored shear V_u does not exceed $0.75\phi V_c$, at an edge support, or does not exceed $0.5\phi V_c$ at a corner support, ACI permits neglect of the interaction between shear and moment. In other words, for such situations, the full exterior moment can be considered transferred through flexure (that is, $\gamma_f = 1.0$), and the exterior support factored shear V_u can be considered independently.

(18) Transfer of moment and shear at junction of slab and column

1995 simplified procedure.

- For unbalanced moments at interior supports and for unbalanced moments about an axis transverse to the edge at exterior supports, where factored shear V_u does not exceed O.4ϕV_c, ACI permits increasing by as much as 25% the proportion γ_f of the full exterior moment transferred by flexure.
- When using the simplified procedure, the reinforcement ρ, within the effective slab width defined in ACI, is not permitted to exceed 0.375$\rho_{b.}$ The simplified procedure is not permitted for prestressed concrete systems.

(18) Transfer of moment and shear at junction of slab and column

Stresses representing interaction between flexure and shear.

- The moment M_{uv} transferred by shear acts in addition to the associated shear force V_u at the centroid of the shear area around the critical periphery located at d/2 from the column faces, as shown in Fig.36&37. Referring to that figure, the factored shear stress is

$$v_{u1} = \frac{V_u}{A_c} - \frac{M_{uv} x_1}{J_c} \qquad (32)$$

$$v_{u2} = \frac{V_u}{A_c} - \frac{M_{uv} x_2}{J_c} \qquad (33)$$

(18) Transfer of moment and shear at junction of slab and column

Stresses representing interaction between flexure and shear.

- By using a section property J_c analogous to the polar moment of inertia of the shear area along the critical periphery taken about the z-z axis, it is assumed that there are both horizontal and vertical shear stresses on the shear areas having dimensions b_1 by d in Fig.36&37. The z-z axis is perpendicular to the longitudinal axis of the equivalent frame; that is, in the transverse direction, and located at the centroid of the shear area.

(18) Transfer of moment and shear at junction of slab and column

Stresses representing interaction between flexure and shear.

- For an exterior column, x_1 and x_2 are obtained by locating the centroid of the channel-shaped vertical shear area represented by the dashed line ($b_1 + b_2 + b_1$) shown in Fig.36, and

$$A_c = (2b_1 + b_2)d \tag{34}$$

$$x_2 = \frac{b_1^2 d}{A_c} \tag{35}$$

$$J_c = d\left[\frac{b_1^3}{3} - (2b_1 + b_2)x_2^2\right] + \frac{b_1 d^3}{6} \tag{36}$$

(18) Transfer of moment and shear at junction of slab and column

Stresses representing interaction between flexure and shear.

For interior column, referring to Fig.37

$$A_c = 2(b_1 + b_2)d \qquad (37)$$

$$J_c = d\left[\frac{b_1^3}{6} + \frac{b_2 b_1^2}{2}\right] + \frac{b_1 d^3}{6} \qquad (38)$$

- Eq.(34) to (38) are derivable by letting the shear stress at any location resulting from M_{uv} alone be proportional to the distance from the centroidal axis z-z to the two shear areas b_1 by d, and either (a) to the one shear area b_2 by d for an exterior column as shown in Fig.38, or (b) to the two shear areas b_2 by d for the interior column.

(18) Transfer of moment and shear at junction of slab and column

Stresses representing interaction between flexure and shear.

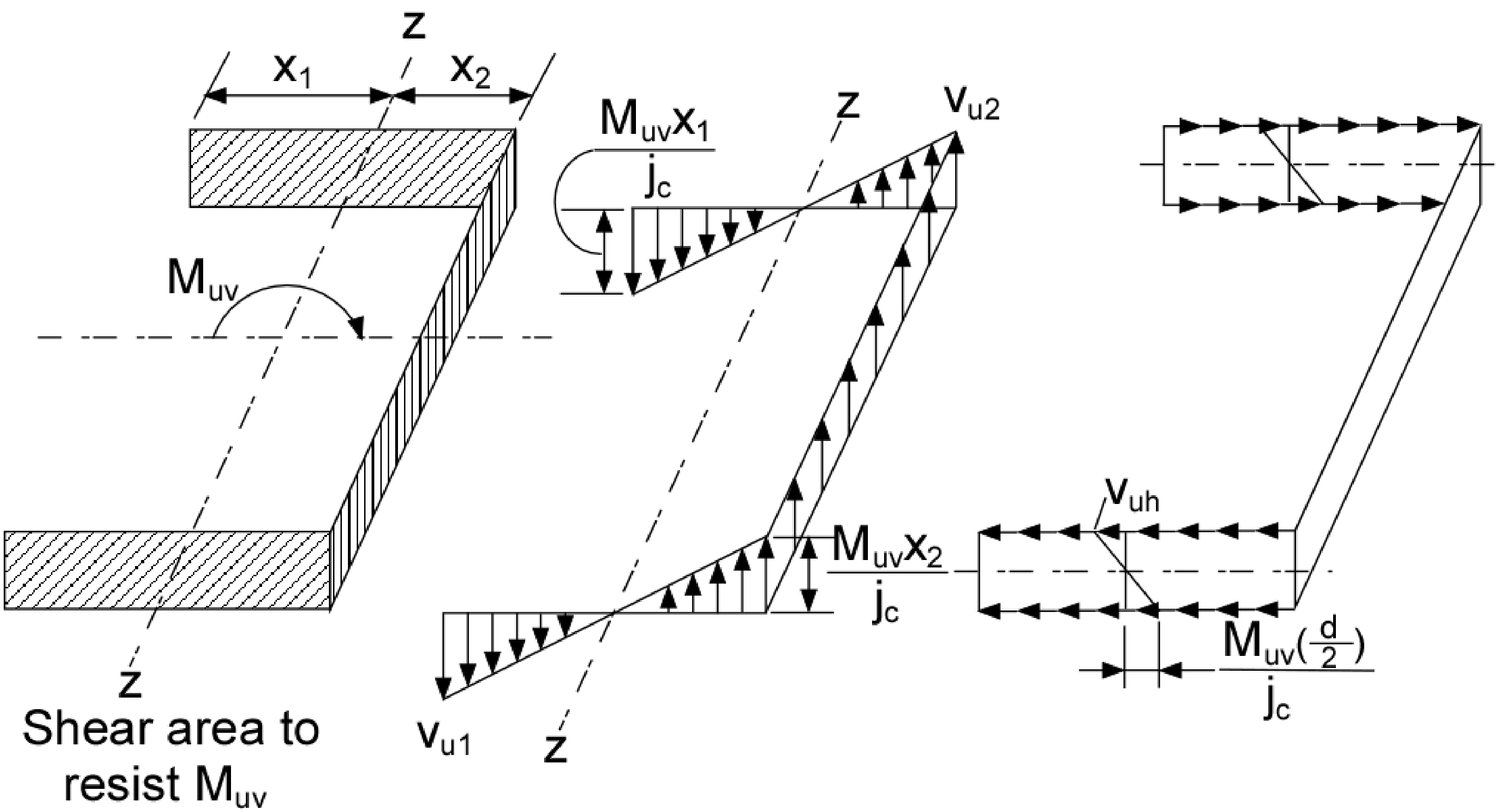


Fig.38(a)Shear areas to resist M_{uv}

Fig.38(b)Vertical resisting shear

Fig.38(c)Horizontal resisting shear stresses

(18) Transfer of moment and shear at junction of slab and column

Stresses representing interaction between flexure and shear.

- According to ACI, the larger factored shear stress v_{u2} shown in Fig.37 must not exceed the stress $\phi v_n = \phi V_c / b_o d$ obtained from Eqs.(21,22 & 23), otherwise shear reinforcement as described in Section-16 is required.

(18) Transfer of moment and shear at junction of slab and column

- **Moment transfer from flat plate to column when shear heads are used**

- Shear stresses computed for factored loads at the critical section distance d/2 from the column face are appropriate for transfer of $M_{uv}=M_u-M_{ub}$ as describe earlier, even when shearheads are used. However, the critical section for V_u is at a periphery passing through points at ¾ $(L_v-c_1/2)$ from but no closer than d/2 to the column faces.

(18) Transfer of moment and shear at junction of slab and column

- ***Moment transfer from flat plate to column when shear heads are used***

- When there are both V_u and M_u to be transferred, ACI requires that the sum of the shear stresses computed for M_{uv} and V_u at their respective locations not exceed $\phi\left(4\sqrt{f_c'}\right)$. The reason for this apparent inconsistency is that these two critical sections are in close proximity at the column corners where the failures initiate.

(19) Openings and corner connections in flat slabs

- When openings and corner connections are present in flat slabs floors, designers must make sure that adequate provisions are made for them. ACI first prescribes in general that openings of any size may be provided if it can be shown by analysis that all strength and serviceability conditions, including the limits on the deflections, are satisfied. However, in common situations (ACI) a special analysis need not be made for slab systems not having beams when (i) openings are within the middle half of the span in each direction, provided the total amount of reinforcement required for the panel without the opening is maintained;

(19) Openings and corner connections in flat slabs

- (ii) openings in the area common to two column strips do not interrupt more than one-eighth of the column strip width in either span, and the equivalent of reinforcement interrupted is added on all sides of the openings; (iii) openings in the area common to one column strip and one middle strip do not interrupt more than one-fourth of the reinforcement in either strip, and the equivalent of reinforcement interrupted is added on all sides of the openings

(19) Openings and corner connections in flat slabs

- In regard to nominal shear strength in two-way action, the critical section for slabs without shearhead is not to include that part of the periphery which is enclosed by radial projections of the openings to the center of the column (ACI).
- For slabs with shearhead, the critical periphery is to be reduced only by one-half of what is cut away by the radial lines from the center of the column to the edges of the opening. Some critical sections with cutaways by openings are shown in Fig.39.

(19) Openings and corner connections in flat slabs

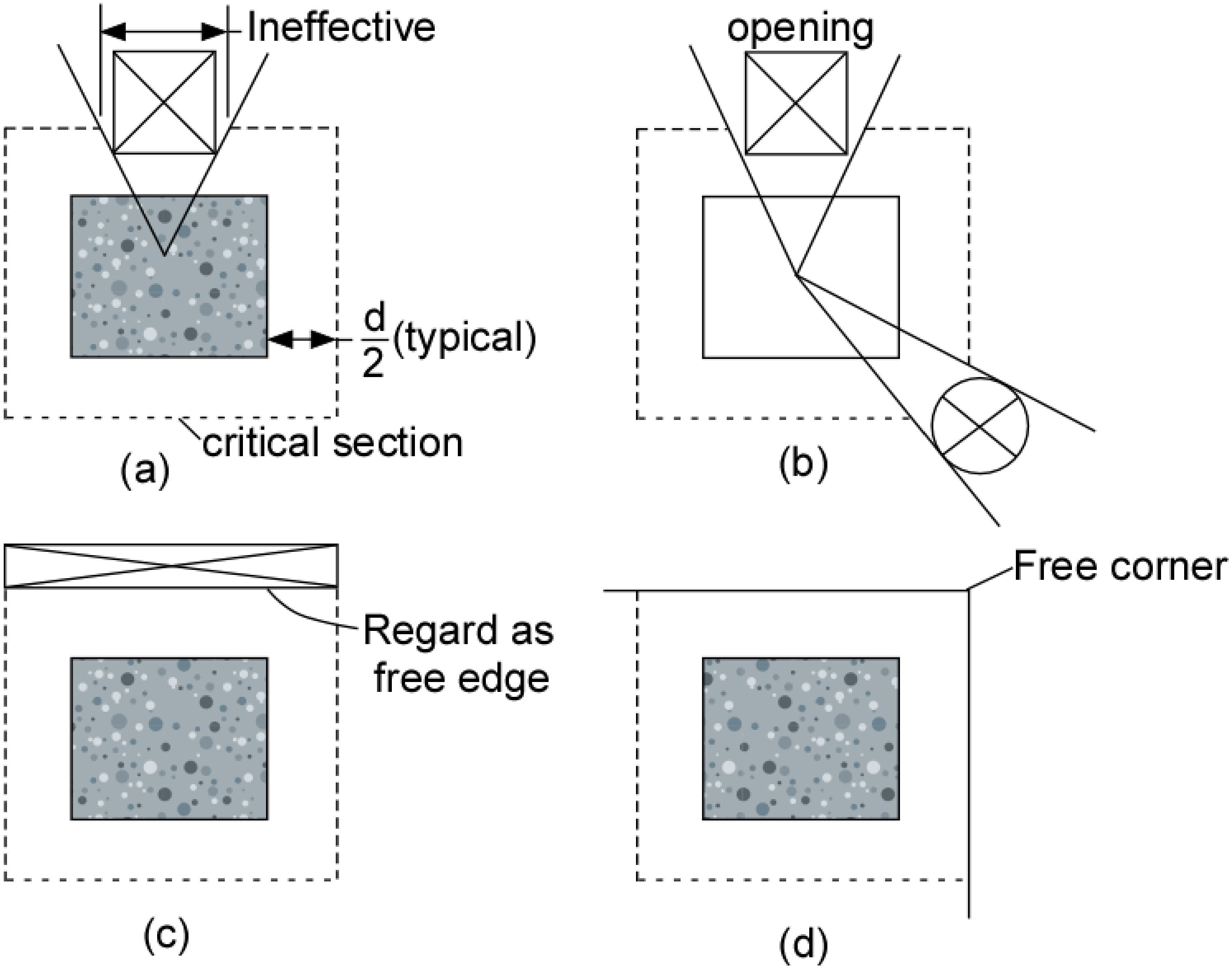


Fig.39