CHAPTER 4

BEAM

4.1 INTRODUCTION

Where ever we visualize an engineering construction work Beam is a very common term, which needs to be meticulously planned and designed. Beams are normally horizontal sections, which carries loads to the columns. Normally an engineer designs the Beam for flexure shear but in fact Beams sustains many other forces such as diagonal tension, torsion e.t.c. This chapter will includes design of different types of Beams for flexure, design of web reinforcement of shear and diagonal tensions, design for torsion, bond, anchorage and development length and the serviceability of the Beam.

4.2 BENDING OF REINFORCED CONCRETE BEAM

Reinforced concrete beam if subjected to a vertical load will of course bend to the direction of the load as shown in Figure 4.1 (b). It is also well understood that if we take a section along A-A and N.A is line showing the neutral axis the upper portion of the neutral axis will face compression and lower one will face tension. If the beam section behaves elastically and uncracked then the Figure 4.1 (c) will be the exact pressure diagram at the particular face. But it is also confirm that a reinforced concrete section cannot be elastic. Therefore the stress distribution will be somewhat like Figure 4.1 (d).

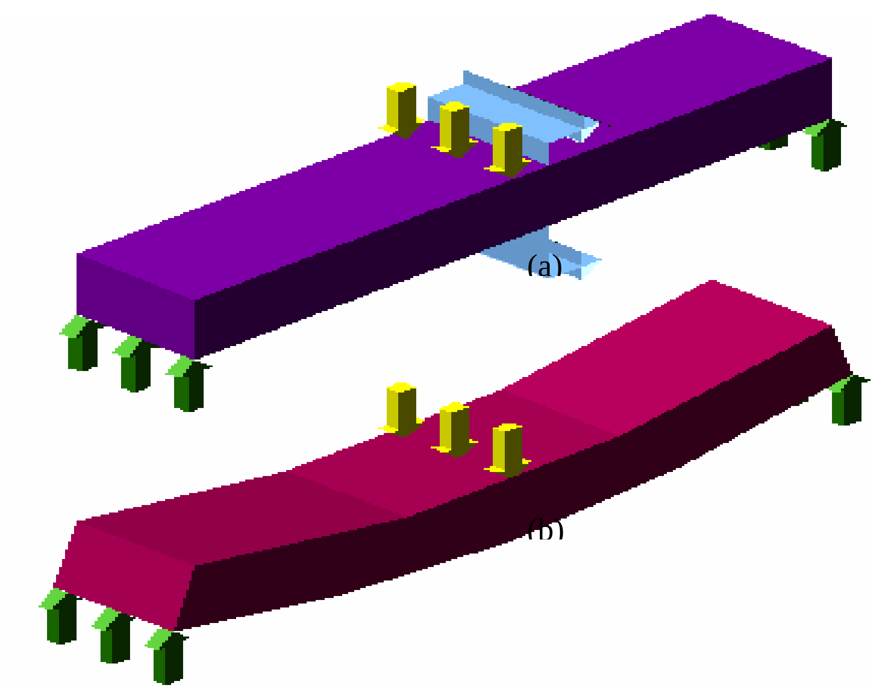


Figure 4.1: Bending of beam

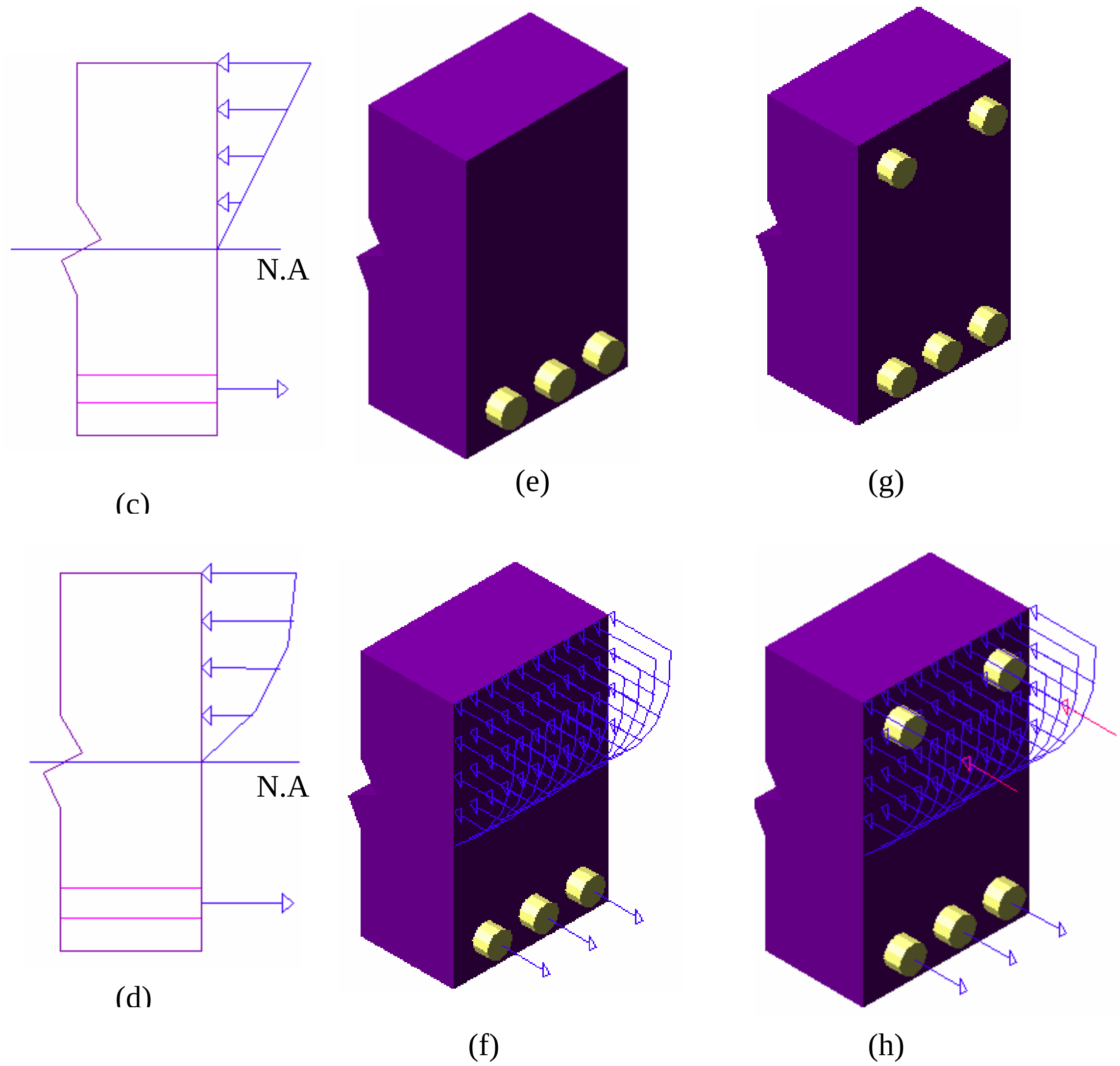


Figure 4.1: Bending of beam (continued)

If we look at section A-A and if it is singly reinforced the compression will be taken entirely by the concrete and tension will be carried by the reinforcement provided. The concrete portion below the neutral axis will not carry any tension what so ever and will be cracked ultimately. This portion of the concrete acts as a part of section, cover for reinforcement, carries shear and diagonal forces and keeps the reinforcement in place.

If we consider the beam to be doubly reinforced (figure 4.1d & 4.1h) then in addition with the earlier case the reinforcement above the neutral axis will carry the compression with the upper portion of the concrete. Now as an engineer if anyone has to design a beam he will have a certain moment to be sustained by his reinforced concrete section. To find out the extreme stresses at the end fibers the following equation can be used

$$f = \frac{My}{I} \quad (4\text{-}1)$$

Where, f = bending stress at a distance y from neutral axis

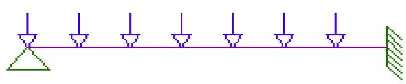
M = external bending moment at the section.

I = moment of inertia of cross section about neutral axis.

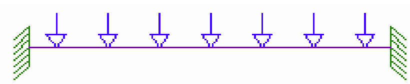
Therefore it is evident to find out the neutral axis of a section, the depth of the reinforcement provided which is termed as the effective depth denoted by d, and as a reinforced concrete section acts in elastically we have to find out the stress distribution at ultimate load.

Beams of different end conditions and load condition:

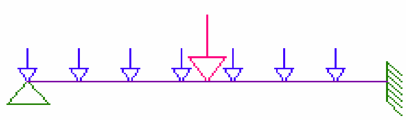
In the subsequent sections the analysis and design of different types of Beam sections are discussed. Beams can be of different end conditions and load condition. Few cases are shown below.



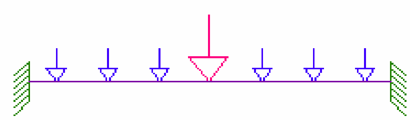
(a) Hinge and fixed with distributed load



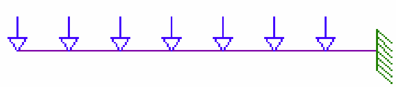
(b) Fixed and fixed with distributed load



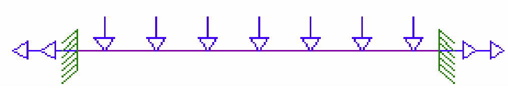
(c) Hinged and fixed with distributed and concentric load



(d) Fixed and fixed with distributed and concentric load



(e) Cantilever with distributed load



(f) Fixed and fixed with both distributed and axial load

Figure 4.2 : End condition and load condition of beam

4.3 DESIGN METHOD

Step 1: Determination of Factored Load

Two types of load will act on the beam. One is the dead load which might come from the dead load of the slab and load from the wall above the beam. The other load is the live load both from slab and on the Beam. Then the ultimate load U is calculated from the following basic formula from *ACI Code 9.2*:

U=1.4 D + 1.7 L (ACI Code-00) (4-2)

U=1.2 D + 1.6 L (ACI Code-02)

Dead load D = self weight of Beam +Portion of slab weight shared by Beam

Live load L = Live load estimated to be faced by the beam

Now the factored load U = 1.4 D+ 1.7 L which will be denoted as w_u in the next step.

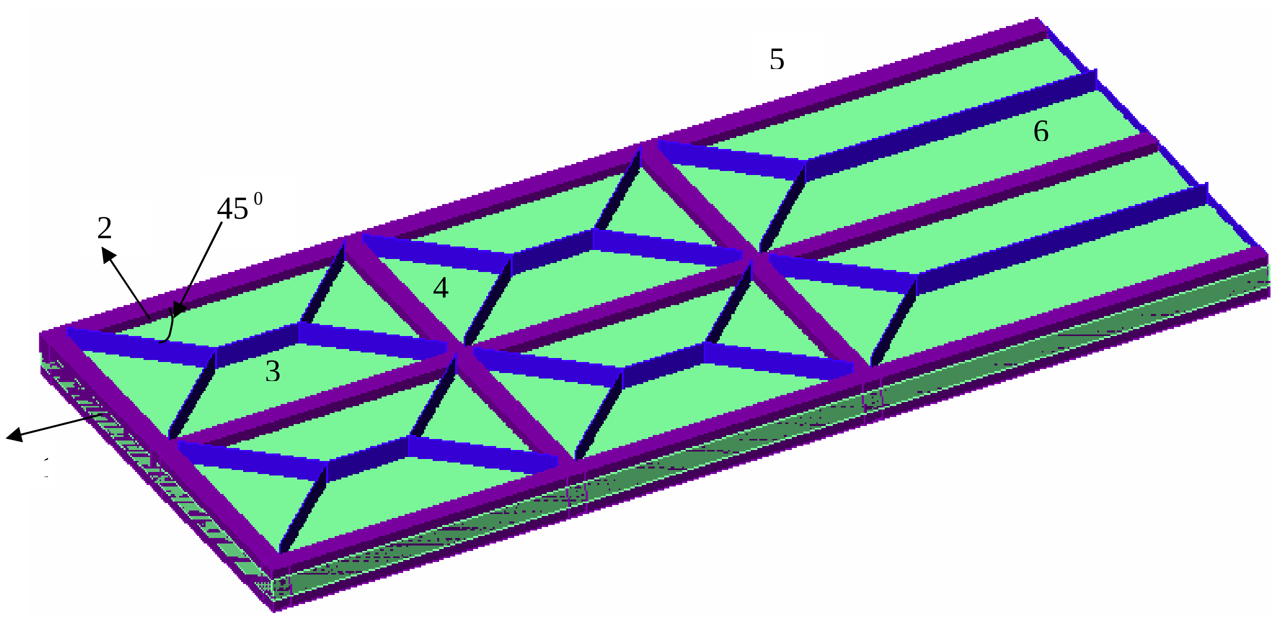
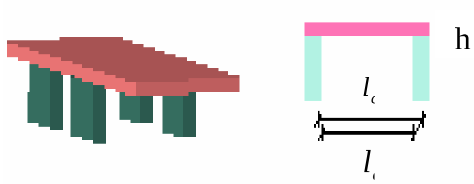


Figure 4.3: Portion of slab affecting the beam loading

Step 2: Determination of Approximate Moments

From Table 3.4 (Chapter-3) coefficients for approximate moment can be obtained.

Span length l_n *(ACI Code 8.7)* :

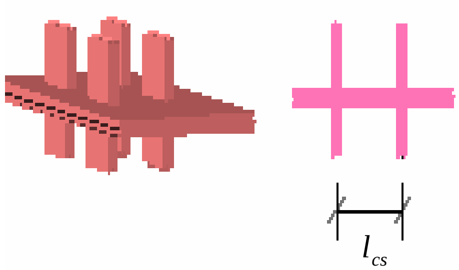


$$l_n = l_c + h \le l_{cs}$$

Span length of members not built integrally with supports shall be considered as the clear span plus the depth of the member, But not exceeding

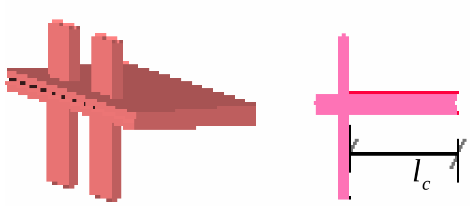
$$l_n = l_c$$

For beams built integrally with supports design on the basis of moments at faces of support shall be permitted.



$$l_n = l_{cs}$$

In analysis of frames or continuous construction for determination of moments, span length shall be taken as the distance center to center of supports.



$$l_n = l_c$$

In case of cantilever the clear span of the beam can be considered as the span length.

Figure 4.4 : Span length

Step 3: Selection of The Cross Sectional Dimension of The Beam

Minimum thickness of h (Figure4.5) is to be selected from Table 4.1.

Table 4.1: Minimum thickness of beam (*ACI Code 9.5.2.1); Table 9.5(a)*

		Minimum thickness h		
	Simply supported	One end continuous	Both end continuous	Cantilever
Member	Members not supporting or attached to portions or other construction likely to be damaged by large deflections			
Beams	l_n /16	l_n /18.5	l_n /21	l_n /8

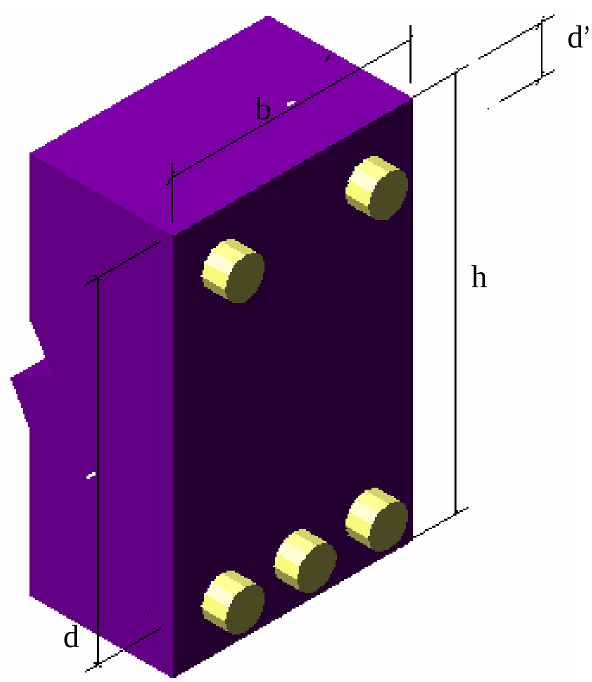


Figure 4.5 : Beam Cross Section

Here, d= h- centroidal axis of tension reinforcement from the extreme tensile edge

And, d′ = distance between the extreme compression fiber to the centroidal axis of the compression steel.

Now b (Figure 4.5) can be calculated from the following equation

$$M_n = \rho f_y bd^2\left(1-0.59\frac{\rho f_y}{f_{c'}}\right) \tag{4-3}$$

Using, M_n = approximate moment

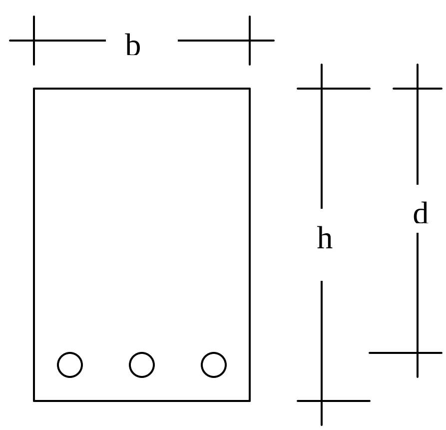
$$\rho = \rho_{max} = \rho_b \times 0.75$$
$$where, \rho_b = 0.85\beta_1\frac{f_{c'}}{f_y}\frac{87000}{87000+f_y} \tag{4-4}$$

The relationship between β_1 and $f_{c'}$ can be shown as

$$\beta_1 = 0.85-0.05\frac{f_{c'}-4000}{1000} \tag{4-5}$$

And $0.65 \le \beta_1 \le 0.85$

Step 4 : Determination whether the Beam has to be Designed as singly or doubly reinforced



Assuming singly reinforced beam,

Steel area, $A_s = \rho bd$

Here,

$$\rho = \rho_{\max} = \rho_b \times 0.75$$

$$where, \rho_b = 0.85\beta_1 \frac{f_{c'}}{f_y}\frac{87000}{87000 + f_y}$$

Now

$$a = \frac{A_s f_y}{0.85 f_{c'} b}$$

And

$M_{n} = A_s f_y (d\text{-}a/2)$

If this M_n > Design M_n, then the beam is to design as singly reinforced beam.

For singly reinforced beam

$$A_s = \frac{M_u}{\phi f_y\left(d - \frac{a}{2}\right)}$$

And $a = \dfrac{\rho f_y d}{0.85 f_{c'}}$

By iteration reinforcement is calculated.

Otherwise doubly reinforced beam. For doubly reinforced beam design see step 5.

Step 5: Determination whether the Beam has to be Designed as Rectangular or a T Beam

Here $a = \dfrac{\rho f_y d}{0.85 f_{c'}}$ (4-6)

If $a < h_f$ then the Beam will act as a rectangular Beam where effective flange width b will be the width of the beam. b_w

If $a > h_f$ then T Beam analysis will be required.

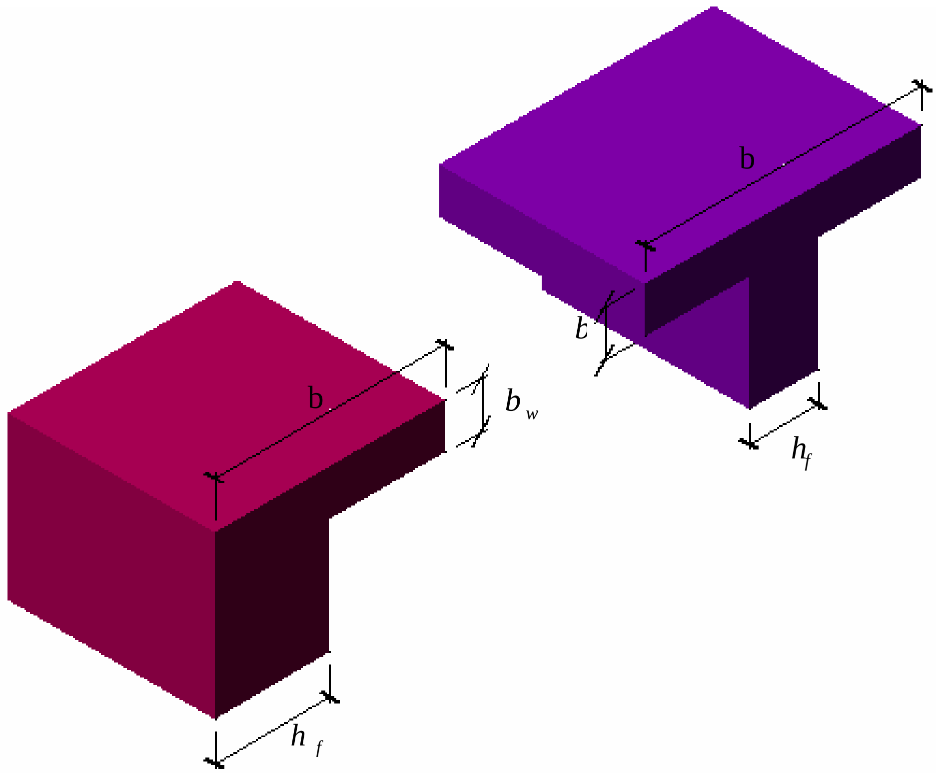


Figure 4.6: T Beam and L Beam Cross Section

Step 6: Rectangular Beam Design

- $A_s = \rho_{\max} bd = A_{s_2}$

  Tension moment, $M_2 = M_n = A_s f_y \left(d - \frac{a}{2} \right)$

  $where, a = \frac{A_s f_y}{0.85 f_{c'} b}$

- Excess extreme moment that has to be resisted is

  Compression moment, $M_1 = \frac{M_u}{\phi} - M_2$

- If $f_{s'} = f_y$ then $A_{s'} = \frac{M_1}{f_y (d - d')} = A_{s_1}$ compression steel area.
- Total tensile steel area $A_s = A_{s_1} + A_{s_2}$

- Then find out

$$\rho_{cy} = 0.85\beta_1 \frac{f_{c'}}{f_y}\frac{d'}{d}\frac{87000}{87000 - f_y} + \rho' \tag{4-7}$$

If $\rho < \overline{\rho}_{cy}$ then compression steel area must be increased. This can be done by the following.

$$a = \frac{\left(A_s - A_{s'}\right) f_y}{0.85 f_{c'} b}$$

$$c = \frac{a}{\beta_1}$$

Now, $f_{s'} = \varepsilon_u E_s \frac{c - d'}{c}$ (4-8)

$$A_{s'revised} = A_{s'trial}\frac{f_y}{f_{s'}} \tag{4-9}$$

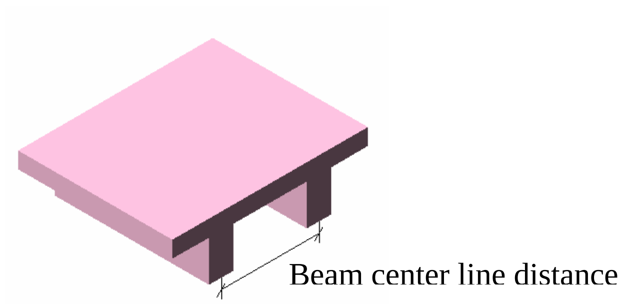
The tensile steel area need not to be revised.

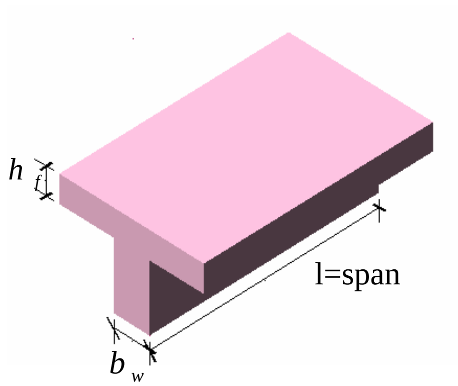
4.4 T Beam Design

The sequence of design for a T Beam is as follows:

- Determining the effective flange width from the lowest governing length from the following conditions. *(ACI Code 8.10.2)*

Ser.No	ACI Code-00	ACI Code-02
1.	$16h_f + b_w$	$8h_f$
2.	$\frac{span}{4}$	$\frac{span}{4}$
3.	½(Centerline Beam spacing)	½(Clear distance to the next web)





b_f = (Beam center line distance)/2

$b_f = 16 h_f + b_w$

b_f =span/4

Figure 4.7: Effective Flange Width

- Web dimension b_w and d are selected through negative bending requirements and shear requirements.
- First assumptions is a= h_f , then the Beam will act as rectangular

Now, $$A_s = \frac{\phi M_u}{\phi f_y \left(d - \frac{a}{2} \right)} \qquad (4\text{-}10)$$

And $$a = \frac{\rho f_y d}{0.85 f_{c'}}$$

Now if $a > h_f$ then T Beam analysis is required.

T and Doubly Beam

- Now using the following equations

$$A_{st} = \frac{0.85 f_{c'} (b - b_w) h_f}{f_y}$$

$$\phi M_{n_1} = \phi A_{sf} f_y \left(d - \frac{h_f}{2} \right)$$

$$\phi M_{n_2} = M_u - \phi M_{n_1}$$

$$A_s - A_{sf} = \frac{\phi M_{n_2}}{\phi f_y \left(d - \frac{a}{2} \right)}$$

T and Singly Beam

- Now using the following equations

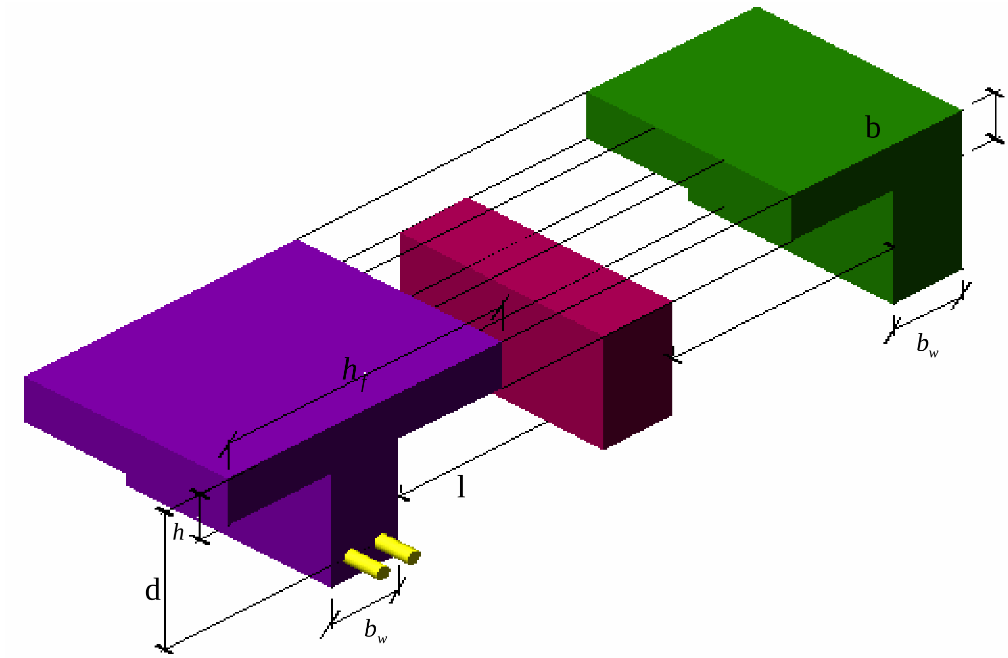
$$A_s = \frac{\phi M_u}{\phi f_y \left(d - \frac{a}{2} \right)}$$

$$a = \frac{\rho f_y d}{0.85 f_{c'}}$$

Where ‘a’ is assumed and by iteration of value of the T Beam is designed.

- Then $\rho_{\max} and \rho_{\min}$ are checked.

Effective flange width



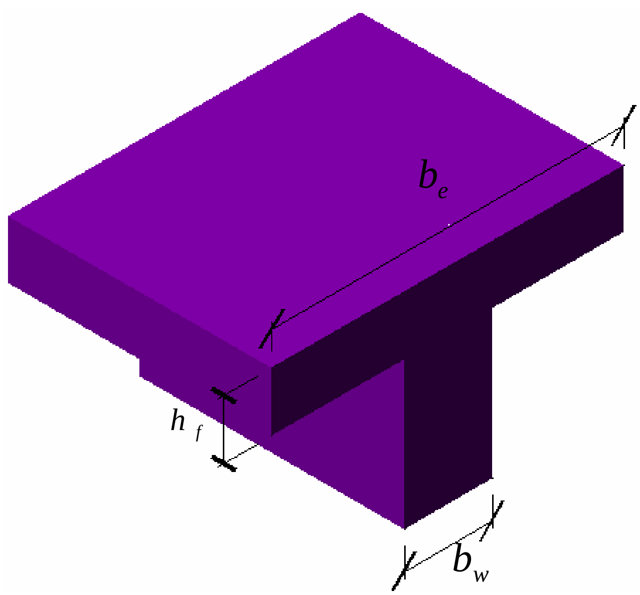


Figure 4.8 : Effective flange width for isolated beam

The effective flange width can be determined by the following relationships from *ACI Code 8.10.*

Beam	'T' beam (Smallest of)	'L' beam (Smallest of)	Isolated beam
ACI Code-00	$b_e \leq \frac{l}{4}; l =$ span length $or, b_e \leq 16h_f + b_w$ $or, b_e \leq$ c/c distance	$b_e \leq \frac{l}{12} + b_w$ $or, b_e \leq 6h_f + b_w$ $or, b_e \leq$ ½(clear distance)	$h_f \geq \frac{b_w}{2}$ $b_f \leq 4b_w$
ACI Code-02	$b_e \leq \frac{l}{4}; l =$ span length $or, b_e \leq 8h_f$ $or, b_e \leq$ ½(clear distance)	$b_e \leq \frac{l}{12}$ $or, b_e \leq 6h_f$ $or, b_e \leq$ ½(clear distance)	$h_f \geq \frac{b_w}{2}$ $b_f \leq 4b_w$

Step 6: Shear Reinforcement

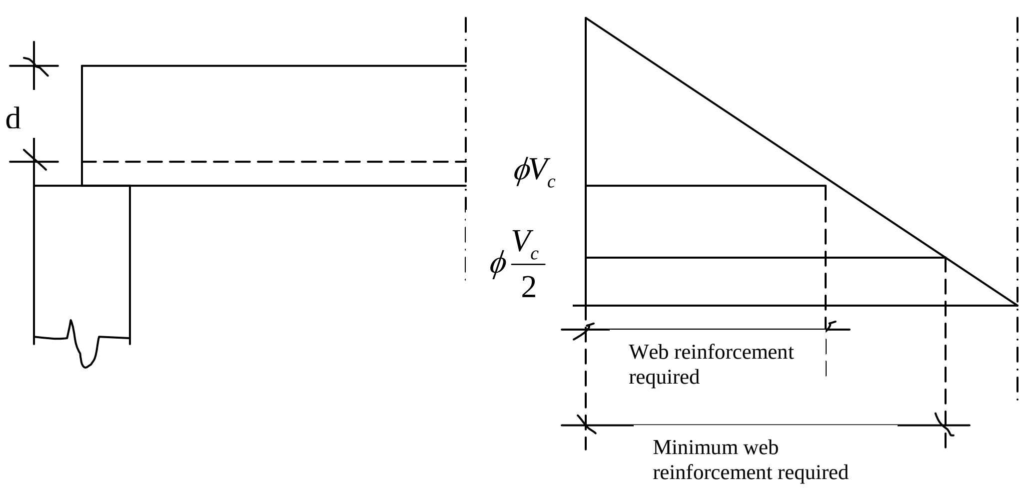


Figure 4.9: Design for Shear

Shear force $V_u = \dfrac{wl}{2}$ is maximum at the edge of the support. The critical section for shear is at distance d from the edge of the support. Concrete can sustain shear up to

$$V_c = 2\sqrt{f_{c'}}\,b_w d \tag{4-11}$$

To get the ultimate shear that can be sustained by the concrete this shear strength need to be multiplied by a reduction factor $\phi = 0.75$ *(ACI Code-9.3.2.3 (02)).*

Now wherever of the Beam the V_u exceeds ϕV_c shear reinforcement need to be provided. The portion of the Beam where V_u exceeds ϕV_c can be determined by superimposing shear diagram and ϕV_c line. Minimum web reinforcement is required wherever shear force exceeds $\dfrac{\phi V_c}{2}$.

Spacing between the shear reinforcements can be determined by the following relationship:

$$s = \frac{\phi A_v f_y d}{V_u - \phi V_c} \tag{4-12}$$

For bent bars:

$$s = \frac{\phi A_v f_y d(\sin\alpha + \cos\alpha)}{V_u - \phi V_c} \tag{4-13}$$

Maximum spacing $s_{\max}$ *(ACI Code-11.5.4.1)* can be determined from the following relations where $V_s \le 4\sqrt{f_{c'}}\,b_w d$

$s_{\max} = 0.75h$

$$s_{\max} = \frac{d}{2}$$

$$s_{\max} = 24 inch$$

$$s_{\max} = \frac{A_v f_y}{50 b_w}$$

The first stirrup is usually placed at s/2 distance from the edge of the support.

Step 7: Design for Torsion (if needed)

Primary Torsion

Primary Torsion also called as equilibrium Torsion exists when the external load has no alternative load path other than being supported by Torsion reinforcement. A cantilever slab is the perfect example for this and is discussed in the followings.

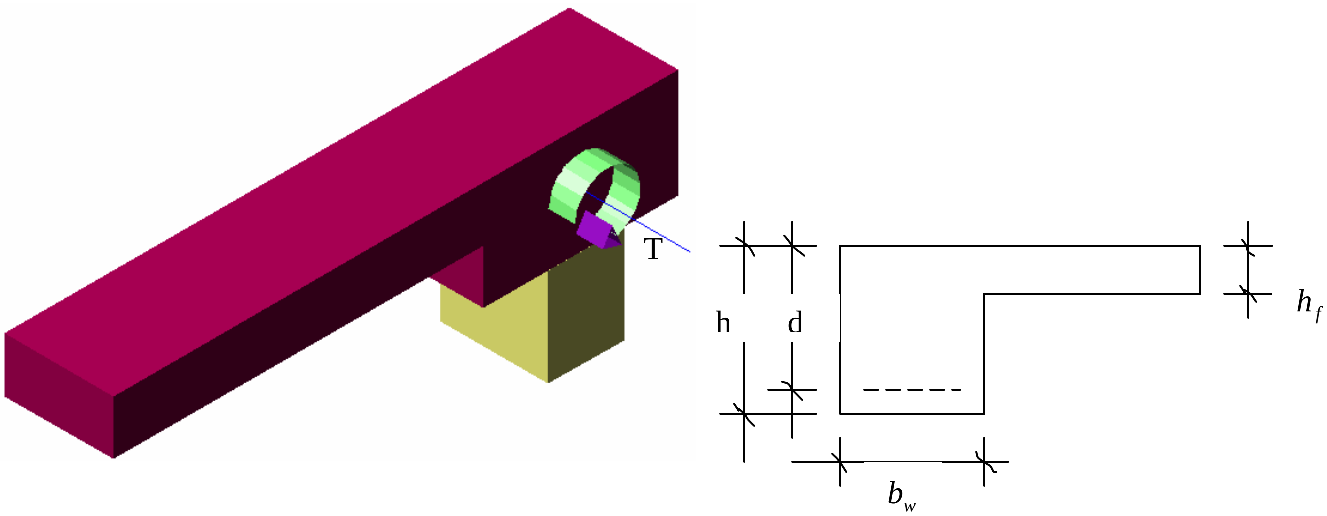


Figure 4.10: Primary Torsion for a Cantilever slab

Slab load

Slab load is not acted eccentrically on the beam. Therefore it creates a torque.

Torque= Slab load× Eccentricity

Here,

Slab load=1.4 w_d +1.7 w_l *(ACI Code-00)*

Slab load=1.2 w_d +1.6 w_l *(ACI Code-02)*

Beam load, w = Slab load + beams own load.

Now at the face of the column the design shear force

$V_u = \dfrac{wl}{2}$ (Here l is the span length of the Beam) (4-14)

At the same location the design Torsional Moment is

$$T_u = \frac{Torque \times l}{2} \tag{4-15}$$

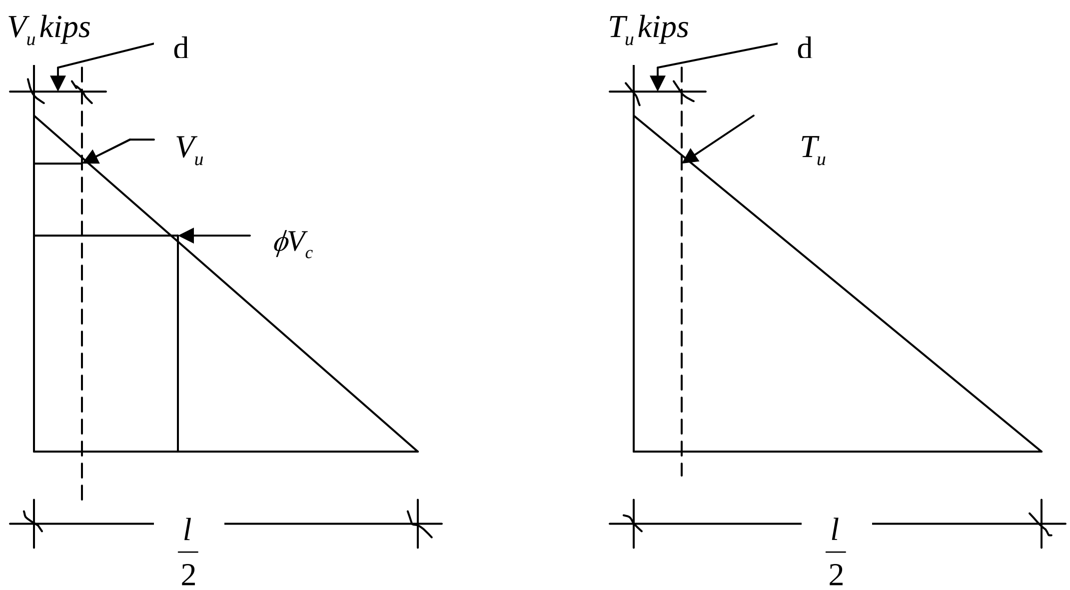


Figure 4.11: Variation of V_u and T_u with distance

The variation of V_u and T_u with distance from the face of the supporting column is given in the figure above. The values of V_u and T_u at the critical design section is at a distance d from the column face.

As per *ACI Code 11.6.1* Torsion can be neglected if the following condition shown in the Table 4.2 is fulfilled.

Table 4.2 Threshold Torsion

Member	Condition (when T_u is less then)
For nonprestressed members	$\phi\sqrt{f_{c'}}\left(\frac{A_{cp}{}^2}{P_{cp}}\right)$
For prestressed members	$\phi\sqrt{f_{c'}}\left(\frac{A_{cp}{}^2}{P_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\sqrt{f_{c'}}}}$
For nonprestressed members subjected to an axial tensile or compressive force	$\phi\sqrt{f_{c'}}\left(\frac{A_{cp}{}^2}{P_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f_{c'}}}}$

Here, A_{cp} = area enclosed by outside perimeter of concrete cross section, in^2

P_{cp} = outside perimeter of the concrete cross section, in.

According to *ACI Code 11.6.1* for L Beam and T Beam some width of the flange contributes in A_{cp} and P_{cp} . This contributing width is the smaller of:

- The projection of the beam above or below the slab, whichever is greater.
- Four times the slab thickness.

If $T_u > \phi\sqrt{f_{c'}}\left(\frac{A_{cp}{}^2}{P_{cp}}\right)$ Torsion must clearly be considered.

Before designing the torsional reinforcement the section need to be checked for adequacy with the following equation.

- For solid sections *(ACI Code- 11.6 3.1)*

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u P_h}{1.7A_{oh}{}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d}+8\sqrt{f_{c'}}\right) \qquad (4.16)$$

- For hollow sections *(ACI Code- 11.6 3.1)*

$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u P_h}{1.7A_{oh}{}^2}\right) \le \phi\left(\frac{V_c}{b_w d}+8\sqrt{f_{c'}}\right) \qquad (4\text{-}16a)$$

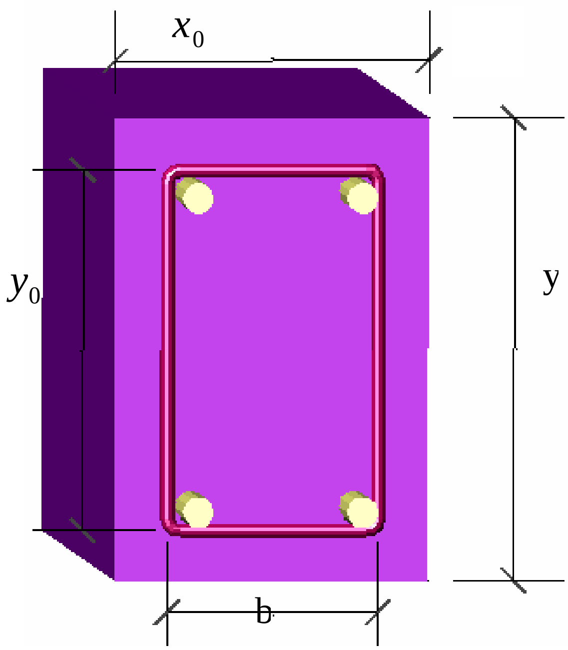


Figure 4.12: Area enclosed by reinforcement

A_{oh} = area enclosed by centerline of the outer most closed transverse torsional reinforcement, $in^2 = x_0 \times y_0$

P_h = perimeter of centerline of outermost closed transverse torsional reinforcement, in

$= 2(x_0 + y_0)$

If the equation satisfies then the cross section is of adequate size. If not the concrete cross section need to be changed.

<u>Transverse tortional reinforcement</u>

Now the value of A_t and A_v will be calculated at the column face from the following equations as per *ACI Code 11.6.3.7*.

$$A_t = \frac{T_u s}{2\phi A_0 f_{yv} \cot\theta}$$ = for one leg of a closed vertical stirrup (4-17)

$$A_v = \frac{(V_u - \phi V_c)s}{\phi f_{yv} d} = \text{To be provided in two vertical legs} \tag{4-18}$$

Here, $A_0 = 0.85A_{oh}$ (4-19)

$30^0 \le \theta \le 60^0$ (45 degree for non-prestressed members) (4-20)

The calculated value of A_t will decrease linearly to zero at the mid span, and value of A_v will decrease linearly to zero up to the point at which $V_u = \phi V_c$.

From this relation the total area to be provided by the two vertical legs can be shown as

$$2A_t + A_v = k_1 s\left(1 - \frac{x}{l/2}\right) + k_2 s\left(1 - \frac{x}{l_1}\right) \quad \text{for } 0 \le x \le l_1 \tag{4-21}$$

$$2A_t + A_v = k_1 s\left(1 - \frac{x}{\frac{l}{2}}\right) \quad \text{for } l_1 \le x \le \frac{l}{2} \tag{4-22}$$

Here, $k_1 = \dfrac{2T_u}{2\phi A_0 f_{yv} \cot\theta}$

$k_2 = \dfrac{(V_u - \phi V_c)}{\phi f_{yv} d}$

x = The distance from the face of the support.

l = Span length of the Beam.

l_1 = Distance from the face of the support up to the point at which $V_u = \phi V_c$.

s = Spacing.

From the above relations the spacing at any required interval can be determined.

As per *ACI Code 11.6.6* the maximum spacing is the smaller of $\frac{p_h}{8}$ or 12 in. On the other hand maximum spacing for shear reinforcement is $\frac{d}{2} or \le 24in$. The most restrictive provision should be maintained.

According to the *ACI Code 11.6.6.3* stirrups may be discontinued at the point where $V_u < \phi \frac{V_c}{2}$ or $(b_t + d)$ beyond the point at which $T_u < \phi \sqrt{f_{c'}} \left(\frac{{A_{cp}}^2}{p_{cp}} \right)$. The minimum web reinforcement should be $50 \frac{b_w s}{f_y}$.

Longitudinal Torsional Reinforcement

$$A_l = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yl}} \right) \cot^2 \theta \qquad \text{From } \textit{ACI Code 11.6.3.7} \qquad (4\text{-}23)$$

With a Total not less than

$$A_{l\,\min} = \frac{5\sqrt{f_{c'}} A_{cp}}{f_{yl}} - \left(\frac{A_t}{s} \right) p_h \frac{f_{yv}}{f_{yl}} \qquad \text{From } \textit{ACI Code 11.6.5.3} \qquad (4\text{-}24)$$

Where, $\frac{A_t}{s}$ shall not be taken less than $25 \frac{b_w}{f_{yv}}$.

According to *ACI Code 11.6.6* the spacing must not exceed 12 in. Bars shall have a diameter at least 0.042 times the stirrup spacing, but not less than a No.3 bar. Reinforcement should be placed at the top, mid depth, and bottom of the member.

Secondary Torsion

Secondary Torsion also called Compatibility Torsion arises from the requirements of continuity.

For the secondary Torsion As per *ACI Code 11.6.2* torsion can be neglected if the following condition shown in the table 4.3 is fulfilled.

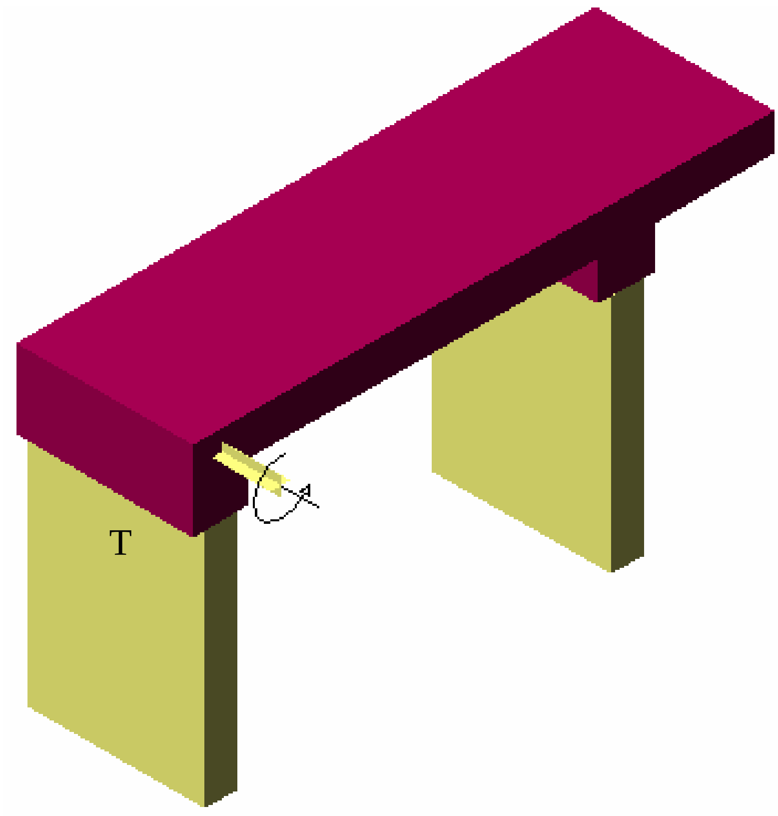


Figure 4.13 : Secondary Torsion from Continuous Slab

Table 4.3 Threshold Torsion for Statically Indeterminate Structure

Member	Condition(when T_u is less then)
For nonprestressed members	$\phi 4\sqrt{f_{c'}}\left(\frac{A_{cp}{}^2}{P_{cp}}\right)$
For prestressed members	$\phi 4\sqrt{f_{c'}}\left(\frac{A_{cp}{}^2}{P_{cp}}\right)\sqrt{1+\frac{f_{pc}}{4\sqrt{f_{c'}}}}$
For nonprestressed members subjected to an axial tensile or compressive force	$\phi 4\sqrt{f_{c'}}\left(\frac{A_{cp}{}^2}{P_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f_{c'}}}}$

If the conditions do not satisfy then an internal adjustment of the cross section of the Beam is carried out to reduce the maximum torque to fulfill the following condition

$$T_u \leq \phi 4\sqrt{f_{c'}}\left(\frac{{A_{cp}}^2}{p_{cp}}\right) \qquad (4\text{-}25)$$

with the moments and the shears in the supported members adjusted accordingly.

Step 8: Checks for Serviceability

Cracking in Beams

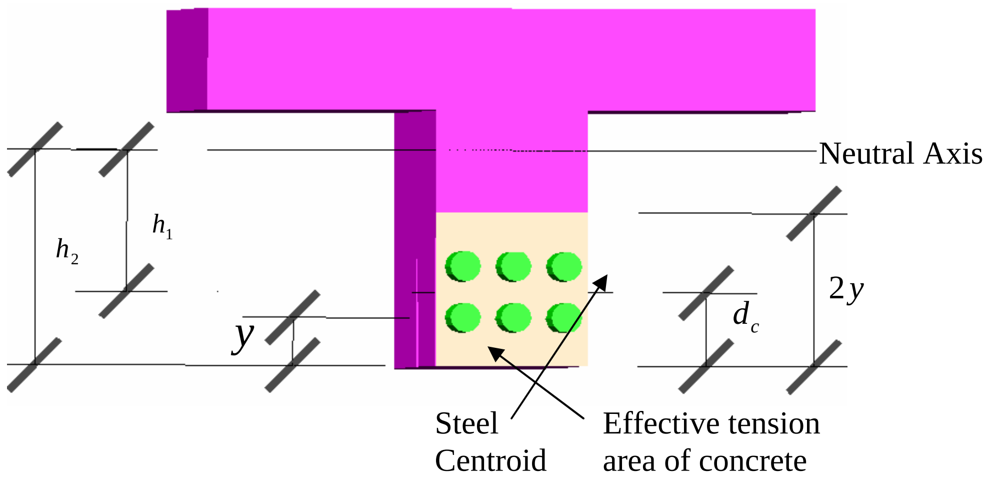
Width of the crack of a Beam at the tension face as per Gergely- Lutz equation is

$$w = 0.076\beta f_s \sqrt[3]{d_c A} \qquad (4\text{-}26)$$

Where, d_c = Thickness of concrete cover measured from tension face to center of bar closest to that face, in.

β = Ratio of distances from tension face and from steel centroid to neutral axis equal to $\frac{h_2}{h}$.

A= Concrete area surrounding one bar equal to total effective tension area of concrete surrounding reinforcement and having same centroid, divided by number of bars, in^2.



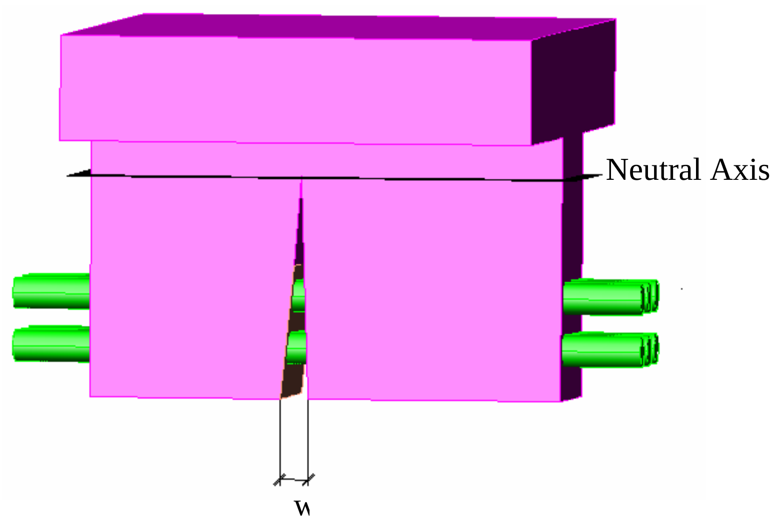


Figure 4.14: Geometric basis of crack width calculations

Table 4.4 Tolerable crack width for reinforced concrete ("Control of cracking in concrete structure" ACI committee 224, 1995)

Exposure Condition	Tolerable crack width	
	In	mm
Dry air or protective membrane	0.016	0.41
Humidity, Moist air, soil	0.012	0.30
Deicing chemicals	0.007	0.18
Sea water and sea water spray, wetting and drying	0.006	0.15
Water retaining structures excluding non pressure pipes	0.004	0.10

Deflection

Deflection of the Beam can be sustained by providing certain minimum thickness. The following table describes the requirements.

Table 4.5 Minimum thickness of non pre stressed beams (*ACI Code 9.5.2.1*)

		Minimum thickness		
	Simply supported	One end continuous	Both end continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one way slab	$\frac{l_n}{20}$	$\frac{l_n}{24}$	$\frac{l_n}{28}$	$\frac{l_n}{10}$
Beams or ribbed one way slab	$\frac{l_n}{16}$	$\frac{l_n}{18.5}$	$\frac{l_n}{21}$	$\frac{l_n}{8}$

Notes:

1. Span length l is in inches.

2. Values given shall be used directly for members with normal weight concrete (w_c =145 lb/ ft^3) and Grade 60 reinforcement. For other conditions the values shall be modified as follows:

a) For structural lightweight concrete having unit weight in the range 90-120 lb/ft^3 the values should be multiplied by (1.65-0.005 w_c) but not less than 1.09, where w_c is the unit weight in lb/ft^3.

b) For f_y other than 60000 psi, the values shall be multiplied by $(0.4+\frac{f_y}{100,000})$

3. If l = l_n then follow step-2 and figure 4.4

Step 9: Detailing

Detailing of the reinforcement in the beam is one of the most important portions of Beam designing. The following figure describes the Bar cut off requirements. According to *ACI Fig. R12.10.2*

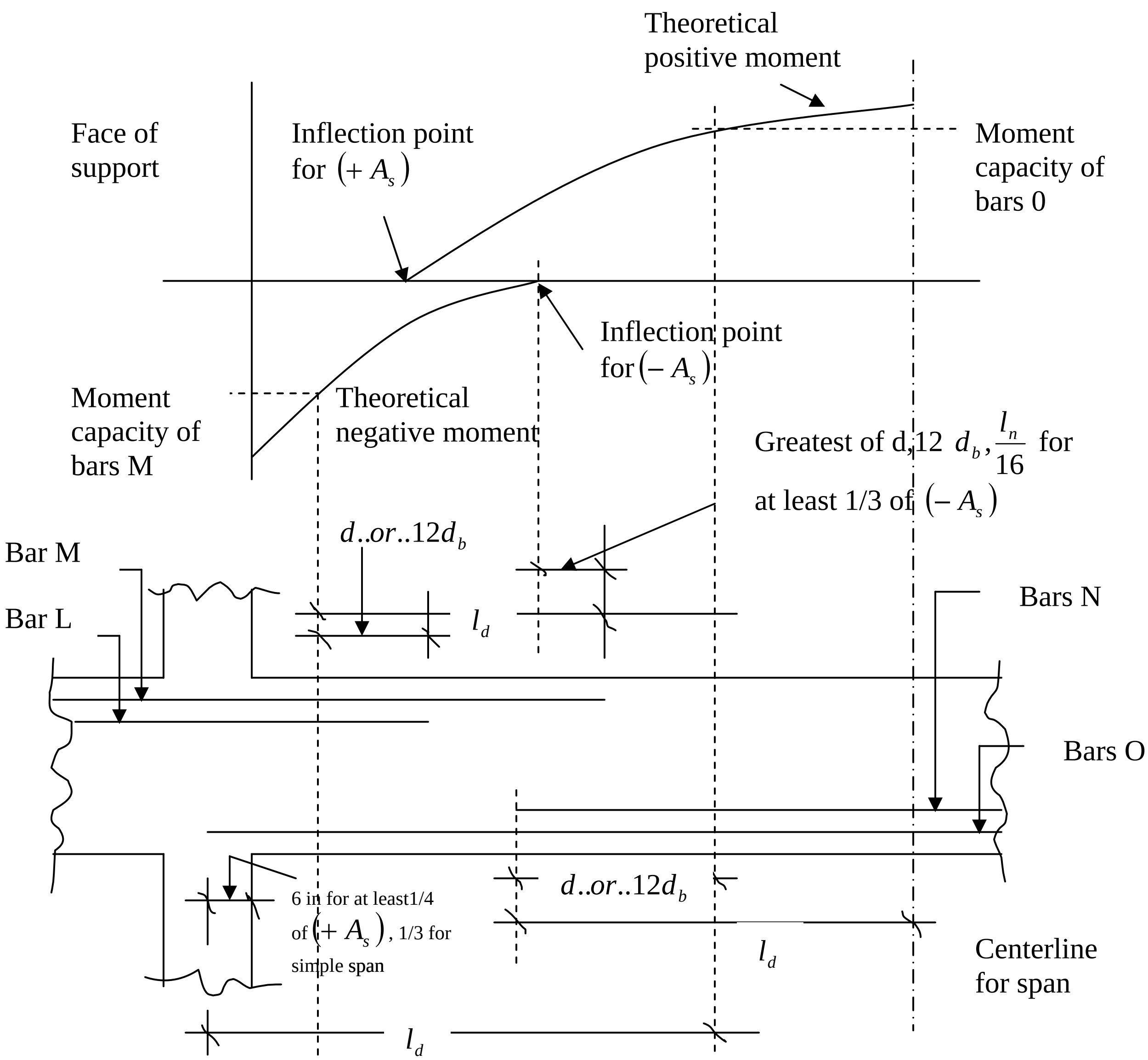


Figure 4.15 (a) Bar cut off requirements

The following text will guide the readers in accurate detailing.

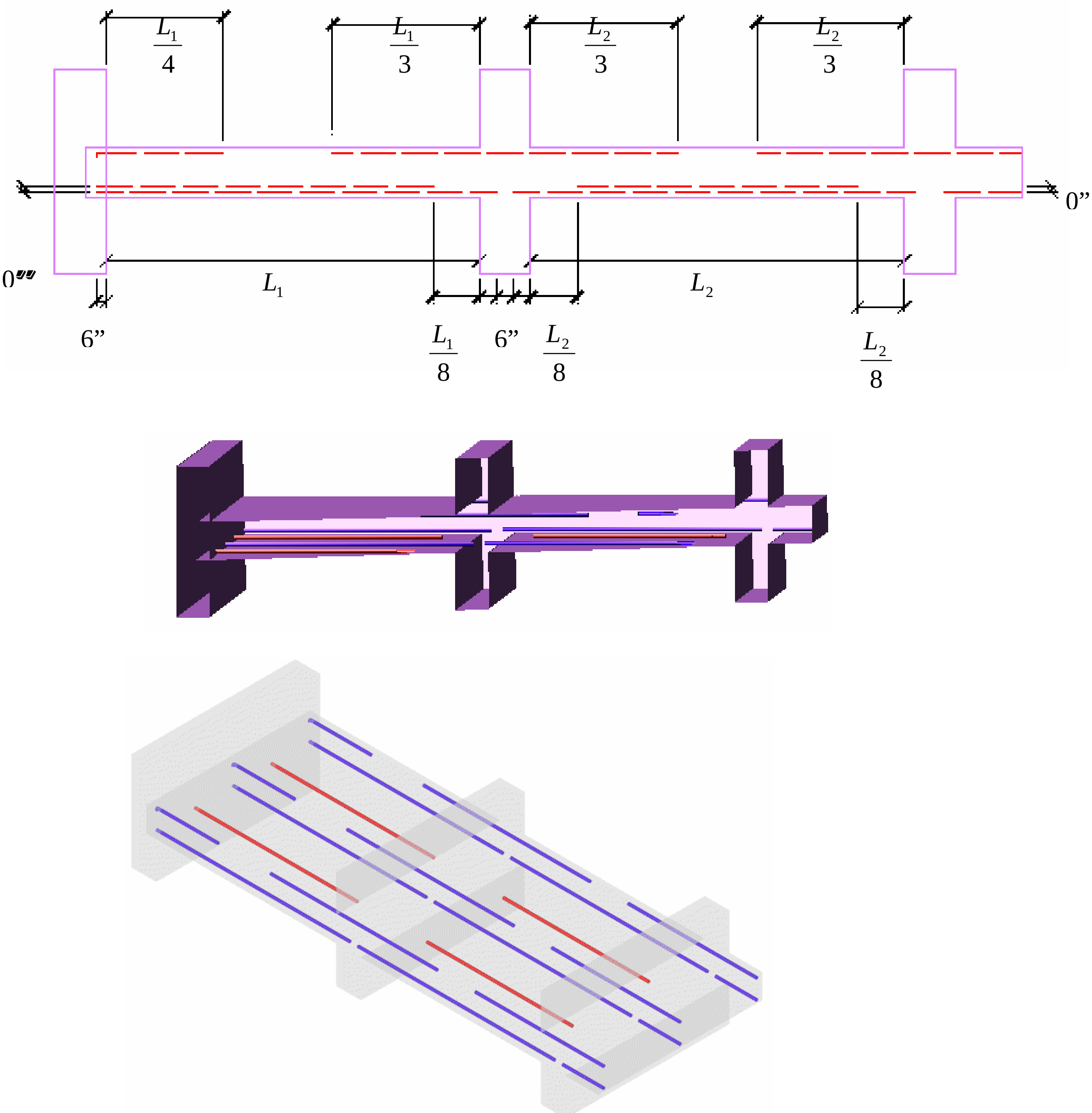


Figure 4.15(b): Standard cutoff points for bars in approximately equal spans with uniformly distributed load

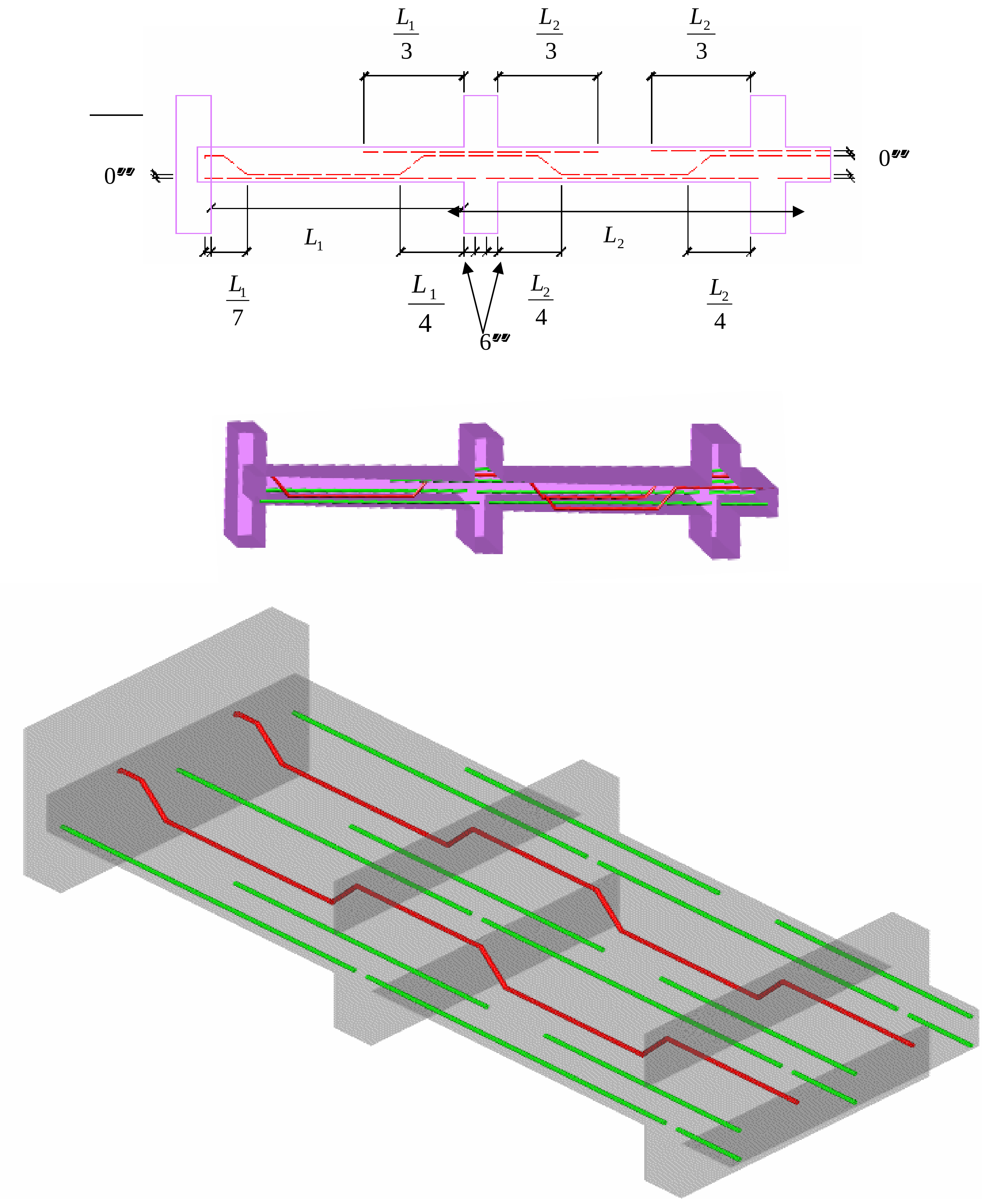
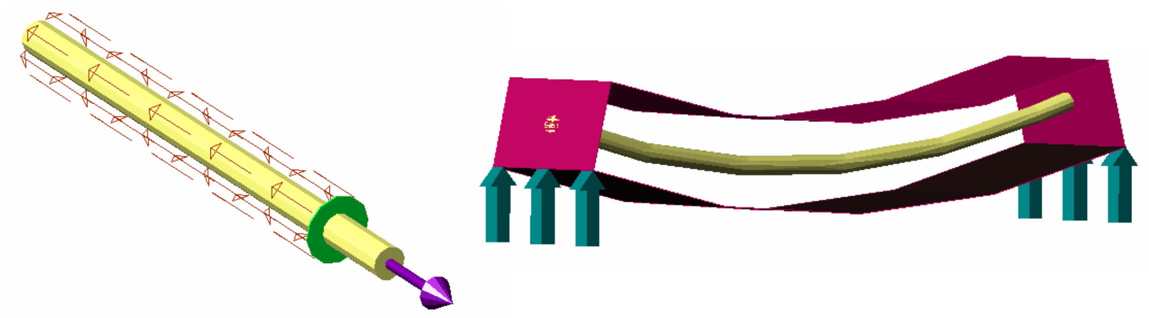


Figure 4.15(c) Standard bend points for bars in approximately equal spans with uniformly distributed

Ultimate Bond Strength



T= $A_b f_s$

Figure 4.16 (a): Bond Strength

Steels are very rarely pulled directly rather it is stressed due to the flexural bending of the Beam. When the steel is stressed in tension the surrounding concrete at the near vicinity of the steel tries to resist this tension. But at certain limit the surrounding concrete gets crushed and the steel slips out and results the ultimate failure. This is the ultimate bond strength of the concrete Beam.

Development Length

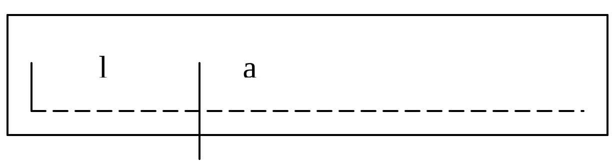


Figure 4.16 (b): Development Length

At point ‘a’ in Figure 4.16 the steel stress is $A_b f_s$ which is transferred to the concrete by distance l. But for ultimate condition steel stress at point a will be $A_b f_y$ the length has to be equal to a development length which is already established by several tests. The length is denoted by l_d and called as development length.

- Development length can be determined from the following relation *(ACI Code-12.2.3; ACI Code 00)*

$$\frac{l_d}{d_b} = \frac{3}{40}\frac{f_y}{\sqrt{f_{c'}}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)} \qquad (4\text{-}27a)$$

- Development length can be determined from the following relation *(ACI Code-12.2.3; ACI Code 02)*

$$\frac{l_d}{d_b} = \frac{9}{10}\frac{f_y}{\sqrt{f_{c'}}}\frac{\alpha\beta\gamma\lambda}{\left(\frac{c+k_{tr}}{d_b}\right)} \qquad (4\text{-}27b)$$

Here, $\frac{(c+k_{tr})}{d_b} \le 2.5$ And, $l_d \ge 12$ inch

Specified tension development length in bar diameter according to *ACI Code 12.2.2*

	No. 6 and smaller bars and deformed wires	No. 7 and larger bars
Clear spacing of bars being developed or spliced not less than d_b clear cover not less than d_b and stirrups or ties throughout l_b not less than the code minimum or Clear spacing of bars being developed or spliced not less than $2d_b$ and clear cover not lea than d_b	$\left(\frac{f_y'\alpha\beta\gamma}{25\sqrt{f_c'}}\right)d_b$	$\left(\frac{f_y'\alpha\beta\gamma}{20\sqrt{f_c'}}\right)d_b$
Other cases	$\left(\frac{3f_y'\alpha\beta\gamma}{50\sqrt{f_c'}}\right)d_b$	$\left(\frac{3f_y'\alpha\beta\gamma}{40\sqrt{f_c'}}\right)d_b$

Table 4.6: Different types of factors

Factors	Conditions	Values
α = Reinforcement location factor	Horizontal reinforcement so placed that more then 12 in of fresh concrete is cast in the member below the development length osplice	1.3
	Other reinforcement	1.0
β = Coating Factor	Epoxy- coated bars or wires with cover less than $3d_b$ or clear spacing less than $6d_b$	1.5
	All other epoxy-coated bar or wires	1.2
	Uncoated reinforcement	1.0
γ =Reinforcement size factor	No 6 and smaller bars and deformed wires	0.8
	No 7 and larger bars	1.0
λ =Light weight aggregate concrete factor	When light weight aggregate concrete is used	1.3
	However, when f_{ct} is specified λ shall be permitted to be taken as $6.7\frac{\sqrt{f_{c'}}}{f_{ct}}$, But not less than	1.0
	When normal weight concrete is used	1.0

However the product $\alpha\beta$ need not be taken greater than 1.7.

C=Spacing or cover dimension, in.

Use the smaller of either the distance from the center of the bar or wire to the nearest concrete surface or one half of the center-to-center spacing of the bars or wires being developed.

$$k_{tr} = \text{Transverse reinforcement index.} = \frac{A_{tr} f_y t}{1500sn} \quad (4\text{-}28)$$

It shall be permitted to use $k_{tr}=0$ as a design simplification even if transverse reinforcement is present.

Anchorage *(ACI Code-12.5)*

When the bars cannot be developed by bond alone, anchorage is provided by bending the bars at the ends. Few standard hooks as per *ACI code 7.1* is shown in the figure below:

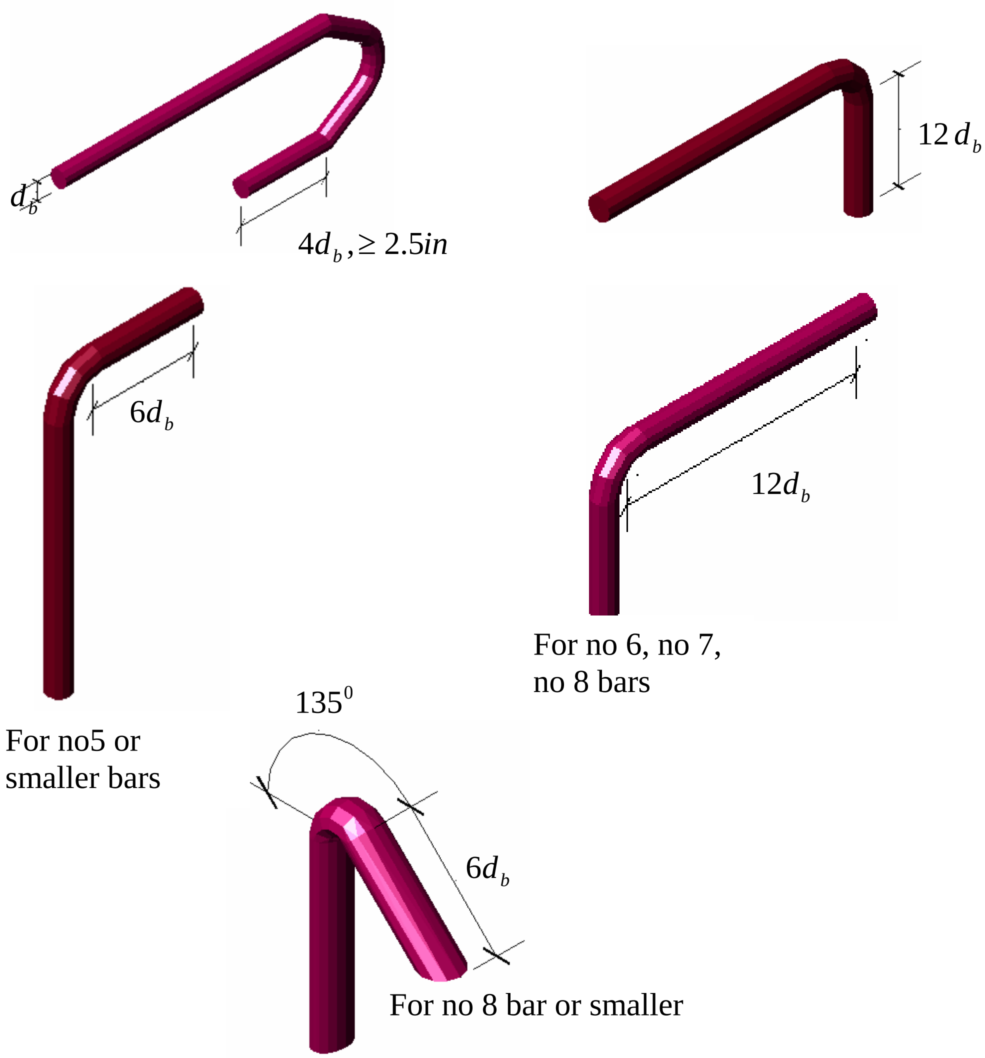


Figure 4.17: Different Types of Anchorage

Diameter of the bends can be determined from the following table.

Table 4.7 Minimum diameter of the bends *(ACI Code-7.2.3)*

Bar size	Minimum diameter
No. 3 Through No.8	$6d_b$
No.9, No.10, and No.11	$8d_b$
No.14 and No.18	$10d_b$

Bar Cut Off

To gain economy the reinforcement in a beam can be curtailed where it is not required. This can be found out from the following relations.

$$T = A_s f_s = \frac{M}{Z} \tag{4-29}$$

From the equation above percentage of A_s required can be calculated and then by using the figure below the point of Bar cut off can be determined.

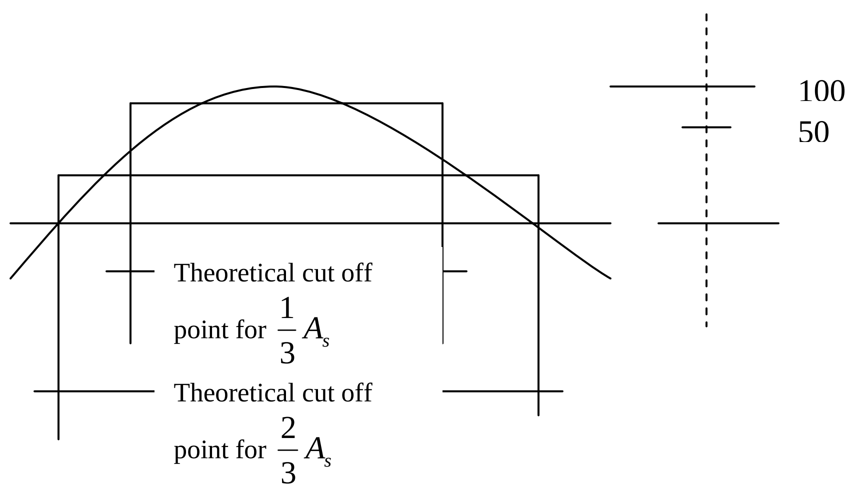
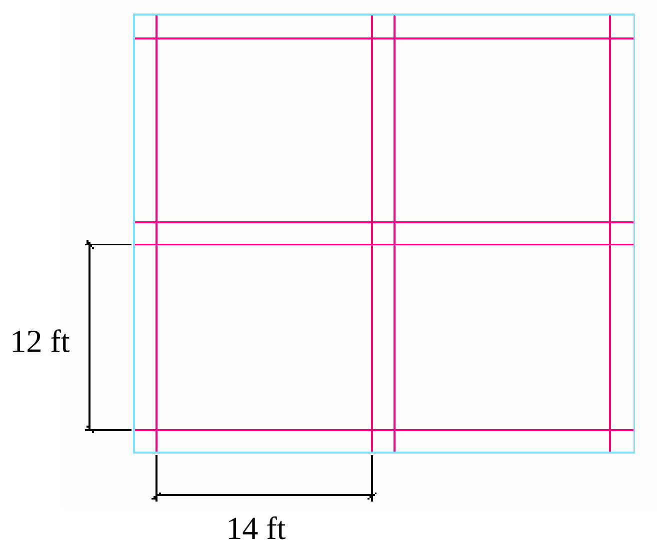


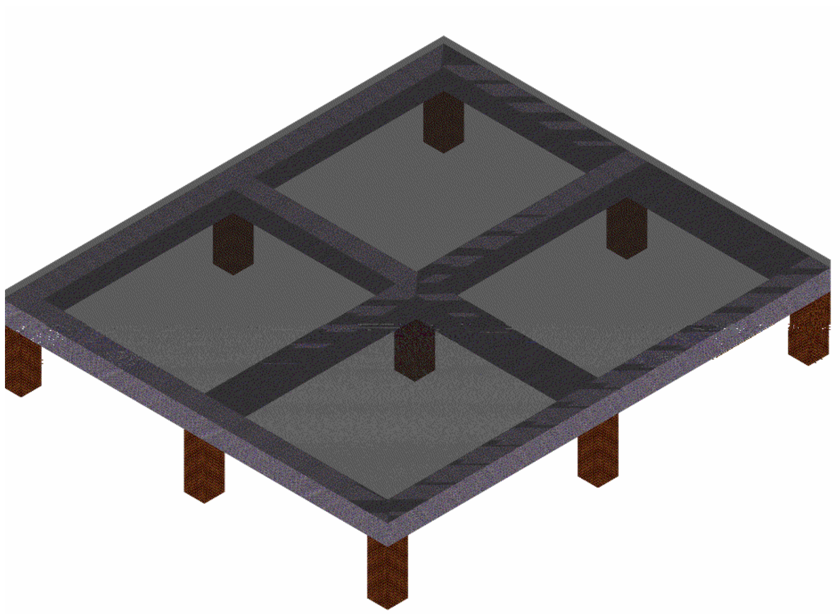
Figure 4.18: Bar cut off points

For practical point of view the bars should be extended beyond the theoretical bar cutoff points.

4.5 PROBLEM ON DESIGN OF BEAM



Plan



Three dimensional view

Figure 4.19: Beam dimensions

The figures of the previous page have depicted a typical beam used in a multi- storied building. The following steps can be followed to reach a reasonable solution for design of the said beam.

Given data:

- Length of beam = 14 ft.
- End condition = One end continuous.
- Live load on slab = 100 psf.
- $f_{c'}$ = 3000 psi.
- f_y =60000 psi.

Step 1: Determination of load on the Beam

Normally a beam will carry the following loads:

- Its self weight. The unit weight of concrete will depend upon the aggregates used in the mixture.

For determining the volume of the beam the cross section of the beam need to be assumed. As a thumb rule 1 inch x 1 inch section for each feet of length is assumed.

So, for this beam let assume a 14 inch X 14 inch section

Volume of beam $= \dfrac{14}{12} \times \dfrac{14}{12} \times 14 = 19.05 cft$

$\therefore deadload = 19.05 \times 145 \times 1.4 = 3867.15\, pcf$ *(ACI Code 00)*

$= 19.05 \times 145 \times 1.2 = 3314.70\, pcf$ *(ACI Code 02)*

Now load per feet $= \dfrac{3867.15}{14 \times 1000} = 0.28$ k/ft *(ACI Code 00)*

$= \dfrac{3314.70}{14 \times 1000} = 0.23$ k/ft *(ACI Code 02)*

Table 4.8 Factored load combination for determining required

Strength U in the *ACI Code 9.2.1*

Condition	Factored load or load effect U *(ACI Code-02)*	Factored load or load effect U *(ACI Code)*
Basic	U=1.2D+1.6L	U=1.4D+1.7L
Winds	U=1.2D+1.6(R or S)+0.8W U=0.9D+1.6W+1.6H U=1.2D+1.6W+1.0L+0.5(S or R)	U=0.75(1.4D+1.7L+1.7W) And include consideration of L=0 U=0.9D+1.3W U=1.4D+1.7L
Earthquake	U= 1.2D+1.0L+1.0E+0.2S U=0.9D+1.0E+1.6H	U=0.75(1.4D+1.7L+1.87E) And include consideration of L=0 U=0.9D+1.43E U=1.4D=1.7L

Therefore Self weight of this Beam = 0.28 k/ft *(ACI Code 00)*

=0.23 k/ft *(ACI Code 02)*

- Slab load can be calculated by adding the dead load of the slab and live load on it. Then the portion of the load that will be carried by one beam can be determined. This is described in a pictorial form in the following figure. The longer arm is shown as b and the shorter is by a. The corners are divided into two each as 45 degree. Then a triangular form along the shorter side and a trapezoidal form along the longer side will form. Now the volume of the portions of the slab is calculated and dead load is determined. The live loads can be determined from table 3.3.

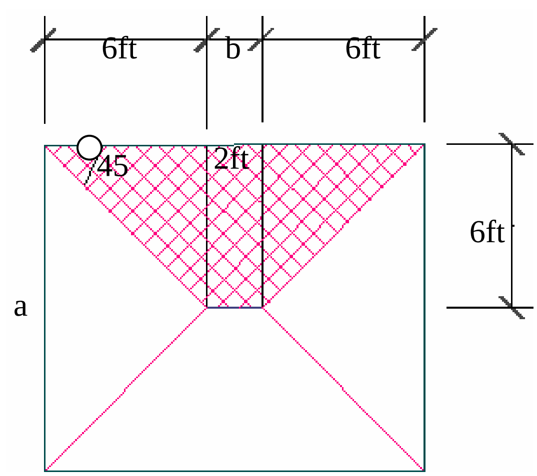


Figure 4.20: Slab Portion that affects the Beam

Portion of slab area= $2 \times 6 + 2 \times \frac{1}{2} \times 6 \times 6 = 48$ sft

Slab thickness= 6 inch=0.5 ft

Volume of the slab portion= $48 \times 0.5 = 24$ cft

So dead load= $24 \times 145 = 3480$ lbs

Service live load from table 3.3 for residential dwellings (adding all type of live loads) =100 psf (from Appendix B)

Total live load= $48 \times 100 = 4800$ lbs

Factored load U=1.4D+1.7L= $1.4 \times 3480 + 1.7 \times 4800 = 13032$ lbs *(ACI Code 00)*

=1.2D+1.6L = $1.2 \times 3480 + 1.6 \times 4800 = 11856$ lbs *(ACI Code 02)*

Now the uniformly distributed load for the Beam is

$$w_u = \frac{13032}{14 \times 1000} = 0.93 \text{ k/ft.}$$ *(ACI Code 00)*

$$w_u = \frac{11856}{14 \times 1000} = 0.85 \text{ k/ft.}$$ *(ACI Code 02)*

- Wall load: Normally the beams of the interior floors will carry the loads of the walls those are built on the slabs and those walls constructed just on the beam.

Wall load =Area of the wall *load in psf * load factor (dead load)

Now total Load=Self weight of the beam+ slab load+ wall load

For this case

Total load, w_u = 0.28+0.93=1.21k/ft *(ACI Code 00)*

= 0.23+0.85=1.08k/ft *(ACI Code 02)*

Step 2: Determination of approximate moment

Span Length

$l_n = l_c + h$ =Clear distance + depth of slab

$$\therefore l_n = 14 + \frac{6}{12} = 14.5$$

Approximate Moment (From Table 3.4)

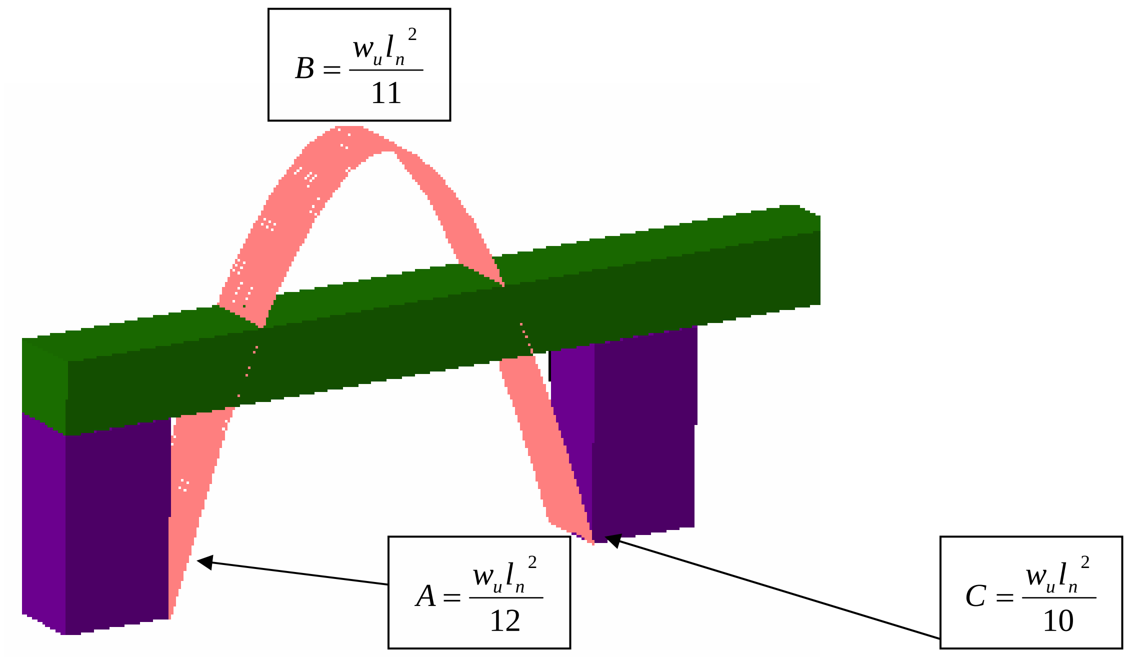


Figure 4.21: Approximate Moment for the Problem

Moment at A, $M_A = -\frac{w_u l_n^2}{12} = -\frac{1.21 \times 14.5^2}{12} = -21.2$ ft-kips. *(ACI Code 00)*

$$M_A = -\frac{w_u l_n^2}{12} = -\frac{1.08 \times 14.5^2}{12} = -18.92 \text{ ft-kips.} \quad (ACI\ Code\ 02)$$

Moment at B, $M_B = \frac{w_u l_n^{\,2}}{11} = \frac{1.21 \times 14.5^2}{11} = 23.13$ ft-kips. *(ACI Code 00)*

$M_B = \frac{w_u l_n^{\,2}}{11} = \frac{1.08 \times 14.5^2}{11} = 20.64$ ft-kips. *(ACI Code 02)*

Moment at C, $M_c = -\frac{w_u l_n^{\,2}}{10} = -\frac{1.21 \times 14.5^2}{10} = -25.44$ ft-kips. *(ACI Code 00)*

$M_c = -\frac{w_u l_n^{\,2}}{10} = -\frac{1.08 \times 14.5^2}{10} = -22.70$ ft-kips. *(ACI Code 02)*

The Beam should be designed for highest negative moment at C, that is 25.23 ft-kips *(ACI Code 00)* and 22.70 *(ACI Code 02).*

$\therefore M_u = 25.44 \times 12 = 305$ in-kips. *(ACI Code 00)*

$\therefore M_u = 22.70 \times 12 = 272$ in-kips. *(ACI Code 02)*

Step 3: Selection of the cross sectional dimension of the Beam

Thickness

Minimum thickness from table 4.5

$\frac{l}{18.5} = \frac{14.5x12}{18.5} = 9.40$ in = 10 in.

Width

Here,

M_u = 305 in-kips *(ACI Code 00)*

= 272 in-kips *(ACI Code 02)*

$\rho = \rho_{\max} = 0.75 \times \rho_b$

$= 0.75 \times 0.85 \times \beta_1 \times \frac{f_{c'}}{f_y} \times \frac{87000}{87000 + f_y}$

$$= 0.75 \times 0.85 \times 0.85 \times \frac{3000}{60000} \times \frac{87000}{87000 + 60000} \text{ (For } f_{c'} = 3000 \text{ psi. } \beta_1 = 0.85)$$

From equation

$$M_u = \phi \rho f_y b d^2 \left(1 - 0.59 \rho \frac{f_y}{f_{c'}}\right)$$

$$\therefore b = \frac{M_u}{\phi \rho f_y d^2 \left(1 - 0.59 \rho \frac{f_y}{f_{c'}}\right)}$$

$$= \frac{305}{0.9 \times .016 \times 60 \times 7.5^2 \left(1 - 0.59 \times 0.016 \frac{60}{3}\right)}$$

$= 7.73 \approx 8$ in. *(ACI Code 00)*

$$= \frac{272}{0.9 \times .016 \times 60 \times 7.5^2 \left(1 - 0.59 \times 0.016 \frac{60}{3}\right)}$$

$= 6.90 \approx 8$ in. *(ACI Code 02)*

Effective depth

Effective depth d=Thickness-clear cover=10-2.5=7.5 in.

If we add the slab thickness h_f =6 in with the minimum Beam thickness 10 in,

Then we get total depth, h=16 in

Effective depth=13.5 in.

And web width, b=8 in.

Step 4: Determination whether the beam has to be designed as Rectangular Beam or a T Beam

<u>Depth of the centroidal axis</u>

$$a = \frac{\rho f_y d}{0.85 f_{c'}} = \frac{0.016 \times 60 \times 13.5}{0.85 \times 3} = 5.08 \text{ in.}$$

as $a < h_f$

So the beam will be designed as rectangular beam which would have the following cross section.

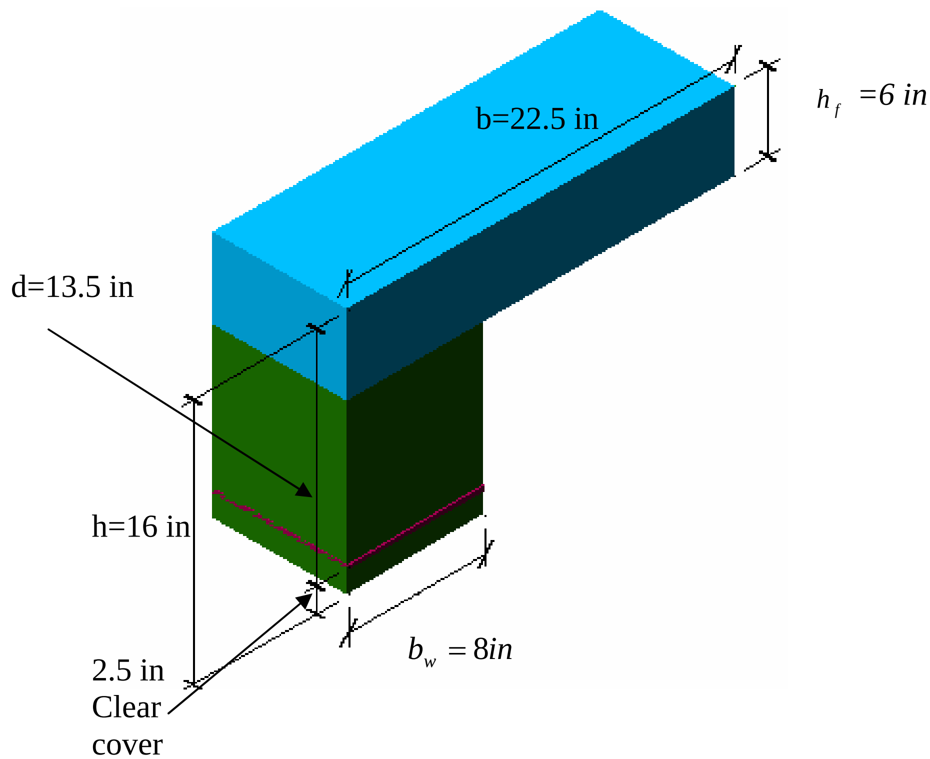


Figure 4.22: Beam Cross Section *(ACI Code 00)*

Effective flange width b

According to *ACI Code 00* smallest of the following three conditions will determine the effective flange width b.

a. $b = b_w + \frac{l}{12} = 8 + \frac{14.5}{12} \times 12 = 22.5$ in.

b. $b = 6h_f + b_w = 6 \times 6 + 8 = 44$ in.

c. *b = ½(Clear distance) =1/2x14 x 12 = 84 in.*

So effective flange width b=22.5 in.

According to *ACI Code 02* smallest of the following three conditions will determine the effective flange width b.

a. $b = \frac{l}{12} = \frac{14.5}{12} \times 12 = 14.5$ in.

b. $b = 6h_f = 6 \times 6 = 36$ in.

c. *b = ½(Clear distance) =1/2x14 x 12 = 84 in.*

So effective flange width b=14.5 in

Step 5: Rectangular Beam Design

$A_s = \rho_{max} bd$

$A_s = \rho_{max} bd = 0.016 \times 22.5 \times 13.5 = 4.86 in^2$ *(ACI Code 00)*

$A_s = \rho_{max} bd = 0.016 \times 14.5 \times 13.5 = 3.13 in^2$ *(ACI Code 02)*

Now $M_n = A_s f_y \left(d - \frac{a}{2} \right)$

$M_n = 4.86 \times 60 \left(13.5 - \frac{5.08}{2} \right) = 3196$ in-kips. *(ACI Code 00)*

$M_n = 3.13 \times 60 \left(13.5 - \frac{5.08}{2} \right) = 2058$ in-kips. *(ACI Code 02)*

Therefore $A_s = 4.86in^2$ or 4 #10 bar *(ACI Code 00)* and $A_s = 3.13in^2$ *(ACI Code 02)* is provided the beam would not require any compression steel. But minimum reinforcement may be provided as stirrup support bars, let 2 # 10 bar.

Step 6: Shear Reinforcement

$$V_c = 2\sqrt{f_{c'}}b_w d$$

$$= 2\times\sqrt{3000}\times 8\times 13.5 = 1183lb = 11.83 \text{ kips}$$

$$V_u = \frac{wl}{2} = \frac{1.21\times 14.5}{2} = 8.7 \text{ kips.}$$

$$\phi V_c = 0.85\times 11.83 = 10.05 \text{ kips.}$$

Now wherever of the beam the V_u exceeds ϕV_c we need to provide shear reinforcement.

For this case nowhere of the beam the shear exceeds ϕV_c but minimum reinforcement required where shear force exceeds $\frac{\phi V_c}{2} = \frac{10.05}{2} = 5.02$ kips.

As $V_s \le 4\sqrt{f_{c'}}b_w d$

$$S_{\max} = \frac{d}{2} = \frac{13.5}{2} = 6.75in$$

$$S_{\max} = 24in$$

$$S_{\max} = \frac{A_v f_y}{50b_w} = \frac{0.22\times 60000}{50\times 8} = 33in \text{(Using no 3 bar)}$$

So $S_{\max}$ = 6.75 in

First stirrup at $\frac{s}{2} = \frac{6.75}{2} = 3.38$ in.

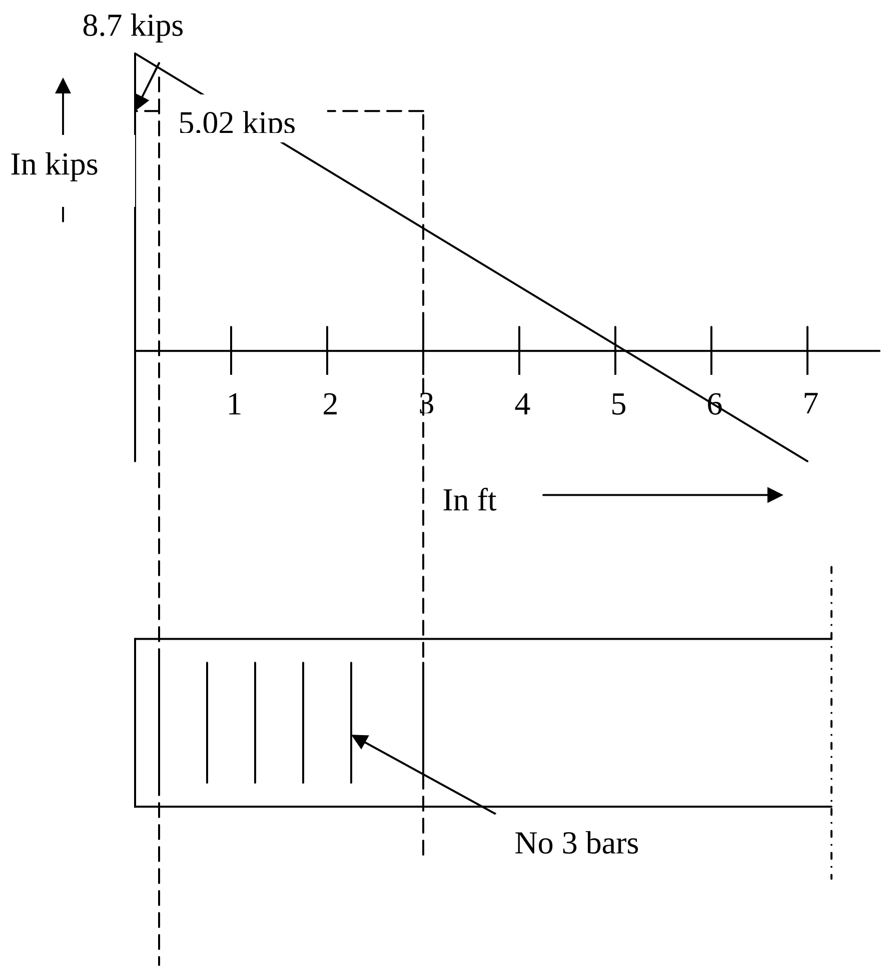


Figure 4.23 : Stirrup Detailing

Step 7: Detailing

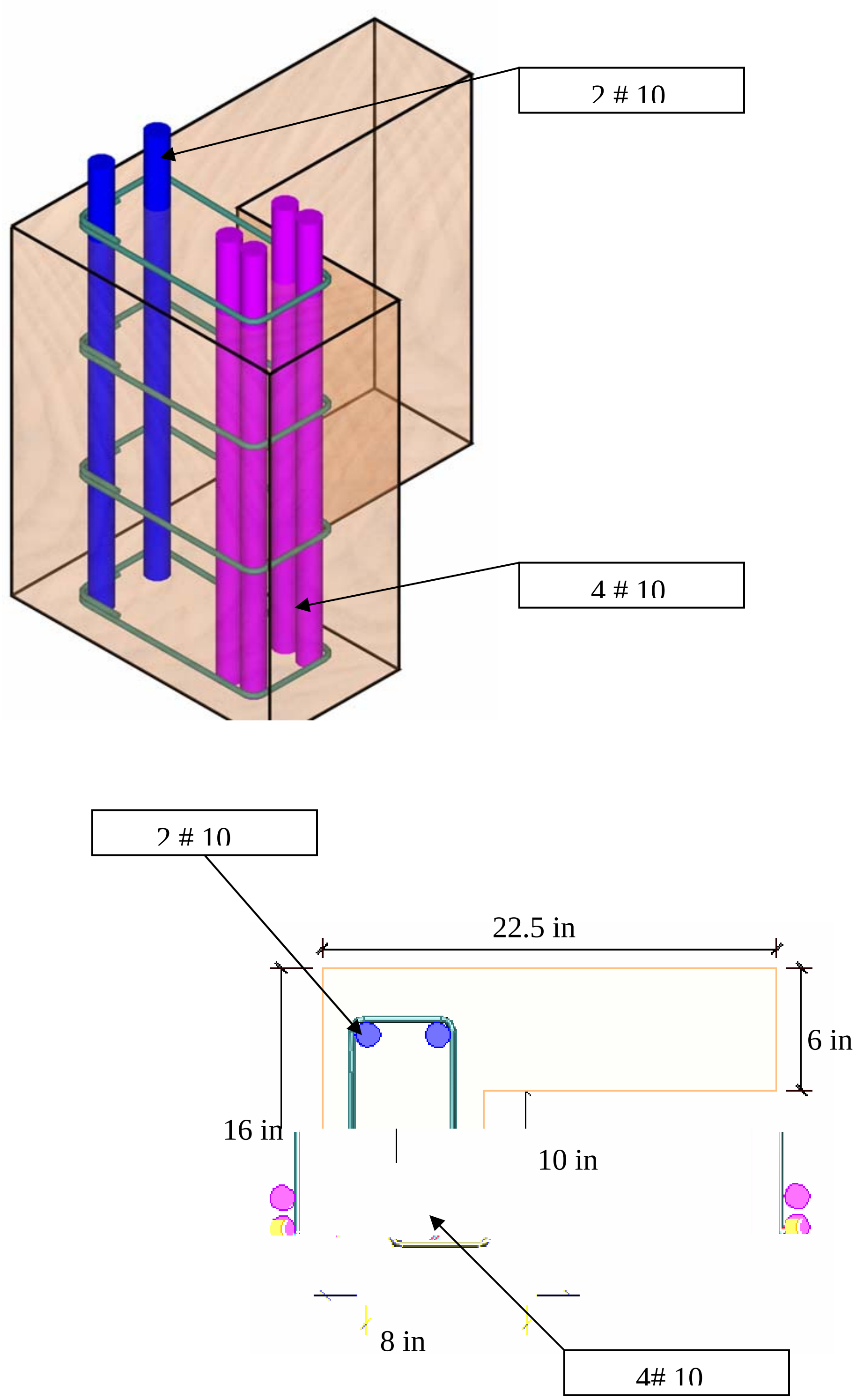


Figure 4.24 : Detailing of example