Chapter 6

Seismic Soil-Structure Interaction

6.1 Introduction

The scales of socio–economic damages caused by an earthquake depend to a great extent on the characteristics of the strong ground motion. It has been well known that earthquake ground motions results primarily from the three factors, namely, source characteristics, propagation path of waves, and local site conditions. Also, the Soil-Structure Interaction (SSI) problem has become an important feature of Structural Engineering with the advent of massive constructions on soft soils such as nuclear power plants, concrete and earth dams. Buildings, bridges, tunnels and underground structures may also require particular attention to be given to the problems of SSI. If a lightweight flexible structure is built on a very stiff rock foundation, a valid assumption is that the input motion at the base of the structure is the same as the free-field earthquake motion. If the structure is very massive and stiff, and the foundation is relatively soft, the motion at the base of the structure may be significantly different than the free-field surface motion. For code design buildings it is important to consider the effect of the SSI. The objective of this chapter is to understand the basic concept of the Soil-Structure Interaction, following the different methods of analysis with some solved examples.

6.2 Free Field Motion and Fixed Base Structures

Ground motions that are not influenced by the presence of structure are referred as free field motions.

Structures founded on rock are considered as fixed base structures. When a structure founded on solid rock is subjected to an earthquake, the extremely high stiffness of the rock constrains the rock motion to be very close to the free field motion.

6.3 Soil-Structure Interaction

If the structure is supported on soft soil deposit, the inability of the foundation to conform to the deformations of the free field motion would cause the motion of the base of the structure to deviate from the free field motion. Also the dynamic response of the structure itself would induce deformation of the supporting soil. This process, in which the response of the soil influences the motion of the structure and the response of the structure influences the motion of the soil, is referred as SSI as shown in Figure. 6.1.

These effects are more significant for stiff and/ or heavy structures supported on relatively soft soils. For soft and /or light structures founded on stiff soil these effects are generally small. It is also significant for closely spaced structure that may subject to pounding, when the relative displacement is large.

In order to understand the SSI problem properly, it is necessary to have some information of the earthquake wave propagation through the soil medium for two main reasons. Firstly, when the seismic waves propagates through the soil as an input ground motion, their dynamic characteristics depends on the modification of the bedrock motion. Secondly, the knowledge of the vibration characteristics of the soil medium is very helpful in determining the soil impedance functions and fixing the boundaries for a semi-infinite soil medium, when the wave propagation analysis is performed by using numerical techniques. To understand the influence of local soil conditions in modifying the nature of free field ground motion it is very essential to understand the terminology of local site effect. Therefore, in this chapter, the terminology of local site effect is discussed first and then, seismic SSI problems are presented.

The first significant structure where the dynamic effect of soil was considered in the analysis in industry in India was the 500MW turbine foundation for Singrauli (Chowdhary, 2009).

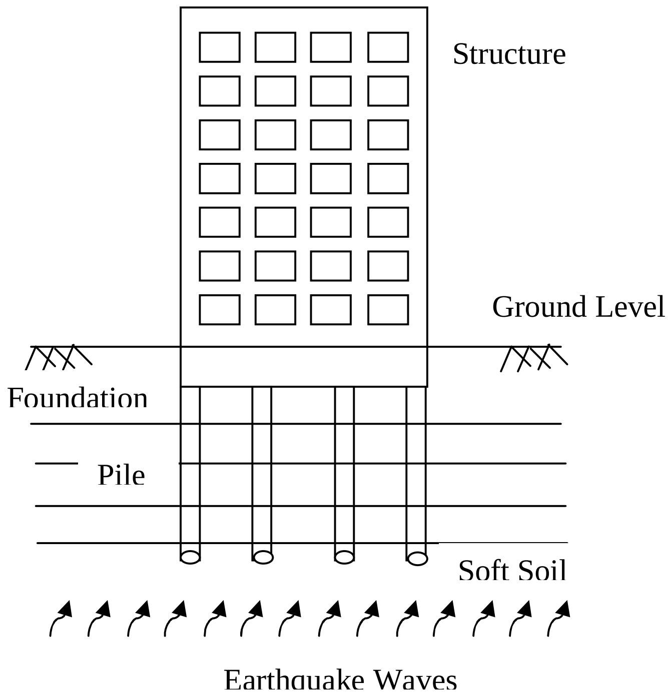

Figure 6.1: Seismic Soil-Structure Interaction.

6.4 Terminology of Local Site Effects

6.4.1 Basin /soil effect on the ground motion characteristics

6.4.1.1 Impedance contrast

Seismic waves travels faster in hard rocks in compare to softer rocks and sediments. As the waves passes from harder to softer rocks they become slow and must get bigger in amplitude to carry the same amount of energy. Thus, shaking tends to be stronger at sites with softer surface layers, where seismic waves move more slowly. Impedance contrast defined as the product of velocity and density of the material (Pisal, 2006).

6.4.1.2 Resonance

When the signal frequency matches with the fundamental frequency or higher harmonics of the soil layer, we say that they are in resonance with one another. This results in to tremendous increase in ground motion amplification. Various spectral peaks characterize resonance patterns. The frequencies of these peaks are related to the surface layer's thickness and velocities. Further, the amplitudes of spectral peaks are related mainly to

- The impedance contrast between the surficial layer and the underlying bedrock.
- To sediment damping.
- To a somewhat lesser extent, to the characteristics of the incident wave-field.

6.4.1.3 Damping in Soil

Absorption of energy occurs due to imperfect elastic properties of medium in which the particle of a medium do not react perfectly elastically with their neighbor and a part of the energy in the waves is lost instead of being transferred through medium, after each cycle. This type of attenuation of the seismic wave is described by a parameter called as quality factor (Q). It is defined as the fractional loss of energy per cycle

$$\frac{\pi}{Q} = \frac{\Delta E}{E} \tag{6.1}$$

where ΔE is the energy lost in one cycle and E is the total elastic energy stored in the wave. If we consider the damping of a seismic wave as a function of the distance and the amplitude of seismic wave, we have

$$A = A_0 \exp\left(\frac{-\pi r}{Q \lambda}\right) = A_0 \exp\left(-\alpha r\right) \tag{6.2}$$

where α is called the absorption coefficient and is inversely proportional to quality factor Q. Damping of soil mainly affects the amplitude of surface waves (Narayan, 2005).

6.4.1.4 Basin Edge Effect

When the seismic waves incident near the basin edge, it enter the basin from its edge and travel in the direction in which the basin is thickening. Figure 6.2 shows that when the wave can become trapped within the basin, if post critical incident angles develop. Interference of trapped waves generates surface waves, which propagate across the basin. The generation of surface waves near the basin is known as basin-edge effect (Bard and Bouchon 1980 a & b, Bakir et al. 2002, Graves et al., 1998, Hatyama et al.1995, Pitarka et al., 1998, Narayan, 2005) . Waves that become trapped in deep sedimentary basins can produce stronger amplitudes at intermediate and low frequencies than those recorded on comparable surface material outside basins, and their durations can be twice as long. This basin edge effect can amplify long period components of ground motion and significantly increases the duration of strong shaking. Basin induced surface waves cause intense damage which is confined in a narrow strip parallel to the edge.

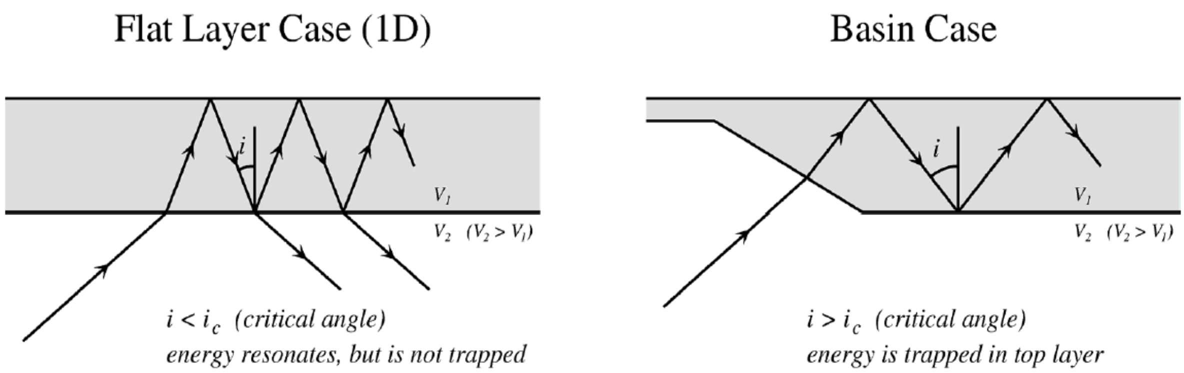

Figure 6.2: Schematic diagram showing that seismic waves entering a sedimentary layer from below will resonate within the layer but escape if the layer is flat (left) but become trapped in the layer if it has varying thickness and the wave enters the layer through its edge (right) (After Grave, 1998).

6.4.1.5 Basement Topography

Irregular basement topography when subjects to body wave incidence below, results in focusing and defocusing effects. This effects are strongly depends on the azimuth and angle of incident waves.

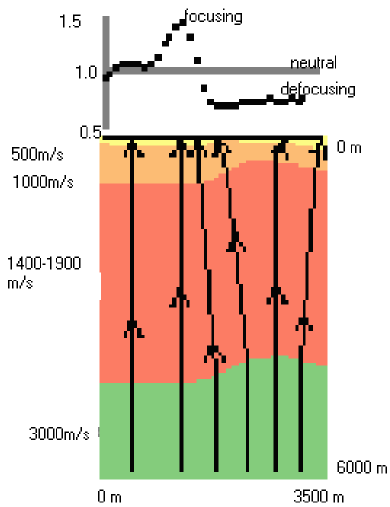

Figure 6.3, Shows seismic waves traveling in the upward direction from depth may be redirected by subtle irregularities at geological interfaces. As wave pass from the deeper unit across the curved interface, their velocity and direction changes, and once again changes at the unit nearest to the surface. Sometimes they meet at certain points on the surface. At these points, the amplification and de-amplification caused due to focusing and defocusing phenomenon (After USGS, http://pubs.usgs.gov/of/1996/ofr-96-0263/localeff.htm).

The damage pattern caused by the Northridge earthquake, Sherman Oaks and Santa Monica reveals effect of basement topography very well.

6.4.1.6 Trapping of Waves

Due to the large impedance contrast between the soft sediments and underlying bedrock, seismic waves trapped over soft sediments. This results in increase in the duration of ground motion.

When layers are horizontal this trapping affects only body waves. While in case of lateral heterogeneities this trapping also affect the surface waves. Interference of these waves also leads to resonance pattern. As discussed earlier, the basin edge effect causes the total reflection of the wave at the base of the layer, making them potentially very damaging. As reported by Kawase (1996) this type of effect was also observed in the 17 January 1995 Hyogo-ken Nabu earthquake, which was the most destructive earthquake in Japan even though of moderate magnitude (M=6).

6.4.2 Effect of Surface Topography

Surface topography considerably affects the amplitude, the frequency content and duration of ground motion (Celebi, 1987 and Geli et al., 1988).

6.4.2.1 Effect of Ridge

The ridge causes strong generation of surface wave near the top of the ridge and their propagation towards the base of the ridge, Narayan and Rao (2003). Amplification of the ground motion depends on the slope and the elevation of the ridge.

In India it had observed when it had damaged very badly the village of Kutri and at Sajan Garh fort, constructed on a hill near the city of Udaipur.

6.4.2.2 Effect of Valley

It has been predicted numerically that in the valley, due to defocusing effect de-amplification of the amplitude of motion takes place. The intensity in a valley may be 1-2 scales lesser as compared with the surrounding, if it is free from the soil deposits.

The effect of valley was observed in the Mandal valley and Pingala Pani, Unali and Chandrapuri villages. The damage in the Mandal Proper village and the Khalla village was lesser as compared to the other villages of the Mandal valley, since these villages are situated at the base of the valley. The houses of the other villages (Siroli, Makroli and Gondi), which were situated at some elevation suffered much more damage.

6.4.2.3 Slope Effect

Hills with variable slope revels complicated damage patterns. The houses situated on or near the bank of a steeply sloping hills suffers much more damages as compare to the houses which were at some distance away from the steep portion or are on the gentle sloping part of the same hill.

6.4.3 Strong Lateral Discontinuity Effect

Lateral discontinuities are nothing but the areas where a softer material lies besides a more rigid one (for instance, ancient faults, anomalous contacts, debris zones, etc.)

The best example of damage caused by strong lateral discontinuity (softer rock sandwiched between hard rocks) was observed in the Bhatwari- Sonar village during Chamoli earthquake of 1999. The village situated on a sloping hill at the left bank of river Mandakini received greater damage. The hill mass is composed of rounded pebbles and young soil and is surrounded by hard older quartzite rocks. Amplitude amplification, generation of local surface waves in the softer medium and larger differential motion caused by shorter wavelength of the surface waves may be reason behind the greater structural damage.

6.5 Degree of Influence of SSI

The degree of Influence of SSI on response of structure depends on the following factors

- Stiffness of soil.

- Dynamic Characteristics of structure itself i.e. Natural Period and damping factor.
- Stiffness and mass of structure.

6.6 Interaction between Ground and Structure during Earthquake

When the seismic wave E_0 generated by an earthquake fault reaches the bottom of the foundation, they are divided into two types as shown in Figure 6.4:

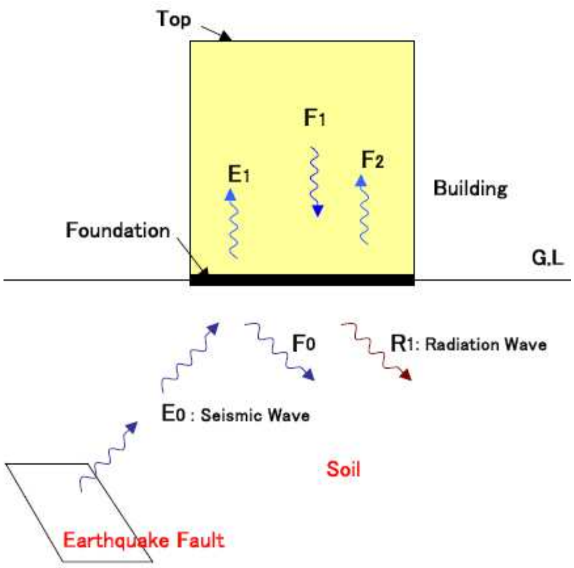

Figure 6.4: Wave propagation during SSI (Miura, 2011)

Transmission Waves which are entering in the building shown as E_1 and **Reflection Waves** which are reflected back in to the ground shown as $F_{0.}$

When the transmission wave enters in to the building they travels in upward direction due to which the structure subjects to vibration. And then they are reflected at the top and travel back down to the foundation of the structure shown as F_1. At this stage Soil-Structure Interaction phenomenon takes place. Again a part of wave are transmitted into the ground, while the rest is reflected back again and starts to move upwards through the sructure shown as F_2. The wave which are transmitted to the ground known as **Radiation Waves** shown as R_1. When the radiated waves are in small amount, the seismic waves once transmitted into the structure continue to trapped in the building, and the sructure starts to vibrate continuously for a long time, similar to the lightly damped structure.

The damping caused by radiation waves is popularly known as **Radiation Damping** of the soil. The radiation damping results in increase of total damping of the soil-structure system in compare to the structure itself. Also, under the influence of SSI the natural frequency of a soil-structure system shall be lower than the natural frequency of the soil.

These interactions results not only in reducing the demands on the structure but also increasing the overall displacement of the structure as due to these interactions foundation can translate and rotate. Basically the dynamic soil-structure interaction consists of two interactions, namely, kinematic interaction and inertial interaction.

6.7 Kinematic Interaction

The SSI effect which is associated with the stiffness of the structure is termed as kinematic interaction. It is explained with the help of Figure 6.5 (a–d). In Figure 6.5 (a), the massless mat foundation restricts the vertical movement of the ground motion because of its flexural stiffness. Due to this, instead of free field ground motion the mat foundation moves differently (that is, the ground motion is away from the foundation) along with the change in nature of ground motion in the close vicinity and below the foundation. Similar examples of kinematic interaction are shown in Figure 6.5 (b and c). In Figure 6.5 (b), a vertically propagating shear wave is confined by the embedded foundation. In Figure 6.5 (c), the axial stiffness of the foundation slab prevents the incoherent ground motion produced below the foundation. For vertically propagating purely S-waves, the rotational movement induced in foundation due to kinematic interaction is shown in Figure 6.5 (d).

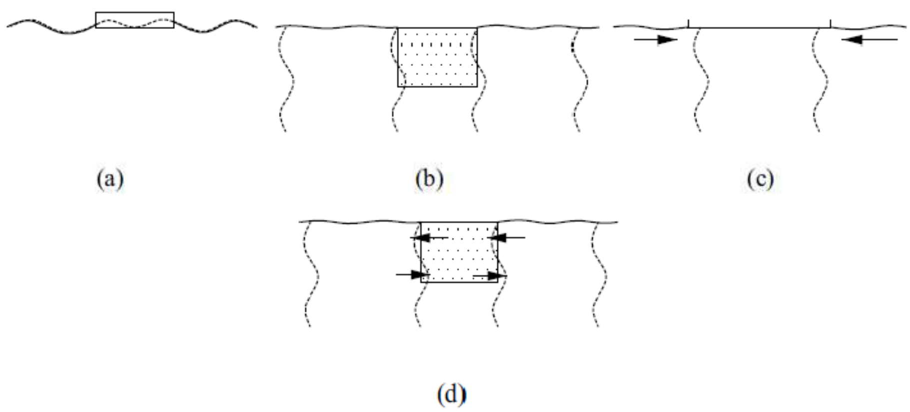

Figure 6.5: Kinematic interaction: (a) vertical motion modified; (b) horizontal motion modified; (c) incoherent ground motion prevented; and (d) rocking motion introduced (Datta, 2010).

The tau (τ) effect, derived by Clough and Penzien (1993), explains the kinematic interaction due to translational excitation with reference to the rigid slab. In Figure 6.6, the shear wave moving in the y-direction produces ground motion in the x-direction which varies with y. At the site of slab where the free field earthquake motion varies significantly, due to the rigidity of slab these motions are constrained to some extent.

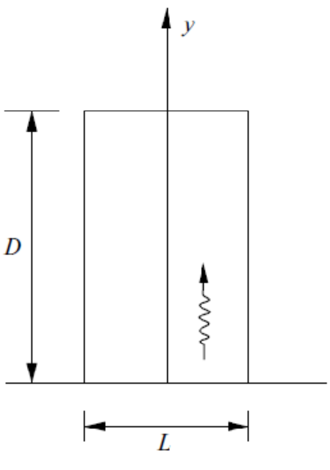

Figure 6.6: Horizontally propagating shear wave in the y-direction below the rigid slab of a large structure. (Clough and Penzien, 1993)

If τ is defined as the ratio of amplitude of harmonic component of translational motion to the amplitude of harmonic component of respective free field motion, then it is shown that

$$\tau = \frac{1}{\alpha}\sqrt{2(1-\cos\alpha)} \tag{6.3}$$

$$\alpha = \frac{\omega D}{V_a} = \frac{2\pi D}{\lambda(\omega)} \tag{6.4}$$

where,

$\lambda(\omega) = \dfrac{2\pi V_a}{\omega}$ is the wavelength.

D = Dimension of the base in the y-Direction.

V_a = Apparent wave velocity.

Also the values of τ decrease from unity at $\alpha = 0$ and $\lambda \to \infty$ to zero at $\alpha = 2\pi$ and $\lambda = D$. This means that if the base dimension of the foundation is very small compared with the wavelength of the ground motion, then the τ effect is negligible (i.e. the slab will exert little constraint on the soil and the slab motions will be essentially the same as the free field motions at that location). On the other hand, if the base dimension of the foundation is fairly large in compared to the wavelength of the ground motion, then the τ effect should be considered and the base motion could be much smaller than the free field ground motion.

whenever the stiffness of the foundation system obstructs the development of the free-field motion, kinematic interaction takes place. When foundation subjects to vertically propagating S-waves of wavelength equal to the depth of embedment, the kinematic interaction induces rocking and torsion modes of vibration in the structure, which are not present in case of free field motion. The deformation caused by kinematic interaction alone can be computed by assuming that the structure and foundation has stiffness but no mass as shown in Figure 6.7. The equation of motion for this case is

$$[M_{soil}]\{\ddot{u}_{KI}\} + [K^*]\{u_{KI}\} = -[M_{soil}]\ \ddot{u}_b(t) \tag{6.5}$$

where $[M_{soil}]$ is the mass matrix assuming that the structure and foundation are massless and $\{u_{KI}\}$ is the foundation input motion, $[K^*]$ is the stiffness matrix and $\ddot{u}_b$ is the acceleration at the boundary (Kramer, 1996).

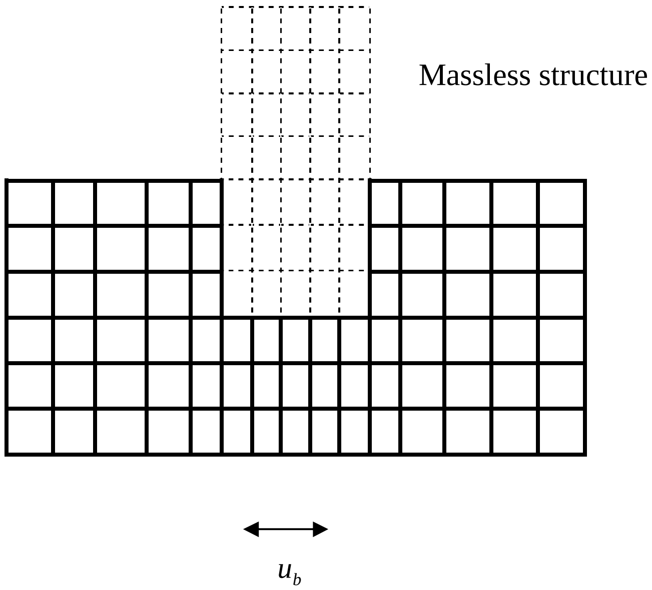

Figure 6.7: Kinematic interaction analysis (Kramer, 1996).

6.8 Inertial Interaction

The mass of structure and foundation causes them to respond dynamically. The SSI effect which is associated with the mass of the structure is termed as inertial interaction. It is purely caused by the inertia forces (seismic acceleration times mass of the structure) generated in the structure due to the movement of masses of the structure during vibration. The inertial loads applied to the structure lead to an overturning moment and a transverse shear. If the supporting soil is compliant, the inertial force transmits dynamic forces to the foundation causing its dynamic displacement that would not occur in case of a fixed-base structure. The deformations due to inertial interaction can be computed from the equation of motion (Kramer,1996).

$$[M]\{\ddot{u}_{\mathrm{II}}\}+[K^{*}]\{u_{\mathrm{II}}\}=-[M_{structure}]\{\ddot{u}_{k\mathrm{I}}(t)+\ddot{u}_{b}(t)\} \tag{6.6}$$

where $[M_{structure}]$ is the mass matrix assuming that the soil is massless as shown in Figure 6.8. Note that the right hand side of equation (6.6) shows the inertial loading on the structure foundation system which depends on base motion and foundation input motion including kinematic interaction effect.

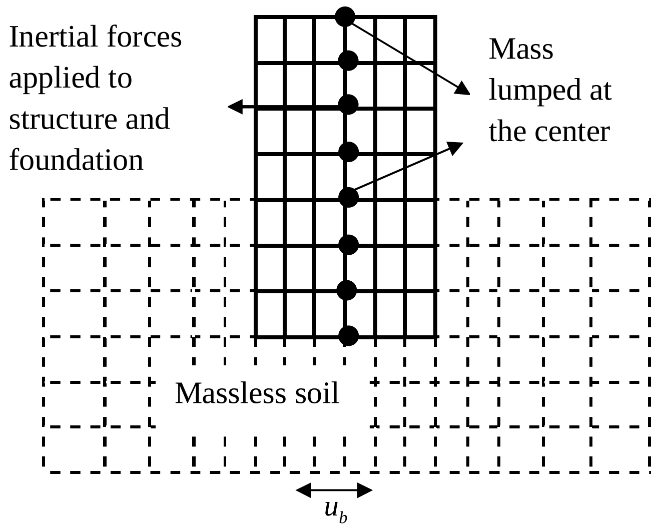

Figure 6.8: Inertial interaction analysis

6.9 Illustration of Soil-Structure Interaction Effects

The soil-structure interaction is illustrated by a simple analysis following the approach of Wolf (1985). Consider a Single degree of freedom system (SDOF) of mass m, stiffness k, and damping coefficient c, connected to a massless rigid, L-shaped foundation of height h as shown in Figure 6.9 (a). The system is subjected to a horizontal excitation of amplitude u_g. If the material supporting the foundation is rigid, the natural frequency ω_0 of the resulting fixed-base system will be

$$\omega_0 = \sqrt{\frac{k}{m}} \tag{6.7}$$

and the hysteretic damping ration ξ of the structure will be

$$\xi = \frac{c\,\omega_0}{2\,k} \tag{6.8}$$

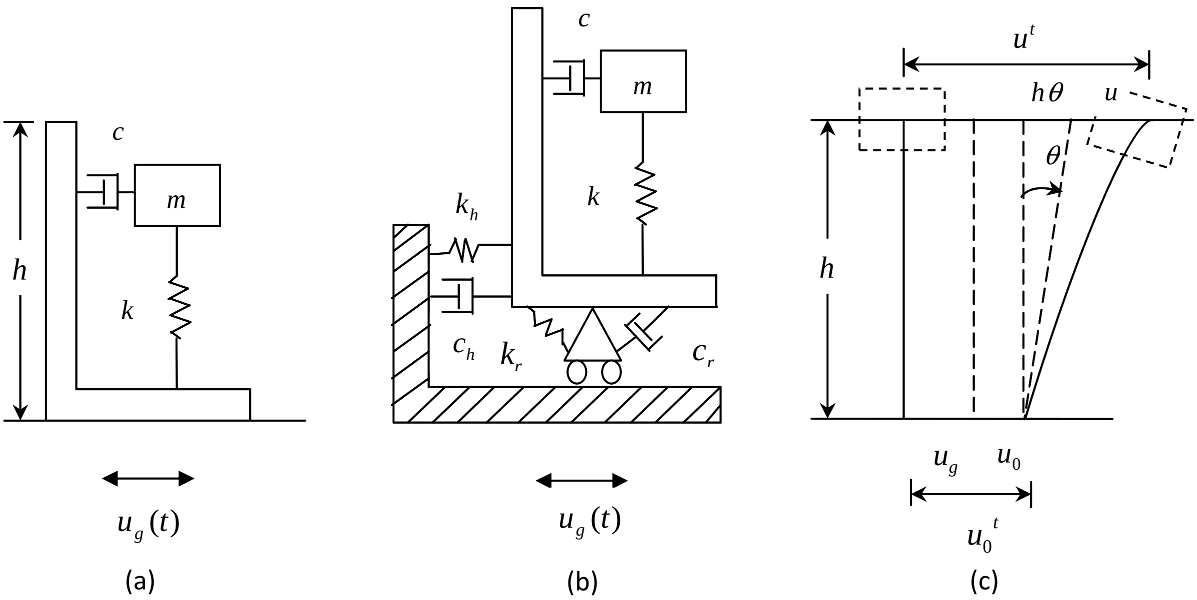

Figure 6.9: Model with compliant base material having one dynamic degree of freedom: (a) SDOF system on a compliant supporting material; (b) idealized discrete system in which the compliance of base is shown by translational and rotational springs and dashpots and (c) total displacements of base and mass.

If the supporting material is compliant, the foundation can translate and rotate. The stiffness and damping characteristic of the compliant soil-foundation system can be represented by k_h and c_h in the horizontal (translational) direction and by k_r and c_r in the rotational (rocking) direction.

The total displacement of the mass u^t and the base of the structure $u_0{}^t$ can split into their individual components as

$$u^t = u_g + u_0 + h\theta + u \tag{6.9}$$

$$u_0{}^t = u_g + u_0 \tag{6.10}$$

where,

u_g = Amplitude of horizontal excitation or free field motion.

u_0 = Amplitude of base relative to the free field motion.

$h\theta$ = Rigid body component due to the base rotation (rocking) of the structure by an angle θ.

u = Amplitude of the relative displacement of the mass with respect to the moving frame attached to the rigid base. It is equal to the structural deformation.

θ = Angle of base rotation (rocking).

For a soil without material damping ($\xi_s = 0$), the horizontal force amplitude p_h acting on it is written as

$$p_x = k_x u_0 + c_x \dot{u}_0 \tag{6.11}$$

where the subscript x denotes the horizontal direction for a purely elastic soil ($\xi_s = 0$) .While for a soil with material damping, the corresponding equation is written as

$$p_h = k_h u_0 + c_h \dot{u}_0 \tag{6.12}$$

For a frequency dependent harmonic excitation

$$\dot{u}_0 = i\omega\, u_0 \tag{6.13}$$

applying in equation (6.11), leads to

$$p_x = k_x \left(1 + \frac{c_x}{k_x}\, i\, \omega\right) u_0 = k_x \left(1 + 2\xi_x\, i\right) u_0 \tag{6.14}$$

where ξ_x represents the ratio of the viscous radiation damping in the horizontal direction.

The material damping can be introduced in an approximate manner by multiplying the spring coefficient k_x (for frequency ω) with the factor $\left(1 + 2\,\xi_s\, i\right)$, where ξ_s is the hysteretic damping ratio, and substituting equation (6.13) in equation (6.12), gives

$$p_h = k_x \left(1 + 2\xi_s\, i + 2\xi_x\, i\right) u_0 \tag{6.15}$$

Comparing equation (6.12) and equation (6.15) and using equation (6.13), the obtained horizontal stiffness and damping coefficient are

$$\begin{aligned} k_h &= k_x \\ c_h &= c_x + \frac{2}{\omega}\xi_s\, k_x \end{aligned} \tag{6.16}$$

The first term on the right side of equation (6.16) corresponds to radiation damping and the second term to the material damping. If the structure is assumed to be rigid $(k = \infty)$ and the foundation unable to rock or rotate $(k_r = \infty)$, the natural frequency for translational vibration would be

$$\omega_h = \sqrt{\frac{k_h}{m}} \tag{6.17}$$

Similarly the moment amplitude M_r acting on the soil, considering rotational (rocking) degree of freedom can be written as

$$M_r = k_r\theta + c_r\theta \tag{6.18}$$

Also

$$M_r = k_\theta \left(1 + 2\xi_s\, i + 2\xi_\theta\, i\ \right)\theta \tag{6.19}$$

Comparing equation (6.18) and equation (6.19), the obtained rotational stiffness and damping coefficient are

$$k_r = k_\theta \tag{6.20}$$

$$c_r = c_\theta\ + \frac{2}{\omega}\xi_s\, k_\theta$$

If the structure is assumed to be rigid $(k = \infty)$ and the foundation unable to translate $(k_h = \infty)$, the natural frequency for rotational vibration would be

$$\omega_r = \sqrt{\frac{k_r}{m\,h^2}} \tag{6.21}$$

To illustrate the soil-structure interaction, an equivalent SDOF system of same mass m is considered. Its properties like natural frequency ω_e, ratio of hysteretic damping ξ_e are selected such that when excited by the equivalent seismic input motion U_g it will respond in essentially the same way as the system shown in Figure 6.9. The subscript e is used to describe the properties of this equivalent system. For harmonic motion, the equation of motion for the equivalent system can be written as

$$\left(-m\omega^2 + i\omega c_e + k_e\right)u = m\omega^2 U_g \tag{6.22}$$

$$\omega_e = \sqrt{\frac{k_e}{m}} \tag{6.23}$$

$$\xi_e = \frac{c_e \omega}{2k_e} \tag{6.24}$$

The response of the equivalent system goes to infinity at its natural frequency for an undamped system (i.e. $\xi_e = 0$). This occurs when

$$\frac{1}{\omega_e^2} = \frac{1}{\omega_0^2} + \frac{1}{\omega_h^2} + \frac{1}{\omega_r^2} \tag{6.25}$$

Substituting the value of ω_0 , ω_h and ω_r in above eq. and solving leads to

$$\omega_e = \frac{\omega_0}{\sqrt{1 + k/k_h + kh^2/k_r}} \tag{6.26}$$

It reveals that the fundamental frequency ω_e of the soil-structure (equivalent) system is always lesser than the frequency ω_0 of the fixed base structure. It shows that the considering the soil-structure interaction is important from the point of view to reduce the natural frequency of the soil-structure system to a value lower than that of the structure with a fixed base condition. For resonance condition (i.e. $\omega_0 = \omega_e$) the hysteretic damping ratio can be formulated as

$$\xi_e = \frac{\omega_e^2}{\omega_0^2}\xi + \left(1 - \frac{\omega_e^2}{\omega_0^2}\right)\xi_s + \frac{\omega_e^2}{\omega_h^2}\xi_x + \frac{\omega_e^2}{\omega_r^2}\xi_\theta \tag{6.27}$$

If no radiation damping occurs in the horizontal and translation direction, $\xi_x = \xi_\theta = 0$ and if the damping of the structure is equal to the damping of the soil, $\xi = \xi_s$ than above equation results in $\xi_e = \xi$. As under normal conditions ξ_s will not be smaller than ξ, the equivalent damping ξ_e will be larger than the damping of the structure. It shows that the SSI increases the effective damping ratio to a value greater than that of the structure.

For the fixed base structure, translation and rotation of the base is not possible. The base translation, base rotation and motion of the mass of the equivalent system with respect to the free field motion (which is given by sum of the base displacement u_0, the displacement of the top of the structure due to rotation of the base $h\theta$, and the displacement due to the distortion of the structure u) can be shown as

$$u_0 = \frac{\omega_0^2}{\omega_h^2}(1 + 2\xi i - 2\xi_x i - 2\xi_s i)u \tag{6.28}$$

$$h\theta = \frac{\omega_0^2}{\omega_r^2}(1 + 2\xi i - 2\xi_\theta i - 2\xi_s i)u \tag{6.29}$$

$$u + u_0 + h\theta = \omega_0^2\left(\frac{1}{\omega_e^2} + 2(\xi - \xi_s)i\left(\frac{1}{\omega_e^2} - \frac{1}{\omega_0^2}\right) - \frac{2\xi_x i}{\omega_h^2} - \frac{2\xi_\theta i}{\omega_r^2}\right)u \tag{6.30}$$

Following dimensionless parameters are to be considered to see the effect of the soil-structure interaction:

- Stiffness ratio defines as the ratio of the stiffness of the structure to that of the soil.

$$\bar{s} = \frac{\omega_0 h}{v_s} \tag{6.31}$$

where v_s is the shear wave velocity of the soil.

- Slenderness ratio $\bar{h} = \dfrac{h}{a}$

  where a is the characteristic length of the rigid foundation(e.g., the radius for a circular basement).

- Mass ratio $\bar{m} = \dfrac{m}{\rho a^3}$

  where ρ is the mass density of the soil.

- Poisson's ratio ν of the soil.
- Hysteretic damping ratios of the structure ξ and soil ξ_s.

If the stiffness ratio is zero, it shows the fixed base condition. If the value of stiffness ratio is very large, it shows that a relatively stiff structure rests on a relatively soft soil. In actual conditions the stiffness and damping coefficient of the foundation are frequency dependent. To illustrate the effect of SSI, the following frequency independent approximate expressions (for the undamped soil) can be used to estimate the stiffness and damping coefficient of a rigid circular footing of radius a (Wolf, 1989)

$$k_x = \frac{8Ga}{2-\nu} \tag{6.32}$$

$$c_x = \frac{4.6}{2-\nu}\rho v_s a^2 \tag{6.33}$$

$$k_\theta = \frac{8Ga^3}{3(1-\nu)} \tag{6.34}$$

$$c_\theta = \frac{0.4}{1-\nu}\rho v_s a^4 \tag{6.35}$$

Expressing the frequency ω_e and damping coefficient ξ_e calculated in equation (6.26) and equation (6.27) of a rigid circular footing using the above mentioned dimensionless parameters leads to

$$\frac{\omega_e{}^2}{\omega_0{}^2} = \frac{1}{1 + \frac{\overline{m}\,\overline{s}^2}{8}\left(\frac{2-\nu}{\overline{h}^2} + 3(1-\nu)\right)} \tag{6.36}$$

$$\xi_e = \frac{\omega_e{}^2}{\omega_0{}^2}\xi + \left(1 - \frac{\omega_e{}^2}{\omega_0{}^2}\right)\xi_s + \frac{\omega_e{}^3}{\omega_0{}^3}\frac{\overline{s}^3\overline{m}}{\overline{h}}\left(0.036\frac{2-\nu}{\overline{h}^2} + 0.028(1-\nu)\right) \tag{6.37}$$

The graphs in Figure 6.10 shows the effect of SSI on the natural frequency and damping ratio of equivalent SDOF system by comparing its response with the fixed base system. Figure 6.10 (a) reveals that when the stiffness ratio is high (i.e. the stiffness of the structure is larger than the stiffness of the soil), the natural frequency of the Equivalent SDOF system reduces. It means that the effect of soil-structure interaction on natural frequency is high at high stiffness ratios. Thus the SSI consideration is important for stiff structures with a large mass supported on flexible soil. In a similar way when the stiffness ratio is low (i.e. stiffness of soil is larger than the stiffness of the structure), the natural frequency of equivalent system increases. It shows the effect of SSI on the natural frequency is small at low stiffness ratios and is important to consider for the flexible (tall) structures supported on stiff soil. Also when the stiffness ratio is zero (i.e. fixed base condition), the natural frequency of the equivalent SDOF is equal to the fixed base natural frequency.

Figure 6.10 (b) reveals that at high stiffness ratio the damping of the equivalent SDOF system is high. It means at high stiffness ratio the effect of radiation damping and soil damping become more apparent and the structural damping represents a small part of the overall damping of the system. Also at the fixed base condition the damping of the equivalent SDOF system will be same as that of the damping of fixed base structure.

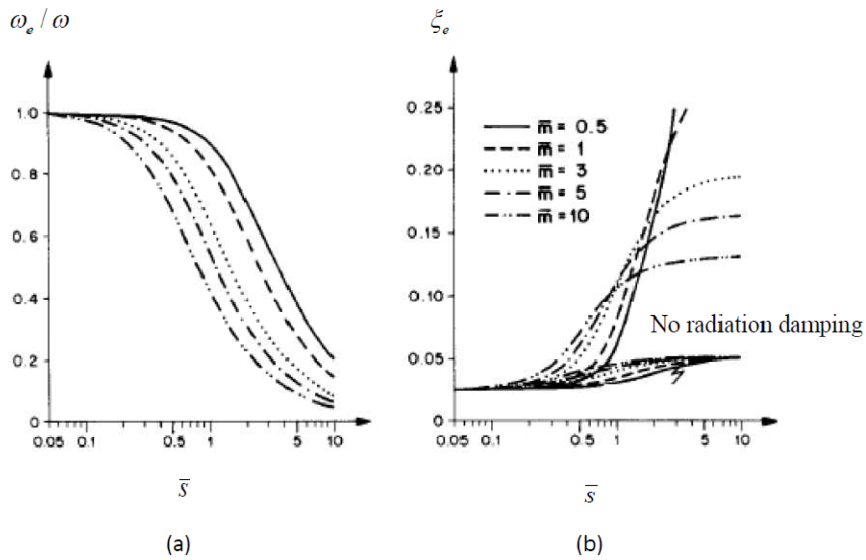

Figure 6.10: Effect of stiffness ratio and mass ratio on (a) natural frequency and (b) damping ratio of soil-structure system ($\bar{h} = 1, \nu = 0.33, \xi = 0.025, \xi_s = 0.05$) (Wolf, 1985).

The graphs in Figure 6.11 show the effect of SSI on the structural distortion and displacement of mass with respect to the free field of an equivalent SDOF system by using an artificial input motion. The maximum responses are for the used artificial motion that produced an NRC response spectrum normalized to $a_{\max} = 1.0g$. Figure 6.11 (a) reveals that as the stiffness ratio is increases, the structural deformation decreases. It means that the considering effect of SSI results in reducing the distortion of the structure. On the other hand Figure 6.11 (b) shows that as the stiffness ratio increases, the overall displacement of the mass relative to the free field increases. It means that considering SSI effect results in increasing the overall displacement of the mass. Finally on one side the SSI tends to reduce the demand on the structure and on the other side as the foundation can rotate and translate, it increases the overall displacement.

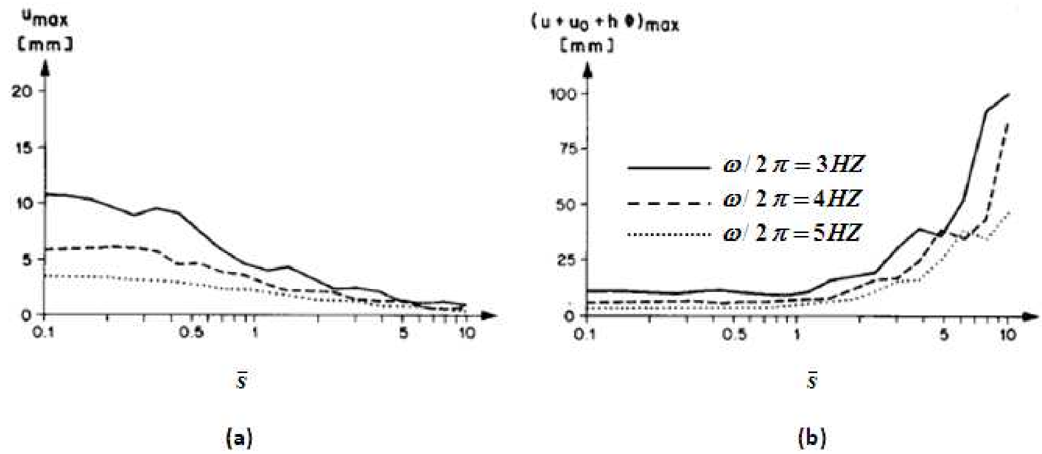

Figure 6.11: Response of equivalent SDOF system to artificial time history, considering SSI ($\bar{h} = 1, \bar{m} = 3, \nu = 0.33, \xi = 0.025, \xi_s = 0.05$): (a) maximum structural displacement; (b) maximum displacement of mass relative to free field (Wolf, 1985).

6.10 Direct Method

In the direct method the soil, structure and foundation is modeled together using finite element method (FEM) and analyzed in single step. The ground motion is specified as free field motion and is applied at all boundaries. The soil domain with some material damping is limited by a fictitious exterior boundary, which is placed so far away from the structure that during the total earthquake excitation, the waves generated along the soil-structure interface does not reach it. The nodes along the soil-structure interface are denoted by subscript f (foundation). The nodes of the structure are denoted by st . The nodes along the interior foundation medium/soil are denoted by s .

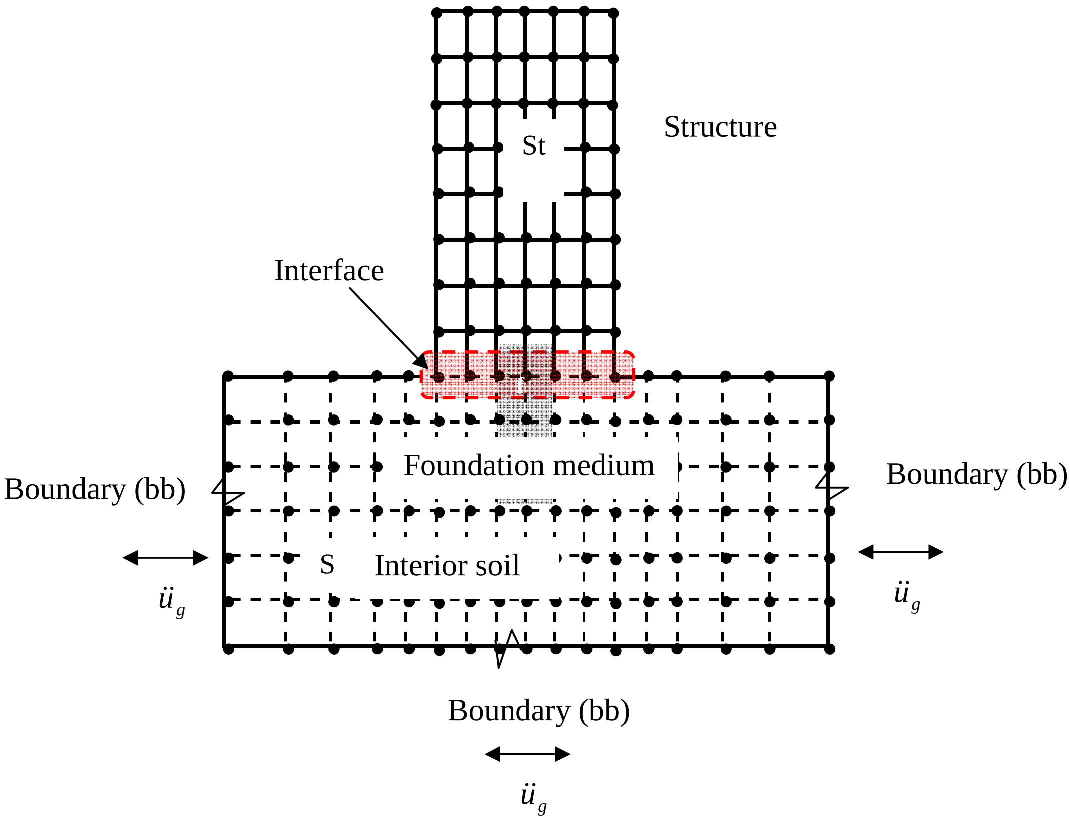

Figure 6.12: Finite element model of soil-structure system for direct method of analysis.

In the above figure the soil is modeled as an assemblage of rectangular plane strain elements having two translational degrees of freedom at each node, while the structure is modeled as an assemblage of beam elements. It is assumed that kinematic interaction is insignificant and the foundation block will move with free field ground motion. The inertia forces acting on the structure produces the vibration of structure, foundation and soil at the soil-structure interface and at the soil below it. The equation of motion for total system shown in Figure 6.12 in time domain can be written as

$$M\ddot{u}+C\dot{u}+ku=-M_{st}I\ddot{u}_g \tag{6.38}$$

where,

M = Mass matrix for the entire Structure, foundation and the soil

$$= \begin{bmatrix} [M_{st\,st}] & [M_{st\,f}] & 0 \\ [M_{f\,st}] & [M_{ff}^{st}]+[\overline{M}_{ff}^{s}] & [\overline{M}_{f\,s}] \\ 0 & [\overline{M}_{s\,f}] & [\overline{M}_{s\,s}] \end{bmatrix}$$

C = Damping matrix (Material) of the structure and the soil

$$= \begin{bmatrix} [C_{st\,st}] & [C_{st\,f}] & 0 \\ [C_{f\,st}] & [C_{ff}^{st}]+[\overline{C}_{ff}^{s}] & [\overline{C}_{f\,s}] \\ 0 & [\overline{C}_{s\,f}] & [\overline{C}_{s\,s}] \end{bmatrix}$$

Note: Here, the damping matrix is generated by constructing the damping matrix of soil and structure separately from their modal damping ratio using Rayleigh damping. Then they are combined together to form final damping matrix shown above. It is assumed that the coupling term between the soil and structure is zero but at the interface of soil and structure they are non-zero.

K = Stiffness matrix of total system, which can be generated using standard assembling procedure.

$$= \begin{bmatrix} [K_{st\,st}] & [K_{st\,f}] & 0 \\ [K_{f\,st}] & [K_{ff}^{st}]+[\overline{K}_{ff}^{s}] & [\overline{K}_{f\,s}] \\ 0 & [\overline{K}_{s\,f}] & [\overline{K}_{s\,s}] \end{bmatrix}$$

M_{st} = Mass matrix having non-zero masses for the structural degree of freedom

$$= \begin{bmatrix} [M_{st\,st}] & [M_{st\,f}] & 0 \\ [M_{f\,st}] & [M_{ff}^{st}] & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

I = Mass matrix having non-zero masses for the structural degree of freedom

$\ddot{u}_g$ = Free field ground acceleration (can be calculated by doing one simple one-dimensional analysis of site model, prior to the soil-structure analysis)

u = The vector of the relative displacement with respect to the base / foundation.

The right hand side of equation (6.38) shows the inertia force, which tends to deform the soil at the soil-structure interface, when transferred to the base (foundation) in the form of shear force and moment. The material damping of soil contributes the response reduction of the structure-soil on system is very insignificant and can be neglected. The deformation of soil due to inertia forces at the interface propagates in the form of radiation waves giving radiation damping which mostly affects the structure-soil foundation response. If the radiation damping will not die out or reflect back from the boundary, some error in the solution may introduce and also the problem may become very large. In order to reduce the size of the problem, the concept of absorbing boundaries has been introduced in the FEM.

By using the direct method of analysis, like time domain method problem can also be solved in frequency domain method using Fourier transform function for a specific free field ground motion. If the time histories of the ground motion are different at different supports, then problem can be solved by modifying the influence coefficient vector $\{I\}$ used in equation (6.38).

The direct method is well suited for non-linear material laws of the soil to be taken into account. To solve the dynamic SSI problem by direct method, computer programs can be used. There are few shortcomings of the direct method of analysis; some of them are listed below.

- The good representation of damping matrix is difficult.
- If the superstructure is modeled as 3D system, the problem size becomes very large and the modeling of soil/foundation – structure interface becomes complex.

6.11 Sub-Structure Method

Sub-structure method is computationally more efficient than the direct method as most of the disadvantages of the direct method can be removed, if the substructure method is employed. In this method the effective input motion is expressed in terms of free-field motions of the soil layer initially. In continuation to this step, the soil/foundation medium and the structure are represented as two independent mathematical models or substructures as shown in Figure 6.13. The connection between them is provided by interaction forces of equal amplitude, acting in opposite directions of the two sub-structures. The total motions developed at the interface are the sum of the free-field motions at the interface of the soil without the added structure and the additional motions resulting from the interaction. As it is explained in this paragraph, the substructure method is advantageous as it allows to break down the complicated soil-structure system into more manageable parts which can be more easily solved and checked. As the stiffness and damping properties of the soil are frequency dependent, it is most convenient to carry out the earthquake response analysis in the frequency domain, then to obtain the response history and again transform it in the time domain.

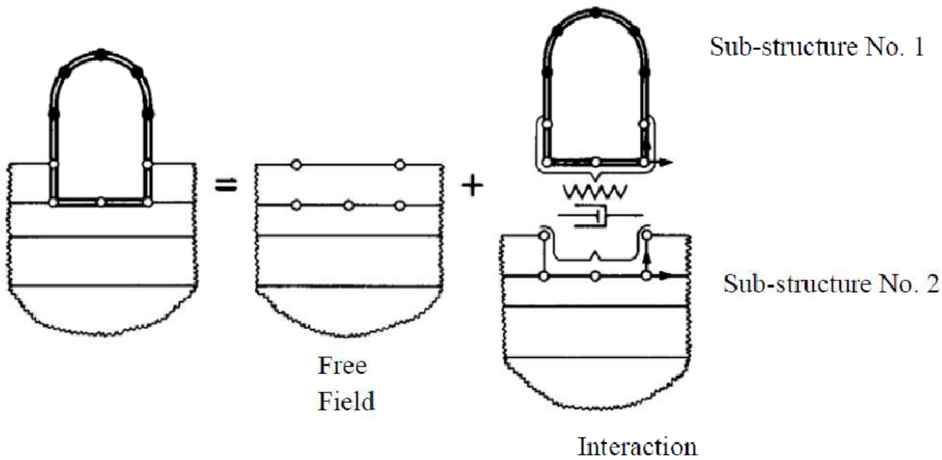

Figure 6.13: Seismic soil-structure interaction with substructure method. (Wolf, 1985)

Note – In case of soil/foundation medium modeling of some structures, a portion of the soil may be included in the superstructure as shown in Figure 6.14(c).

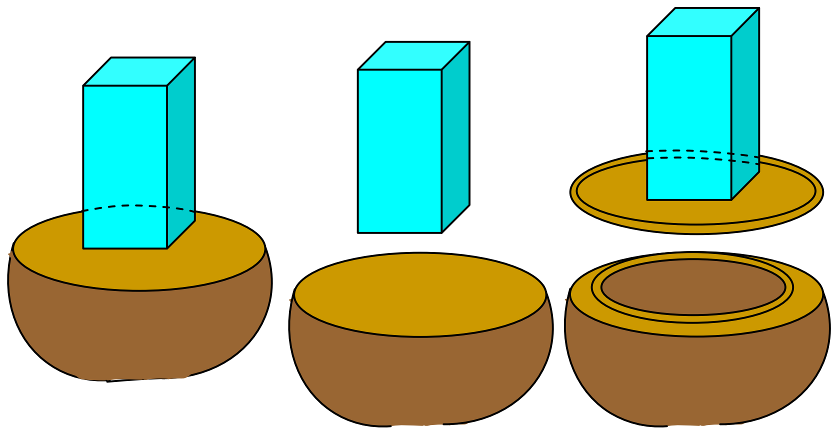

Figure 6.14: Seismic soil-structure interaction with substructure method: (a) SDOF system resting on a half space; (b) modeling superstructure and soil medium separately; (c) some portion of the soil is included in the superstructure model.

For such structures two interfaces exists, one at the free ground surface and the other at the surface between the superstructure and the soil/foundation medium.

The substructure method of analysis can be explained in detail with SDOF structure supported by a rigid foundation slab resting on an elastic half space.

6.11.1 SDOF System Considering SSI

Consider a SDOF system, supported on a rigid base of mass m_b and mass moment of inertia I_{mb}, resting on a half-space as shown in Figure 6.15 (a). To make the τ effect negligible, the horizontal dimensions of the base are assumed as sufficiently small.

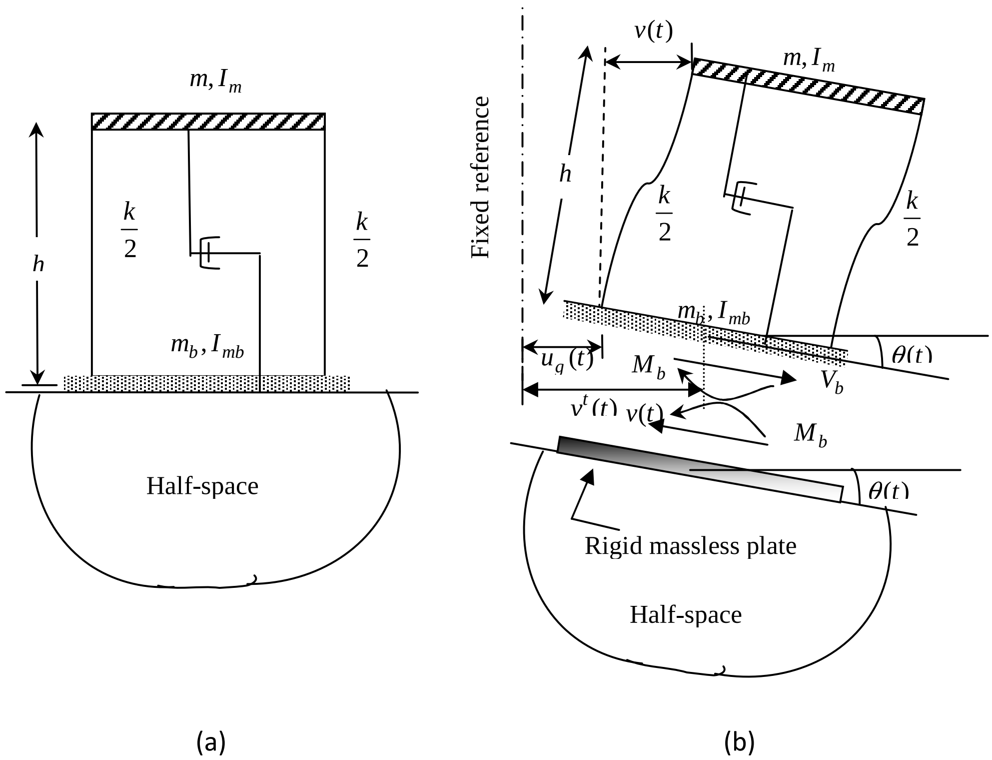

Figure 6.15: Seismic soil-structure interaction analysis using substructure method for SDOF system.

The uniform free-field ground acceleration $\ddot{u}_g(t)$ at the half-space surface will introduce the foundation forces at the interface between the base of the structure and the half-space. Under the influence of these forces i.e. horizontal base shear forces and moment at the base of SDOF system will translate and rotate the system due the SSI phenomenon, as shown in Figure 6.15 (b). It is also assumed that a rigid massless plate is present on the surface of the half-space to ensure its displacement compatibility with the lower surface of the rigid base.

The total base displacement of the SDOF system shown in Figure 6.15 $v^t(t)$ will be

$$v^t(t) = u_g(t) + v(t) \tag{6.39}$$

where,

$u_g(t)$ = Free field ground displacement.

$v(t)$ = Added displacement (or base displacement) caused by SSI.

Also $\theta(t)$ represents the base rotation caused by SSI. It is also noted that as the SSI results in translation and rotation of the base of SDOF system, it introduces the $v(t)$ and $\theta(t)$ displacement of the system and thus the overall system has 3 DOF. The equation of motion for substructure no.1 (i.e. the top mass of the SDOF system) may be written as

$$m\ddot{u} + 2m\xi\omega_0\dot{u} + ku + mh\ddot{\theta} + m\ddot{v}^t = 0 \tag{6.40}$$

where,

u = Relative displacement of the top mass with respect to the base.

m = Lumped mass at the top.

ξ = Percentage critical damping.

ω_0 = Natural frequency of the SDOF system.

k = Total lateral stiffness of the mass with respect to the base.

h = Height of the column.

θ = Displacement due to rotation of base of SDOF system.

v^t = Total displacement of the base.

Considering the equilibrium of the substructure no. 1, we will get the base interaction forces V_b and M_b developed between the super-structure and the half space.

$$m\ddot{u} + mh\ddot{\theta} + (m + m_b)\ddot{v}^t = V_b \tag{6.41}$$

$$mh\ddot{u} + (mh^2 + I_m + I_{mb})\ddot{\theta} + mh\ddot{v}^t = M_b \tag{6.42}$$

where,

m_b = Mass of the base.

I_{mb} = Mass moment of inertia of the base.

Equation (6.40), (6.41) and (6.42) can also be written in the frequency domain by using a fourier transform function as

$$g(\omega)u(\omega) - m h^2 \omega^2 \theta(\omega) - m\omega^2 v^t(\omega) = 0 \tag{6.43}$$

$$-m\omega^2 u(\omega) - m h \ \omega^2 \theta(\omega) - (m + m_b)\omega^2 v^t(\omega) = V_b(\omega) \tag{6.44}$$

$$-m h \omega^2 u(\omega) - \bar{I}_m \omega^2 \theta(\omega) - m h \omega^2 v^t(\omega) = M_b(\omega) \tag{6.45}$$

where,

$\bar{I}_m = I_m + I_{mb} + m h^2.$

$g(\omega)$ = Inverse of complex frequency response functions of a SDOF system.

The complex frequency response functions forms the dynamic stiffness (i.e. impedance function) for the rigid massless circular footing of radius r resting on an isotropic homogeneous half space for translational and rotational degrees of freedom as shown below (Datta, 2010),

$$G_d(\omega) = \begin{bmatrix} G_{vv} & G_{v\theta} \\ G_{\theta v} & G_{\theta\theta} \end{bmatrix} \tag{6.46}$$

where, G_{vv}, $G_{v\theta}$, $G_{\theta v}$, $G_{\theta\theta}$ are the complex frequency response functions. These functions have real and imaginary parts and can be written as

$$G(ia) = G^R(a) + iG^I(a) \tag{6.47}$$

In the above expression R denotes the real part which represents the soil resistance (stiffness) and I denotes the imaginary part which represents the radiation damping of the soil. Also a represents the non-dimensional frequency, which can be given as

$$a = \frac{r\,\omega}{v_s} \tag{6.48}$$

Where, v_s is the shear wave velocity for the material of the uniform half-space. Plots of the $G^R(a)$ and $G^I(a)$ in the non dimensional form for the elements of the $G_d(\omega)$ matrix are available in many publications of various investigators, in the form of graphs. (Note: - for rectangular footings, approximate expressions for impedance functions may be derived from those expressions which are available for the equivalent area of circular footings.) The resulting displacements of the degrees of freedom of the plate are obtained as complex number and are arranged in a column to form a flexibility matrix. The inverse of flexibility matrix gives the impedance matrix as shown in equation (6.46). These impedance functions are the key parameters for the substructure method of analysis.

Equation (6.43), (6.44), (6.45) can be rearrange as (Datta, 2010)

$$K_g(\omega)\, d(\omega) = M\, \ddot{u}_g(\omega) \tag{6.49}$$

where,

$K_g(\omega)$ = Frequency dependent complex stiffness matrix of the soil-structure system.

$d(\omega)$ = Complex frequency components of the displacement vector (i.e. degrees of freedom) $\{u \quad v \quad \theta\}^T$.

$\ddot{u}_g(\omega)$ = Complex frequency components of the free field ground acceleration.

$M = -\{m \quad (m+m_b) \quad m h\}^T$

Also $V_b(\omega)$ and $M_b(\omega)$ can be written in terms of the impedance matrix $G_d(\omega)$ as

$$\begin{Bmatrix} V_b \\ M_b \end{Bmatrix} = G_d\ (\omega) \begin{Bmatrix} u(\omega) \\ \theta(\omega) \end{Bmatrix} \tag{6.50}$$

And

$$\omega^2 v^t(\omega) = \omega^2 v(\omega) + \ddot{u}_g(\omega) \tag{6.51}$$

The elements of matrix $K_g(\omega)$ are given as

$$\begin{aligned}
K_{g11} &= g(\omega) \\
K_{g12} &= K_{g21} = -\omega^2 m \\
K_{g13} &= K_{g31} = -\omega^2 m\, h \\
K_{g22} &= -\omega^2 (m + m_b) + G_{uu}(\omega) \\
K_{g23} &= K_{g32} = -\omega^2 m\, h + G_{u\theta}(\omega) \\
K_{g33} &= -\omega^2 \bar{I}_m + G_{\theta\theta}(\omega)
\end{aligned} \tag{6.52}$$

Equation (6.49) can be solved for discrete value of ω, which gives the response vector $d(\omega)$ in the frequency domain. Fourier transform of ground acceleration gives $\ddot{u}_g(\omega)$. Also the inverse Fourier transform of $d(\omega)$ gives the response $u(t),\ v(t),\ \theta(t)$ in time domain in the form of time histories.

6.11.2 MDOF System with Multi-Support Excitation Considering SSI

The basic principles involved in the analysis of SDOF system are same and applicable for the MDOF system except that the formulation of the equation in case of MDOF system with multi-support excitation is more complex. The situation of multi-support excitation can occur in case of large structures such as bridges and arch dams etc, where the free-field motion at all points of contact of structure and foundation are not constant. The approach normally used to solve this kind of problem is to define a quasi-static component of the response in the total or absolute response of different degrees of freedom. The total displacement of the system can be given as combination of two quasi-static components of displacement and a dynamic displacement.

$$\{u^t\}=\{u\}+\{u_r\}+\{u_d\} \tag{6.53}$$

where,

$\{u^t\}$= Total/absolute displacement of the system from a fixed reference.

$\{u\}$ =The vector of the displacements produced at all non-supported degrees of freedom produced due to the ground displacements at the supports.

$\{u_r\}$ =The vector of the displacements at the supports for maintaining elastic compatibility between the foundation and the soil.

$\{u_d\}$ =The vector of the relative dynamic displacements produced at all non-supported degrees of freedom due to the inertial actions.

The quasi-static displacement involves the stiffness of the soil-structure system only. Initially the free field ground motion tends to move the supports with the same distance with which the soil supporting the support moves. The different supports have different ground motion (as it is not constant for large structures), due to this the relative motion between the supports takes place. This results in the development of the elastic forces in the structure. Due to these elastic forces a set of equal and opposite reactions develops at the interface between the substructure no.1 and substructure no.2. This equal and opposite reactions produces the deformations in the interface and induces compatible displacement in the structure and soil. Also the inertia forces developed at the masses related to each degree of freedom of the structure intodoces a pair of equal and opposite dynamic forces at the foundation and soil interface. This results in the development of the compatible dynamic displacements in the structure and the soil. The dynamic displacements caused in the soil, propagates in the form of a wave within the soil giving rise to radiation damping in the soil-structure interaction. To formulate the governing equations of motion for the general soil-structure system, consider a MDOF system as shown in Figure 6.16.

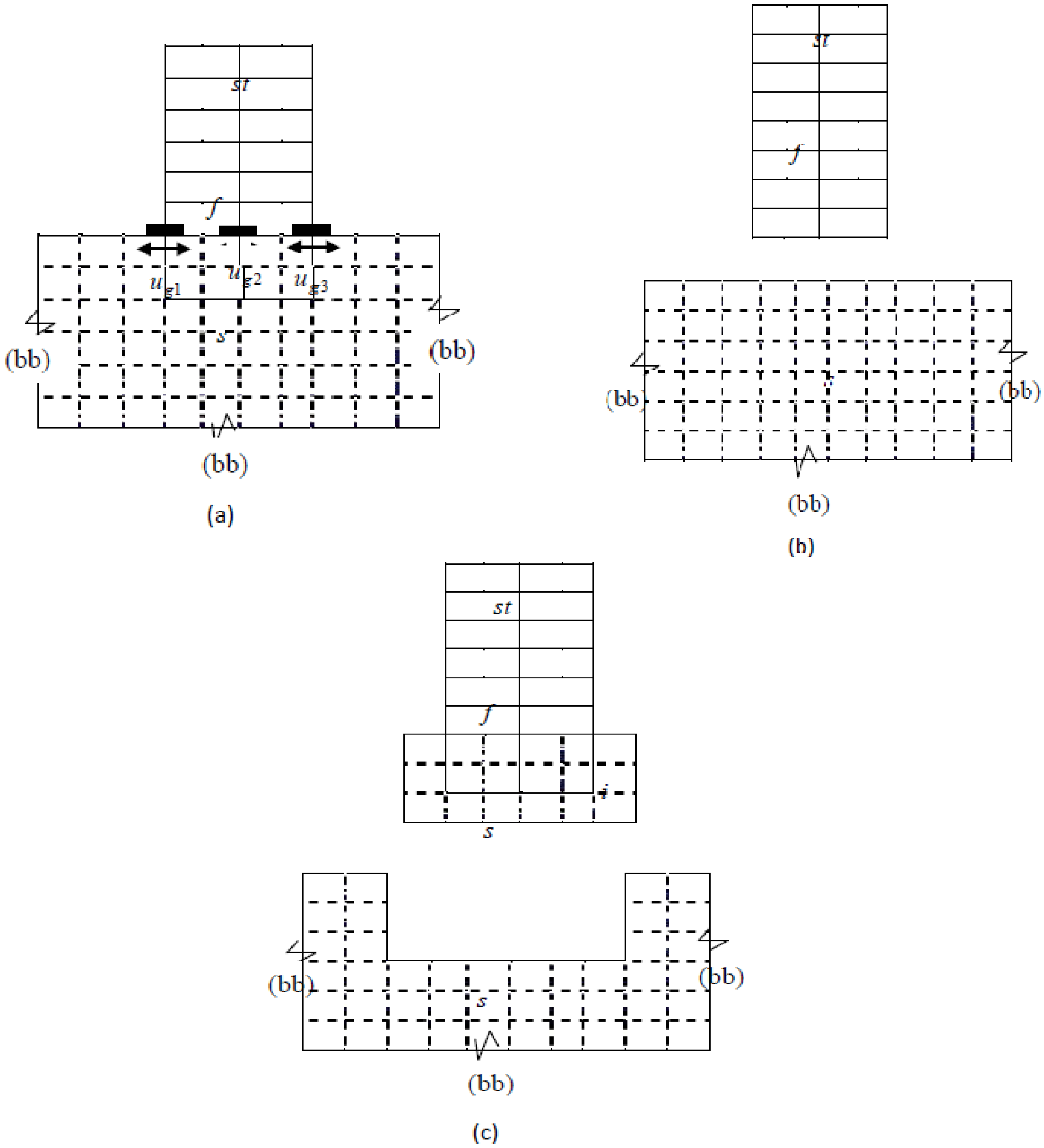

Figure 6.16: Seismic SSI with substructure method: (a) MDOF system with multi-support excitation; (b) modeling superstructure (substructure no. 1) and soil medium (i.e. substructure no. 2) separately; (c) some portion of the soil is included in the superstructure model referred as substructure no.1 and remaining soil as substructure no. 2.

The equation of motion of the system shown in Figure 6.16 (a), can be written as

$$\left[\begin{array}{cc:c} M_{stst} & M_{stf} & 0 \\ M_{fst} & M_{ff} & M_{fs} \\ \hdashline 0 & M_{sf} & M_{ss} \end{array}\right]\left\{\begin{array}{c} \ddot{u}^t_{st} \\ \ddot{u}^t_f \\ \hdashline \ddot{u}^t_s \end{array}\right\}+\left[\begin{array}{cc:c} C_{stst} & C_{stf} & 0 \\ C_{fst} & C_{ff} & C_{fs} \\ \hdashline 0 & C_{sf} & C_{ss} \end{array}\right]\left\{\begin{array}{c} \dot{u}^t_{st} \\ \dot{u}^t_f \\ \hdashline \dot{u}^t_s \end{array}\right\}+\left[\begin{array}{cc:c} K_{stst} & K_{stf} & 0 \\ K_{fst} & K_{ff} & K_{fs} \\ \hdashline 0 & K_{sf} & K_{ss} \end{array}\right]\left\{\begin{array}{c} u^t_{st} \\ u^t_f \\ \hdashline u^t_s \end{array}\right\}=\left\{\begin{array}{c} 0 \\ P^t_f \\ \hdashline P^t_s=-P^t_f \end{array}\right\} \tag{6.54}$$

Partitioning the equation of motion, will lead to the two sets of equation of motion of both substructures, shown in Figure 6.16(b)

The equation of motion of the substructure no. 1 would take the form

$$\begin{bmatrix} M_{stst} & M_{stf} \\ M_{fst} & M_{ff} \end{bmatrix}\begin{Bmatrix} \ddot{u}^t_{st} \\ \ddot{u}^t_f \end{Bmatrix}+\begin{bmatrix} C_{stst} & C_{stf} \\ C_{fst} & C_{ff} \end{bmatrix}\begin{Bmatrix} \dot{u}^t_{st} \\ \dot{u}^t_f \end{Bmatrix}+\begin{bmatrix} K_{stst} & K_{stf} \\ K_{fst} & K_{ff} \end{bmatrix}\begin{Bmatrix} u^t_{st} \\ u^t_f \end{Bmatrix}=\begin{Bmatrix} 0 \\ P^t_f \end{Bmatrix} \tag{6.55}$$

In a similar way the equation of motion of substructure no. 2 is

$$[M_{ss}]\{\ddot{u}^t{}_s\}+[C_{ss}]\{\dot{u}^t{}_s\}+[K_{ss}]\{u^t{}_s\}=\{P_s{}^t\}-[M_{sf}]\{\ddot{u}^t{}_f\}-[C_{sf}]\{\dot{u}^t{}_f\}-[K_{sf}]\{u^t{}_f\} \tag{6.56}$$

where,

st : Represents the nodes of structure.

f : Represents the nodes of foundation or common interface between substructure 1 & 2.

s : Represents the nodes of soil i.e. substructure no. 2.

For the model (c) shown in Figure 6.16, the equation of motion will be

$$\left[\begin{array}{ccc:c} M_{stst} & M_{stf} & 0 & 0 \\ M_{fst} & M_{ff} & M_{fi} & 0 \\ 0 & M_{if} & M_{ii} & M_{is} \\ \hdashline 0 & 0 & M_{si} & M_{ss} \end{array}\right]\left\{\begin{array}{c} \ddot{u}^t_{st} \\ \ddot{u}^t_f \\ \ddot{u}^t_i \\ \hdashline \ddot{u}^t_s \end{array}\right\}+\left[\begin{array}{ccc:c} C_{stst} & C_{stf} & 0 & 0 \\ C_{fst} & C_{ff} & C_{fi} & 0 \\ 0 & C_{if} & C_{ii} & C_{is} \\ \hdashline 0 & 0 & C_{si} & C_{ss} \end{array}\right]\left\{\begin{array}{c} \dot{u}^t_{st} \\ \dot{u}^t_f \\ \dot{u}^t_i \\ \hdashline \dot{u}^t_s \end{array}\right\}+\left[\begin{array}{ccc:c} K_{stst} & K_{stf} & 0 & 0 \\ K_{fst} & K_{ff} & K_{fi} & 0 \\ 0 & K_{if} & K_{ii} & K_{is} \\ \hdashline 0 & 0 & K_{si} & K_{ss} \end{array}\right]\left\{\begin{array}{c} u^t_{st} \\ u^t_f \\ u^t_i \\ \hdashline u^t_s \end{array}\right\}=\left\{\begin{array}{c} 0 \\ 0 \\ 0 \\ \hdashline P^t_s \end{array}\right\} \tag{6.57}$$

where,

st : Represents the nodes of structure.

f : Represents the nodes (DOF) of base/foundation i.e. soil-structure interface but Excluding s_t and s.

i : Represents the nodes in the soil region excluding the nodes of f and s.

s : Represents the nodes at the interface of substructure no. 1 & 2 and also the nodes of substructure no. 2.

Partitioning this equation as indicated gives the equation of similar form of equation (6.55).

where,

$[M_{stst}]$ of equation (6.55) and $\begin{bmatrix} M_{stst} & M_{st\,f} & 0 \\ M_{f\,st} & M_{ff} & M_{fi} \\ 0 & M_{if} & M_{ii} \end{bmatrix}$ of equation (6.57) shows the mass matrix for the nodes/DOF of the substructureno.1 excluding the nodes of the interface between the substructure no. 1 and 2. $[M_{st\,f}]$ and $\begin{bmatrix} 0 \\ 0 \\ M_{fi} \end{bmatrix}$ of equation (6.55) and equation (6.57) respectively shows the mass of the DOF of the substructure no.1 and the interface of substructure no. 1.

Similar is the case of $[M_{f\,st}]$ and $\lfloor 0 \quad 0 \quad M_{if} \rfloor$.

$[M_{f\,f}]$ of equation (6.55) and $[M_{ii}]$ of equation (6.57) represents the mass of the DOF at the interface of substructure no. 1 and 2, excluding the DOF inside the substructure no. 2.

$\{P_f^{\,t}\}$ of equation (6.55) and $\{P_s^{\,t}\}$ of equation (6.57) respectively shows the nodal forces developed at the interface of substructure 1 and substructure 2. In a similar way the damping and stiffness matrix are having same relation. So now onwards the solution of equation (6.55) and

(6.57) will follow the same steps as mention below though the steps are written, by keeping equation (6.55) in mind.

In order to obtain the quasi-static components of the responses i.e. u and u_r, only stiffness terms of the equation of motion are considered. Let the quasi-static response of non support degrees of freedom due to free field ground motion at the support be denoted by u^{st} and the free field ground motion at the support be denoted by $u^f = u_g$. Also the quasi-static displacements at non support degrees of freedom produced due to the compatible displacements at the soil foundation interface be denoted by u_r^{st} and the compatible displacement at the supports be denoted by u_r^f. Then the equilibrium of forces at the soil-structure interface written in the frequency domain is given as

$$K_{f\,st}\left(u^{st} + u_r^{st}\right) + K_{ff}\left(u^f + u_r^f\right) + G_{ff}\,u_r^f = 0 \tag{6.58}$$

In which G_{ff} is the impedance matrix for the soil corresponding to the interface degrees of freedom. As this equation is written only for quasi-static motion, the imaginary part of the impedance matrix is not included in it.

After simplifying this equation, we will get

$$K_{f\,st}\,u_r^{st} + \left(K_{ff} + G_{ff}\right)u_r^f = -K_{f\,st}\,u^{st} - K_{ff}\,u^f = -p_f \tag{6.59}$$

If the displacement, due to the free-field ground motion of the non support degrees of freedom the supports and the ground motions at the support are only considered, then

$$K_{st\,st}\,u^{st} + K_{st\,f}\,u^f = 0 \tag{6.60}$$

$$u^{st} = -K_{st\,st}{}^{-1} K_{st\,f} \;\; u^{f}$$

Or

$$= -K_{st\,st}{}^{-1} K_{st\,f} \;\; u_g \tag{6.61}$$

$$= \frac{1}{\omega^2} K_{st\,st}{}^{-1} K_{st\,f} \;\; \ddot{u}_g$$

Substituting equation (6.61) in the R.H.S. equation (6.59), leads to

$$p_f = -\frac{1}{\omega^2}\left(K_{ff} - K_{f\,st} K_{st\,st}{}^{-1} K_{st\,f}\right) \ddot{u}_g \tag{6.62}$$

If the displacements at the non support degrees of freedom produced due to the compatible displacements at the soil foundation interface and the compatible displacements at the supports are only considered then

$$K_{st\,st} \;\; u_r{}^{st} + K_{st\,f} \;\; u_r{}^{f} = 0 \tag{6.63}$$

Adding equation (6.63) to the L.H.S. of equation (6.59) (as there is no external set of forces acting on the structure), the following expression for u_r is obtained.

$$\begin{bmatrix} K_{st\,st} & K_{st\,f} \\ K_{f\,st} & K_{ff} + G_{ff} \end{bmatrix} \begin{Bmatrix} u_r{}^{st} \\ u_r{}^{f} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -p_f \end{Bmatrix} \tag{6.64}$$

Equation (6.53) can also be written as

$$
\begin{aligned}
&\left\{u^t\right\}=\left\{u_a\right\}+\left\{u_d\right\} \\
&where, \left\{u_a\right\}=\left\{u\right\}+\left\{u_r\right\} \\
&and \quad \left\{u_a{}^{st}\right\}=\left\{u^{st}\right\}+\left\{u_r{}^{st}\right\} \\
&also \quad \left\{u_a{}^{f}\right\}=\left\{u^{f}\right\}+\left\{u_r{}^{f}\right\}
\end{aligned}
\tag{6.65}
$$

Further substituting equation (6.65) in equation (6.55) and rearranging it, we will get the expression for u_d.

$$
\begin{aligned}
&\begin{bmatrix} M_{st\,st} & M_{st\,f} \\ M_{f\,st} & M_{ff} \end{bmatrix}\begin{Bmatrix} \ddot{u}_d^{st} \\ \ddot{u}_d^{f} \end{Bmatrix}+\begin{bmatrix} C_{st\,st} & C_{st\,f} \\ C_{f\,st} & C_{ff} \end{bmatrix}\begin{Bmatrix} \dot{u}_d^{st} \\ \dot{u}_d^{f} \end{Bmatrix}+\begin{bmatrix} K_{st\,st} & K_{st\,f} \\ K_{f\,st} & K_{ff} \end{bmatrix}\begin{Bmatrix} u_d^{st} \\ u_d^{f} \end{Bmatrix}= \\
&-\begin{bmatrix} M_{st\,st} & M_{st\,f} \\ M_{f\,st} & M_{ff} \end{bmatrix}\begin{Bmatrix} \ddot{u}_a^{st} \\ \ddot{u}_a^{f} \end{Bmatrix}-\begin{bmatrix} C_{st\,st} & C_{st\,f} \\ C_{f\,st} & C_{ff} \end{bmatrix}\begin{Bmatrix} \dot{u}_a^{st} \\ \dot{u}_a^{f} \end{Bmatrix}-\begin{bmatrix} K_{st\,st} & K_{st\,f} \\ K_{f\,st} & K_{ff} \end{bmatrix}\begin{Bmatrix} u_a^{st} \\ u_a^{f} \end{Bmatrix}+\begin{Bmatrix} 0 \\ P_f^t \end{Bmatrix}
\end{aligned}
\tag{6.66}
$$

In the above expression the damping terms of the R.H.S. makes little contribution to the effective load of a relatively low damped system, say $\xi \prec 0.1$, and can be neglected. Using equation (6.58), equation (6.66) can be written in frequency domain by performing Fourier transform, as

$$
\begin{aligned}
&\left\{-\omega^2\begin{bmatrix} M_{st\,st} & M_{st\,f} \\ M_{f\,st} & M_{ff} \end{bmatrix}+i\omega\begin{bmatrix} C_{st\,st} & C_{st\,f} \\ C_{f\,st} & C_{ff} \end{bmatrix}+\begin{bmatrix} K_{st\,st} & K_{st\,f} \\ K_{f\,st} & K_{ff} \end{bmatrix}\right\}\begin{Bmatrix} u_d^{st} \\ u_d^{f} \end{Bmatrix}= \\
&\omega^2\begin{bmatrix} M_{st\,st} & M_{st\,f} \\ M_{f\,st} & M_{ff} \end{bmatrix}\begin{Bmatrix} u_a^{st} \\ u_a^{f} \end{Bmatrix}+\begin{Bmatrix} 0 \\ P_f^t - p_{rf} \end{Bmatrix}
\end{aligned}
\tag{6.67}
$$

where, $p_{rf} = -G_{ff}\, u_r{}^f$. Also $P_f^t - p_{rf}$ represents the dynamic component of the loading at the foundation which is arise due to dynamic characteristic of displacement at the interface. It may be obtained in a similar way as that of quasi-static displacement (equation 6.58) i.e.

$$
P_f^t - p_{rf} = p_f{}^d = -G_{ff}\, u_d{}^f \tag{6.68}
$$

In this equation G_{ff} have both real and imaginary components. The imaginary component denotes the radiation damping due to which overall damping of the system increases.

Substituting equation (6.68) in equation (6.67), gives

$$\left\{-\omega^2\begin{bmatrix} M_{st\,st} & M_{st\,f} \\ M_{f\,st} & M_{ff}\end{bmatrix} + i\omega\begin{bmatrix} C_{st\,st} & C_{st\,f} \\ C_{f\,st} & C_{ff}\end{bmatrix} + \begin{bmatrix} K_{st\,st} & K_{st\,f} \\ K_{f\,st} & K_{ff}+G_{ff}\end{bmatrix}\right\}\begin{Bmatrix} u_d^{st} \\ u_d^{f}\end{Bmatrix} = \omega^2\begin{bmatrix} M_{st\,st} & M_{st\,f} \\ M_{f\,st} & M_{ff}\end{bmatrix}\begin{Bmatrix} u_a^{st} \\ u_a^{f}\end{Bmatrix} \quad (6.69)$$

${u_d}^{st}$ and ${u_d}^{f}$ can be determine by solving equation (6.69) and inversing complex matrix for each value of ω. Using these results u^t can be obtained. Inverse Fourier transform of u^t gives the desire response in time domain.

6.12 Solution of SSI Problem Using ABAQUS Software

The SSI problem can be solved using ABAQUS software by following the steps mentioned below

1. Part Module: Forming the geometry of the structure and soil.
2. Property Module: Generating the property of the structure and soil.
3. Assembly Module: Assembly of the structure and soil into common platform.
4. Step Module: Define the analysis type.
5. Interaction Module: Define the interaction between the structure and soil medium.
6. BC Module: Define the boundary condition in the structure.
7. Mesh Module: Meshing of the structure and soil.
8. Job Module: Submission of the Job for the analysis.
9. Visualization Module: Viewing the result.

Example 6.1

Analyze the frame shown in Figure 6.17 by performing soil-structure interaction analysis in ABAQUS by

- Direct Method.
- Sub-Structure Method.

The frame is supported by two isolated footings having properties as mentioned below.

1. Structural Configuration

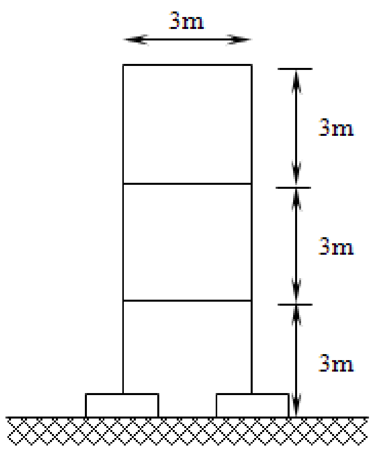

Figure 6.17: Frame to be analyzed by Seismic soil-structure interaction analysis.

2. Properties of structure

Size of Beams	= 400 mm x 400 mm
Size of columns	= 400 mm x 400 mm
Size of foundation	= 750 mm radius

3. Material properties of structure

Density ρ_{st}	= 2500.00 kg/m^3
Modulus of Elasticity E_{st}	= 2500.00 N/m^2
Poisson's ratio μ_{st}	= 0.15

Damping ξ_{st} = 5.00 %

4. Properties of Soil

Density ρ_s = 1800.00 kg/m^3

Shear Velocity v_s = 200.00 $m/\sec$

Poisson's ratio μ_s = 0.3

Damping ξ_s = 20.00 %

5. Input Time History = El Centro Earthquake Time History as shown in figure 6.18

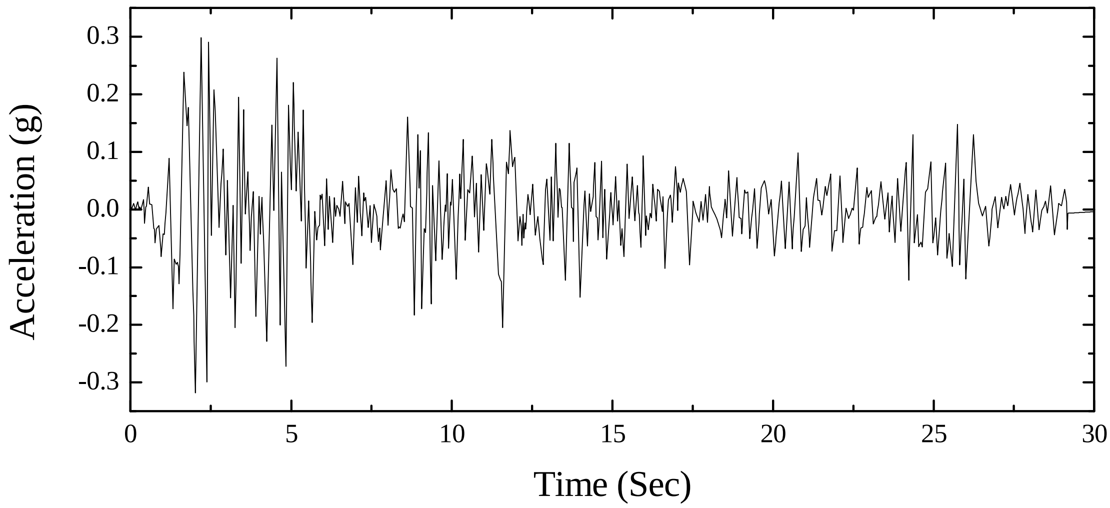

Figure 6.18: Input El-Centro Earthquake Time - history.

6. Problem requirement and = Find the time histories of relative acceleration

Rotational acceleration at the top floor of the given frame.

Soution:

A. <u>Procedure by Direct Method</u>

1. Modeling of structure geometry

- Structure is modeled with beam elements and soil is modeled with plain strain elements.
- Mesh size for beam elements is 1m and mesh size for plain strain elements is 3m x 3m.
- To avoid reflecting effect of wave sufficient amount of soil beyond the structure i.e. 30m is modeled.
- Abaqus Model of structure along with soil is shown in the Figure 6.19 below

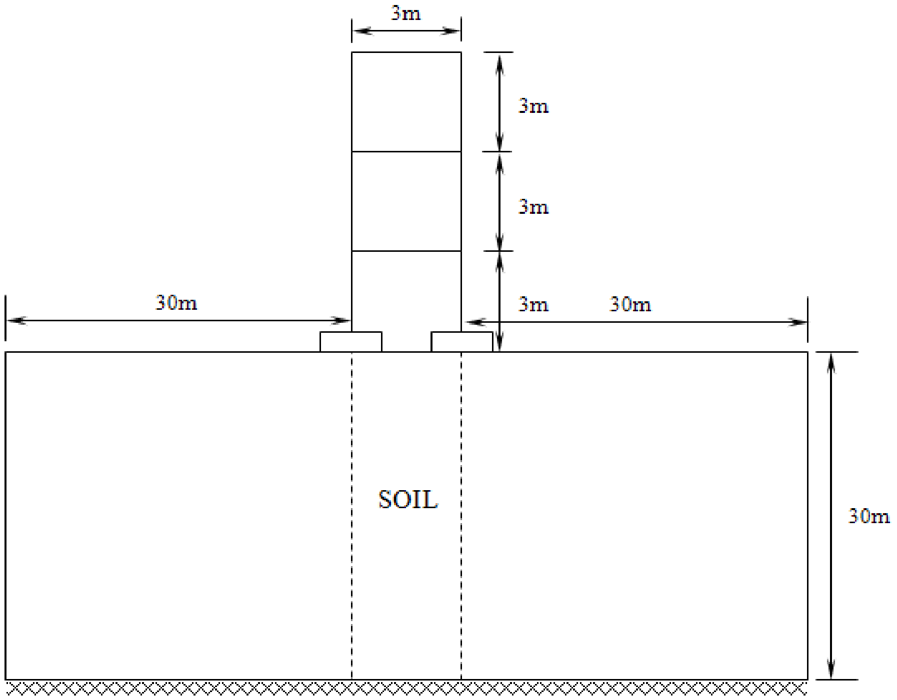

Figure 6.19: ABAQUS model of structure with soil.

2. Soil Structure interaction and support conditions
    - Appropriate boundaries for the soil medium are assumed - Support conditions at bed rock level are assumed to be fixed.
    - Interaction between structure / footing with soil is modeled with tie elements.
    - Contact surface between footing and soil is defined as
        - Hard contact in vertical direction.
        - Friction contact in tangential direction.
        - There is no separation in vertical direction.

3. Analysis of structure and results

- Structure is analyzed in ABAQUS and following results are presented.

❖ Relative acceleration time history at top floor

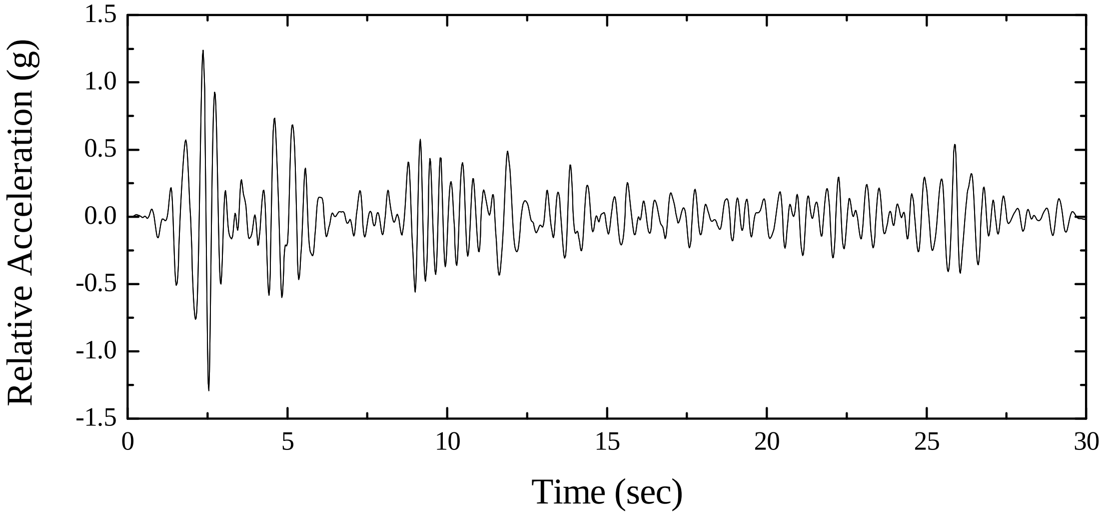

Figure 6.20: Relative acceleration time - history at top floor of frame.

❖ Rotational acceleration time history at top floor

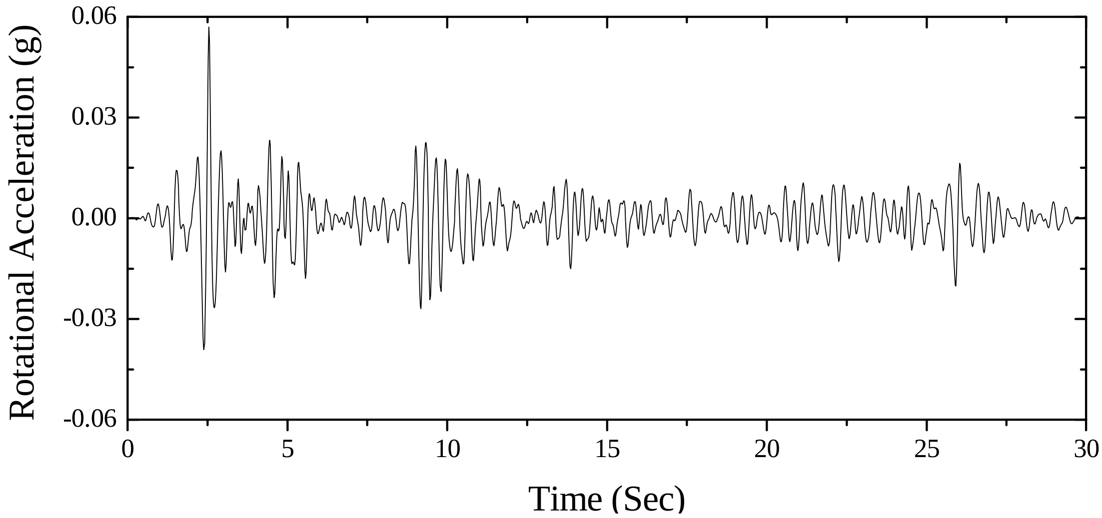

Figure 6.21: Rotational acceleration time - history at top floor of frame.

B. <u>Procedure by Sub-Structure Method</u>

Basically there are 3 steps for soil structure interaction analysis by sub-structure method using ABAQUS.

1. Input time history is at bedrock level and we need the time history at foundation level.
   - In first step time history at the bedrock level is converted to time history at foundation level by kinematic interaction analysis.
   - Procedure - Massless structure (i.e. structure with stiffness only) is modeled along with soil and time history analysis is carried out by applying the time history at the bed rock level.
   - The modified time history at the foundation level is shown in Figure 6.22. By comparing time history at bed rock level shown in Figure 6.18 and time history at foundation level shown in Figure 6.22, one can notice that the foundation level time history is having some more peaks but small value of acceleration amplitude.

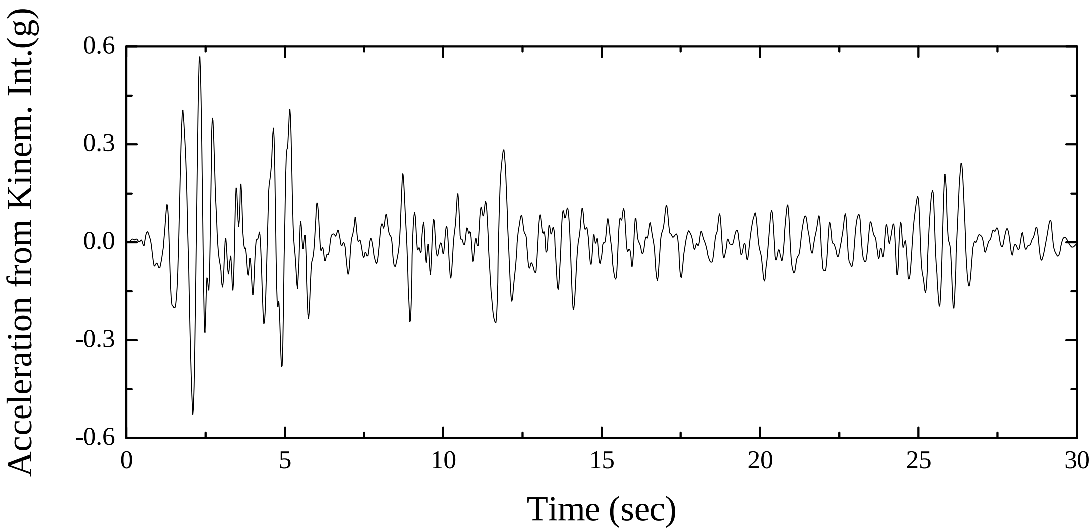

Figure 6.22: Time -history (modified) after performing kinematic interaction analysis.

2. Estimation of spring constants for soil-spring constants are estimated for foundation by considering properties of soil.

   Shear wave velocity $\quad v_s = \sqrt{G / \rho_S}$

Shear modulus $G = \rho_s \times v_s^2$

$$= 1800 \times 200^2$$

$$= 7.2 \times 10^7 \ N/m^2$$

Spring constants are given by

$$K_x = 32\,(1-\mu_s)\,G\,R/(7-8\mu_s)$$

$$= 32\,(1-0.3)\times 7.2\times 10^7 \times 0.75/(7-8\times 0.3)$$

$$= 2.63\times 10^8 \ N/m$$

$$K_z = 4\,G\,R/(1-\mu_s)$$

$$= 4\times 7.2\times 10^7 \times 0.75/(1-0.3)$$

$$= 3.08\times 10^8 \ N/m$$

$$K_r = 8\,G\,R^3/[3(1-\mu_s)]$$

$$= 8\times 7.2\times 10^7 \times 0.75^3/[3\times(1-0.3)]$$

$$= 1.16\times 10^8 \ N/m$$

3. Modeling of structure

- Superstructure is modeled as per the requirement of the problem.
- Support conditions are modeled by spring constants and values of spring constants are considered as estimated in step 2.
- Time history analysis is carried out for the structure, using modified time history obtained in step 1(i.e. after performing kinematic interaction analysis). This procedure is called as inertial interaction analysis.
- Model of the structure along with spring supports is shown in Figure 6.23.

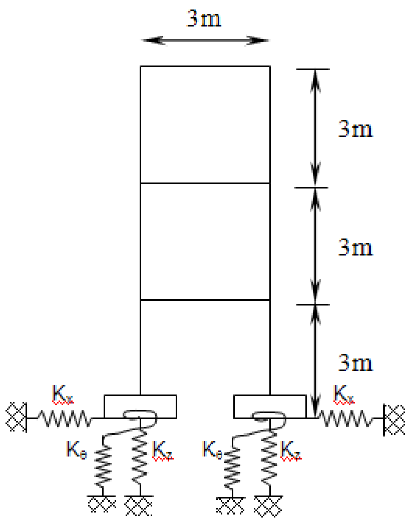

Figure 6.23 : Model of structure only, with spring supports.

4. Analysis of structure and results

- Structure is analyzed in ABAQUS and the following results are presented.

❖ Relative acceleration time history at top floor.

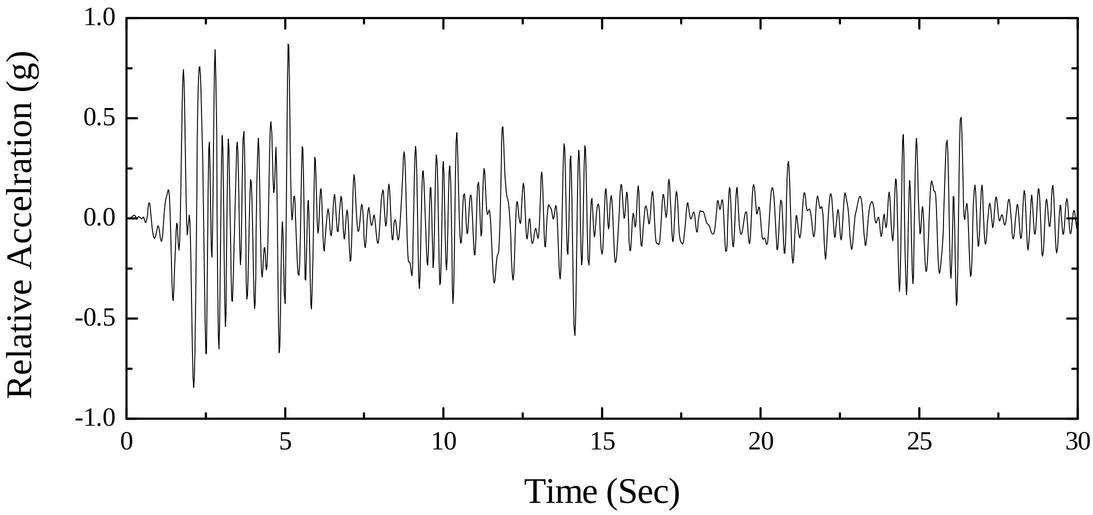

Figure 6.24: Relative acceleration time - history at top floor of frame.

❖ Rotational acceleration time history at top floor.

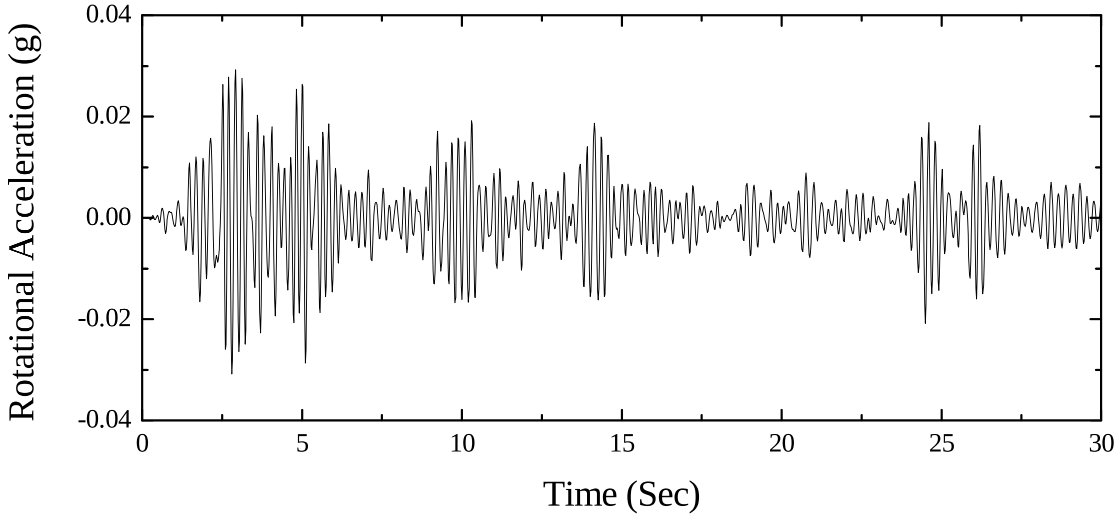

Figure 6.25: Rotational acceleration time - history at top floor of frame.

Discussion

By comparing the obtained results of the acceleration response of the system, we can see there is difference in the acceleration response time history of the two analysis. The reason for the difference between the results of the two analysis is that in direct method we are applying original time history (with higher amplitude) directly at the foundation level of the structure. While in case of substructure method firstly we are applying time history at the bed rock level and then modifying it to the foundation level. Due to which the modified time history is having somewhat small acceleration amplitude in compare to the original time history. Further in this method we are modeling soil also, due to which the two results are differing.

6.13 Tutorial Problems

Q1. Analyze the frame shown in Figure 6.26 by performing soil-structure interaction analysis in ABAQUS by

- Direct Method.
- Sub-Structure Method.

The frame is supported by two isolated footings having properties as mentioned below.

1. Structural Configuration

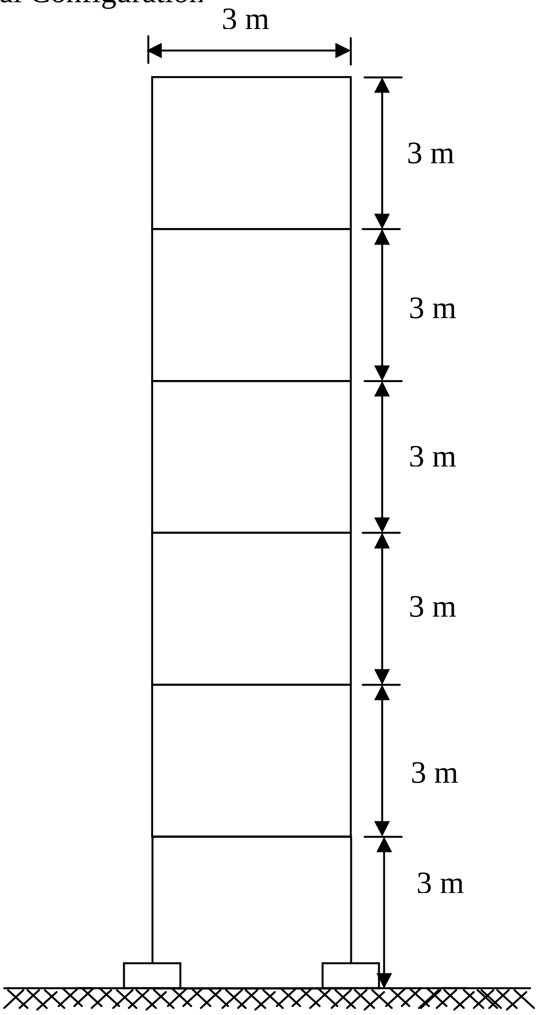

Figure 6.26: Frame to be analyzed by Seismic soil-structure interaction analysis.

2. Properties of structure

Size of Beams	= 450 mm x 450 mm
Size of columns	= 450 mm x 450 mm
Size of foundation	= 900 mm radius

Material properties of structure and soil are same as taken in solved example 6.12. Find the time histories of relative acceleration and Rotational acceleration at the top floor of the given frame when it is subjected to El-Centro Earthquake time history.

Q2. Analyze the frame shown in Figure 6.26 by performing soil-structure interaction analysis in ABAQUS by

- Direct Method.
- Sub-Structure Method.

The frame is supported by two isolated footings having properties as mentioned below. Structural configuration, Properties of structure and material properties of structure are same as mentioned in the previous problem. The properties of soil are as mentioned below

Density ρ_s	= 2000.00 kg/m^3
Shear Velocity v_s	= 600.00 $m/\sec$
Poisson's ratio μ_s	= 0.3
Damping ξ_s	= 20.00 $\%$

Find the time histories of relative acceleration and Rotational acceleration at the top floor of the given frame when it is subjected to El-Centro Earthquake time history.

6.14 Answer to Tutorial Problems

Q1

- **Direct Method**

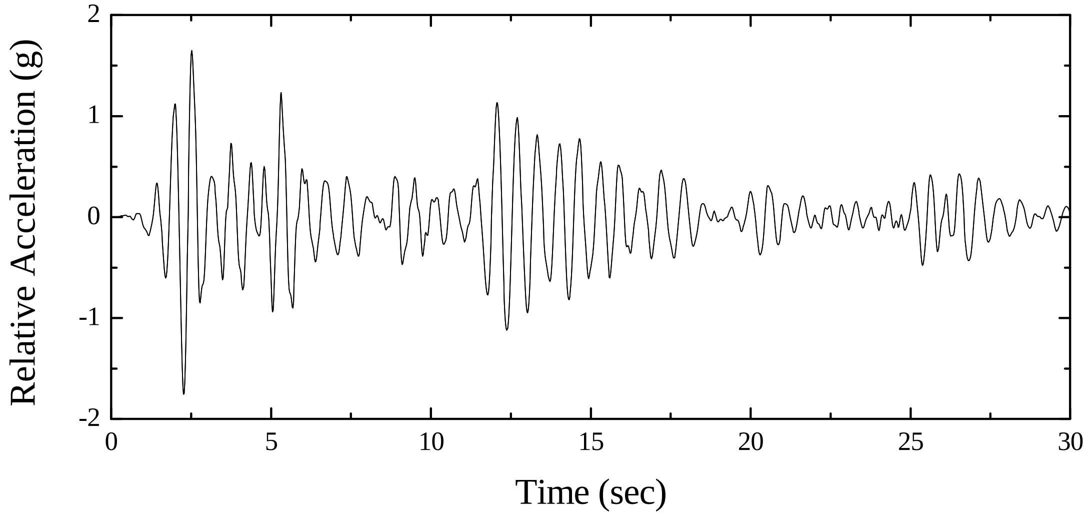

Figure 6.27: Relative acceleration time - history at top floor of frame.

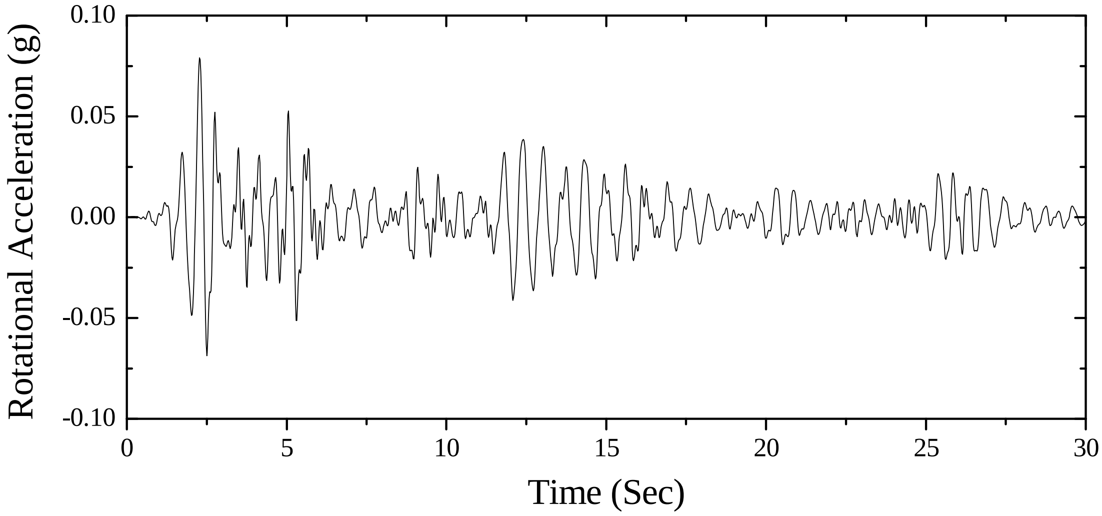

Figure 6.28: Rotational acceleration time - history at top floor of frame.

- **Sub-Structure Method**

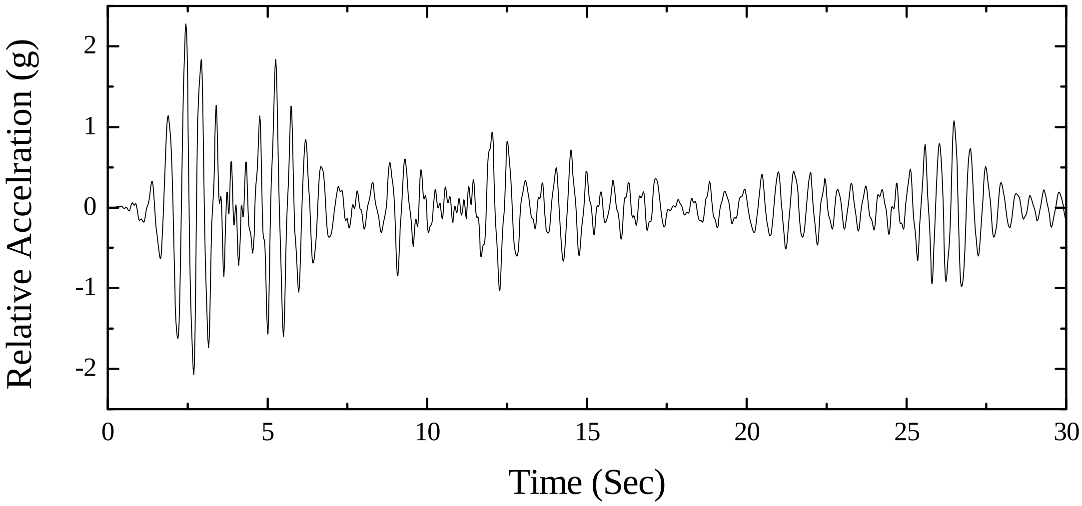

Figure 6.29: Relative acceleration time - history at top floor of frame.

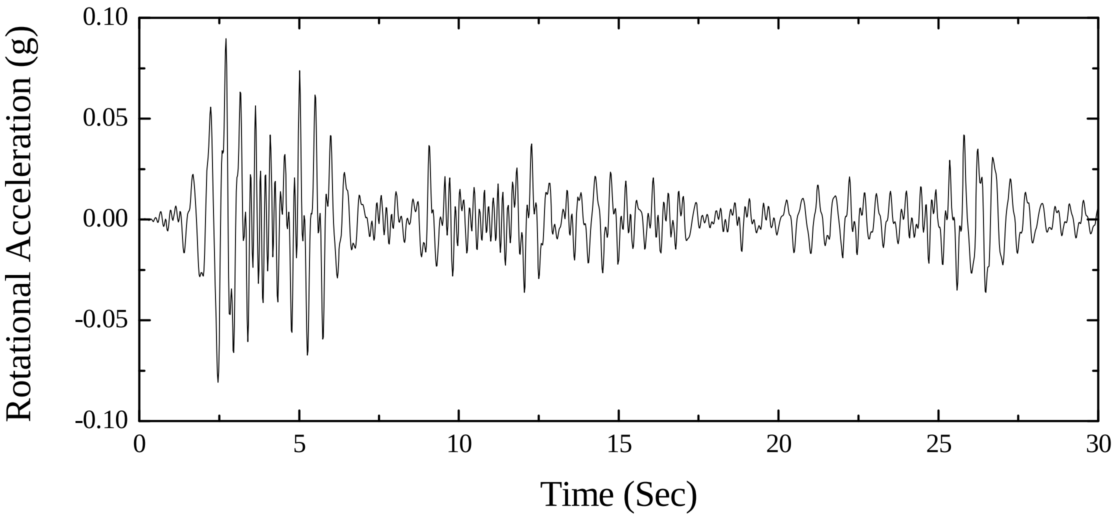

Figure 6.30: Rotational acceleration time - history at top floor of frame.

Q2.

- **Direct Method**

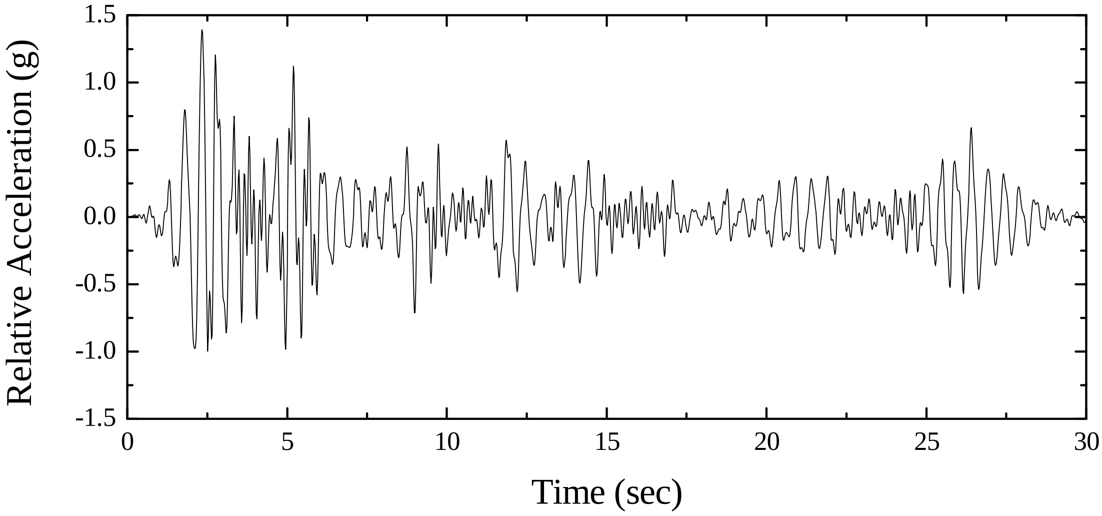

Figure 6.31: Relative acceleration time - history at top floor of frame.

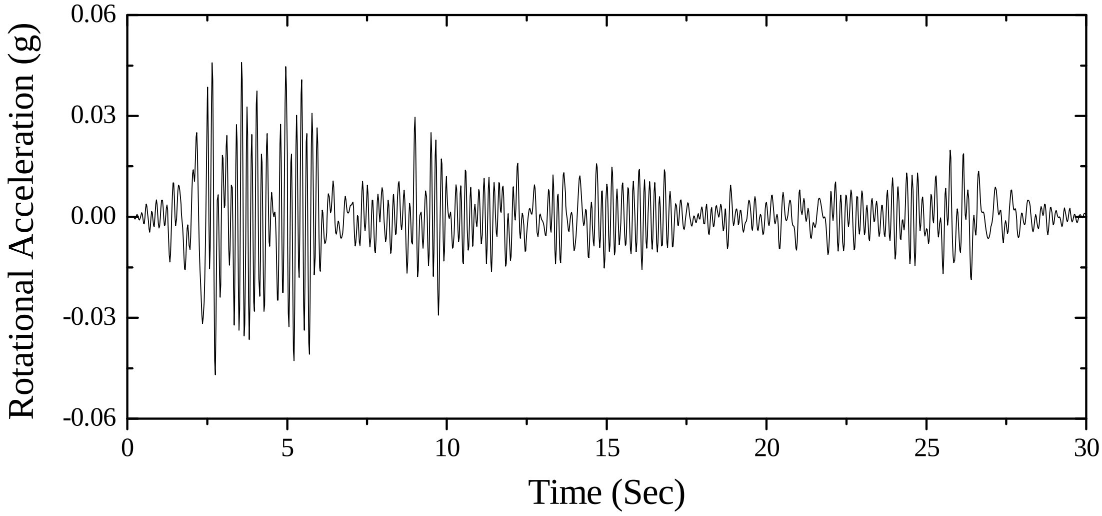

Figure 6.32: Rotational acceleration time - history at top floor of frame.

- **Sub-Structure Method**

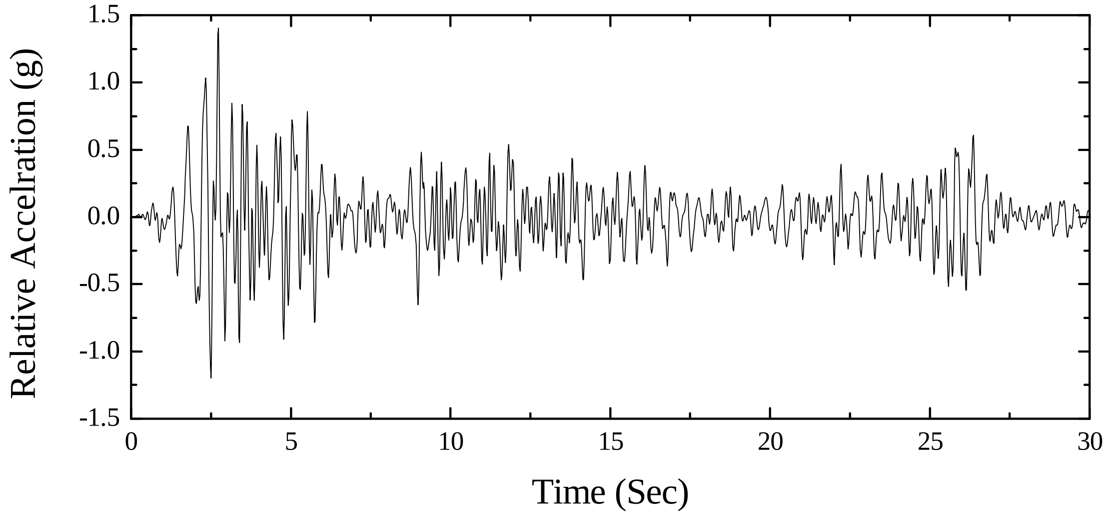

Figure 6.33: Relative acceleration time - history at top floor of frame.

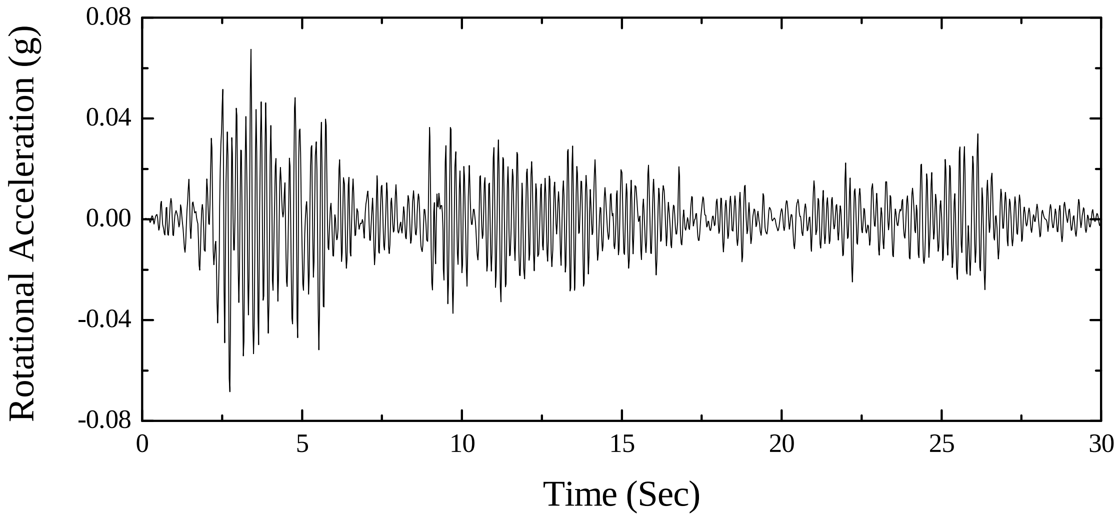

Figure 6.34: Rotational acceleration time - history at top floor of frame.